**Problem 1:** Consider the prestressed concrete beam shown below, which is to be used in an industrial building construction. The beam is only reinforced with prestressing steel and no mild reinforcement is used. What is the maximum live load that this beam can carry (in addition to the self weight) considering only the allowable section stresses at Midspan?

The material properties and prestressing are as follow:

- $f'_c = 40 \text{MPa}$
- $f'_{ci} = 35 \text{MPa}$
- $f_{pu} = 1860 \text{MPa}$
- $f_{pi} = 1120 \text{MPa}$
- $\Delta f_{PT} = 140 \text{MPa}$ (total losses)
- $A_{ps} = 8 \phi_{13} = 8 \times 99 \text{mm}^2/\text{strand}$

**SOLUTION:**

At service limit state

$W_D = (0.3 \times 0.6)(25) = 4.5 \text{kN/m}$

$P_e = (8 \times 99)(1120 - 140)/1000 = 776.16 \text{kN}$

$e = 300 - 70 = 230 \text{mm}; A = 300 \times 600 = 180,000 \text{mm}^2; I = (300)(600)^3/12 = 5.4 \times 10^9 \text{mm}^4$

$r^2 = 5.4 \times 10^9/180,000 = 30,000 \text{mm}^2$

$S^t = S^b = 5.4 \times 10^9/300 = 18 \times 10^6 \text{mm}^3$

\[
f^t = \frac{-P_e}{A_c} \left(1 - \frac{e c^t}{r^2}\right) - \frac{M_T}{S^t} - 0.45 \times 40 = \frac{-776,160}{180,000} \left(1 - \frac{(230)(300)}{30,000}\right) - \frac{M_T}{18 \times 10^6}
\]

$\therefore M_T = 424.9 \text{kN.m}, m = (W_{LL} + 4.5)(15)^2/8$

$\therefore W_{LL} = 10.61 \text{kN/m}$

\[
f^b = \frac{-P_e}{A_c} \left(1 + \frac{e c^t}{r^2}\right) + \frac{M_T}{S^t}
\]

$0.5\sqrt{40} = \frac{-776,160}{180,000} \left(1 + \frac{(230)(300)}{30,000}\right) + \frac{M_T}{18 \times 10^6}$

$\therefore M_T = 313.1 \text{kN.m}, m = (W_{LL} + 4.5)(15)^2/8$

$\therefore W_{LL} = 6.631 \text{kN/m}$

$\therefore W_{LL} = 6.631 \text{kN/m}$
Problem 2: Answer the following three questions with short and precise answers:

(a) What do you understand from the term “prestressing transfer length”

The distance over which the strand should be bonded to the concrete to develop the effective prestress after losses

(b) If allowable stress limits are not satisfied at the end of a beam with straight strands, write down at least two options that can be used to satisfy these limits.

1. Use Raised-up tendons near the ends
2. Use Debonding some tendons near the ends
3. Use Supplementary nonprestressed steel

(c) In case the tension limit is exceeded, which of the following is true:

1. Extra tension force resulting from extra tension stressed should be carried by mild steel
2. Concrete cover should be increased by 30%
3. All tension force resulting from the entire tension stresses should be carried by mild steel
4. This case is not allowed to occur in the design by standard codes.
Problem 2: Design, for service load condition, a post-tensioned T-section to carry a total service load of 15 kN/m (not including self weight) on a 12m simply supported span. Design the section for zero-tension, for $f_{ci} = 12.5MPa$ and $f_c = 11.0MPa$ at transfer and service conditions, respectively. Assume that the sectional properties are $b_f = 0.5h$, $h_f = 0.2h$, and $b_w = 0.25h$ and use multiples of 50mm for $h$.

Assume the self weight of the beam is 6.0 kN/m and the following prestressing data:

- $f'_c = 35MPa$
- $f'_{ci} = 30MPa$
- $f_{pu} = 1860 MPa$
- $f_{pi} = 1300MPa$
- $\Delta f_{pt} = 250MPa$
- $A_{ps} = 1\phi_{s13} = 99 \text{ mm}^2$/strand

The prestressed concrete section is designed for the service load condition. The solution involves calculating the moments and forces at different stages, considering the effects of prestressing as well as the transfer and service conditions. The sectional properties are determined to ensure the beam meets the design requirements.

SOLUTION:

1. **Service Load Condition:**
   - $M_D = 6 \times 12^2/8 = 108kN.m$
   - $M_{SD} + M_L = 15 \times 12^2/8 = 270kN.m$

   \[ S^t = \frac{(1 - \gamma)M_D + M_{SD} + M_L}{\gamma f_{ci} - f_c} \]

   \[ S^t = \frac{(1 - 0.192)(108) + (270)}{0 - (-11)} \times 10^6 = 26.431 \times 10^6 \text{mm}^3 \]

2. **Transfer Load Condition:**

   \[ S^b = \frac{(1 - \gamma)M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}} = \frac{(0.192)(108) + (270)}{0 - (0.808)(-12.5)} \times 10^6 = 28.786 \times 10^6 \text{mm}^3 \]

   \[ l = \frac{(0.5h)(0.2h)^3}{12} + (0.5h)(0.2h)(0.33h)^2 + \frac{(0.25h)(0.8h)^3}{12} + (0.25h)(0.8h)(0.17h)^2 = 0.02767h^4 \]

   \[ S^t = \frac{0.02767h^4}{0.43h} = 26.431 \times 10^6 \rightarrow h = 743.3mm ; \]

   \[ S^b = \frac{0.02767h^4}{0.57h} = 28.786 \times 10^6 \rightarrow h = 840mm ; \]

   - **Use** $h = 850mm$; $A_{total} = 216,750\text{mm}^2$; $I = 1.444 \times 10^{10}\text{mm}^4$; $S^t = 39.20 \times 10^6 \text{ mm}^3$
   - $S^b = 30.00 \times 10^6 \text{ mm}^3$; $r^2 = 66,621 \text{ mm}^2$; $M_D = 97.54 \text{ kN.m}$

**Analysis at Initial Stage:**

- $f_{cgc} = \frac{-12.5}{850} \times (0.43 \times 850) = -5.375MPa$
- $P_t = 5.375 \times 216,750/1000 = 1,165kN$
- $A_{ps} = 1,165 \times 10^3/1300 = 896.2\text{mm}^2$  ➔ USE $9\phi_{s13} = 891\text{mm}^2$
- $P_t = 891 \times 1300/1000 = 1,158.3kN$
- $e_c = (0 + 5.375)\frac{39.20\times10^6}{1.158300} + \frac{97.54\times10^6}{1.158300} = 266.00mm$

   - **Use** $9\phi_{s13} = 891\text{mm}^2$; $P_t = 1,158.3 \text{ kN}$; $e_c = 265 \text{ mm}$; $P_e = 935.91 \text{ kN}$

Page 3 of 4
Check Stresses:

1. At initial stage:

\[ f^t = -\frac{P_i}{A_c} \left( 1 - \frac{e c^t}{r^2} \right) - \frac{M_D}{S^t} \]
\[ f^t = -\frac{1,158,300}{216,750} \left( 1 - \frac{(265)(365.5)}{66,621} \right) = \frac{97.54 \times 10^6}{39.2 \times 10^6} \]
\[ f^t = +2.425 - 2.488 = -0.063 \text{ MPa} \approx 0.0 \text{ OKAY!} \]

\[ f^b = -\frac{P_i}{A_c} \left( 1 + \frac{e c^b}{r^2} \right) + \frac{M_D}{S^t} \]
\[ f^b = -\frac{1,158,300}{216,750} \left( 1 + \frac{(265)(484.5)}{66,621} \right) = \frac{97.54 \times 10^6}{30.0 \times 10^6} \]
\[ f^b = -15.643 + 3.251 = -12.392 \text{ MPa} < 12.5 \text{ MPa} \text{ OKAY!} \]

2. At final stage:

\[ f^t = -\frac{P_i}{A_c} \left( 1 - \frac{e c^t}{r^2} \right) - \frac{M_T}{S^t} \]
\[ f^t = -\frac{935,910}{216,750} \left( 1 - \frac{(265)(365.5)}{66,621} \right) = \frac{367.54 \times 10^6}{39.2 \times 10^6} \]
\[ f^t = +1.960 - 9.376 = -7.416 \text{ MPa} < 11 \text{ MPa} \text{ OKAY!} \]

\[ f^b = -\frac{P_i}{A_c} \left( 1 + \frac{e c^b}{r^2} \right) + \frac{M_T}{S^t} \]
\[ f^b = -\frac{935,910}{216,750} \left( 1 + \frac{(265)(484.5)}{66,621} \right) = \frac{367.54 \times 10^6}{30.0 \times 10^6} \]
\[ f^b = -12.639 + 12.251 = 0.388 \text{ MPa} \approx 0.0 \text{ OKAY!} \]