Section 2.6, page 89

1. a) $\frac{\partial z}{\partial x} = \frac{-2xy}{(x^2+y^2)^2}, \frac{\partial z}{\partial y} = \frac{x^2-y^2}{(x^2+y^2)^2}$; **b)** $\frac{\partial z}{\partial x} = y^2 \cos xy, \frac{\partial z}{\partial y} = \sin xy + xy \cos xy$;
c) $\frac{\partial z}{\partial x} = \frac{3x^2+2xy-2xz}{x^2-3z^2}, \frac{\partial z}{\partial y} = \frac{x^2}{x^2-3z^2}$; **d)** $\frac{\partial z}{\partial x} = \frac{e^{x+2y}}{2\sqrt{e^{x+2y}-y^2}}, \frac{\partial z}{\partial y} = \frac{e^{x+2y}-y}{\sqrt{e^{x+2y}-y^2}}$.
e) $z_x = 3x(x^2+y^2)^{1/2}, z_y = 3y(x^2+y^2)^{1/2}$; **f)** $z_x = [1-(x+2y)^2]^{-1/2}$,
$z_y = 2[1-(x+2y)^2]^{-1/2}$; **g)** $z_x = e^x(e^z+1)^{-1}, z_y = 2e^y(e^z+1)^{-1}$;
h) $z_x = -(y+z)(x+2z)^{-1}, z_y = -(2xy+z^2+xz)(2yz+xy)^{-1}$.
2. Approximately $f_x(1,1) = \frac{f(2,1)-f(1,1)}{1} = 3$,
or $f_x(1,1) = \frac{f(1,1)-f(0,1)}{1} = 1$, or $f_x(1,1) = \frac{f(2,1)-f(0,1)}{2} = 2$.
It can be shown that the last value is "in general" the best estimate. For $f_y(1,1)$ the analogous formulas give the estimates $-2, -2, -2$. This topic is discussed in Problem 9 following Section 6.18.
3. a) $\left(\frac{\partial u}{\partial x}\right)_y = 2x, \left(\frac{\partial v}{\partial y}\right)_x = -2$; **b)** $\left(\frac{\partial x}{\partial u}\right)_v = e^u \cos v, \left(\frac{\partial y}{\partial v}\right)_u = e^u \cos v$;
c) $\left(\frac{\partial x}{\partial u}\right)_y = 1, \left(\frac{\partial y}{\partial v}\right)_u = -\frac{1}{2}$; **d)** $\left(\frac{\partial r}{\partial x}\right)_y = x(x^2+y^2)^{-1/2}, \left(\frac{\partial r}{\partial \theta}\right)_x = x \sec\theta \tan\theta$.
4. a) $\frac{y\,dx - x\,dy}{y^2}$; **b)** $\frac{x\,dx+y\,dy}{x^2+y^2}$; **c)** $\frac{(y-y^2)\,dx+(x-x^2)\,dy}{(1-x-y)^2}$;
d) $(x-2y)^4 e^{xy}[(xy-2y^2+5)\,dx+(x^2-2xy-10)\,dy]$; **e)** $\frac{-y\,dx+x\,dy}{x^2+y^2}$,
f) $\frac{-(x\,dx+y\,dy+z\,dz)}{(x^2+y^2+z^2)^{3/2}}$. **5. a)** $\Delta z = 4\Delta x + 2\Delta y + \overline{\Delta x}^2 + 2\Delta x\Delta y, dz = 4\Delta x + 2\Delta y$;
b) $\Delta z = \frac{\Delta x - \Delta y}{2(2+\Delta x+\Delta y)}, dz = \frac{\Delta x - \Delta y}{4}$, so $\Delta z = dz - \frac{(\Delta x-\Delta y)(\Delta x+\Delta y)}{4(2+\Delta x+\Delta y)}$.
6. 2.2, 2.4, 2.6.

Section 2.7, page 95

1. a) $\begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix}$; **b)** $\begin{bmatrix} 4x_1 & 2x_2 \\ 3x_2 & 3x_1 \end{bmatrix}$; **c)** $\begin{bmatrix} x_2x_3 & x_1x_3 & x_1x_2 \\ 2x_1x_3 & 0 & x_1^2 \end{bmatrix}$;

d) $\begin{bmatrix} \cos y & -x\sin y \\ \sin y & x\cos y \\ 2x & 0 \end{bmatrix}$; **e)** $(2xyz, x^2z, x^2y)$; **f)** $(2x, 2y, -2z)$;

g) col $(2t, 3t^2, 4t^3)$. **2. a)** $\begin{bmatrix} dy_1 \\ dy_2 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix}$, (5.18, 2.06);

b) $\begin{bmatrix} dy_1 \\ dy_2 \end{bmatrix} = \begin{bmatrix} 2 & 3 & -2 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix}$, (4.93, 9.09);

c) $\begin{bmatrix} du \\ dv \\ dw \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix}$, (−0.03, 1.1, 2.2);

d) $d\mathbf{y} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 2 & 0 & \cdots & 0 \\ 2 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 2 & 0 & \cdots & 0 \end{bmatrix} d\mathbf{x}$, (0, 1, …, 1).

3. a) $9(x^2+y^2)^2$, **b)** 0; **c)** $2u^3v^2w$; **d)** $4z^2(x^2-y)$.
4. a) $e^2 = 7.39$; **b)** 7.44; **c)** $du = e\,dx; dv = e\,dy, e^2 = 7.39$.

Section 2.8, page 100

2. $\frac{dy}{dx} = vu^{v-1}\frac{du}{dx} + u^v \log u \frac{dv}{dx}$. **3.** $\frac{dy}{dx} = -\frac{\log v}{u\log^2 u}\frac{du}{dx} + \frac{1}{v\log u}\frac{dv}{dx}$.
4. 1. **5.** 336. **6.** 197.

10. In all cases $dz = \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy$: **a)** $dz = 2\cot(x^2y^2 - 1)(xy^2\,dx + x^2y\,dy)$;
b) $dz = \dfrac{(2xy^2 - 3x^3y^2 - 2xy^4)\,dx + (2x^2y - 3x^2y^3 - 2x^4y)\,dy}{\sqrt{1 - x^2 - y^2}}$; **c)** $dz = \dfrac{x\,dx + 2y\,dy}{z}$.

Section 2.9, page 104

1. a) $\begin{bmatrix} u_2 - 3 & u_1 \\ 2u_2 + 2 & 2u_2 + 2u_1 - 1 \end{bmatrix}\begin{bmatrix} \cos 3x_2 & -3x_1 \sin 3x_2 \\ \sin 3x_2 & 3x_1 \cos 3x_2 \end{bmatrix}, \begin{bmatrix} -3 & 0 \\ 2 & 0 \end{bmatrix}$;

b) $\begin{bmatrix} 2u_1 - 3 & 2u_2 & 1 \\ 2u_1 + 2 & -2u_2 & -3 \end{bmatrix}\begin{bmatrix} x_2x_3^2 & x_1x_3^2 & 2x_1x_2x_3 \\ x_2^2x_3 & 2x_1x_2x_3 & x_1x_2^2 \\ 2x_1x_2x_3 & x_1^2x_3 & x_1^2x_2 \end{bmatrix}, \begin{bmatrix} 3 & 4 & 1 \\ -4 & -3 & 3 \end{bmatrix}$;

c) $\begin{bmatrix} e^{u_2} & u_1e^{u_2} \\ e^{-u_2} & -u_1e^{-u_2} \\ 2u_1 & 0 \end{bmatrix}\begin{bmatrix} 2x_1 & 1 \\ 4x_1 & -1 \end{bmatrix}, \begin{bmatrix} 6e^2 & 0 \\ -2e^{-2} & 2e^{-2} \\ 4 & 2 \end{bmatrix}$;

d) $\begin{bmatrix} 0 & 2u_2 & \dots & 2u_n \\ 2u_1 & 0 & \dots & 2u_n \\ \vdots & \vdots & & \vdots \\ 2u_1 & & \dots & 0 \end{bmatrix}\begin{bmatrix} 2x_1 + x_2 & x_1 \\ 2x_1 + 2x_2 & 2x_1 \\ \vdots & \vdots \\ 2x_1 + nx_2 & nx_1 \end{bmatrix}, \begin{bmatrix} 4(n-1) & n^2 + n - 2 \\ 4(n-1) & n^2 + n - 4 \\ \vdots & \vdots \\ 4(n-1) & n^2 + n - 2n \end{bmatrix}$.

2. a) 31; **b)** $-\frac{3}{2}$. **4.** $\begin{bmatrix} 13 & 26 \\ -8 & 1 \end{bmatrix}$. **5.** $\begin{bmatrix} -5 & -9 \\ 16 & 2 \end{bmatrix}$.

Section 2.11, page 116

1. a) $2x/z,\ y/z$; **b)** $-(yz + 4xz + 3z^2)/(xy + 2x^2 + 6xz),\ -z/(y + 2x + 6z)$;
c) $-z/(3z^2 + x + 2y),\ -2z/(3z^2 + x + 2y)$;
d) $-ze^{xz}/(xe^{xz} + ye^{yz} + 1),\ -ze^{yz}/(xe^{xz} + ye^{yz} + 1)$.
2. $\left(\frac{\partial x}{\partial y}\right)_z = -\frac{5}{4}, \left(\frac{\partial y}{\partial x}\right)_u = -\frac{5}{7}, \left(\frac{\partial z}{\partial u}\right)_x = -\frac{5}{7}, \left(\frac{\partial y}{\partial z}\right)_x = \frac{1}{5}$. **3. a)** $3x/u,\ y/u$;
b) $(xu + vy - ue^v)/(e^{u+v} - xe^u + xe^v - x^2 + y^2),\ (ve^v - xv - yu)/$
$(e^{u+v} - xe^u + xe^v - x^2 + y^2)$.
c) $-(4xy + 2yu + u^2 - 2yuv)/(2xy + 2vy + xu - 2xyv),\ (2uv - 2yv^2)/$
$(2xy + 2vy + xu - 2xyv)$. **4. a)** $du = \frac{1}{9}(dx + 3\,dy + 2\,dz),\ dv = -\frac{1}{3}(dx + 2\,dz)$;
b) $\left(\frac{\partial u}{\partial x}\right)_{y,z} = \frac{1}{9}, \left(\frac{\partial v}{\partial y}\right)_{x,z} = 0$; **c)** $u = 3.033,\ v = 2.1$.
5. $(3yu - 4xu + 4x^2 + 9xy + 8xv)^{-1}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $a = 8uv + 4u^2 - 8xy - 4xu$,
$b = -3u^2 - 8x^2 - 9xu,\ c = 4xy - 3y^2 - 6yu - 4xv - 9yv - 4uv - 8v^2$,
$d = 4x^2 - 3xy + 6xu + 3uv$. **6. a)** $\frac{1}{2}, -1$; **b)** $-1, \frac{3}{2}$; **c)** $-1, -1$.

Section 2.12, page 121

2. a) $u = \frac{1}{5}(x + 2y),\ v = \frac{1}{5}(y - 2x)$; **b)** $J = 5$, J for inverse $= \frac{1}{5}$.
3. a) $J = 4(u^2 + v^2)$, **b)** $\left(\frac{\partial u}{\partial x}\right)_y = \frac{u}{2(u^2 + v^2)}, \left(\frac{\partial v}{\partial x}\right)_y = -\frac{v}{2(u^2 + v^2)}$. **6. a)** $J = \rho^2 \sin\phi$;
b) $\frac{\partial \rho}{\partial y} = \sin\phi \sin\theta,\ \frac{\partial \phi}{\partial z} = -\frac{\sin\phi}{\rho},\ \frac{\partial \theta}{\partial x} = -\frac{\sin\theta}{\rho \sin\phi}$.

Section 2.13, page 127

1. b) $x = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(t - \frac{\pi}{3}\right),\ y = \frac{1}{2} - \frac{\sqrt{3}}{2}\left(t - \frac{\pi}{3}\right),\ z = \frac{3}{4} + \frac{\sqrt{3}}{2}\left(t - \frac{\pi}{3}\right)$;
c) $x - \sqrt{3}y + \sqrt{3}z = 3\sqrt{3}/4$. **2. b)** $\sqrt{3}x + 2y + z = 13/4$.
5. b) $3\sqrt{3}x - y - 4z = 1$. **8. a)** $2x + 2y + z = 9,\ \frac{x - 2}{2} = \frac{y - 2}{2} = \frac{z - 1}{1}$;

b) $z = 1, x = 0, y = 0$; **c)** $2x - y - z = 0, \frac{x-1}{2} = \frac{y-1}{-1} = \frac{z-1}{-1}$; **d)** none;
e) $y_1x + x_1y - z = x_1y_1$; **f)** $(y_1 + z_1)x + (x_1 + z_1)y + (x_1 + y_1)z = 2$.
10. a) $z - 2 = 2(x - 1) + 2(y - 1), \frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{-1}$;
b) $z - \frac{1}{3} = -2\left(x - \frac{2}{3}\right) - 2\left(y - \frac{2}{3}\right), \frac{x-\frac{2}{3}}{2} = \frac{y-\frac{2}{3}}{2} = \frac{z-\frac{1}{3}}{1}$;
c) $z = x - 2y + 2, x = 2 + t, y = 1 - 2t, z = 2 - t$;
d) $5z = 6x + 8y - 10, x = \frac{3}{5} + \frac{6}{5}t, y = \frac{4}{5} + \frac{8}{5}t, z = -t$.
11. a) $x = 3 + 4t, y = 1 - 5t, z = 1 + 3t$; **b)** $x = 2 + t, y = 2 - t, z = 1$;
c) $x = 1, y = -1 + t, z = -t$.

13. $$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} & \frac{\partial G}{\partial z} \end{vmatrix} = 0, \begin{vmatrix} x-2 & y-2 & z-1 \\ 4 & 4 & 2 \\ 4 & 4 & -16 \end{vmatrix} = 0,$$

$$\begin{vmatrix} x-1 & y & z+1 \\ 2 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 0.$$

14. $y + 2z = 0$. **15. a)** $2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$; **b)** $4x\mathbf{i} + 2y\mathbf{j}$. **16. c)** $x + y + \sqrt{2}z = 2$.
17. $\frac{x - x_1}{\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}} = \frac{y - y_1}{\frac{\partial f}{\partial x} - \frac{\partial g}{\partial x}} = \frac{z - z_1}{\frac{\partial(f,g)}{\partial(x,y)}}$. **18.** $\frac{x - x_1}{\frac{\partial(F,G,H)}{\partial(t,y,z)}} = \frac{y - y_1}{\frac{\partial(F,G,H)}{\partial(x,t,z)}} = \frac{z - z_1}{\frac{\partial(F,G,H)}{\partial(x,y,t)}}$.

Section 2.14, page 134
1. a) $-\sqrt{22}$; **b)** 0; **c)** $\frac{1}{2}$; **d)** $-\frac{4}{\sqrt{5}}$; **e)** $-\frac{2}{3}$; **f)** 0. **2. a)** $x^2 - y^2$;
b) $\frac{7xyz}{\sqrt{x^2 + 4y^2 + 16z^2}}$. **6.** $du/ds = -2xy$. Minimum is -4 at $(0, \pm 2)$.
8. Maximum is 6 in direction $\mathbf{i}$ at (1, 0), in direction $-\mathbf{i}$ at $(-1, 0)$. **9.** 3, $\frac{7}{\sqrt{2}}$, $\frac{15}{\sqrt{3}}$.

Section 2.18, page 142
1. a) $\frac{2x^2 - y^2}{(x^2 + y^2)^{\frac{5}{2}}}, \frac{2y^2 - x^2}{(x^2 + y^2)^{\frac{5}{2}}}$; **b)** $\frac{2xy}{(x^2 + y^2)^2}, \frac{-2xy}{(x^2 + y^2)^2}$;
c) $4xe^{x^2-y^2}(2y^2 - 1), -4ye^{x^2-y^2}(2x^2 + 1)$; **d)** $m!n!$. **4. c)** $a = -c$;
d) $c = -3a, b = -3d$.
6. $\frac{\partial z}{\partial x}\frac{\partial^2 x}{\partial u \partial v} + \frac{\partial z}{\partial y}\frac{\partial^2 y}{\partial u \partial v} + \frac{\partial^2 z}{\partial x^2}\frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial^2 z}{\partial x \partial y}\left(\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}\right) + \frac{\partial^2 z}{\partial y^2}\frac{\partial y}{\partial u}\frac{\partial y}{\partial v}$.
10. $\frac{2(u^2 - y^2)}{(1 - 2ux)^3}(2u - 3u^2x - xy^2)$. **12. a)** $\frac{dy}{du} = \frac{1}{y+u}$; **b)** $\frac{dv}{du} = \frac{2u - v}{u + v}$; **c)** $\frac{d^3y}{dt^3} = 0$;
d) $\frac{d^2x}{dy^2} - 1 = 0$; **e)** $\frac{d^2v}{dx^2} - x^2v = 0$; **f)** $\frac{\partial u}{\partial w} = 0$; **g)** $\frac{\partial^2 u}{\partial z \partial w} = 0$.

Section 2.21, page 158
1. a) max. at -1, min. at 1; **b)** max. at $\pi/3 + 2n\pi$, min. at $-\pi/3 + 2n\pi$, horiz. infl. at $\pi + 2n\pi$ $(n = 0, \pm 1, \pm 2, \ldots)$; **c)** max. at log 2.
2. min. for $n = 2, 4, 6, \ldots$, horiz. infl. for $n = 3, 5, 7, \ldots$.
3. a) max. = 1, min. = 0; **b)** max. = 0; **c)** no max. or min.;
d) max. = $\frac{1}{2}$, min. = $-\frac{1}{2}$. **4. a)** max. at (0, 0); **b)** min. at (0, 0);
c) saddle point at (1, 1); **d)** saddle point at (0, 0); **e)** critical point at every point of the line $y = x$, each point giving a relative min.; **f)** triple saddle point at (0, 0);
g) min. at $\left(\sqrt{2}, \frac{\pi}{4} + 2n\pi\right), \left(-\sqrt{2}, \frac{5\pi}{4} + 2n\pi\right)$, neither at $\left(0, n\pi - \frac{\pi}{4}\right)$;
h) min. at $\left(\frac{1}{3}, \frac{1}{3}\right)$, saddle points at (0, 0), (1, 0), (0, 1); **i)** (0, 0), neither max. nor min.;
j) min. at (0, 0); **k)** min. at $(0, \pm 1)$, saddle point at (0, 0).
5. a) abs. max. at (0, 0); **b)** saddle point at (0, 0);

c) saddle points at $\pi/2 + 2n\pi$ $(n = 0, \pm 1, \pm 2, \ldots)$;
d) no critical points [discontinuity at (0, 0)]; **e)** abs. min. at (0, 0);
f) abs. max. at $(\sqrt{3}/3, \sqrt{3}/3)$. **6. a)** abs. min. at $(-3/5, -4/5)$, abs. max. at $(3/5, 4/5)$;
b) abs. max. at $(\pm 2^{-1/4}, \pm 2^{-1/4})$ (four points), abs. min. at $(\pm 1, 0)$ and $(0, \pm 1)$;
c) max. at $(\pm 3, \pm 4)$, min. at $(\pm 4, \mp 3)$; **d)** max. at $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$, min. at $\left(-\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$;
e) max. at $\left(\pm\sqrt{\frac{2}{3}}, \sqrt{\frac{1}{3}}, \pm\sqrt{\frac{2}{3}}\right)$ and $(0, -1, 0)$, min. at $\left(\pm\sqrt{\frac{2}{3}}, -\sqrt{\frac{1}{3}}, \pm\sqrt{\frac{2}{3}}\right)$ and $(0, 1, 0)$.
f) Abs. min. at $(1 - \sqrt{2}/2, 1 - \sqrt{2}/2, \sqrt{2} - 1)$, rel. min. at $(1 + \sqrt{2}/2, 1 + \sqrt{2}/2, -\sqrt{2} - 1)$, no abs. max. **7.** $(0, \pm 1, 0)$ and $(\pm 1, 0, 0)$.
8. a) max. $= 1$; **b)** max. $= \frac{1}{2}$, min. $= -\frac{1}{2}$; **c)** max. $= 3$, min. $= -3$; **d)** max. $= 1$.
9. (a) and (c) are positive definite. **11.** $a = \frac{1}{14}(2e_1 - e_2 - 2e_3 - e_4 + 2e_5)$,
$b = \frac{1}{10}(-2e_1 - e_2 + e_4 + 2e_5)$, $c = \frac{1}{35}(-3e_1 + 12e_2 + 17e_3 + 12e_4 - 3e_5)$. **12.** $\sqrt{3}/3$.

Section 2.22, page 166
1. $1, 0, \sin x$. **3.** $ax + by + c$, a, b, c arbitrary constants.
5. $f(x, y) = g(x) + h(y)$, $g(x)$ and $h(y)$ being "arbitrary functions."

CHAPTER 3

Section 3.3, page 180
9. a) $\begin{bmatrix} 6xy & 3x^2 & 0 \\ 3x^2 & -6yz & -3y^2 \\ 0 & -3y^2 & 0 \end{bmatrix}$, $\begin{bmatrix} 2 & 4 & 10 \\ 4 & 8 & 2 \\ 10 & 2 & 4 \end{bmatrix}$.

Section 3.6, page 185
5. a) $x^2yz + \text{const}$; **b)** $(2y + z^2)e^{xy} + \text{const}$. **7. a)** $yz\mathbf{i} - 2xz\mathbf{j} + \operatorname{grad} f$, f arbitrary;
b) $\frac{1}{2}[z^2\mathbf{i} + (x^2 - 2yz)\mathbf{j}] + \operatorname{grad} f$, f arbitrary. **12. b)** $\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y}$; **c)** $2x^2\mathbf{i} + 2y^2\mathbf{j}$.
15. $-\frac{2}{3}$. **16.** $\operatorname{div} \mathbf{v} = 0$, $\operatorname{curl} \mathbf{v} = 2\boldsymbol{\omega}$. **17.** Vol. $= 1$. **18.** Vol. $= e$.

Section 3.11, page 210
1. a) $u_1 = -36x^1x^1 + 51x^1x^2 - 18x^2x^2 - 28x^1 + 20x^2$,
$u_2 = 24x^1x^1 - 34x^1x^2 + 12x^2x^2 + 21x^1 - 15x^2$;
b) $v^1 = (3x^1 - 2x^2)[3\cos(-4x^1 + 3x^2) + 2\sin(-4x^1 + 3x^2)]$,
$v^2 = (3x^1 - 2x^2)[(4\cos(-4x^1 + 3x^2) + 3\sin(-4x^1 + 3x^2)]$;
c) $w_{11} = w_{22} = 0$, $w_{12} = -w_{21} = -12x^1x^1 + 17x^1x^2 - 6x^2x^2$;
d) $z_1^1 = -71x^1 + 49x^2$, $z_2^1 = 53x^1 - 36x^2$, $z_1^2 = -103x^1 + 72x^2$, $z_2^2 = 77x^1 - 53x^2$.
2. In standard coordinates, all reduce to δ_{ij}. In (x^i), $g_{11} = g^{22} = 25$,
$g_{12} = g_{21} = -g^{12} = -g^{21} = -18$, $g_{22} = g^{11} = 13$, $g^i_j = \delta_{ij}$ always.
3. a) $u^1 = 13u_1 + 18u_2$, $u^2 = 18u_1 + 25u_2$, where u_1, u_2 are as in the answer to Problem 1(a); **b)** $v_1 = 25v^1 - 18v^2$, $v^2 = -18v^1 + 13v^2$, where v^1, v^2 are as in the answer to Problem 1(b); **c)** $w^{11} = w^{22} = 0$, $w^{12} = -w^{21} = w_{12}$, where w_{12} is as in the answer to Problem 1(c).

CHAPTER 4

Section 4.1, page 219
1. a) $2x\sin x - (x^2 - 2)\cos x + c$; **b)** $\frac{1}{2}\arctan x^2 + c$; **c)** $\log\frac{2 - x}{x - 1} + C$;
d) $2(\sqrt{x - 1} - \log(1 + \sqrt{x - 1})) + C$. **2. a)** $\pi/4$; **b)** 0; **c)** $3e - 8$;
d) $(\pi/4) + \log(1/\sqrt{2})$. **3. a)** $\pi/2$; **b)** 1; **c)** -1; **d)** $\log(1 + \sqrt{2})$; **e)** 2; **f)** 1.

4. a) 0; **b)** div; **c)** $\pi/2$; **d)** div; **e)** div; **f)** $\log 2$. **5. a)** $4/3$; **b)** 1; **c)** $14-\pi-6\sqrt{3}$. **6. a)** $2/\pi$; **b)** $-2/\pi$; **c)** $1/2$, **d)** $b+\frac{1}{2}a(x_1+x_2)$.
8. a) value 0.3095, error at most 0.0082 (worst error at $x=1$); **b)** value 0.7667, error at most 0.1321 (worst error at $x=1$).

Section 4.5, page 234
1. a) $1/3$; **b)** $\pi/48$; **c)** $(-15\sqrt{2})/8$; **d)** 1. **2. a)** $e-1$; **b)** $(2-5e^{-1})(1-e^{-2})$; **c)** $3/4$; **d)** $\pi/6$. **3. a)** $\int_1^2\int_{1-x}^{1+x} f\,dy\,dx$ or $\int_{-1}^{3}\int_{\phi(y)}^{2} f\,dx\,dy$, where $\phi(y)=|1-y|$ for $-1\le y\le 0$ and for $2\le y\le 3$, $\phi(y)=1$ for $0\le y\le 2$; **b)** $\int_0^1\int_{-\phi(x)}^{\phi(x)} f\,dy\,dx$ or $\int_{-1/2}^{1/2}\int_{1/2-\psi(y)}^{1/2+\psi(y)} f\,dx\,dy$, where $\phi(x)=(x-x^2)^{1/2}$, $\psi(y)=\frac{1}{2}(1-4y^2)^{1/2}$.
4. a) $(8/35)(9\sqrt{3}-8\sqrt{2}+1)$; **b)** $2/5$. **5. a)** $\int_0^{1/2}\int_{1/2}^{1-y} f\,dx\,dy$;
b) $\int_0^1\int_0^{\sqrt{1-y^2}} f\,dx\,dy$; **c)** $\int_{-1}^{0}\int_0^{x+1} f\,dy\,dx$; **d)** $\int_0^2\int_{|y-1|}^{1} f\,dx\,dy$.
10. a) $\frac{1}{3}\mathbf{i}-(e-1)\mathbf{j}+\log 2\mathbf{k}$; **b)** $\frac{1}{60}(\mathbf{i}+\mathbf{j})$.

Section 4.6, page 241
1. a) $\frac{3}{16}\pi$; **b)** $2\sqrt{2}-2+2\log(2\sqrt{2}-2)$; **c)** $\sqrt{2}\left(\tan^{-1}\sqrt{2}-\tan^{-1}\frac{1}{\sqrt{2}}\right)$;
d) $\log c+1-c$, $c=\pi/(4\sqrt{2})$. **3. b)** $\cosh 1-1$; **c)** $\log 2$.
4. a) $\pi/2$; **b)** $(14\sqrt{2}-7)/9$; **c)** $\pi^4/3$; **d)** $\frac{11}{18}-\frac{1}{3}\log\frac{3}{2}$; **e)** $\frac{2\pi}{9}$.
5. $\int_0^e\int_{f(u)}^{g(u)} \frac{1}{1+u^2v^2}\,dv\,du$, where $f(u)=\sqrt{1-u^2}$ for $0\le u\le 1$, $f(u)=0$ for $1\le u\le e$, and $g(u)=\sqrt{e^2-u^2}$ for $0\le u\le e$.
7. a) $2\int_0^{\frac{1}{2}}\int_v^{1-v}\log(1+2u^2+2v^2)\,du\,dv$; **b)** $\int_0^1\int_{1-2u}^{1}\sqrt{1+u^2(u+v)^2}\,dv\,du$.
11. a) $\int_0^{2\pi}\int_0^1\int_0^1 r^4\cos^2\theta\sin\theta\,dz\,dr\,d\theta$; **b)** $\int_0^{\pi/2}\int_0^1\int_0^{1+r(\cos\theta+\sin\theta)} r^3\cos 2\theta\,dz\,dr\,d\theta$.
12. a) $\int_0^{2\pi}\int_0^{\pi}\int_0^{a}\rho^5\sin^4\phi\cos^2\theta\sin\theta\,d\rho\,d\phi\,d\theta$; **b)** $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sec\phi}\rho^4\sin\phi\,d\rho\,d\phi\,d\theta$.

Section 4.7, page 248
3. a) $4\pi^2ab$; **b)** $\pi(5^{3/2}-1)/24$.

Section 4.8, page 252
2. $-\pi/2$. **3. a)** $4\pi/(3-p)$; **b)** $4\pi/(p-3)$.
4. a) div; **b)** conv; **c)** div; **d)** conv; **e)** conv.

Section 4.9, page 256
1. a) $-\int_{\pi/2}^{\pi}\sin(xt)\,dx$; **b)** $\int_1^2\frac{2x^3}{(1-tx)^3}\,dx$; **c)** $\int_1^2\frac{1}{u}\,dx$; **d)** $n!\int_1^2\frac{\sin x}{(x-y)^{n+1}}\,dx$.
2. a) x^2; **b)** $2t\sin t^4$; **c)** $-3t^2\log(1+t^6)$; **d)** $\sec^2 x e^{-\tan^2 x}-e^{-x^2}$.
4. a) $\frac{-1}{(n+1)^2}$; **b)** $\frac{n!}{a^{n+1}}$; **c)** $\frac{\pi}{2}\frac{1\cdot 3\cdots(2n-3)}{2\cdot 4\cdots(2n-2)}\frac{1}{x^{2n-1}}$, $x>0$.

Section 4.11, page 264
1. b) uniformly continuous. **2.** All but b) are uniformly continuous.
4. a) f must be uniformly continuous; **b)** f need not be uniformly continuous.

CHAPTER 5

Section 5.3, page 278
1. a) $\frac{8}{3}$; **b)** $-\frac{3}{2}$; **c)** 0. **2. a)** $\frac{4}{3}$; **b)** 8; **c)** $-\pi/2$.
3. a) 0; **b)** -2π; **c)** $-\frac{1}{4}$. **4. a)** 0; **b)** $\sqrt{2}/2$; **c)** $\frac{1}{2}\sqrt{5}+\frac{1}{4}\log(2+\sqrt{5})$.
6. a) 7; **b)** 5; **c)** 8; **d)** 5.5; **e)** 0.

Section 5.5, page 286
1. a) $\frac{4}{3}$; **b)** $\frac{4}{3}$; **c)** $\frac{4}{3}$. **2. a)** 0; **b)** $\frac{1}{3}$; **c)** $\frac{4}{3}$.
5. a) $(b-a)$ times area enclosed by C; **b)** 0; **c)** $\frac{3}{2}\pi$; **d)** 0; **e)** 0; **f)** 4π; **g)** 0; **h)** 0.

Section 5.7, page 300
1. a) $F = x^2y$, integral $= 1$; **b)** $F = e^{xy}$, integral $= 0$; **c)** $F = -1/\sqrt{x^2+y^2}$, integral $= 1 - e^{-2\pi}$. **2. a)** $-\frac{10}{3}$; **b)** $\frac{1}{3}$; **c)** 2; **d)** 1.
3. a) 0; **b)** -2π; **c)** -2; **d)** 12π; **e)** 0; **f)** 0.
4. $\frac{\pi}{4} + 2n\pi$ $(n = 0, \pm 1, \pm 2, \ldots)$. **5. a)** 0; **b)** 2π; **c)** 4π; **d)** 0.
6. a) $x^2y - \frac{1}{3}(y^3+2)$; **b)** $x \sin y$. **7.** $-\pi$. **8.** $K + n_1\alpha_1 + n_2\alpha_2 + \cdots + n_k\alpha_k$, where the numbers $n_1, \ldots, n_k$ are positive or negative integers or 0. **9.** 7. **10. a)** -60; **b)** 0.

Section 5.10, page 312
1. a) 3π; **b)** $-\frac{5}{3}$; **c)** $\frac{1}{2}$; **d)** 0; **e)** $-163/15$. **5. a)** $\frac{1}{2}$; **b)** π; **c)** 2π; **d)** $\frac{\pi}{2}$.
7. a) $\pi/48$; **b)** $-(e-1) - \frac{3\pi}{4}(e^2-1)$; **c)** $\pi/8$; **d)** $(e^2-1)/2$; **e)** 0.

Section 5.11, page 319
1. a) 4π; **b)** 3; **c)** 0; **d)** $6V$; **e)** 0; **f)** 0.

Section 5.13, page 330
1. a) 6π; **b)** 0. **2. a)** -2; **b)** 0. **4.** $\frac{\pi}{2} \pm 2n\pi$. **5. b)** $\mathbf{v} + \operatorname{grad} f$, where $\mathbf{v} = x^2y^2z\mathbf{i} - xy^3z\mathbf{j} + xy^2z^2\mathbf{k}$.

Section 5.14, page 336
1. a) $\int_0^1 \int_0^u (u^2+v^2)\,dv\,du$; **b)** $\iint_{R_{uv}} (u-v)(1-2u-2v)\,du\,dv$;
c) $4\iint_{R_{uv}} uv(u^4-v^4)\,du\,dv$. **3. a)** -1; **b)** 2; **c)** 3. **6. a)** -1; **b)** 2.

Section 5.15, page 347
1. a) Potential energy is $\frac{1}{2}k^2x^2$; $\frac{1}{2}mv^2 + \frac{1}{2}k^2x^2 = \text{const.}$ **b)** Potential energy is $\frac{1}{2}(a^2x^2+b^2y^2)$; $\frac{1}{2}(mv^2+a^2x^2+b^2y^2) = \text{const.}$ **5.** $T = T_1 + \frac{T_2-T_1}{d}x$.

Section 5.18, page 363
3. (a) and (b) $2\pi ab \log 1/a$ for $R < a$, $2\pi ab \log 1/R$ for $R > a$;
c) $\mathbf{0}$ for $R < a$, $-2\pi abR^{-2}\mathbf{R}$ for $R > a$. **4. a)** $-2\pi\gamma$ for $R < a$, 0 for $R > a$;
b) normal derivatives have limits 0 everywhere. **6.** $U = \pi k(4a^3 - R^3)/3$ for $R \le a$, $U = \pi ka^4/R$ for $R \ge a$.

Section 5.22, page 373
1. a) $x^1(x^4)^2\,dx^1 - (x^1)^2x^4\,dx^4$; **b)** $-x^1\,dx^2\,dx^3 - x^2\,dx^3\,dx^1 - x^3\,dx^1\,dx^2$;
c) $2x^1x^4\,dx^1\,dx^2\,dx^3 - 2(x^1)^2\,dx^2\,dx^3\,dx^4 + 2x^1x^2\,dx^1\,dx^3\,dx^4 - 2x^1x^3\,dx^1\,dx^2\,dx^4$;
d) 0 (4-form); **e)** $x^4\,dx^1 + x^1\,dx^4$; **f)** 0; **g)** $3\,dx^1\,dx^2\,dx^3$.
5. a) $(\bar{x}^3+3\bar{x}^4)\,d\bar{x}^1\,d\bar{x}^2 + (6\bar{x}^4-\bar{x}^2)\,d\bar{x}^1\,d\bar{x}^3 + (-3\bar{x}^2-6\bar{x}^3)\,d\bar{x}^1\,d\bar{x}^4 + (\bar{x}^1+6\bar{x}^4)\,d\bar{x}^2\,d\bar{x}^3 + (3\bar{x}^1-6\bar{x}^3)\,d\bar{x}^2\,d\bar{x}^4 + (6\bar{x}^1+6\bar{x}^2)\,d\bar{x}^3\,d\bar{x}^4$;
b) $(\bar{x}^1)^3\bar{x}^3\,d\bar{x}^1\,d\bar{x}^2 - (\bar{x}^1)^3\bar{x}^2\,d\bar{x}^1\,d\bar{x}^3 + (\bar{x}^1)^4\,d\bar{x}^2\,d\bar{x}^3$.
6. a) $\int (\overline{X}_1+\overline{X}_2+\overline{X}_3)\,d\bar{x}^1 + \overline{X}_2\,d\bar{x}^2 + \overline{X}_3\,d\bar{x}^3$ on $\overline{C}$;
b) $\iint \overline{X}_1\,d\bar{x}^2\,d\bar{x}^3 + (\overline{X}_2-\overline{X}_1)\,d\bar{x}^3\,d\bar{x}^1 + (\overline{X}_3-\overline{X}_1)\,d\bar{x}^1\,d\bar{x}^2$ on $\overline{S}$.
8. $C_{ijk} = A_iB_{jk} + A_kB_{ij} + A_jB_{ki}$.