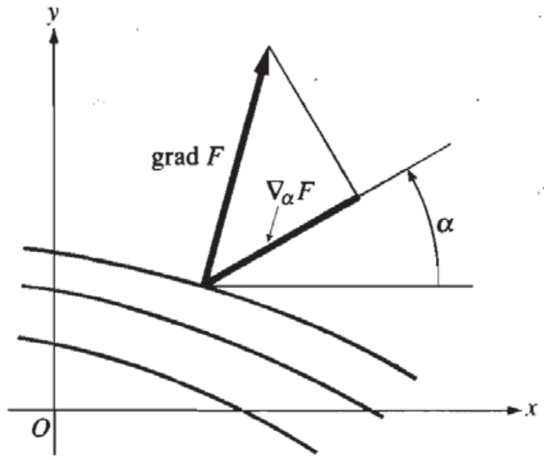

Figure 2.14 Gradient of $F(x, y)$ and curves $F(x, y) = \text{const.}$

for $\mathbf{v}$ has direction cosines $\cos\alpha$ and $\cos\beta = \cos(\frac{1}{2}\pi - \alpha) = \sin\alpha$. Again the directional derivative is the component of grad $F = (\partial F/\partial x)\mathbf{i} + (\partial F/\partial y)\mathbf{j}$ in the given direction; the directional derivative at a given point has its maximum in the direction of grad F, the value being

$$|\nabla F| = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2}. \tag{2.122}$$

The directional derivative is zero along a level curve of F, as suggested in the accompanying Fig. 2.14.

If the level curves are interpreted as contour lines of a landscape, that is, of the surface $z = F(x, y)$, then the directional derivative means simply the rate of climb in the given direction. The rate of climb in the direction of steepest ascent is the *gradient*, precisely the term in common use. The bicyclist zigzagging up a hill is taking advantage of the component rule to reduce the directional derivative.

PROBLEMS

1. Evaluate the directional derivatives of the following functions for the points and directions given:

a) $F(x, y, z) = 2x^2 - y^2 + z^2$ at $(1, 2, 3)$ in the direction of the line from $(1, 2, 3)$ to $(3, 5, 0)$;

b) $F(x, y, z) = x^2 + y^2$ at $(0, 0, 0)$ in the direction of the vector $\mathbf{u} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$; discuss the significance of the result;

c) $F(x, y) = e^x \cos y$ at $(0, 0)$ in a direction making an angle of $60°$ with the x axis;

d) $F(x, y) = 2x - 3y$ at $(1, 1)$ along the curve $y = x^2$ in the direction of increasing x;

e) $F(x, y, z) = 3x - 5y + 2z$ at $(2, 2, 1)$ in the direction of the outer normal of the surface $x^2 + y^2 + z^2 = 9$;

f) $F(x, y, z) = x^2 + y^2 - z^2$ at $(3, 4, 5)$ along the curve $x^2 + y^2 - z^2 = 0$, $2x^2 + 2y^2 - z^2 = 25$ in the direction of increasing x; explain the answer.