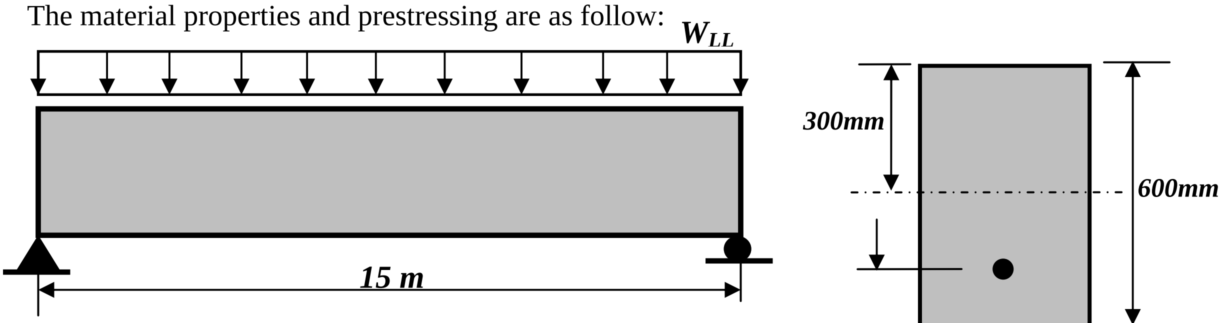
Name:………………………….. ID.:………………..

(7 points)

Problem 1: Consider the prestressed concrete beam shown below, which is to be used in an industrial building construction. The beam is only reinforced with prestressing steel and no mild reinforcement is used. What is the maximum live load that this beam can carry (in addition to the self weight) considering only the allowable section stresses at Midspan?

The material properties and prestressing are as follow:



f'_c = 40MPa f'_{ci} = 35MPa

f_{pu} = 1860 MPa f_{pi} = 1120MPa

Δf_{pT} = 140MPa (total losses) $A_{ps} = 8\phi_s 13 = 8 \times 99mm^2$/strand

$f_t = 0.5\sqrt{f'_c}$ and $f_c = 0.45 f'_c$

SOLUTION:

At service limit state

$W_D = (0.3 \times 0.6)(25) = 4.5kN/m$

$P_e = (8 \times 99)(1120 - 140)/1000 = 776.16kN$

$e = 300 - 70 = 230mm; A = 300 \times 600 = 180{,}000mm^2;\ I = (300)(600)^3/12 = 5.4 \times 10^9\ mm^4$

$r^2 = 5.4 \times 10^9/180{,}000 = 30{,}000mm^2$

$S^t = S^b = 5.4 \times 10^9/300 = 18 \times 10^6 mm^3$

$$f^t = \frac{-P_e}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_T}{S^t}$$

$$-0.45 \times 40 = \frac{-776{,}160}{180{,}000}\left(1 - \frac{(230)(300)}{30{,}000}\right) - \frac{M_T}{18 \times 10^6}$$

$\therefore M_T = 424.9kN.m = (W_{LL} + 4.5)(15)^2/8$

$\therefore W_{LL} = 10.61kN/m$

$$f^b = \frac{-P_e}{A_c}\left(1 + \frac{ec^t}{r^2}\right) + \frac{M_T}{S^t}$$

$$0.5\sqrt{40} = \frac{-776{,}160}{180{,}000}\left(1 + \frac{(230)(300)}{30{,}000}\right) + \frac{M_T}{18 \times 10^6}$$

$\therefore M_T = 313.1kN.m = (W_{LL} + 4.5)(15)^2/8$

$\therefore W_{LL} = 6.631kN/m$

$$\boxed{\therefore W_{LL} = 6.631kN/m}$$

(6 points)

Problem 2: Answer the following three questions with short and precise answers:

(a) What do you understand from the term “prestressing transfer length”

The distance over which the strand should be bonded to the concrete to develop the effective prestress after losses

(b) If allowable stress limits are not satisfied at the end of a beam with straight strands, write down at least two options that can be used to satisfy these limits.

1. Use Raised-up tendons near the ends
2. Use Debonding some tendons near the ends
3. Use Supplementary nonprestressed steel

(c) In case the tension limit is exceeded, which of the following is true:

1. Extra tension force resulting from extra tension stressed should be carried by mild steel
2. Concrete cover should be increased by 30%
3. All tension force resulting from the entire tension stresses should be carried by mild steel
4. This case is not allowed to occur in the design by standard codes.

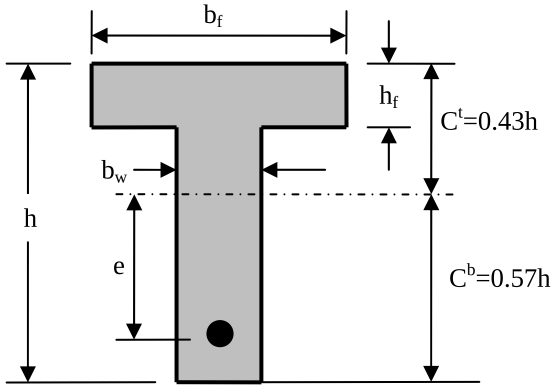
(12 points)

Problem 2: Design, for service load condition, a post-tensioned T-section to carry a total service load of **15 kN/m** (not including self weight) on a **12m** simply supported span. Design the section for zero-tension, for $\mathbf{f_{ci} = 12.5MPa}$ and $\mathbf{f_c = 11.0MPa}$ at transfer and service conditions, respectively. Assume that the sectional properties are $\mathbf{b_f = 0.5h}$, $\mathbf{h_f = 0.2h}$, and $\mathbf{b_w = 0.25h}$ and use multiples of **50mm** for **h**. Assume the self weight of the beam is **6.0 kN/m** and the following prestressing data:

$f'_c = 35MPa$ $\quad$ $f'_{ci} = 30MPa$

$f_{pu} = 1860$ MPa $\quad$ $f_{pi} = 1300MPa$

$\Delta f_{pT} = 250MPa$ $\quad$ $A_{ps} = 1\phi_s13 = 99$ mm^2/strand



SOLUTION:

$(1-\gamma) = 250/1300 = 0.192$

$M_D = 6 \times 12^2/8 = 108\text{kN.m}$

$M_{SD} + M_L = 15 \times 12^2/8 = 270\text{kN.m}$

$$S^t = \frac{(1-\gamma)M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

$$S^t = \frac{(0.192)(108) + (270)}{0-(-11)} \times 10^6 = 26.431 \times 10^6 mm^3$$

$$S^b = \frac{(1-\gamma)M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}} = \frac{(0.192)(108) + (270)}{0-(0.808)(-12.5)} \times 10^6 = 28.786 \times 10^6 mm^3$$

$$I = \frac{(0.5h)(0.2h)^3}{12} + (0.5h)(0.2h)(0.33h)^2 + \frac{(0.25h)(0.8h)^3}{12} + (0.25h)(0.8h)(0.17h)^2 = 0.02767h^4$$

$$S^t = \frac{0.02767h^4}{0.43h} = 26.431 \times 10^6 \rightarrow h = 743.3mm \ ;$$

$$S^b = \frac{0.02767h^4}{0.57h} = 28.786 \times 10^6 \rightarrow h = 840mm \ ;$$

➔ **Use h = 850mm; A=216,750mm^2 I=1.444 x 10^{10}mm^4 ; S^t=39.20 x 10^6 mm^3**
S^b = 30.00 x 10^6 mm^3 ; r^2 = 66,621 mm^2 ; M$_D$ = 97.54 kN.m

Analysis at Initial Stage:

$$f_{cgc} = \frac{-12.5}{850} \times (0.43 \times 850) = -5.375MPa$$
$$P_i = 5.375 \times 216{,}750/1000 = 1{,}165kN$$
$$A_{ps} = 1{,}165 \times 10^3/1300 = 896.2mm^2$$ ➔ USE 9ϕ_s13= 891mm^2
$$P_i = 891 \times 1300/1000 = 1{,}158.3kN$$
$$e_c = (0+5.375)\frac{39.20\times10^6}{1{,}158{,}300} + \frac{97.54\times10^6}{1{,}158{,}300} = 266.00mm$$

➔ **Use 9ϕ_s13= 891mm^2 ; P$_i$ = 1,158.3 kN ; e$_c$ = 265 mm ; P$_e$ = 935.91 kN**

Check Stresses:

1. At initial stage:

$$f^t = \frac{-P_i}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_D}{S^t}$$

$$f^t = \frac{-1{,}158{,}300}{216{,}750}\left(1 - \frac{(265)(365.5)}{66{,}621}\right) - \frac{97.54 \times 10^6}{39.2 \times 10^6}$$

$$f^t = +2.425 - 2.488 = -0.063\ MPa \approx 0.0 \quad \boldsymbol{OKAY}!$$

$$f^b = \frac{-P_i}{A_c}\left(1 + \frac{ec^b}{r^2}\right) + \frac{M_D}{S^t}$$

$$f^b = \frac{-1{,}158{,}300}{216{,}750}\left(1 + \frac{(265)(484.5)}{66{,}621}\right) + \frac{97.54 \times 10^6}{30.0 \times 10^6}$$

$$f^t = -15.643 + 3.251 = -12.392\ MPa < 12.5 MPa \quad \boldsymbol{OKAY}!$$

2. At final stage:

$$f^t = \frac{-P_e}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_T}{S^t}$$

$$f^t = \frac{-935{,}910}{216{,}750}\left(1 - \frac{(265)(365.5)}{66{,}621}\right) - \frac{367.54 \times 10^6}{39.2 \times 10^6}$$

$$f^t = +1.960 - 9.376 = -7.416\ MPa < 11 MPa \quad \boldsymbol{OKAY}!$$

$$f^b = \frac{-P_e}{A_c}\left(1 + \frac{ec^b}{r^2}\right) + \frac{M_T}{S^t}$$

$$f^b = \frac{-935{,}910}{216{,}750}\left(1 + \frac{(265)(484.5)}{66{,}621}\right) + \frac{367.54 \times 10^6}{30.0 \times 10^6}$$

$$f^t = -12.639 + 12.251 = -0.388\ MPa \approx 0.0 \quad \boldsymbol{OKAY}!$$