Solution for Second Exam

**Part one: Complete the following sentences by the final answers only (2 points each).

1. A general solution to $y'' - 6y' + 5y = 0$ is $y = c_1e^{-5x} + c_2e^{-x}$

2. The Wronskian of $2, x$ and x^2 ($W[2, x, x^2]$) equals 4

3. A general solution to $(x+1)^2y'' + 2(x+1)y' - 6y = 0, \quad x > -1$ is $c_1(x+1)^2 + c_2(x+1)^{-3}$ (Hint: $y = (x+1)^2$ is a solution).

4. A general solution to $y^{(4)} - 5y'' - 36y = 0$ is $c_1e^{3x} + c_2e^{-3x} + c_3\cos(2x) + c_4\sin(2x)$

5. Given that $(r+7)^3(r^2-1)^2(r^2+r+1)^2 = 0$ is the auxiliary equation of some linear differential equation with constant coefficients $L[y](x) = 0$,
   then the of the differential equation $L[y](x) = 0$ is 11

6. A **form** of a particular solution (using undetermined coefficients) to
   $y''' - 4y'' + 4y' = -3e^{2x} + 10$ is $A_1x^2e^{2x} + A_3x$

7. A **form** of a particular solution (using undetermined coefficients) to
   $x^2y'' + xy' + 4y = \cos(\ln(x^2)), \;\; x > 0$ is $A_1\ln(x)\cos(\ln(x^2)) + A_2\ln(x)\sin(\ln(x^2))$

8. If y_1 is a solution to $L[y](x) = 3h(x)$ and y_2 is a solution to $L[y](x) = 2h(x)$, then $5y_1 - 4y_2$ is a solution to the differential equation $L[y](x) = 7h(x)$

9. Let y_1, y_2, y_3 be three solutions to $xy''' - 2y'' + p(x)y' + q(x)y = 0, \quad x > 0$.
   Then $W(y_1, y_2, y_3)$ is Cx^2

**Part two: Find a general solution to the following equation (7 points).

$$y'' + 16y = \sec^3(4x)$$

$$y_G = y_h + y_p$$

$$\text{auxiliary equation is } \; r^2 + 16 = 0$$

$$r = \pm i$$

$$y_h = c_1 \cos(4x) + c_2 \sin(4x)$$

$$y_p = v_1(x)\cos(4x) + v_2(x)\sin(4x)$$

$$v_1' = \frac{\begin{vmatrix} 0 & \sin(4x) \\ \sec^3(4x) & 4\cos(4x) \end{vmatrix}}{\begin{vmatrix} \cos(4x) & \sin(4x) \\ -4\sin(4x) & 4\cos(4x) \end{vmatrix}}$$

$$v_1' = \frac{-1}{4}\tan(4x)\sec^2(4x) \quad v_1 = \frac{-1}{32}\tan^2(4x)$$

$$v_2' = \frac{\begin{vmatrix} \cos(4x) & 0 \\ -4\sin(4x) & \sec^3(4x) \end{vmatrix}}{\begin{vmatrix} \cos(4x) & \sin(4x) \\ -4\sin(4x) & 4\cos(4x) \end{vmatrix}}$$

$$v_2' = \frac{1}{4}\sec^2(4x) \quad v_2 = \frac{1}{16}\tan(4x)$$

$$y_G = c_1\cos(4x) + c_2\sin(4x) + \frac{-1}{32}\tan^2(4x)\cos(4x) + \frac{1}{16}\tan(4x)\sin(4x)$$