

# CHAPTER TWENTY THREE

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## *CMOS Inverter*

# Introduction

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Complementary MOS (CMOS) uses a P-channel MOS as a pull-up load device with an N-channel MOS as a pull-down device.

CMOS is widely used in digital circuit technology because it possesses the lowest power dissipation (wrist watches, hand-held calculators, and recently note book computers) and has the highest packing density.

All modern microprocessors are manufactured using CMOS technology including Intel's 80286, 80386, 80486, 8086, 8088 and Motorola's 68010, 68020, 68030, 68040

In this chapter, we will describe the operation of CMOS as an inverter device, and then analyze its VTC, power-dissipation, and fan-out.

# Operation of CMOS Inverter

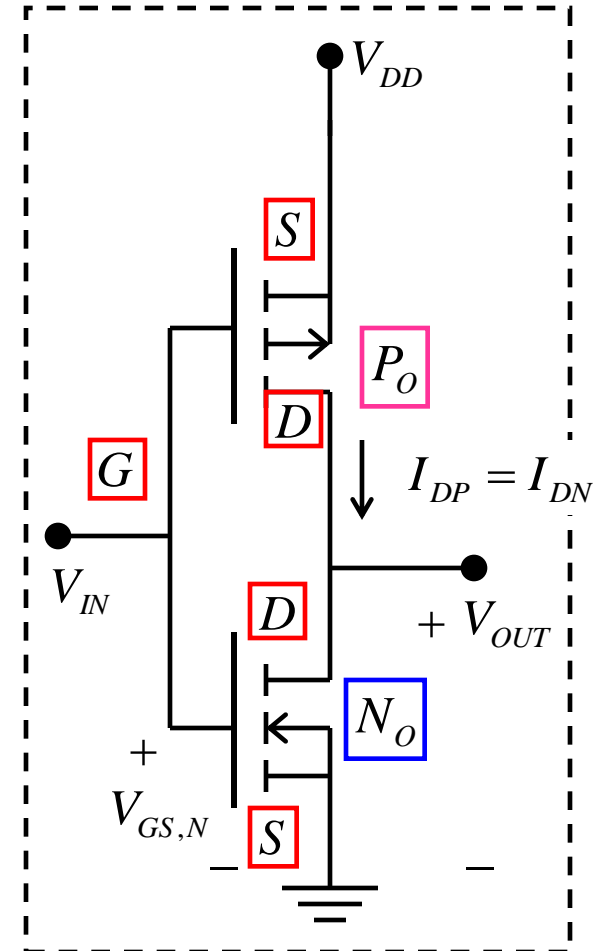
The drain terminal of the NMOS is connected with the drain terminal of the PMOS

$$V_{IN} = V_{GS,N} = V_{DD} - V_{SG,P}$$

The output is taken at the common drain terminal

$$V_{OUT} = V_{DS,N} = V_{DD} - V_{SD,P}$$

Note: either MOS can be considered as a load for the other, therefore the operations of  $N_O$  and  $P_O$  complement each other.



# VTC of CMOS Inverter

$V_{OH}$

When  $V_{IN}=0$ , i.e.  $V_{IN}=V_{GS,N} < V_{TN}$ :  $\rightarrow N_O$  is cut-off

$$I_{D,N} = I_{D,P} = 0$$

$$\underline{P_O} \quad V_{SG,P} = V_{DD} - V_{IN} = V_{DD}$$

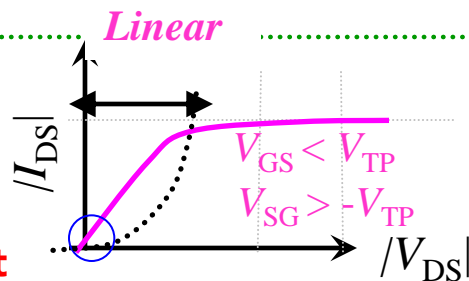
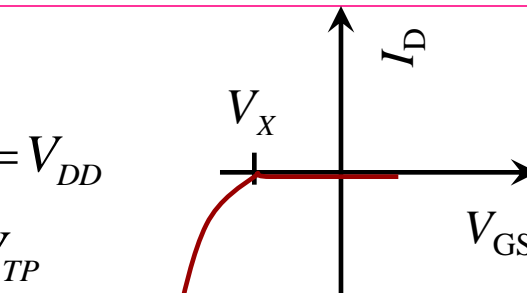
$$V_{GS,P} = -V_{DD} < V_{TP}$$

$\rightarrow$  Channel is created between the source and the drain of  $P_O$  MOS

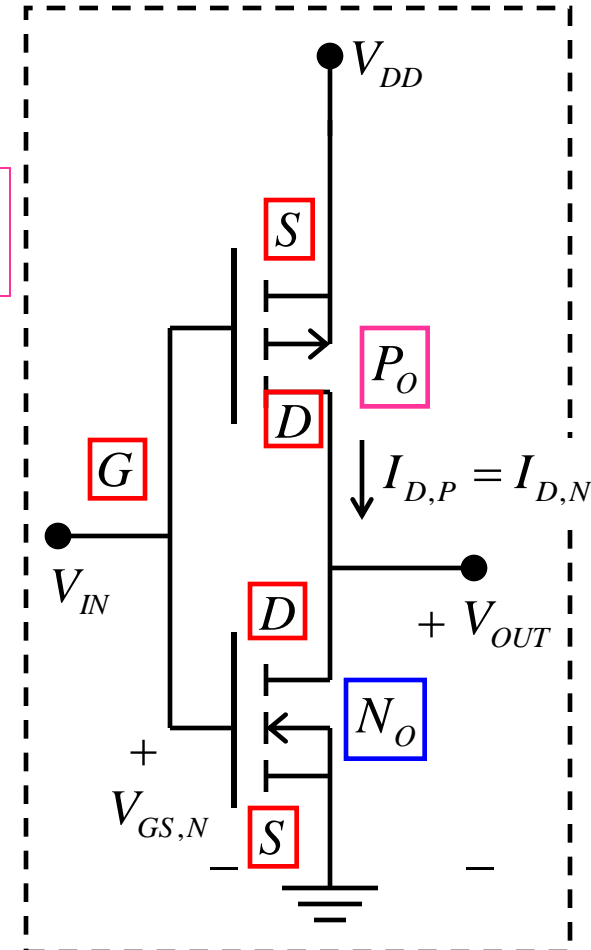
In active mode: ( $V_{GS} \leq V_{TP}$ )

Zero drain current

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$\rightarrow P_O$  is active (linear) mode



# VTC of CMOS Inverter

$V_{OH}$

When  $V_{IN}=0$ , i.e.  $V_{IN}=V_{GS,N} < V_{TN}$ :  $\rightarrow N_O$  is cut-off

$$I_{D,N} = I_{D,P} = 0$$

$\rightarrow P_O$  is active (linear) mode

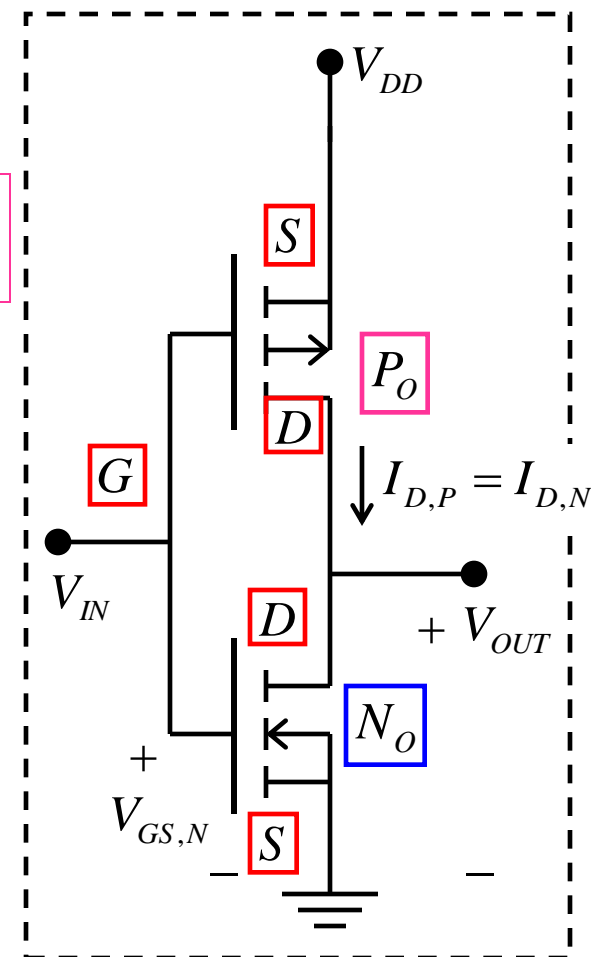
$$I_{SD,P}(LIN) = \frac{K_P}{2} [2 \times (V_{SG,P} + V_{TP}) V_{SD,P} - V_{SD,P}^2] = 0$$

$$V_{SD,P} \{2 \times (V_{SG,P} + V_{TP}) - V_{SD,P}\} = 0$$

$$V_{SD,P} = 0 \quad \checkmark$$

$$V_{SD,P} = 2 \times (V_{SG,P} + V_{TP}) > V_{SG,P} + V_{TP} \quad \times$$

$$V_{OH} = V_{DS,N} = V_{DD} - V_{SD,P} = V_{DD}$$



Invalid since  $V_{SD}$  has to be less than  $(V_{SG} + V_{TP})$

# VTC of CMOS Inverter

$V_{OL}$

For  $V_{IN} = V_{OH} = V_{DD}$  ( $V_{GS,P} = 0 > V_{T,P}^{(-)}$ )

→  $P_O$  is cut-off

Active mode: ( $V_{GS} \geq V_{TN}$ ) **Zero drain current**

→  $N_O$  is active, linear

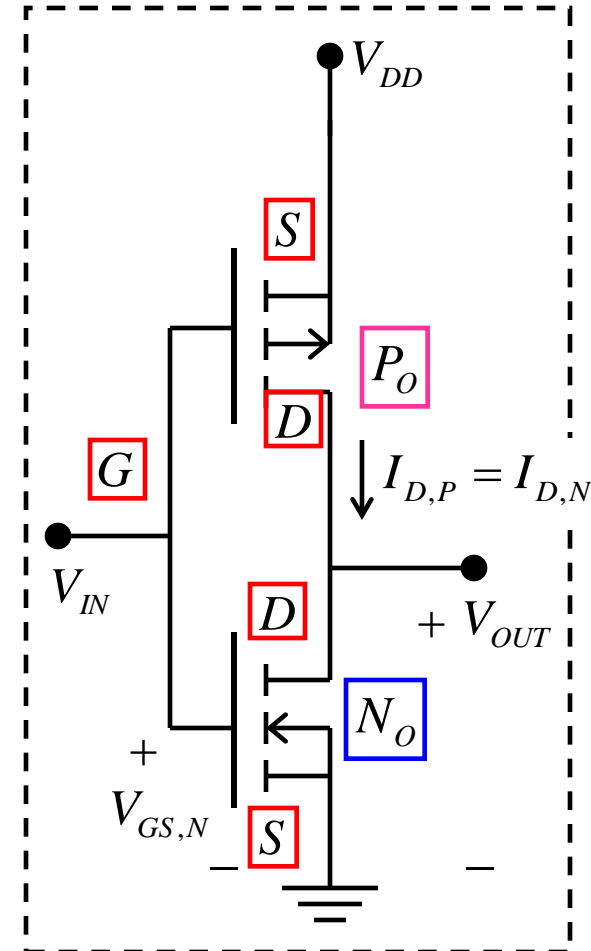
$$I_{DS,N}(LIN) = \frac{K_n}{2} [2 \times (V_{GS,N} - V_{TN}) V_{DS,N} - V_{DS,N}^2] = 0$$

$$V_{DS,N} \{2 \times (V_{GS,N} + V_{TN}) - V_{DS,N}\} = 0$$

$$V_{DS,N} = 0 \quad \checkmark$$

$$V_{DS,N} = 2 \times (V_{GS,N} - V_{TN}) > V_{GS,N} - V_{TN} \quad \times$$

$$V_{OL} = V_{DS,N} = 0$$

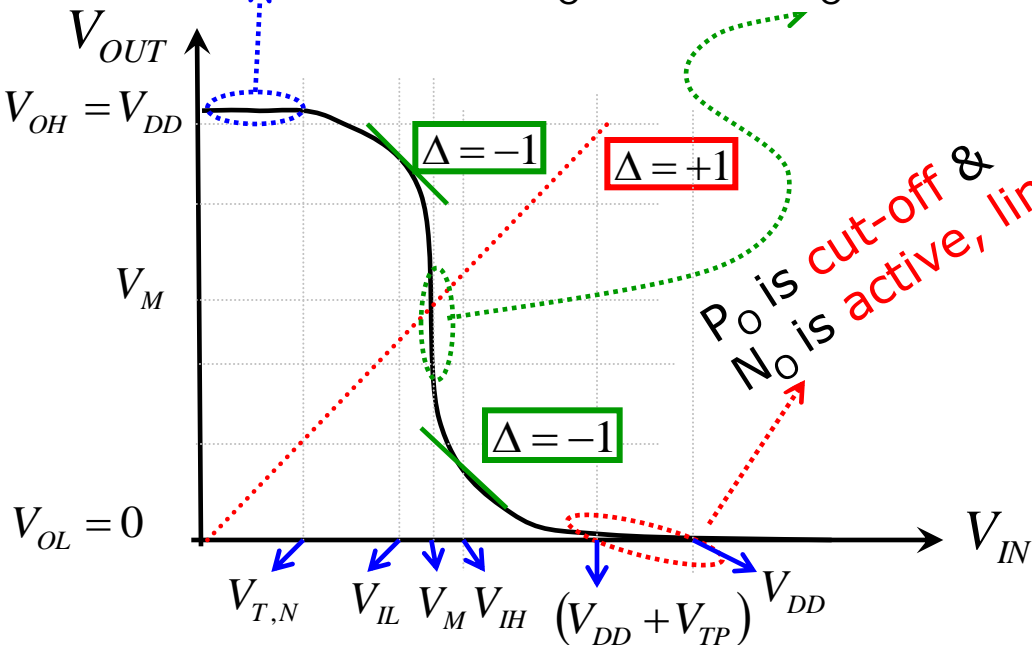


Invalid since  $V_{DS}$  has to be less than  $(V_{GS} - V_{TN})$

# VTC of CMOS Inverter

$N_O$  is cut-off &  $P_O$  is active, linear

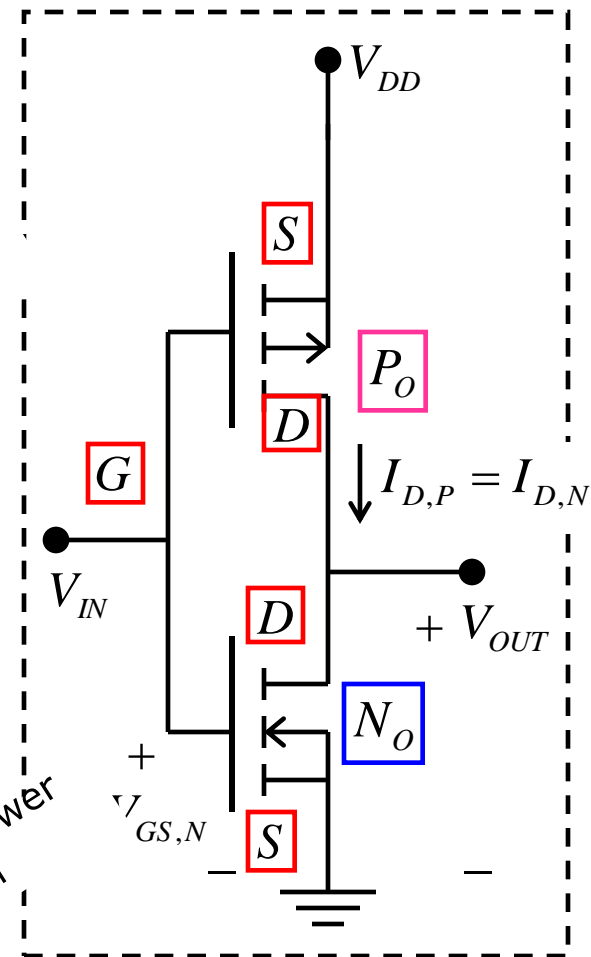
$N_O$  is sat &  $P_O$  is sat




$P_O$  is cut-off &  $N_O$  is active, linear


- for  $V_O = V_{OH}$  ( $V_I < V_{IL}$ ) →  $N_O$  is cut-off &  $P_O$  is linear
- for  $V_I = V_{IL}$  →  $N_O$  is sat &  $P_O$  is linear
- for  $V_I = V_O = V_M$  →  $N_O$  is sat &  $P_O$  is sat
- for  $V_I = V_{IH}$  →  $N_O$  is linear &  $P_O$  is sat
- for  $V_O = V_{OL}$  →  $N_O$  is linear &  $P_O$  is cut-off

Dynamic power dissipation



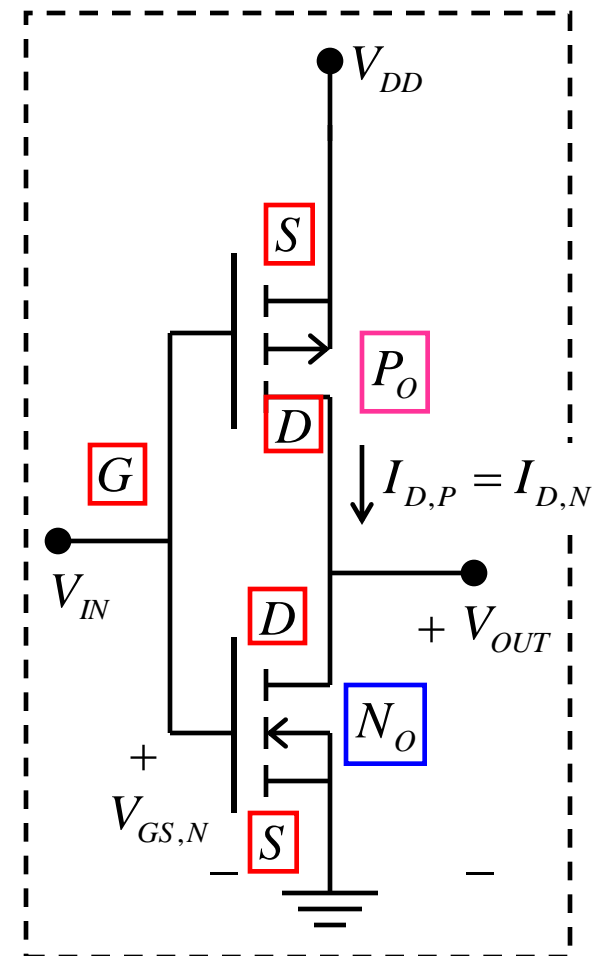
# Power Dissipation CMOS Inverter

$I_{DD}(OL)$    $\rightarrow P_O$  is cut-off

$I_{DD}(OH)$    $\rightarrow N_O$  is cut-off

## Static Power Dissipation $P_{DD}(avg)$

$$P_{DD}(avg) = V_{DD} \left( \frac{I_{DD}(OL) + I_{DD}(OH)}{2} \right) = 0$$





# Power Dissipation CMOS Inverter

## Dynamic Power Dissipation $P_{DD}(\text{dyn})$

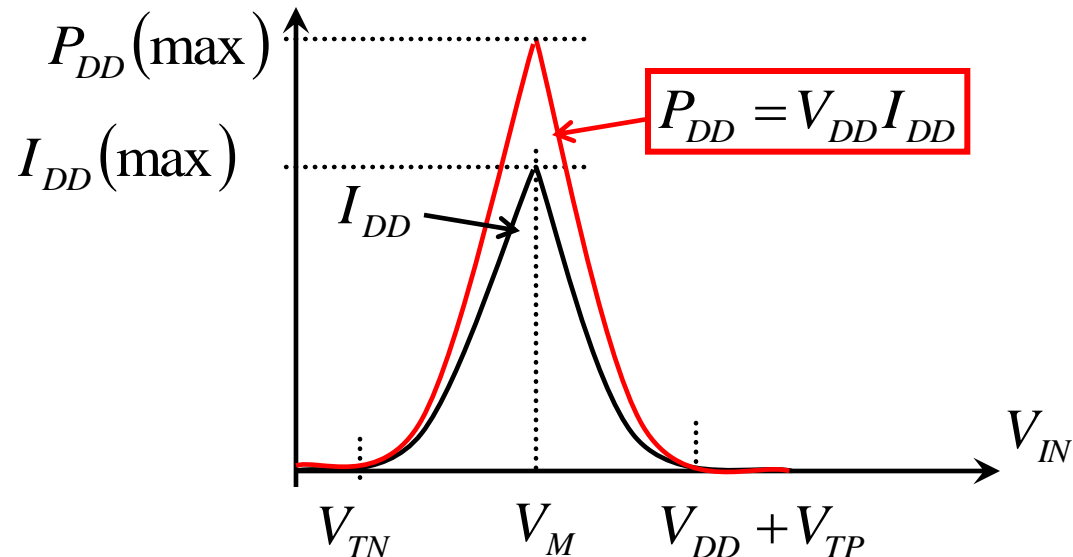
$$P_{DD}(\text{dyn}) = C_L \nu (V_{DD})^2$$

$C_L$  is the total load capacitance at the output of the gate

$\nu$  is the switching frequency of the gate

$$P_{DD}(\text{CMOS}) = P_{DD}(\text{dyn})$$

See example 23.1



# VTC of CMOS Inverter

Analytical determination of VTC :

$$V_{OH}$$

for  $V_O = V_{OH} (V_I < V_{IL}) \rightarrow$

$N_O$  is cut-off &  $P_O$  is linear

$$I_{D,N} = I_{D,P} = 0$$

$$V_{OH} = V_{DD}$$

$$V_{DS,N} = V_{DD}$$

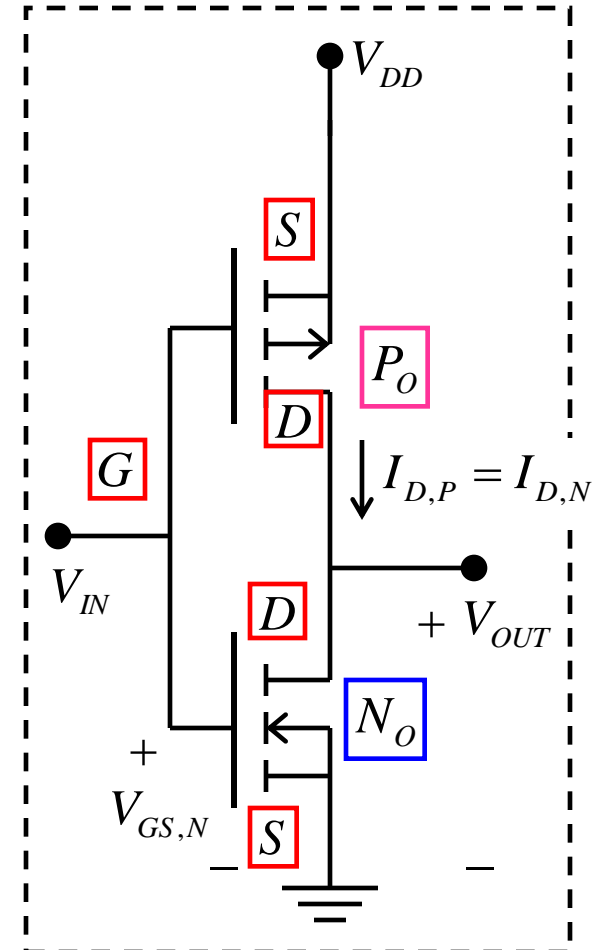
$$V_{OL}$$

for  $V_O = V_{OL} \rightarrow$   $N_O$  is linear &  $P_O$  is cut-off

$$I_{D,N} = I_{D,P} = 0$$

$$V_{OL} = 0$$

$$V_{DS,N} = 0$$



# VTC of CMOS Inverter

Analytical determination of VTC :

$V_{IL}$  for  $V_I = V_{IL} \longrightarrow N_O$  is **sat** &  $P_O$  is **linear**

$$\begin{array}{ll} V_{GS} = V_{IL} & V_{SG,P} = V_{DD} - V_{IL} \\ V_{DS,N} = V_{Out} & V_{SD,P} = V_{DD} - V_{Out} \end{array}$$

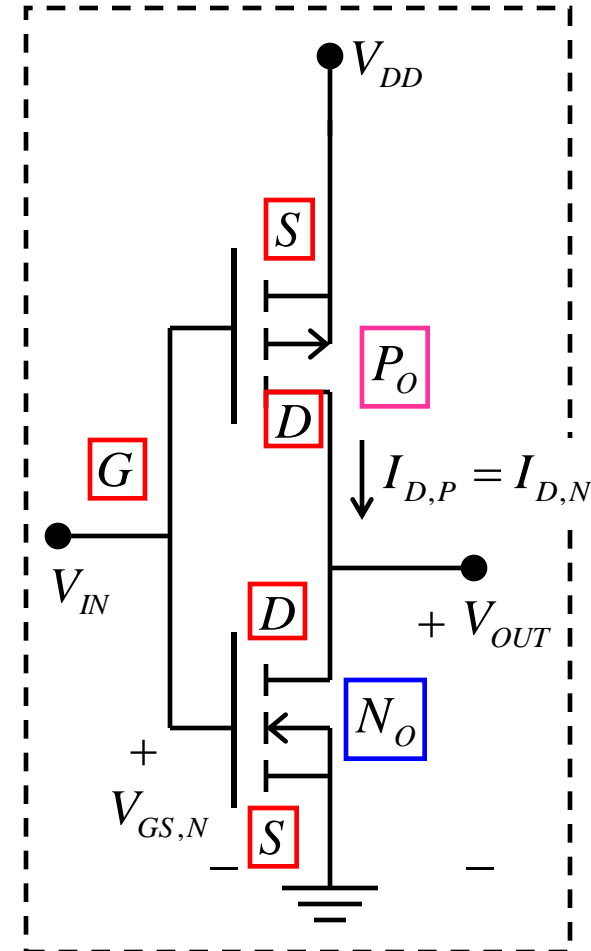
$$I_{D,N} = \frac{K_n}{2} (V_{GS} - V_{TN})^2 \Rightarrow I_{D,N} = \frac{K_n}{2} (V_{IN} - V_{TN})^2$$

$$I_{D,P} = \frac{K_p}{2} [2 \times (V_{SG} + V_{TP}) V_{SD} - V_{SD}^2]$$

$$I_{D,P} = \frac{K_p}{2} [2 \times (V_{DD} - V_{IN} + V_{TP})(V_{DD} - V_{OUT}) - (V_{DD} - V_{OUT})^2]$$

$$I_{D,N} = I_{D,P} \quad \text{or}$$

$$\frac{\partial I_{D,N}}{\partial V_{IN}} = \frac{\partial I_{D,P}}{\partial V_{IN}}$$



# VTC of CMOS Inverter

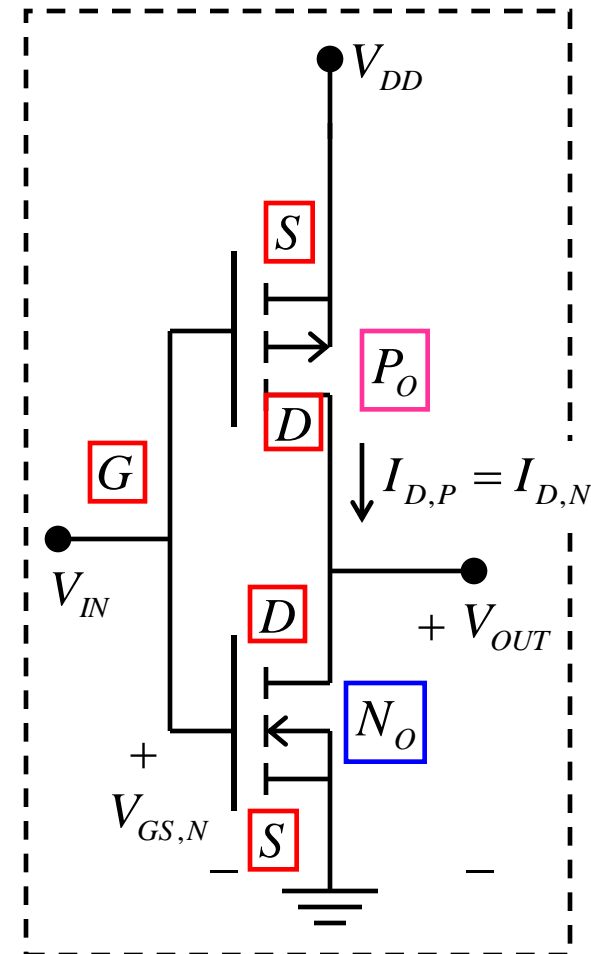
Analytical determination of VTC :

$V_{IL}$  for  $V_I = V_{IL} \longrightarrow N_O$  is **sat** &  $P_O$  is **linear**

$$\begin{array}{ll} V_{GS} = V_{IL} & V_{SG,P} = V_{DD} - V_{IL} \\ V_{DS,N} = V_{Out} & V_{SD,P} = V_{DD} - V_{Out} \end{array}$$

$$\frac{dV_{OUT}}{dV_{IN}} \Big|_{V_{IN}=V_{IL}} = -1$$

$$V_{IL} = \frac{2K_p V_{OUT} - K_p (V_{DD} - V_{TP}) + K_n V_{TN}}{K_n + K_p}$$



# VTC of CMOS Inverter

Analytical determination of VTC :

$V_{IH}$  for  $V_I = V_{IH} \longrightarrow N_O$  is linear &  $P_O$  is sat

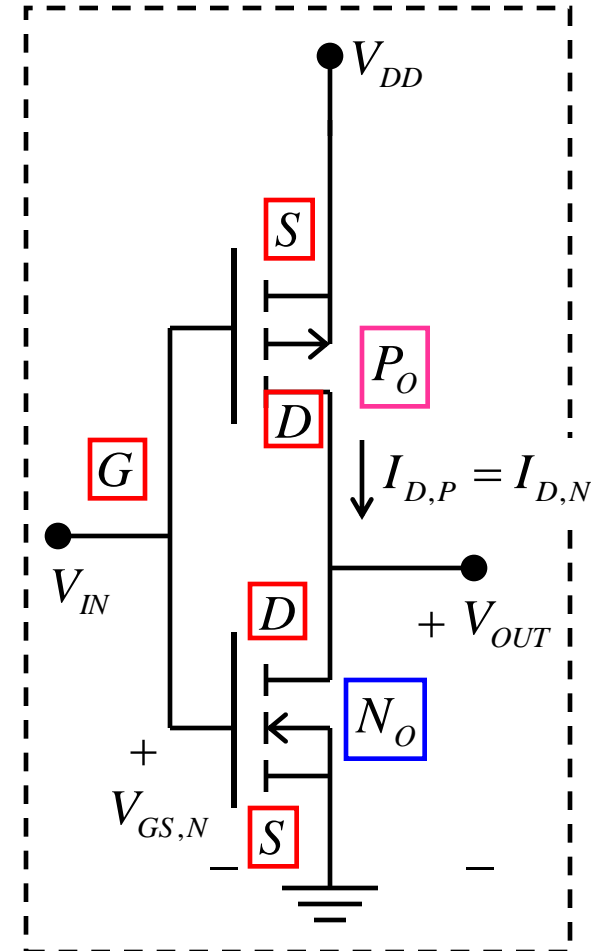
$$\begin{array}{ll} V_{GS} = V_{IH} & V_{SG,P} = V_{DD} - V_{IH} \\ V_{DS,N} = V_{Out} & V_{SD,P} = V_{DD} - V_{Out} \end{array}$$

$$I_{D,n} = \frac{K_n}{2} \left[ 2 \times (V_{IN} - V_{TN}) V_{OUT} - V_{OUT}^2 \right]$$

$$I_{D,p} = \frac{K_p}{2} (V_{DD} - V_{IN} + V_{TP})^2$$

$$I_{D,N} = I_{D,P} \quad \text{or}$$

$$\frac{\partial I_{D,N}}{\partial V_{IN}} = \frac{\partial I_{D,P}}{\partial V_{IN}}$$



# VTC of CMOS Inverter

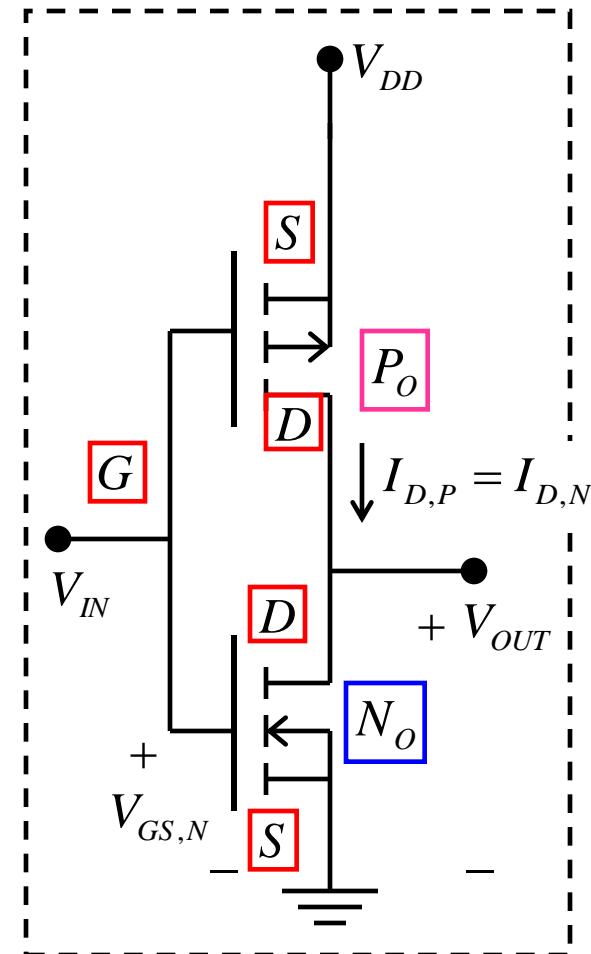
Analytical determination of VTC :

$V_{IH}$  for  $V_I = V_{IH} \longrightarrow N_O$  is linear &  $P_O$  is sat

$$\begin{array}{ll} V_{GS} = V_{IH} & V_{SG,P} = V_{DD} - V_{IH} \\ V_{DS,N} = V_{Out} & V_{SD,P} = V_{DD} - V_{Out} \end{array}$$

$$\frac{dV_{OUT}}{dV_{IN}} \Big|_{V_{IN}=V_{IH}} = -1$$

$$V_{IH} = \frac{2K_n V_{OUT} + K_p (V_{DD} + V_{TP}) + K_n V_{TN}}{K_n + K_p}$$



# VTC of CMOS Inverter

Analytical determination of VTC :

$V_M$  Mid point

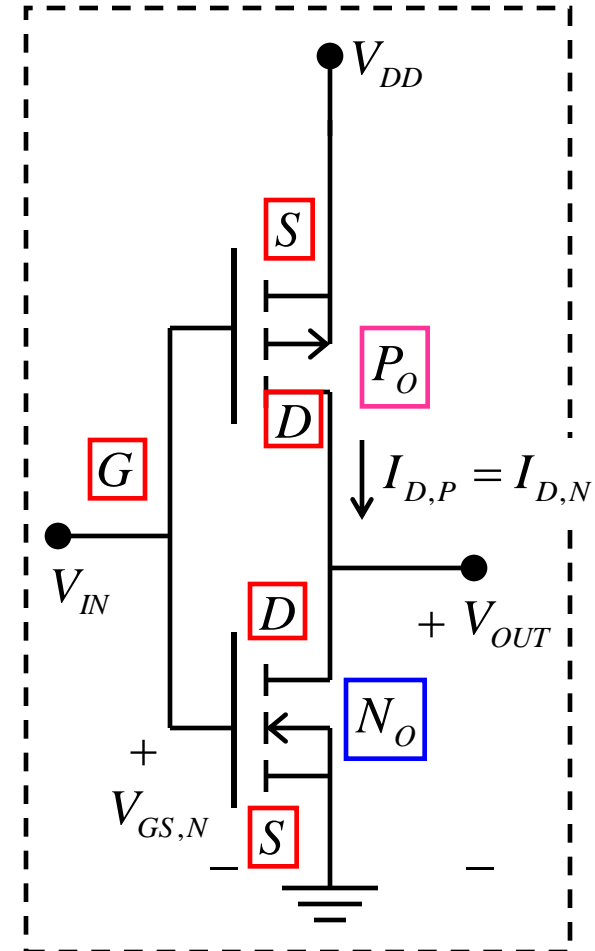
for  $V_I = V_O = V_M$   $\longrightarrow$

$N_O$  is sat &  $P_O$  is sat

$$\frac{K_n}{2} (V_M - V_{TN})^2 = \frac{K_p}{2} ((V_{DD} - V_M) + V_{TP})^2$$

Then solve for  $V_M$

See example on page 342



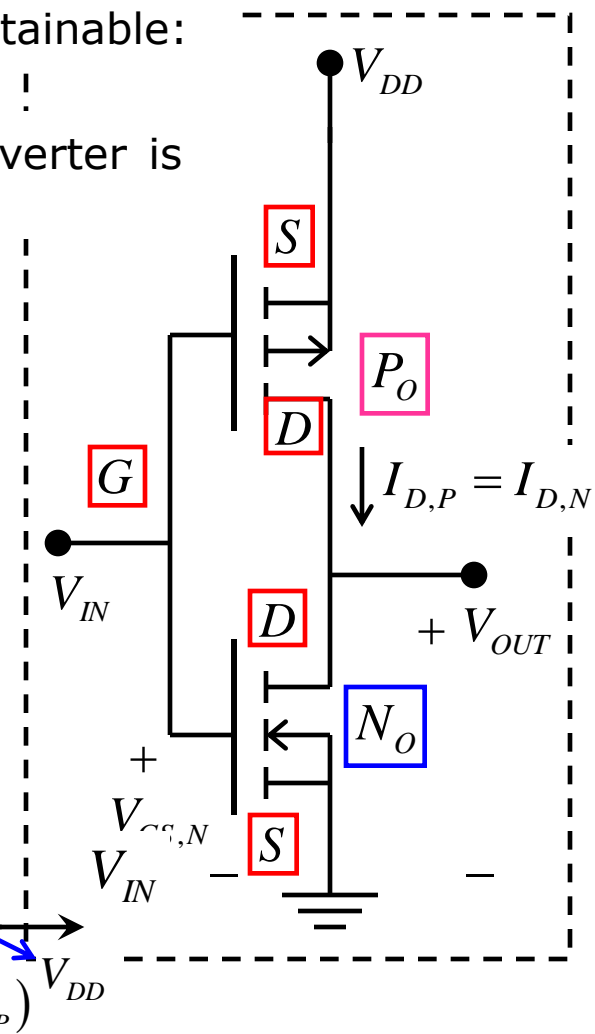
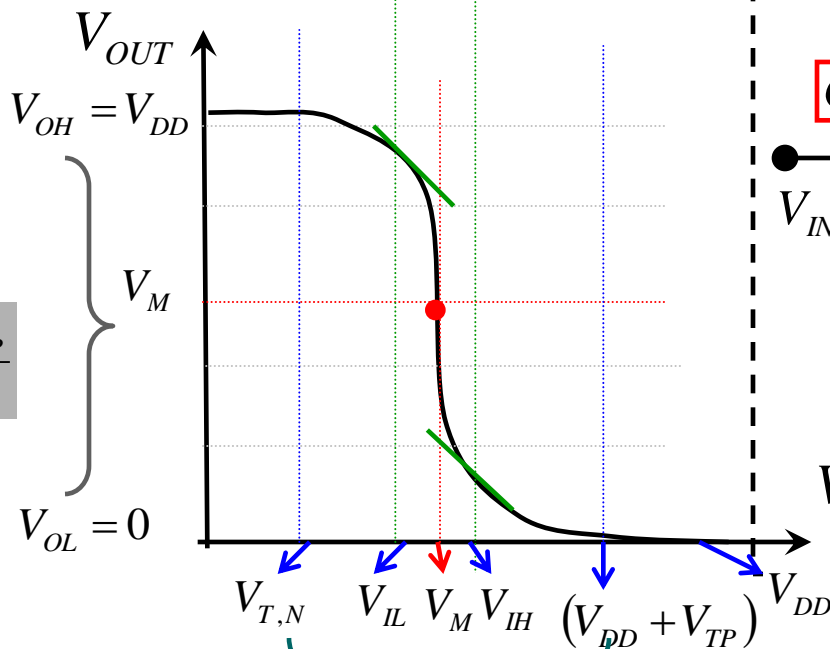
# The Symmetric CMOS Inverter

- The VTC of symmetric CMOS inverter is easily obtainable:
- One reason of designing a symmetric CMOS inverter is to obtain a symmetric transient response:
- ❖ Remember the VTC of CMOS inverter

$$V_M = \frac{V_{OH} + V_{OL}}{2} = \frac{V_{DD}}{2}$$

$$V_M = \frac{V_{IH} + V_{IL}}{2} = \frac{V_{TN} + V_{DD} + V_{TP}}{2}$$

$$V_{TN} = -V_{TP}$$





# Design of Symmetric CMOS Inverter

1. The threshold voltages of N-MOS and P-MOS are made equal in magnitude
2. The requirements for  $K_n = K_p$  must be considered

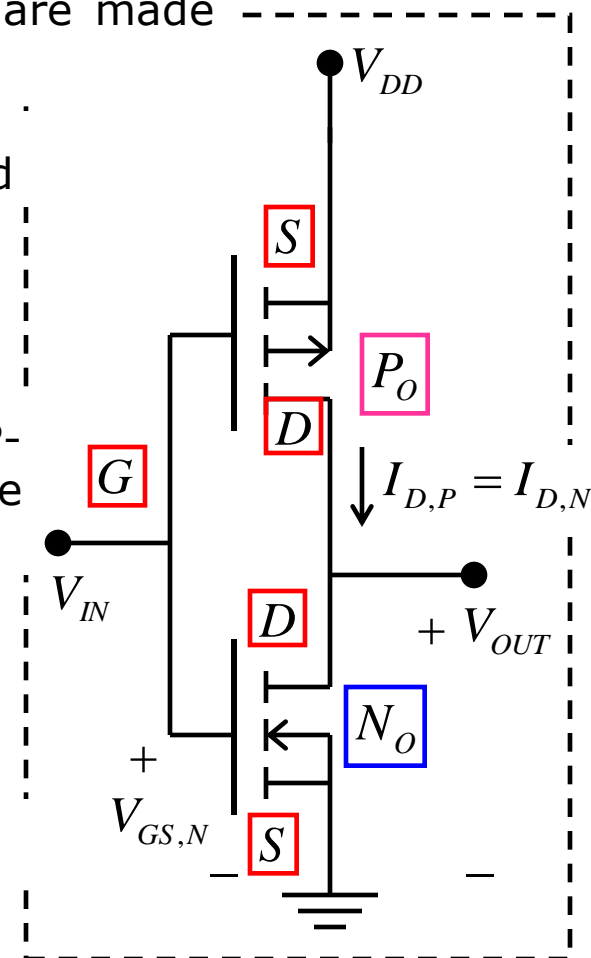
$$K_n = \frac{W_n}{L_n} \mu_n C_{ox} \quad K_p = \frac{W_p}{L_p} \mu_p C_{ox}$$

3. Usually the gate oxide layers of the N-MOS and P-MOS devices are grown simultaneously, and hence have the same thickness  $t_{ox}$

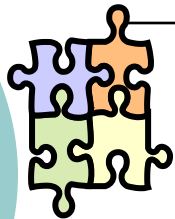
$$\frac{W_n}{L_n} \mu_n = \frac{W_p}{L_p} \mu_p$$

4. Typically  $\mu_n(\text{Si}) = 580 \text{ cm}^2/\text{V}\cdot\text{s}$  and  $\mu_p(\text{Si}) = 230 \text{ cm}^2/\text{V}\cdot\text{s}$

$$\frac{W_p}{L_p} = 2.5 \frac{W_n}{L_n}$$



# Design of Symmetric CMOS Inverter



## ○ Example

Design a symmetrical CMOS inverter assuming:

Ch. 23  $V_{DD}=5V, V_{TN}=1 V, V_{TP}=-1 V, \mu_n C_{ox}=40 \mu A/V^2, \mu_p C_{ox}=16 \mu A/V^2,$   
 $W_n=4 \mu m, L_n=L_p=2 \mu m$

And verify that the midpoint voltage is half of  $V_{DD}$ , and  $V_{IL}$  and  $V_{IH}$  are symmetric about  $V_M$ .

## ○ Solution

For symmetry, the width of the channel of P-MOS is determined from

$$\frac{W_p}{L_p} = 2.5 \frac{W_n}{L_n} \Rightarrow \boxed{W_p = 2.5 \times 4 = 10 \mu m}$$

$$K_n = \frac{W_n}{L_n} \mu_n C_{ox}$$

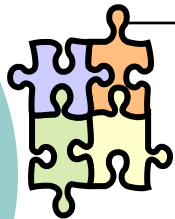
$$K_n = 2 \times 40 = 80 \mu A/V^2 \quad \swarrow$$

$$K_p = \frac{W_p}{L_p} \mu_p C_{ox}$$

$$K_p = 5 \times 16 = 80 \mu A/V^2 \quad \swarrow$$

*Equal values*

# Design of Symmetric CMOS Inverter



## ○ Example

Design a symmetrical CMOS inverter assuming:

Ch. 23  $V_{DD}=5V$ ,  $V_{TN}=1V$ ,  $V_{TP}=-1V$ ,  $\mu_n C_{ox}=40 \mu A/V^2$ ,  $\mu_p C_{ox}=16 \mu A/V^2$ ,  
 $W_n=4 \mu m$ ,  $L_n=L_p=2 \mu m$

And verify that the midpoint voltage is half of  $V_{DD}$ , and  $V_{IL}$  and  $V_{IH}$  are symmetric about  $V_M$ .

## ○ Solution

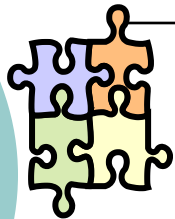
$$\frac{K_n}{2} (V_M - V_{TN})^2 = \frac{K_p}{2} ((V_{DD} - V_M) + V_{TP})^2$$

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$$(V_M - 1)^2 = ((5 - V_M) - 1)^2$$

$$2V_M = 5 \Rightarrow V_M = 2.5V \Rightarrow V_M = \frac{V_{DD}}{2}$$

# Design of Symmetric CMOS Inverter



## ○ Example

Design a symmetrical CMOS inverter assuming:

Ch. 23  $V_{DD}=5V$ ,  $V_{TN}=1V$ ,  $V_{TP}=-1V$ ,  $\mu_n C_{ox}=40 \mu A/V^2$ ,  $\mu_p C_{ox}=16 \mu A/V^2$ ,  
 $W_n=4 \mu m$ ,  $L_n=L_p=2 \mu m$

And verify that the midpoint voltage is half of  $V_{DD}$ , and  $V_{IL}$  and  $V_{IH}$  are symmetric about  $V_M$ .

## ○ Solution

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chapter

$$V_{IL} = \frac{2K_p V_{OUT} - K_p (V_{DD} - V_{TP}) + K_n V_{TN}}{K_n + K_p}$$



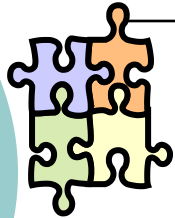
$$V_{IL} = \frac{160V_{OUT} - 80(5+1) + 80}{160} = V_{OUT} - 2.5$$

$$I_{D,N} = \frac{K_n}{2} (V_{IN} - V_{TN})^2 = I_{D,P} = \frac{K_p}{2} [2 \times (V_{SG} + V_{TP}) V_{SD} - V_{SD}^2]$$

$$\frac{K_n}{2} (V_{IL} - V_{TN})^2 = \frac{K_p}{2} [2 \times ((V_{DD} - V_{IL}) + V_{TP})(V_{DD} - V_{OUT}) - (V_{DD} - V_{OUT})^2]$$

$$(V_{IL} - 1)^2 = [2 \times ((5 - V_{IL}) - 1)(5 - V_{IL} - 2.5) - (5 - V_{IL} - 2.5)^2]$$

# Design of Symmetric CMOS Inverter



## ○ Example

Design a symmetrical CMOS inverter assuming:

Ch. 23  $V_{DD}=5V$ ,  $V_{TN}=1 V$ ,  $V_{TP}=-1 V$ ,  $\mu_n C_{ox}=40 \mu A/V^2$ ,  $\mu_p C_{ox}=16 \mu A/V^2$ ,  
 $W_n=4 \mu m$ ,  $L_n=L_p=2 \mu m$

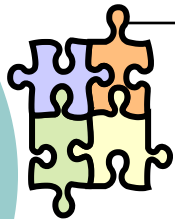
And verify that the midpoint voltage is half of  $V_{DD}$ , and  $V_{IL}$  and  $V_{IH}$  are symmetric about  $V_M$ .

## ○ Solution

$$(V_{IL} - 1)^2 = \left[ 2 \times ((5 - V_{IL}) - 1)(5 - V_{IL} - 2.5) - (5 - V_{IL} - 2.5)^2 \right]$$

$$(V_{IL} - 1)^2 = \left[ 2 \times (4 - V_{IL})(2.5 - V_{IL}) - (2.5 - V_{IL})^2 \right] \Rightarrow V_{IL} = \frac{12.75}{6} = 2.125V$$

# Design of Symmetric CMOS Inverter



## ○ Example

Design a symmetrical CMOS inverter assuming:

Ch. 23  $V_{DD}=5V$ ,  $V_{TN}=1V$ ,  $V_{TP}=-1V$ ,  $\mu_n C_{ox}=40 \mu A/V^2$ ,  $\mu_p C_{ox}=16 \mu A/V^2$ ,  
 $W_n=4 \mu m$ ,  $L_n=L_p=2 \mu m$

And verify that the midpoint voltage is half of  $V_{DD}$ , and  $V_{IL}$  and  $V_{IH}$  are symmetric about  $V_M$ .

## ○ Solution :similarly

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$$V_{IH} = \frac{2K_n V_{OUT} + K_p (V_{DD} + V_{TP}) + K_n V_{TN}}{K_n + K_p}$$



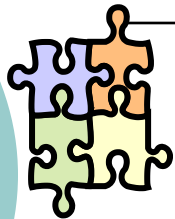
$$V_{IH} = \frac{160V_{OUT} + 80(5-1) + 80}{160} = V_{OUT} + 2.5$$

$$I_{D,N} = \frac{K_n}{2} (V_{IN} - V_{TN})^2 = I_{D,P} = \frac{K_p}{2} [2 \times (V_{SG} + V_{TP}) V_{SD} - V_{SD}^2]$$

$$\frac{K_n}{2} (V_{IH} - V_{TN})^2 = \frac{K_p}{2} [2 \times ((V_{DD} - V_{IH}) + V_{TP})(V_{DD} - V_{OUT}) - (V_{DD} - V_{OUT})^2]$$

$$(V_{IH} - 1)^2 = [2 \times ((5 - V_{IH}) - 1)(5 - V_{IH} + 2.5) - (5 - V_{IH} + 2.5)^2] \Rightarrow V_{IH} = 2.875V$$

# Design of Symmetric CMOS Inverter



## ○ Example

Design a symmetrical CMOS inverter assuming:

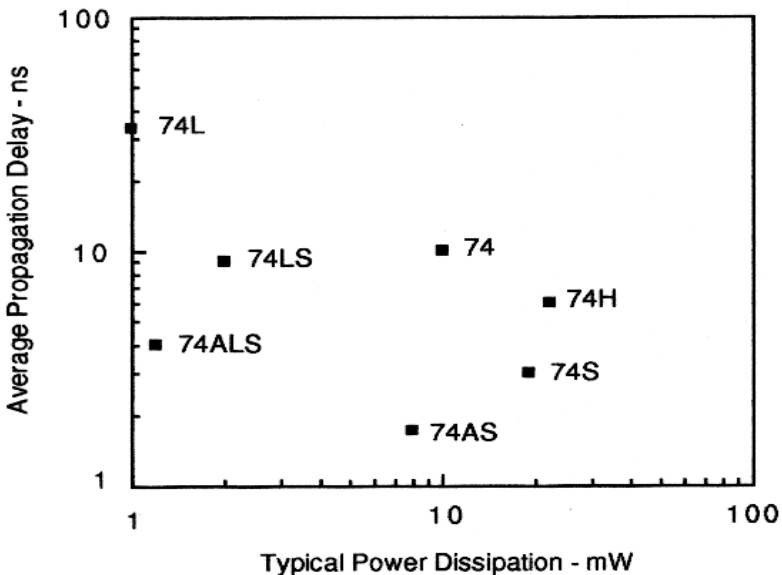
Ch. 23  $V_{DD}=5V$ ,  $V_{TN}=1V$ ,  $V_{TP}=-1V$ ,  $\mu_n C_{ox}=40 \mu A/V^2$ ,  $\mu_p C_{ox}=16 \mu A/V^2$ ,  
 $W_n=4 \mu m$ ,  $L_n=L_p=2 \mu m$

And verify that the midpoint voltage is half of  $V_{DD}$ , and  $V_{IL}$  and  $V_{IH}$  are symmetric about  $V_M$ .

## ○ Solution

$$\begin{array}{l} V_{IL} = 2.125V \\ V_{IH} = 2.875V \end{array} \Rightarrow \frac{V_{IH} + V_{IL}}{2} = \frac{2.875 + 2.125}{2} = 2.5V = V_M$$

$V_{IL}$  and  $V_{IH}$  are symmetric about  $V_M$ .



<u>Family</u>	<u>Characteristics</u>	<u>Example</u>
<b>TTL</b>	<b>Standard transistor/transistor logic</b>	<b>7400</b>
<b>L</b>	<b>Low power TTL</b>	<b>74L00</b>
<b>H</b>	<b>High speed TTL</b>	<b>74H00</b>
<b>S</b>	<b>Schottky TTL - high speed</b>	<b>74S00</b>
<b>LS</b>	<b>Low power Schottky TTL</b>	<b>74LS00</b>
<b>AS</b>	<b>Advanced Schottky</b>	<b>74AS00</b>
<b>ALS</b>	<b>Advanced Low power Schottky</b>	<b>74ALS00</b>
<b>F</b>	<b>Fast Schottky</b>	<b>74F00</b>
<b>HC</b>	<b>High speed CMOS</b>	<b>74HC00</b>
<b>HCT</b>	<b>High speed CMOS, TTL-voltage compatible</b>	<b>74HCT00</b>
<b>AC</b>	<b>Advanced CMOS</b>	<b>74AC00</b>
<b>ACT</b>	<b>Advanced CMOS, TTL-output compatible</b>	<b>74ACT00</b>



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- HW #12: Solve Problems: 23.1-3, 23.22