The Hashemite University | Department of Mathematics | October 24, 2012
Ordinary Differential Equations (1) | First Exam | Time: 60 Minutes
Name (in Arabic): ____________ Student Number: ____________ Serial Number: ____________
Section number or lecture time: ____________ Instructor name: ____________

Question one: Circle the letter that represents the correct answer (2 points each).

1. The differentia equation $yy' = x(\ln y^2 - \ln x^2)$, is

   a. separable b. linear c. homogeneous d. second order

2. A general solution to $y' = (3x+y)^2 - 2$ is

   a. $tan^{-1}(3x+y)+x=c$ b. $tan^{-1}(x+y)-3x=c$
   c. $tan^{-1}(2x+y)+3x=c$ d. $tan^{-1}(3x+y)-x=c$

3. If $\frac{dy}{dt} = -\ln(t^y)$, $y>0$ with $y(1)=e^{-1}$, then $y(2)=$

   a. 4 b. $\frac{1}{4}$ c. $4e$ d. $\frac{1}{4}e^2$

4. If $F(x,y)=c$ is a general solution to $(2x+4y)dx+(y-x)dy=0$, then a general solution to $(2x+4y-2)dx+(y-x-2)dy=0$ is

   a. $F(x-1,y+1)=c$ b. $F(x+1,y-1)=c$ c. $F(x-3,y+2)=c$ d. $F(x+3,y-2)=c$

5. An integrating factor (of the form $\mu(x,y)=x^p+y^q$) for $2ydx+xdy=0$ is

   a. $\mu(x,y)=x+\sqrt[2]{y^{-1}}$ b. $\mu(x,y)=x^{-3}+\sqrt[2]{y^{-3}}$
   c. $\mu(x,y)=\sqrt[2]{x^{-1}}+y$ d. $\mu(x,y)=\sqrt[2]{x^{-3}}+y^{-3}$

6. The particular solution of $(\cos x - x\sin x + y^2)dx + 2xy\,dy = 0$ that satisfies the initial condition $y=1$ when $x=2\pi$ is

   a. $xy^2 - x\cos x = 0$ b. $xy^2 - x\cos x = 2\pi$
   c. $xy^2 + x\cos x = 4\pi$ d. $xy^2 + x\cos x = 0$

Question two : Show that $y^2+(x^2+1)y-3x^3-6=0$ is solution for the following IVP (3 points)

$$2xy^2 - 9x^2y + (2y^2+x^2y+y)\frac{dy}{dx} = 0, \quad y(0)=-3$$

$2yy' + (x^2+1)y' + (2x)y - 9x^2 = 0$ } 1

$(2y+x^2+1)y' + 2xy - 9x^2 = 0$

multip by y $\Rightarrow (2y^2+x^2y+y)y' + 2xy^2 - 9x^2y = 0$ } 1 $\therefore$ soln to D.E.

& satisfies I. Cond.: $(-3)^2+(0^2+1)(-3)-3(0)^3-6$
$9 \quad -3 \quad -6 \quad = 0$ } 1

Question three : Consider the following differential equation (4 points)

$$(2e^{3xy} + 3xye^{3xy})dx + (3x^2e^{3xy} - x^{-1})dy = 0 \tag{1}$$

(a) Show that equation (1) is non-exact.

$$6xe^{3xy} + 3xy(3xe^{3xy}) + (3x)e^{3xy} \neq 3x^2(3ye^{3xy}) + (6x)e^{3xy} + x^{-2}$$

0.5 1 0.5

(b) Find an integrating factor $[\mu(x)]$ of equation (1).

0.5 $$\frac{M_y - N_x}{N} = \frac{3xe^{3xy} - x^{-2}}{3x^2e^{3xy} - x^{-1}} \cdot \frac{x}{x} = \frac{1}{x}$$ 1

$$\therefore \mu = e^{\int \frac{1}{x}dx} = e^{\ln|x|} = |x|$$ 0.5

Question four : Find a general solution to the following equation (6 points)

$$5\frac{dy}{dx} + y = \frac{y^{-4}}{e^{2x}+1} \tag{2}$$

$v = y^{1--4} = y^5$ 1

$v' = 5y^4y'$

$$5y^4y' + y^5 = \frac{1}{e^{2x}+1}$$

$$v' + v = \frac{1}{e^{2x}+1}$$ 1 lin.

$$\mu = e^{\int 1dx} = e^x$$ 1

$$\therefore v = \frac{1}{e^x}\left(\int \frac{1}{e^{2x}+1}\cdot e^x\,dx + c\right)$$ 1

$$v = e^{-x}\left(\tan^{-1}(e^x) + c\right)$$ 1

1 $$y^5 = e^{-x}\left(\tan^{-1}e^x + c\right)$$

Question three : Consider the following differential equation (4 points)

$$(3y^2e^{3xy} - y^{-1})dx + (2e^{3xy} + 3xye^{3xy})dy = 0 \tag{1}$$

(a) Show that equation (1) is non-exact.

$3y^2 \cdot 3xe^{3xy} + 6ye^{3xy} + y^{-2} \neq 6ye^{3xy} + 3xy \cdot 3ye^{3xy} + 3ye^{3xy}$

$\frac{1}{2}$ 1 $\frac{1}{2}$

(b) Find an integrating factor $[\mu(y)]$ of equation (1).

$\frac{1}{2}$ $\frac{N_x - M_y}{M} = \frac{3ye^{3xy} - y^{-2}}{3y^2e^{3xy} - y^{-1}} = \frac{1}{y}$ 1

$\therefore \mu = e^{\int \frac{1}{y}dy} = e^{\ln|y|} = |y|$ $\frac{1}{2}$

Question four : Find a general solution to the following equation (6 points)

$$4\frac{dy}{dx} + y = \frac{y^{-3}}{e^{2x} + 1} \tag{2}$$

$v = y^{1--3} = y^4$ 1

$v' = 4y^3y'$

$4y^3y' + y^4 = \frac{1}{e^{2x}+1}$

$\therefore v' + v = \frac{1}{e^{2x}+1}$ 1

$\therefore \mu = e^{\int 1dx} = e^x$ 1

$\therefore v = \frac{1}{e^x}\left(\int e^x \cdot \frac{1}{e^{2x}+1}dx + c\right)$ 1

1

$v = e^{-x}\left(\tan^{-1}e^x + c\right)$

$\therefore$ 1 $y^4 = e^{-x}\left(\tan^{-1}e^x + c\right)$

The Hashemite University — Department of Mathematics — October 24, 2012

Ordinary Differential Equations (1) — First Exam — Time: 1 hour

Name (in Arabic): ____________ Student Number: ____________ Serial Number: ____________

Section number or lecture time: ____________________ Instructor name: ____________________

Question one: Circle the letter that represents the correct answer (2 points each).

1. The differentia equation $xy' = y(\ln x^2 - \ln y^2)$, is

   a. separable b. linear c. second order d. homogeneous

2. A general solution to $y' = (2x+y)^2 - 1$ is

   a. $tan^{-1}(2x+y) + x = c$ b. $tan^{-1}(x+y) - 2x = c$
   c. $tan^{-1}(2x+y) - x = c$ d. $tan^{-1}(3x+y) + 2x = c$

3. If $\dfrac{dy}{dt} = \ln(t^y)$, $y > 0$ with $y(1) = e$, then $y(2) =$

   a. 4 b. $\frac{1}{4}$ c. $4e^2$ d. $\frac{1}{4}e^2$

4. If $F(x,y) = c$ is a general solution to $(2x+4y)dx + (y-x)dy = 0$, then a general solution to $(2x+4y+2)dx + (y-x+2)dy = 0$ is

   a. $F(x-1, y+1) = c$ b. $F(x+1, y-1) = c$ c. $F(x+2, y-2) = c$ d. $F(2x+4y, y-x) = c$

5. An integrating factor (of the form $\mu(x,y) = x^p + y^q$) for $2ydx - xdy = 0$ is

   a. $\mu(x,y) = x + \sqrt[2]{y^{-1}}$ b. $\mu(x,y) = x^{-3} + \sqrt[2]{y^{-3}}$
   c. $\mu(x,y) = \sqrt[2]{x^{-1}} + y$ d. $\mu(x,y) = \sqrt[2]{x^{-3}} + y^{-3}$

6. The particular solution of $(\cos x - x\sin x + y^2)dx + 2xy\,dy = 0$ that satisfies the initial condition $y = 1$ when $x = \pi$ is

   a. $xy^2 - x\cos x = 0$ b. $xy^2 - x\cos x = 2\pi$
   c. $xy^2 + x\cos x = 4\pi$ d. $xy^2 + x\cos x = 0$

Question two : Show that $y^2 + (x^2+1)y - 3x^3 - 12 = 0$ is solution for the following IVP (3 points)

$$2xy^2 - 9x^2y + (2y^2 + x^2y + y)\frac{dy}{dx} = 0, \quad y(0) = 3$$

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Ordinary Differential Equations (1) | First Exam | Time: 1 hour.
Name (in Arabic): ____________ Student Number: ____________ Serial Number: ____________
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Question one: Circle the letter that represents the correct answer (2 points each).

1. The differentia equation $x^2y' = y^2(\ln x - \ln y)$, is

   a. separable b. homogeneous c. second order d. linear

2. A general solution to $y' = (4x+y)^2 - 3$ is

   a. $tan^{-1}(4x+y) - x = c$ b. $tan^{-1}(x+y) - 4x = c$
   c. $tan^{-1}(5x+y) + 4x = c$ d. $tan^{-1}(4x+y) + x = c$

3. If $\frac{dy}{dt} = \ln(t^y)$, $y > 0$ with $y(1) = e^2$, then $y(2) =$

   a. 4 b. $\frac{1}{4}$ c. $4e$ d. $\frac{1}{4}e^2$

4. If $F(x,y) = c$ is a general solution to $(2x+4y)dx + (y-x)dy = 0$, then a general solution to $(2x+4y+2)dx + (y-x+5)dy = 0$ is

   a. $F(x-1, y+1) = c$ b. $F(x+1, y-1) = c$ c. $F(x-3, y+2) = c$ d. $F(x+3, y-2) = c$

5. An integrating factor (of the form $\mu(x,y) = x^p + y^q$) for $-ydx + 2xdy = 0$ is

   a. $\mu(x,y) = x + \sqrt[2]{y^{-1}}$ b. $\mu(x,y) = x^{-3} + \sqrt[2]{y^{-3}}$
   c. $\mu(x,y) = \sqrt[2]{x^{-1}} + y$ d. $\mu(x,y) = \sqrt[2]{x^{-3}} + y^{-3}$

6. The particular solution of $(\cos x - x\sin x + y^2)dx + 2xy\,dy = 0$ that satisfies the initial condition $y = 2$ when $x = \pi$ is

   a. $xy^2 - x\cos x = 2\pi$ b. $xy^2 + x\cos x = 3\pi$
   c. $xy^2 - x\cos x = 0$ d. $xy^2 + x\cos x = 0$

Question two : Show that $y^2 + (x^2+1)y - 3x^3 - 12 = 0$ is solution for the following IVP (3 points)

$$2xy^2 - 9x^2y + (2y^2 + x^2y + y)\frac{dy}{dx} = 0, \quad y(0) = 3$$

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Ordinary Differential Equations (1) — First Exam — $\boxed{\text{Time: 60 Minutes.}}$
Name (in Arabic): ______ Student Number: ______ Serial Number: ______
Section number or lecture time: ______ Instructor name: ______

Question one: Circle the letter that represents the correct answer (2 points each).

1. The differentia equation $y^2y' = x^2(\ln y - \ln x)$, is

   a. homogeneous b. linear c. separable d. second order

2. A general solution to $y' = (5x+y)^2 - 4$ is

   a. $tan^{-1}(5x+y)+x=c$ b. $tan^{-1}(5x+y)-x=c$
   c. $tan^{-1}(4x+y)+5x=c$ d. $tan^{-1}(x+y)-5x=c$

3. If $\dfrac{dy}{dt} = -\ln(t^y)$, $y>0$ with $y(1)=e$, then $y(2) =$

   a. 4 b. $\frac{1}{4}$ c. $4e$ d. $\frac{1}{4}e^2$

4. If $F(x,y)=c$ is a general solution to $(2x+4y)dx+(y-x)dy=0$, then a general solution to $(2x+4y-2)dx+(y-x-5)dy=0$ is

   a. $F(x-1,y+1)=c$ b. $F(x+1,y-1)=c$ c. $F(x-3,y+2)=c$ d. $F(x+3,y-2)=c$

5. An integrating factor (of the form $\mu(x,y)=x^p+y^q$) for $ydx+2xdy=0$ is

   a. $\mu(x,y)=x+\sqrt[2]{y^{-1}}$ b. $\mu(x,y)=x^{-3}+\sqrt[2]{y^{-3}}$
   c. $\mu(x,y)=\sqrt[2]{x^{-1}}+y$ d. $\mu(x,y)=\sqrt[2]{x^{-3}}+y^{-3}$

6. The particular solution of $(\cos x - x\sin x + y^2)dx + 2xy\,dy = 0$ that satisfies the initial condition $y=2$ when $x=2\pi$ is

   a. $xy^2+x\cos x=10\pi$ b. $xy^2-x\cos x=2\pi$
   c. $xy^2+x\cos x=0$ d. $xy^2-x\cos x=0$

Question two : Show that $y^2+(x^2+1)y-3x^3-6=0$ is solution for the following IVP (3 points)

$$2xy^2-9x^2y+(2y^2+x^2y+y)\frac{dy}{dx}=0, \quad y(0)=-3$$