# Some Hypothesis Testing Examples

## Group 1

One Tailed (Upper Tailed)

An insurance company is reviewing its current policy rates. When originally setting the rates they believed that the average claim amount was \$1,800. They are concerned that the true mean is actually higher than this, because they could potentially lose a lot of money. They randomly select 40 claims, and calculate a sample mean of \$1,950. Assuming that the standard deviation of claims is \$500, and set $\alpha = .05$, test to see if the insurance company should be concerned.

*Solution*

- Step 1: Set the null and alternative hypotheses

$$H_0: \ \mu \leq 1800$$

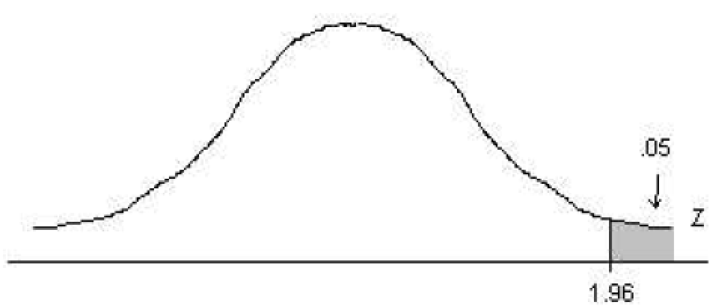
$$H_1: \ \mu > 1800$$

- Step 2: Calculate the test statistic

$$Z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{1950 - 1800}{500/\sqrt{40}} = 1.897$$

- Step 3: Set Rejection Region
  Looking at the the picture below, we need to put all of $\alpha$ in the right tail. Thus,

$$R: Z > 1.96$$




- Step 4: Conclude
  We can see that $1.897 < 1.96$, thus our test statistic is not in the rejection region. Therefore we fail to reject the null hypothesis. We cannot conclude anything statistically significant from this test, and cannot tell the insurance company whether or not they should be concerned about their current policies.

## One Tailed (Lower Tailed)

Trying to encourage people to stop driving to campus, the university claims that on average it takes people 30 minutes to find a parking space on campus. I don't think it takes so long to find a spot. In fact I have a sample of the last five times I drove to campus, and I calculated $\overline{x} = 20$. Assuming that the time it takes to find a parking spot is normal, and that $\sigma = 6$ minutes, then perform a hypothesis test with level $\alpha = .10$ to see if my claim is correct.

*Solution*

- Step 1: Set the null and alternative hypotheses

$$H_0 : \ \mu \geq 30$$

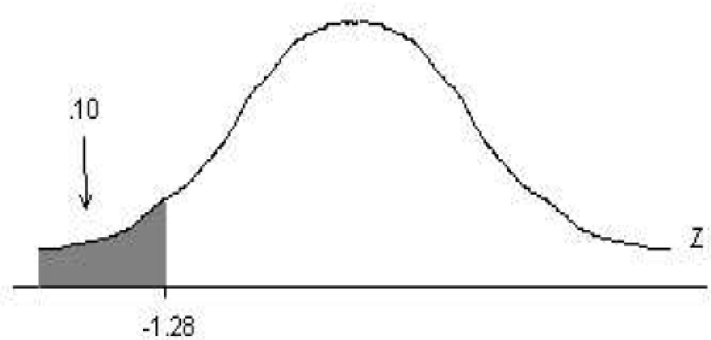
$$H_1 : \ \mu < 30$$

- Step 2: Calculate the test statistic

$$Z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{20 - 30}{6/\sqrt{5}} = -3.727$$

- Step 3: Set Rejection Region
  Looking at the the picture below, we need to put all of $\alpha$ in the left tail. Thus,

$$R : Z < -1.28$$




- Step 4: Conclude
  We can see that $-3.727 < -1.28$, thus our test statistic is in the rejection region. Therefore we reject the null hypothesis in favor of the alternative. We can conclude that the mean is significantly less than 30, thus I have proven that the mean time to find a parking space is less than 30.

## Two Tailed

A sample of 40 sales recepts from a grcery store has $\overline{x} = \$137$ and $\sigma = \$30.2$. Use these values to test whether or not the mean is sales at the grocery store are different from \$150.

*Solution*

- Step 1: Set the null and alternative hypotheses

$$H_0: \ \mu = 150$$

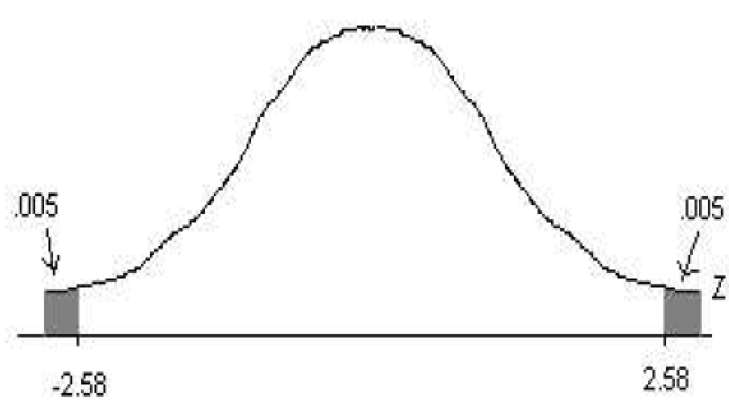
$$H_1: \ \mu \neq 150$$

- Step 2: Calculate the test statistic

$$Z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{137 - 150}{30.2/\sqrt{40}} = -2.722$$

- Step 3: Set Rejection Region
  Looking at the the picture below, we need to put half of $\alpha$ in the left tail, and the other half of $\alpha$ in the right tail. Thus,

$$R: |Z| > 2.58$$




- Step 4: Conclude
  We can see that $|-2.722| = 2.722 > 2.58$, thus our test statistic is in the rejection region. Therefore we reject the null hypothesis in favor of the alternative. We can conclude that the mean is significantly different from \$150, thus I have proven that the mean sales at the grocery store is not \$150.

## Group 2:

Hypothesis Testing Examples

1. Suppose we would like to determine if the typical amount spent per customer for dinner at a new restaurant in town is more than $20.00. A sample of 49 customers over a three-week period was randomly selected and the average amount spent was $22.60. Assume that the standard deviation is known to be $2.50. Using a 0.02 level of significance, would we conclude the typical amount spent per customer is more than $20.00?

2. Suppose an editor of a publishing company claims that the mean time to write a textbook is at most 15 months. A sample of 16 textbook authors is randomly selected and it is found that the mean time taken by them to write a textbook was 12.5. Assume also that the standard deviation is known to be 3.6 months. Assuming the time to write a textbook is normally distributed and using a 0.025 level of significance, would you conclude the editor's claim is true?

3. Suppose, according to a 1990 demographic report, the average U. S. household spends $90 per day. Suppose you recently took a random sample of 30 households in Huntsville and the results revealed a mean of $84.50. Suppose the standard deviation is known to be $14.50. Using a 0.05 level of significance, can it be concluded that the average amount spent per day by U.S. households has decreased?

4. Suppose the mean salary for full professors in the United States is believed to be $61,650. A sample of 36 full professors revealed a mean salary of $69,800. Assuming the standard deviation is $5,000, can it be concluded that the average salary has increased using a 0.02 level of significance?

5. Historically, evening long-distance calls from a particular city have averaged 15.2 minutes per call. In a random sample of 35 calls, the sample mean time was 14.3 minutes. Assume the standard deviation is known to be 5 minutes. Using a 0.05 level of significance, is there sufficient evidence to conclude that the average evening long-distance call has decreased?

6. Suppose a production line operates with a mean filling weight of 16 ounces per container. Since over- or under-filling can be dangerous, a quality control inspector samples 30 items to determine whether or not the filling weight has to be adjusted. The sample revealed a mean of 16.32 ounces. From past data, the standard deviation is known to be .8 ounces. Using a 0.10 level of significance, can it be concluded that the process is out of control (not equal to 16 ounces)?

1. Ho: $\mu = 20$
   Ha: $\mu > 20$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{22.60 - 20}{\frac{2.50}{\sqrt{49}}} = 7.28$$

   Reject Ho if $Z > 2.06$
   Reject Ho
   There is sufficient evidence to conclude the typical amount spent per customer is more than $20.00, $\alpha = 0.02$.

2. Ho: $\mu = 15$
   Ha: $\mu < 15$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{12.5 - 15}{\frac{3.6}{\sqrt{16}}} = -2.78$$

   Reject Ho if $Z < -1.96$
   Reject Ho
   There is sufficient evidence to conclude the editor's claim is true, $\alpha = 0.025$.

3. Ho: $\mu = 90$
   Ha: $\mu < 90$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{84.50 - 90}{\frac{14.50}{\sqrt{30}}} = -2.078$$

   Reject Ho if $Z < -1.65$
   Reject Ho
   There is sufficient evidence to conclude the average amount spent per day by U.S. households has decreased, $\alpha = 0.05$.

4. Ho: $\mu = 61,650$
Ha: $\mu > 61,650$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{69,800 - 61,650}{\frac{5,000}{\sqrt{36}}} = 9.78$$

Reject Ho if Z > 2.06
Reject Ho
There is sufficient evidence to conclude the average salary has increased, $\alpha = 0.02$.

5. Ho: $\mu = 15.2$
Ha: $\mu < 15.2$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{14.3 - 15.2}{\frac{5}{\sqrt{30}}} = 1.065$$

Reject Ho if Z < -1.65
Fail to Reject Ho
There is insufficient evidence to conclude the average evening long-distance call has decreased, $\alpha = 0.05$.

6. Ho: $\mu = 16$
Ha: $\mu \neq 16$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{16.32 - 16}{\frac{0.8}{\sqrt{30}}} = 2.19$$

Reject Ho if Z < - 1.65 OR Z > 1.65
Reject Ho
There is sufficient evidence to conclude the process is out of control, $\alpha = 0.10$.