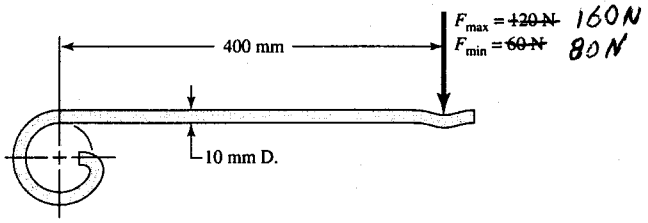
EXAMPLE 6–12 Figure 6–31 shows a formed round-wire cantilever spring subjected to a varying force. The hardness tests made on 25 springs gave a minimum hardness of 380 Brinell. It is apparent from the mounting details that there is no stress concentration. A visual inspection of the springs indicates that the surface finish corresponds closely to a hot-rolled finish. What number of applications is likely to cause failure? Solve using:
(*a*) Modified Goodman criterion.
(*b*) Gerber criterion.

Solution

$$S_{ut} = 3.41(380) = 1295.8 \text{ MPa}$$
$$S'_e = 0.5(1295.8) = 648 \text{ MPa}$$
$$k_a = 57.7(1295.8)^{-0.718} = 0.336$$

For a non-rotating round bar in bending, Eq. (6–24) gives: $d_e = 0.370d = 0.370(10) = 3.7$ mm

$$k_b = \left(\frac{3.7}{7.62}\right)^{-0.107} = 1.08$$
$$S_e = 0.336(1.08)(648) = 235 \text{ MPa}$$
$$F_a = \frac{160 - 80}{2} = 40 \text{N}, \quad F_m = \frac{160 + 80}{2} = 120 \text{N},$$
$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(120)(400)}{\pi(10^3)} = 488.9 \text{ MPa}$$



| Figure 6–31

$$\sigma_a = \frac{32(40)(400)}{\pi(10^3)} = 162.9 \text{ MPa}$$
$$r = \frac{122.2}{366.7} = 0.333$$

(*a*) Modified Goodman, Table 6–6

$$n_f = \frac{1}{(162.9/235) + (488.9/1295.8)} = 0.93 \Rightarrow \text{Finite Life}$$

From Fig. 6–18, for $S_{ut} = 1295.8$ MPa, $f = 0.78$

Eq. (6–14): $a = \frac{[0.78(1295.8)]^2}{235} = 5573$ MPa

Eq. (6–15): $b = -\frac{1}{3}\log\frac{0.78(1295.8)}{235} = -0.211\,19$

$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1 \Rightarrow S_f = \frac{\sigma_a}{1 - (\sigma_m/S_{ut})} = \frac{162.9}{1 - (488.9/1295.8)} = 261.6$ MPa

Eq. (6–16) with $\sigma_a = S_f$

Answer $$N = \left(\frac{261.6}{170.4}\right)^{1/-0.211\,19} = 195\,253 \text{ cycles}$$

(*b*) Gerber, Table 6–7

$$n_f = \frac{1}{2}\left(\frac{1295.8}{488.9}\right)^2\left(\frac{162.9}{235}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(366.7)(235)}{1295.8(162.9)}\right]^2}\right\} = 1.16$$

Answer Thus, infinite life is predicted ($N \geq 10^6$ cycles).