

# CHAPTER NINETEEN

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## *Saturated Enhancement-Only Loaded NMOS Inverter*

# Introduction

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In this chapter, we will describe the saturated enhancement-only loaded NMOS inverter.

The basic inverter consists of two enhancement-only NMOS transistors (size of load resistor is more one thousand larger in size than the MOSFET)

# Operation of Saturated Enhancement-Only Loaded NMOS Inverter

The enhancement NMOS load device operates always in saturation

$$V_{DS,L} = V_{GS,L}$$

$$\begin{aligned} V_{DS,L}(sat) &= V_{GS,L} - V_{TN,L} \\ &= V_{DS,L} - V_{TN,L} \end{aligned}$$

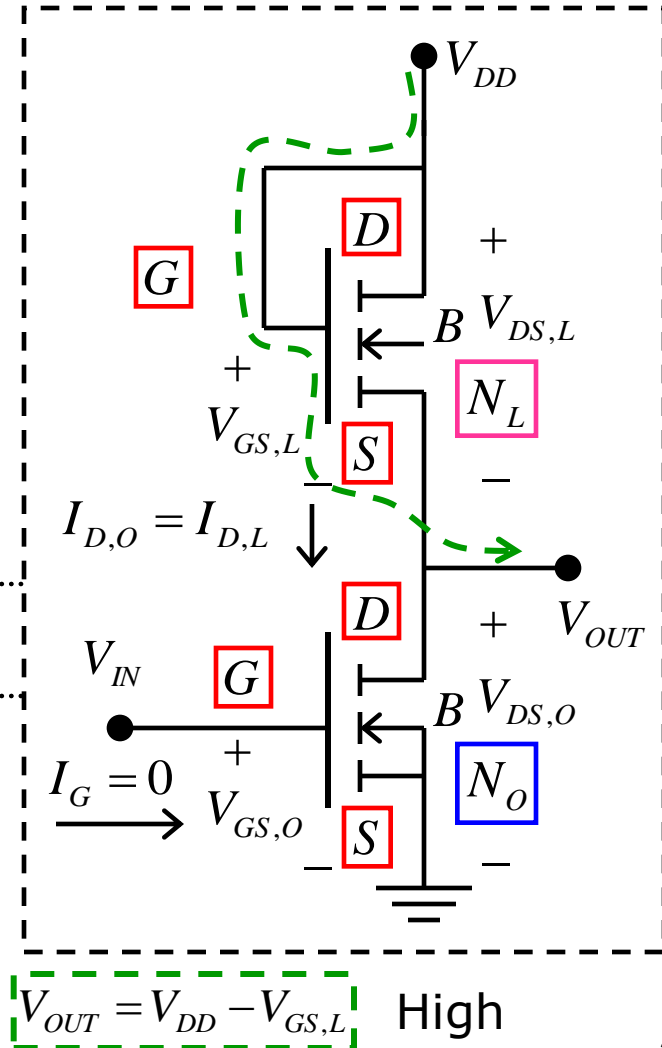
⇒  $V_{DS,L}(sat) < V_{DS,L}$   $N_L$  is in saturation operation

$$V_{IN} = V_{GS,O}$$

$$V_{OUT} = V_{DS,O} = V_{DD} - V_{DS,L}$$

When  $V_{IN} = V_{GS,O} < V_{TN,O}$ : →  $N_O$  is cut-off and it does not conduct current, however  $N_L$  is still in the saturation region

$$I_{D,L} = \frac{K_n}{2} (V_{GS,L} - V_{TN,L})^2 = 0 \Rightarrow V_{GS,L} = V_{TN,L}$$



# Operation of Saturated Enhancement-Only Loaded NMOS Inverter

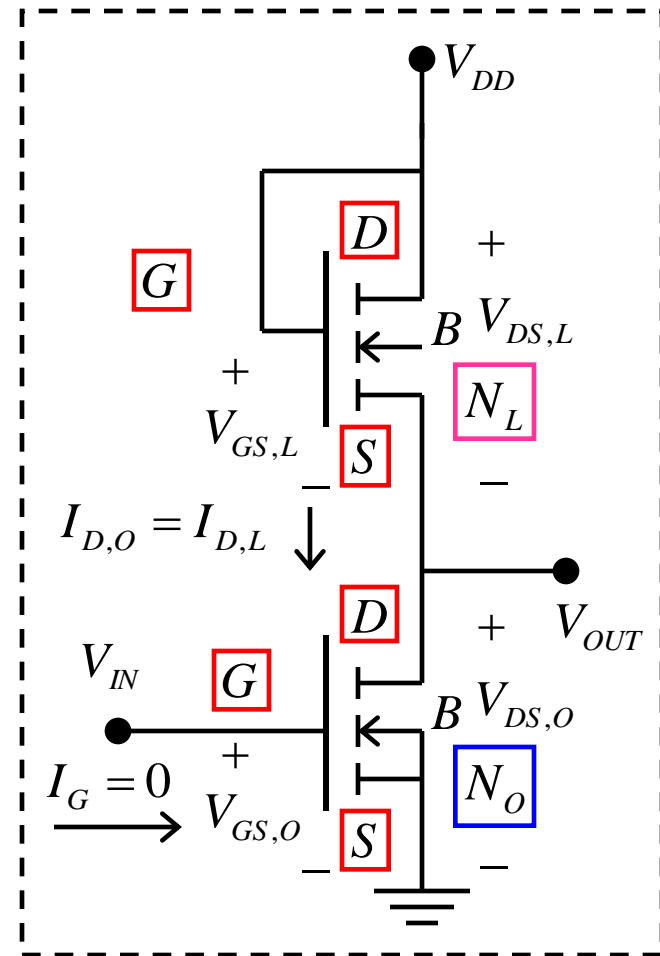
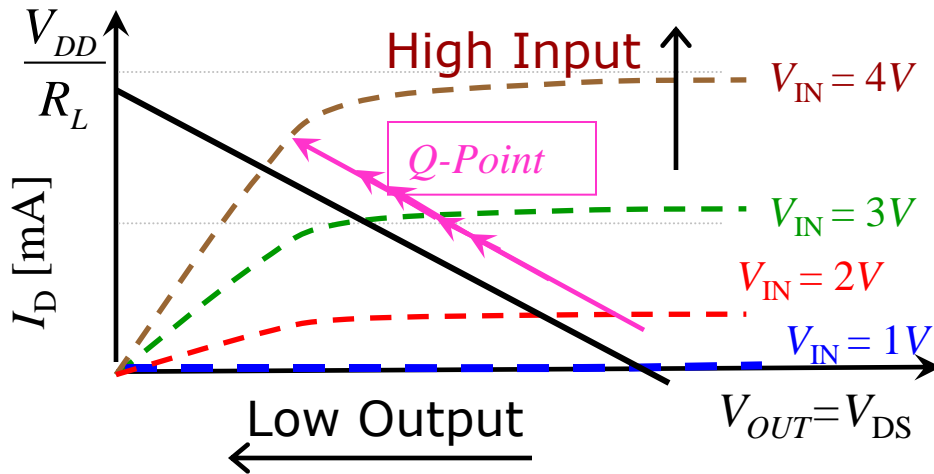
As  $V_{IN} = V_{GS,O} > V_{TN,O}$ :  $\rightarrow N_O$  starts to conduct (saturation), while  $N_L$  is still in the saturation region

$$I_{D,O}(sat) = I_{D,L}(sat)$$

$$\frac{K_{n,O}}{2} (V_{GS,O} - V_{TN,O})^2 = \frac{K_{n,L}}{2} (V_{GS,L} - V_{TN,L})^2$$

$$V_{IN} = V_{GS,O}$$

$$V_{GS,L} = V_{DD} - V_{OUT}$$



# Operation of Saturated Enhancement-Only Loaded NMOS Inverter

As  $V_{IN} = V_{GS,O} > V_{TN,O}$ :  $\rightarrow N_O$  starts to conduct (saturation), while  $N_L$  is still in the saturation region

$$\frac{K_{n,O}}{2} (V_{GS,O} - V_{TN,O})^2 = \frac{K_{n,L}}{2} (V_{GS,L} - V_{TN,L})^2$$

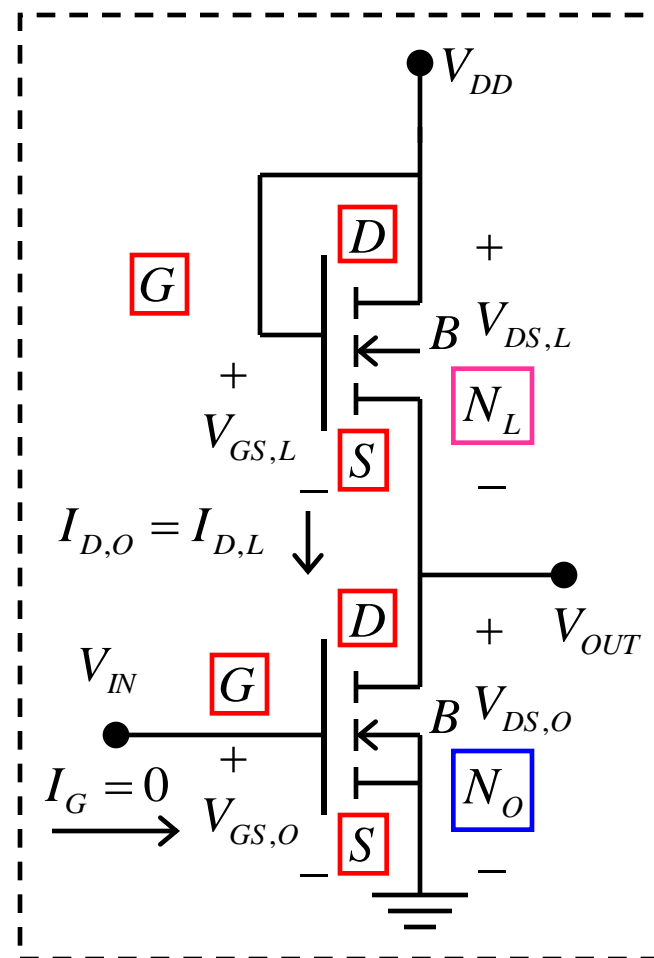
$$V_{IN} = V_{GS,O}$$

$$V_{GS,L} = V_{DD} - V_{OUT}$$

$$\sqrt{\frac{K_{n,O}}{K_{n,L}}} (V_{IN} - V_{TN,O}) = (V_{DD} - V_{OUT} - V_{TN,L})$$

$$\sqrt{\frac{K_{n,O}}{K_{n,L}}} (V_{IN} - V_{TN,O}) = (V_{DD} - V_{TN,L}) - V_{OUT}$$

$$V_{OUT} = -\sqrt{\frac{K_{n,O}}{K_{n,L}}} V_{IN} + \sqrt{\frac{K_{n,O}}{K_{n,L}}} V_{TN,O} + (V_{DD} - V_{TN,L})$$



$$\text{Slope} = -\left(\frac{K_{n,O}}{K_{n,L}}\right)^{1/2}$$

# Operation of Saturated Enhancement-Only Loaded NMOS Inverter

$N_O$  stays in saturation as long as  $V_{OUT} > V_{GS,O} - V_{TN,O}$ , but if  $V_{OUT} = V_{DS,O} < V_{GS,O} - V_{TN,O}$  then  $N_O$  moves into linear region

$$\frac{K_{n,O}}{2} (2(V_{GS,O} - V_{TN,O})V_{DS,O} - V_{DS,O}^2) = \frac{K_{n,L}}{2} (V_{GS,L} - V_{TN,L})^2$$

$$V_{IN} = V_{GS,O}$$

$$V_{GS,L} = V_{DD} - V_{OUT}$$

$$\frac{K_{n,O}}{2} (2(V_{IN} - V_{TN,O})V_{OUT} - V_{OUT}^2) = \frac{K_{n,L}}{2} (V_{DD} - V_{OUT} - V_{TN,L})^2$$

$$2 \frac{K_{n,O}}{K_{n,L}} (V_{IN} - V_{TN,O}) V_{OUT} - \frac{K_{n,O}}{K_{n,L}} V_{OUT}^2 = V_{OUT}^2 - 2(V_{DD} - V_{TN,L}) V_{OUT} + (V_{DD} - V_{TN,L})^2$$

$$\left(1 + \frac{K_{n,O}}{K_{n,L}}\right) V_{OUT}^2 - 2 \left[ (V_{DD} - V_{TN,L}) + \frac{K_{n,O}}{K_{n,L}} (V_{IN} - V_{TN,O}) \right] V_{OUT} + (V_{DD} - V_{TN,L})^2 = 0$$

# Operation of Saturated Enhancement-Only Loaded NMOS Inverter

$$\left(1 + \frac{K_{n,O}}{K_{n,L}}\right) V_{OUT}^2 - 2 \left[ (V_{DD} - V_{TN,L}) + \frac{K_{n,O}}{K_{n,L}} (V_{IN} - V_{TN,O}) \right] V_{OUT} + (V_{DD} - V_{TN,L})^2 = 0$$

$$V_{OUT} = \frac{2 \left[ (V_{DD} - V_{TN,L}) + \frac{K_{n,O}}{K_{n,L}} (V_{IN} - V_{TN,O}) \right]}{2 \left(1 + \frac{K_{n,O}}{K_{n,L}}\right)} \pm \frac{\sqrt{4 \left[ (V_{DD} - V_{TN,L}) + \frac{K_{n,O}}{K_{n,L}} (V_{IN} - V_{TN,O}) \right]^2 - 8 \left(1 + \frac{K_{n,O}}{K_{n,L}}\right) (V_{DD} - V_{TN,L})^2}}{2 \left(1 + \frac{K_{n,O}}{K_{n,L}}\right)}$$

EXACT SOLUTION

$$V_{OUT} = \frac{(V_{DD} - V_{TN,L})^2}{2 \left[ (V_{DD} - V_{TN,L}) + \frac{K_{n,O}}{K_{n,L}} (V_{IN} - V_{TN,O}) \right]}$$

APPROXIMATED SOLUTION

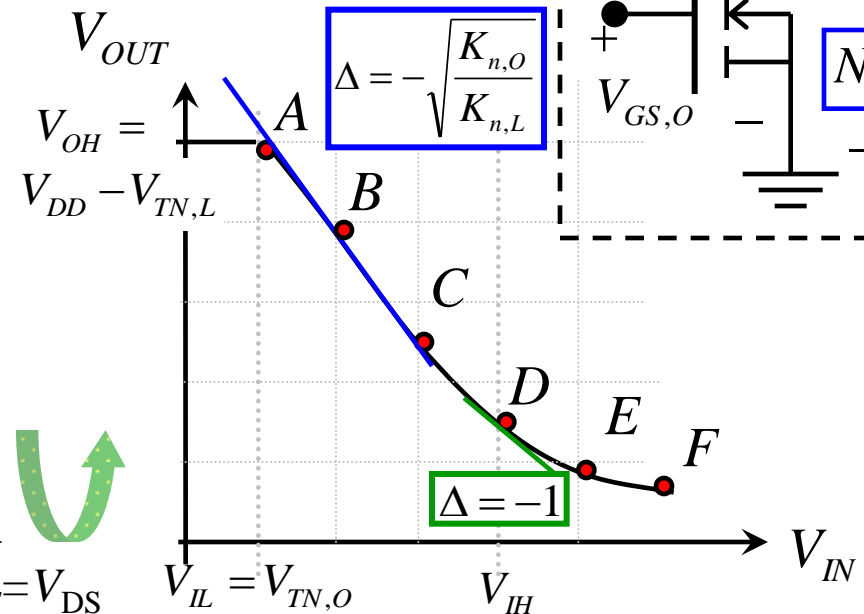
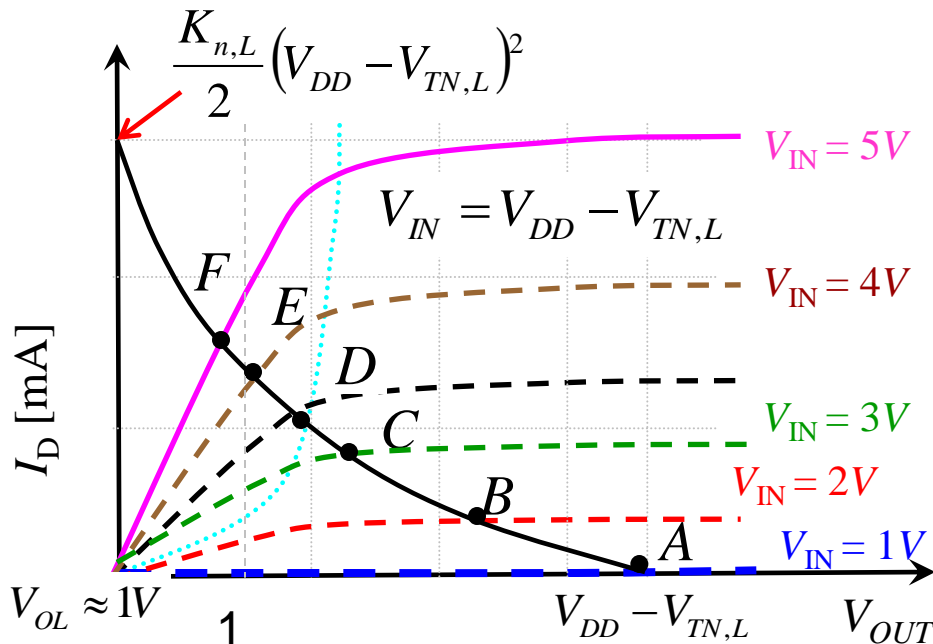
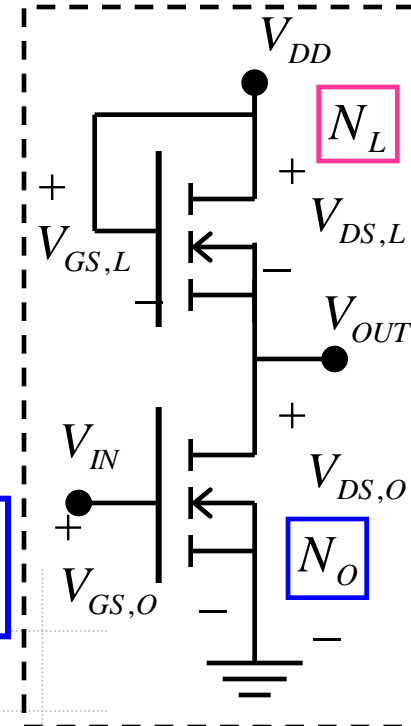
# VTC of Saturated Enhancement-Only Loaded NMOS Inverter

Graphical determination of VTC :

$$V_{IN} = V_{GS} \quad , \quad V_{OUT} = V_{DS}$$

Load line: 
$$I_{D,O} = I_{D,L} = \frac{K_{n,L}}{2} (V_{DD} - V_{DS,O} - V_{TN,L})^2$$

From the intersections between the load line and the current-voltage characteristics, the pairs  $(V_{IN}, V_{OUT})$  can be determined





# VTC of Saturated Enhancement-Only Loaded NMOS Inverter

Analytical determination of VTC :

$$V_{OH}$$

When  $V_{IN}$  is low, i.e.  $V_{IN} = V_{GS} < V_{TN}$ :  $\rightarrow N_O$  is cut-off

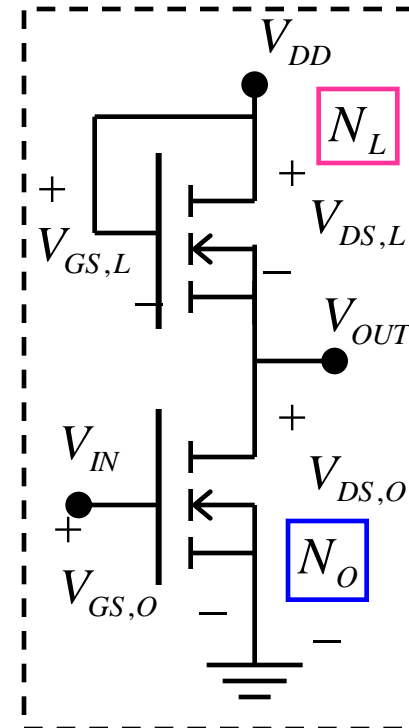
$$I_L = I_D = 0$$

$$V_{OH} = V_{DD} - V_{TN,L}$$

$$V_{IL}$$

$V_{IL}$  in MOSFET is **not** defined from  $dV_{OUT}/dV_{IN} = -1$

$$V_{IL} = V_{TN,O}$$



# VTC of Saturated Enhancement-Only Loaded NMOS Inverter

Analytical determination of VTC :

$$V_{OL}$$

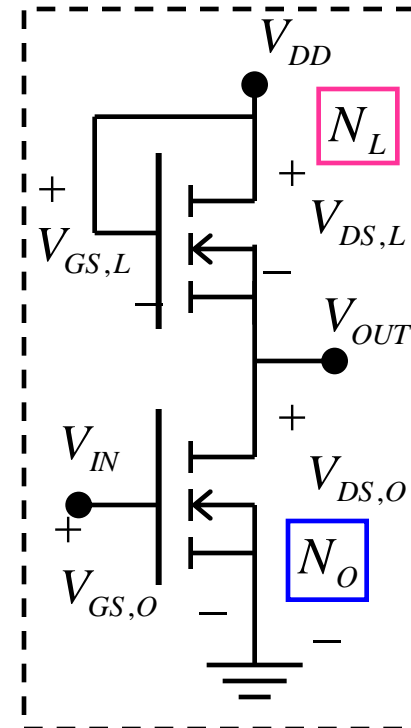
For the low output state, the N-MOS operates in linear mode:

$$I_D = \frac{K_n}{2} [2 \times (V_{GS} - V_{TN}) V_{DS} - V_{DS}^2]$$

Now, just as an assumption that the MOS is driven by a similar gate, i.e.  $V_{IN} = V_{GS,O} = V_{OH} = V_{DD} - V_{TN,L}$ .

$$\frac{K_{n,O}}{2} (2(V_{GS,O} - V_{TN,O}) V_{DS,O} - V_{DS,O}^2) = \frac{K_{n,L}}{2} (V_{GS,L} - V_{TN,L})^2$$

$$\frac{K_{n,O}}{2} (2(V_{DD} - V_{TN,L} - V_{TN,O}) V_{OL} - V_{OL}^2) = \frac{K_{n,L}}{2} (V_{DD} - V_{OL} - V_{TN,L})^2$$



# VTC of Saturated Enhancement-Only Loaded NMOS Inverter

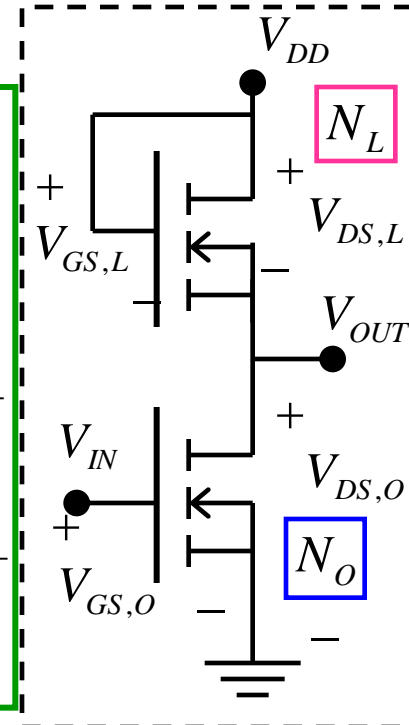
Analytical determination of VTC :

$$V_{OL} = \frac{2 \left[ (V_{DD} - V_{TN,L}) + \frac{K_{n,O}}{K_{n,L}} (V_{DD} - V_{TN,L} - V_{TN,O}) \right] \pm \sqrt{4 \left[ (V_{DD} - V_{TN,L}) + \frac{K_{n,O}}{K_{n,L}} (V_{DD} - V_{TN,L} - V_{TN,O}) \right]^2 - 8 \left( 1 + \frac{K_{n,O}}{K_{n,L}} \right) (V_{DD} - V_{TN,L})^2}}{2 \left( 1 + \frac{K_{n,O}}{K_{n,L}} \right)}$$

EXACT SOLUTION

$$V_{OL} \cong \frac{(V_{DD} - V_{TN,L})^2}{2 \left[ (V_{DD} - V_{TN,L}) + \frac{K_{n,O}}{K_{n,L}} (V_{DD} - V_{TN,L} - V_{TN,O}) \right]}$$

APPROXIMATED SOLUTION



# VTC of Saturated Enhancement-Only Loaded NMOS Inverter

Analytical determination of VTC :

$V_{IH}$

$V_{IH}$  in MOSFET is defined as the input voltage slightly before  $V_{OUT} = V_{OL}$  where **slope = -1** or

$$\frac{dV_{OUT}}{dV_{IN}} = -1$$

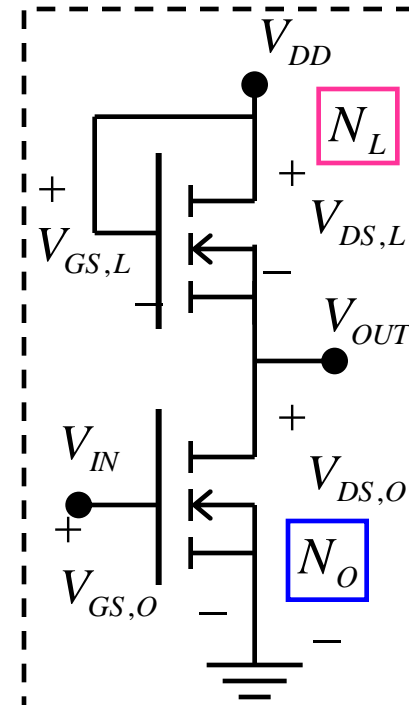
Ch. 19

For high input voltages (i.e.  $V_{OUT} \rightarrow V_{OL}$ ),  $N_O$  is in linear operation

$$\frac{K_{n,O}}{2} (2(V_{IN} - V_{TN,O})V_{OUT} - V_{OUT}^2) = \frac{K_{n,L}}{2} (V_{DD} - V_{OUT} - V_{TN,L})^2$$

$$\frac{K_{n,O}}{2} \left( 2(V_{IN} - V_{TN,O}) \frac{dV_{OUT}}{dV_{IN}} + 2V_{OUT} - 2V_{OUT} \frac{dV_{OUT}}{dV_{IN}} \right) = -K_{n,L} (V_{DD} - V_{OUT} - V_{TN,L}) \frac{dV_{OUT}}{dV_{IN}}$$

$$K_{n,O} (-(V_{IN} - V_{TN,O}) + 2V_{OUT}) = K_{n,L} (V_{DD} - V_{OUT} - V_{TN,L})$$



# VTC of Saturated Enhancement-Only Loaded NMOS Inverter

Analytical determination of VTC :

$V_{IH}$  in MOSFET is defined as the input voltage slightly before  $V_{OUT} = V_{OL}$  where **slope = -1** or

$$\frac{dV_{OUT}}{dV_{IN}} = -1$$

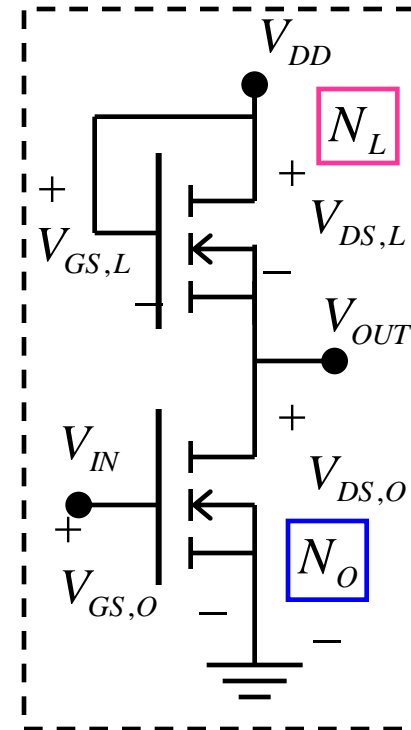
Ch. 19

For high input voltages (i.e.  $V_{OUT} \rightarrow V_{OL}$ ),  $N_O$  is in linear operation

$$V_{OUT}(IH) = \frac{K_{n,O}(V_{IN} - V_{TN,O}) + K_{n,L}(V_{DD} - V_{TN,L})}{2K_{n,O} + K_{n,L}}$$

$$V_{IH} = \frac{V_{DD} - V_{TN,L}}{\frac{1}{2} + \sqrt{\frac{3K_{n,O}}{4K_{n,L}}}} + V_{TN,O}$$

must  $V_{DS} \leq (V_{GS} - V_{TN})$  i.e.  $V_{OUT}(IH) \leq (V_{IH} - V_{TN,O})$



# VTC of Saturated Enhancement-Only Loaded NMOS Inverter

Analytical determination of VTC :

$V_M$  Mid point

$$V_{OUT} = V_{IN} = V_M$$

$$\Rightarrow V_{DS,O} = V_{OUT} = V_M \quad \& \quad V_{GS,O} = V_{IN} = V_M$$

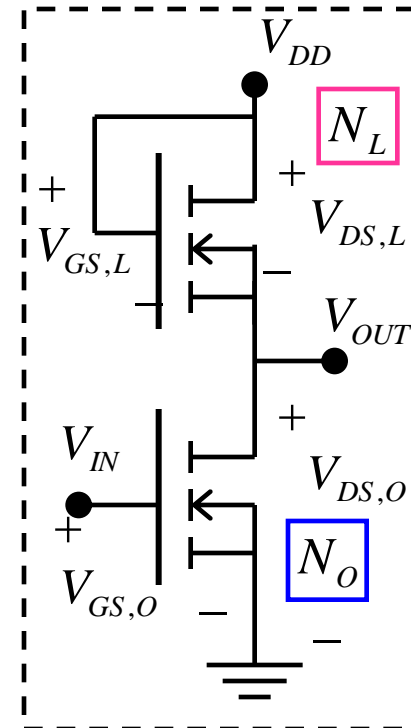
$$\begin{aligned} V_{DS,O}(sat) &= V_{GS,O} - V_{TN,O} \\ &= V_{DS,O} - V_{TN,O} \end{aligned}$$

$\Rightarrow V_{DS,O}(sat) < V_{DS,O}$   $N_O$  is in saturation operation

$$\frac{K_{n,O}}{2} (V_{GS,O} - V_{TN,O})^2 = \frac{K_{n,L}}{2} (V_{GS,L} - V_{TN,L})^2$$

$$\frac{K_{n,O}}{2} (V_M - V_{TN,O})^2 = \frac{K_{n,L}}{2} (V_{DD} - V_M - V_{TN,L})^2$$

Then solve for  $V_M$



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- HW #11: Solve Problems: **19.1-3,**