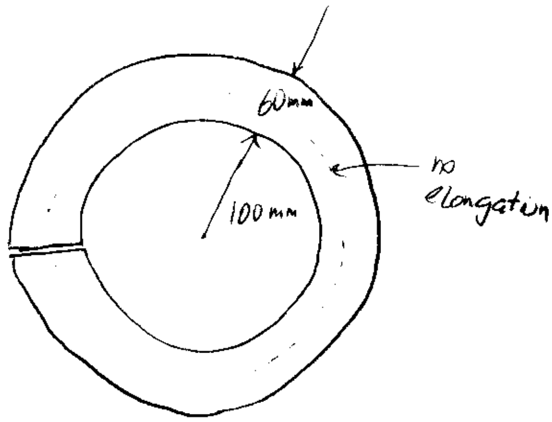
Machine Design I

Dr. Ala Hijazi

Chapter 2 Solved problems

1. A 60 mm thick Aluminum 7075-T6 flat plate (properties are given in Table A-22) is rolled to make a cylinder with an inner radius of 100mm. Assuming that the mid-plane of the plate does not experience any tension or compression (i.e., there is no elongation at the mid-plane). Determine if the plate will fracture during rolling.



strain at the outer surface

$$\epsilon_o = \frac{\Delta L}{L} = \frac{2\pi(160) - 2\pi(130)}{2\pi(130)} = 0.23$$

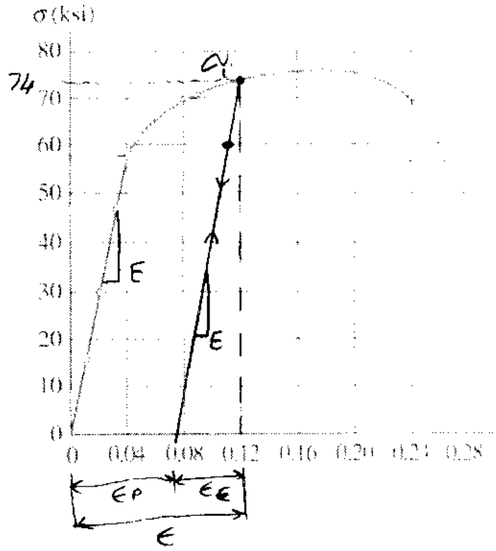
From Table A-22, the fracture strain for Al 7075-T6 : $\epsilon_f = 0.18$

$$0.23 > 0.18$$

$\Rightarrow$ The plate will fracture.

2. The stress-strain diagram for some material is shown in the figure.
A tensile test specimen having a diameter of 0.5 inch and initial length of 5 inches was made form this material. The specimen was mounted on the testing machine and the load was increases until the streets in the specimen reached 74ksi then the specimen was unloaded.

a) Find the new length of the specimen.
b) If the specimen was reloaded again until the stress reached 60ksi, what would be the length of the specimen at this stress level?



a)

$$E = \frac{58\,ksi}{0.04} = 1450\,ksi$$

$$\epsilon_E = \frac{\sigma_i}{E} = \frac{74}{1450} = 0.051$$

from curve $\epsilon = 0.12$

$$\epsilon_p = \epsilon - \epsilon_E = 0.12 - 0.051 = 0.069$$

New length $= L_o(1+\epsilon) = 5(1+0.069)$

$= 5.345$ in

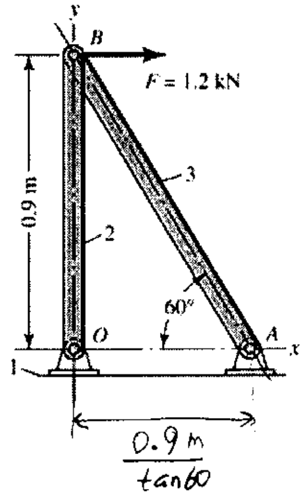
b) $$\epsilon = \epsilon_p + \frac{\sigma}{E} = 0.069 + \frac{60}{1450} = 0.11$$

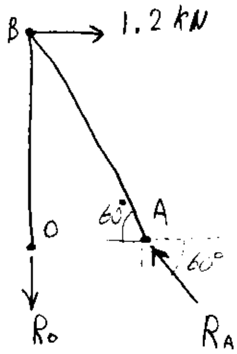
$$\Rightarrow L = l_o(1+\epsilon) = 5(1+0.11) = 5.55 \text{ in}$$

Machine Design I — Dr. Ala Hijazi

Chapter 3 Solved Problems

1. For the shown structure, draw the free body diagram of each element and find the magnitude and direction of each force.





$\circlearrowleft + \; \sum M_A = 0$

$$R_o\left(\frac{0.9}{\tan 60}\right) - 1.2\,(0.9) = 0$$

$$\Rightarrow \; R_o = 1.2 \tan 60 = \underline{2.08 \text{ kN}}$$

$\sum F_x = 0$

$$1.2 - (R_A)_x = 0$$

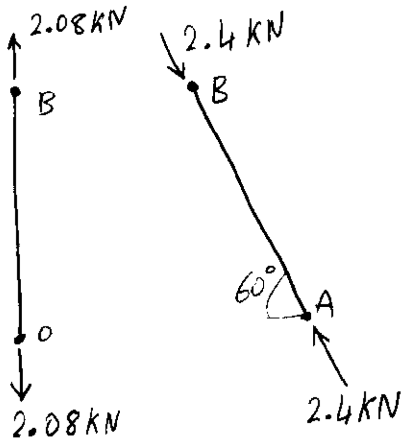
$$\Rightarrow (R_A)_x = \underline{1.2 \text{ kN}}$$

$\sum F_y = 0$

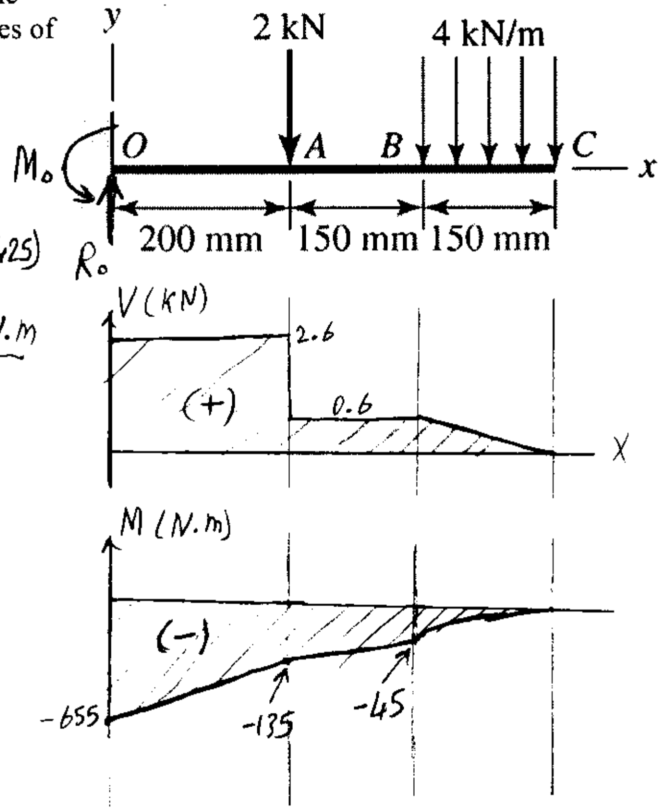
$$(R_A)_y - R_o = 0$$

$$\Rightarrow (R_A)_y = \underline{2.08 \text{ kN}}$$

$$\Rightarrow R_A = \sqrt{(1.2)^2 + (2.08)^2} = \underline{2.4 \text{ kN}}$$



2. Sketch the shear and moment diagrams for the shown beam and indicate the maximum values of the shear force and bending moment.



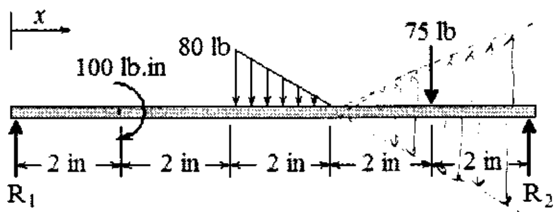
$\sum F_y = 0 \Rightarrow R_o = 2.6\ KN$

$(+)\ \sum M_o = 0, \quad -M_o + 2(0.2) + [4 * 0.15](0.425)$

$\Rightarrow M_o = 0.655\ KN.m = 655\ N.m$

$$\boxed{\begin{aligned} V_{max} &= 2.6\ KN \\ M_{max} &= 655\ N.m \end{aligned}}$$

3. Using singularity functions derive the expressions for the loading, shear force, and bending moment for the beam shown.

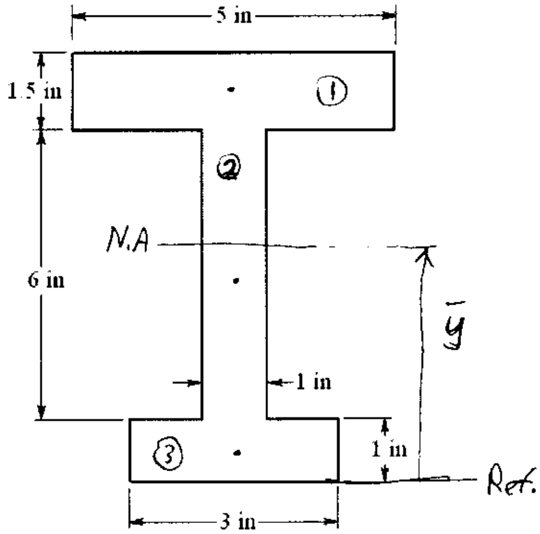


$$q(x) = R_1 \langle x \rangle^{-1} - 100 \langle x-2 \rangle^{-2} - 80 \langle x-4 \rangle^{0} + \frac{80}{2} \langle x-4 \rangle^{1} + \frac{80}{2} \langle x-6 \rangle^{1}$$
$$- 75 \langle x-8 \rangle^{-1} + R_2 \langle x-10 \rangle^{-1}$$

$$V(x) = R_1 \langle x \rangle^{0} - 100 \langle x-2 \rangle^{-1} - 80 \langle x-4 \rangle^{1} + \frac{80}{4} \langle x-4 \rangle^{2} + \frac{80}{4} \langle x-6 \rangle^{2}$$
$$- 75 \langle x-8 \rangle^{0} + R_2 \langle x-10 \rangle^{0}$$

$$M(x) = R_1 \langle x \rangle^{1} - 100 \langle x-2 \rangle^{0} - \frac{80}{2} \langle x-4 \rangle^{2} + \frac{80}{12} \langle x-4 \rangle^{3} + \frac{80}{12} \langle x-6 \rangle^{3}$$
$$- 75 \langle x-8 \rangle^{1} + R_2 \langle x-10 \rangle^{1}$$

4. Locate the centroid of the shown section then find the moment of inertia about the neutral axis and the product of inertia at the neutral axis (Q_{NA}).



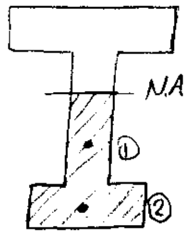
$$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i} = \frac{7.75(5*1.5)+4(1*6)+0.5(3*1)}{(5*1.5)+(1*6)+(3*1)}$$

$$\Rightarrow \bar{y} = 5.07 \text{ in}$$

$$I = \Sigma \bar{I}_i + A_i \bar{y}_i^2$$

$$= \left[\tfrac{1}{12}(5)(1.5)^3 + (5*1.5)(7.75-5.07)^2\right] + \left[\tfrac{1}{12}(1)(6)^3 + (1*6)(5.07-4)^2\right] + \left[\tfrac{1}{12}(3)(1)^3 + (3*1)(5.07-0.5)^2\right] = 55.27 + 24.87 + 62.9$$

$$\Rightarrow I = 143.04 \text{ in}^4$$

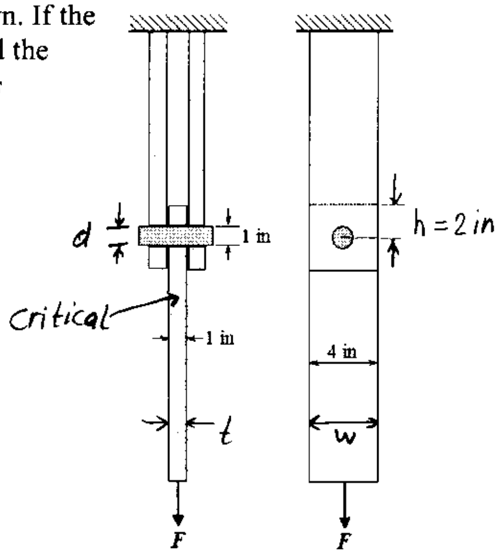


$$Q = \Sigma A_i' \bar{y}_i'$$

$$= \left[(1*4.07)(4.07/2)\right] + \left[(3*1)(5.07-0.5)\right] = 8.28 + 13.71$$

$$\Rightarrow Q_{NA} = 21.99 \text{ in}^3$$

5. A double-lapped joint is connected using a rigid pin as shown. If the maximum allowable stress in the plate material is 60ksi, find the maximum allowable value of the load F. *(consider the stress concentration)*



* From Figure A-15-12

$$\sigma_o = \frac{F}{A} \quad \text{where } A = (w-d)t$$

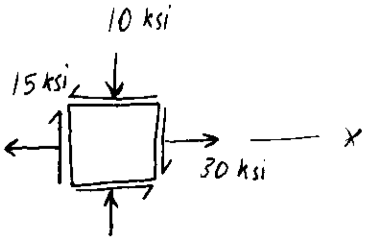
$$\Rightarrow \sigma_o = \frac{F}{(4-1)(1)} = \frac{F}{3}$$

$$h/w = 0.5, \quad d/w = 0.25 \Rightarrow K_t \simeq 4.7$$

$$\sigma_{max} = \sigma_o K_t = 60 \text{ ksi}$$

$$\frac{F}{3}(4.7) = 60 \text{ ksi} \quad \Rightarrow F = \frac{60*3}{4.7} = 38.3 \text{ Kip}$$

$$\Rightarrow \boxed{F_{max} = 38300 \text{ lb}}$$



6. If the state of stress at a point is given as $\sigma_x = 30$ ksi, $\sigma_y = -10$ ksi, $\tau_{xy} = -15$ ksi, using Mohr's circle find:
   a) The principal normal stresses and the maximum shear stress and show each on a stress element.
   b) The state of stress if we perform a rotation of 20° ccw, and show it on a stress element.

a) center: $\sigma_c = \frac{\sigma_x + \sigma_y}{2} = \frac{30-10}{2} = 10$ ksi

Raduis: $R = \sqrt{(30-10)^2 + 15^2} = 25$ ksi

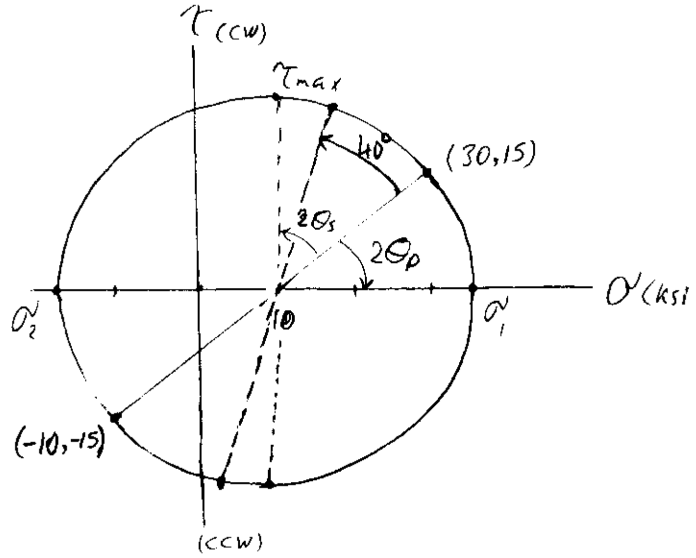
$\sigma_{1,2} = \sigma_c \pm R = 10 \pm 25$

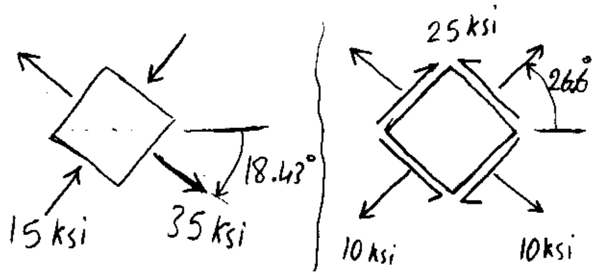
$\Rightarrow \underline{\sigma_1 = 35 \text{ ksi}}$, $\underline{\sigma_2 = -15 \text{ ksi}}$

$\tau_{max} = R = \underline{25 \text{ ksi}}$

$2\theta_p = \tan^{-1}\frac{15}{20} \Rightarrow \theta_p = 18.43°$

$2\theta_s = 90 - 2\theta_p \Rightarrow \theta_s = 26.57$



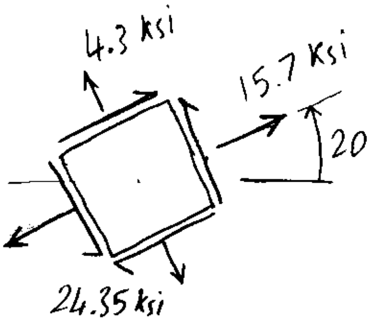


b) 20° rotation = 40° on the circle

$\sigma_x = \sigma_c + R\sin(2\theta_s - 40) = 10 + 25\sin(53.14 - 40) = \underline{15.7 \text{ ksi}}$

$\sigma_y = \sigma_c - R\sin(2\theta_s - 40) = 10 - 25\sin(53.14 - 40) = \underline{4.3 \text{ ksi}}$

$\tau_{xy} = R\cos(2\theta_s - 40) = 25\cos(53.14 - 40) = \underline{24.35 \text{ ksi}}$



7. A close-ended cylinder of internal radius of $r_i = 30$ in and a wall thickness of $t = 1$ in. The pressure inside the cylinder is $P_i = 8$ psi. The cylinder is also subjected to torque of $T = 500$ lb.in applied at both ends. Find the state of stress at a point on the surface of the cylinder and show it on a stress element.

$$\frac{r_i}{t} = 30 > 20 \Rightarrow \text{thin-walled } \& \ \sigma_r = 0$$

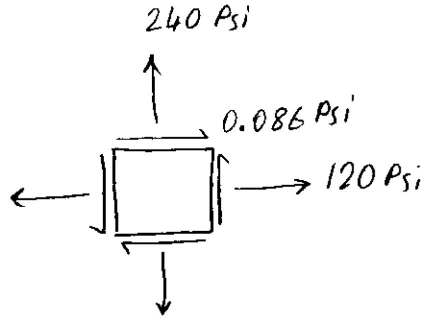
* Due to the internal pressure

$$\sigma_t = \frac{P d_i}{2t} = \frac{8(60)}{2*1} = 240 \text{ Psi}$$

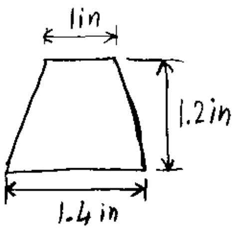
$$\sigma_L = \frac{P d_i}{4t} = \frac{8(60)}{4*1} = 120 \text{ Psi}$$

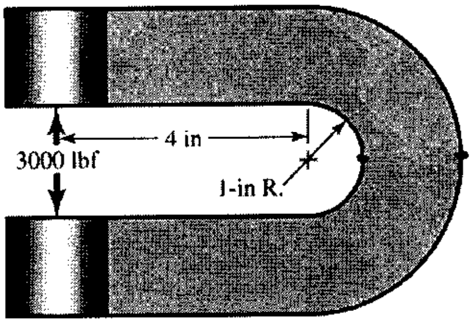
* Due to torque (thin-walled)

$$\tau = \frac{T}{2A_m t} = \frac{500}{2(\pi * 30.5^2)(1)} = 0.086 \text{ Psi}$$



8. The C-frame shown in the figure has a trapezoidal cross-section having the following dimensions $b_o = 1$ in, $b_i = 1.4$ in, $h = 1.2$ in (see Table 4-5). Determine the stresses at the inner and outer surfaces at the throat of the C.





From Table 4-5

$$r_c = 1 + \frac{1.2}{3}\,\frac{1.4+2(1)}{1.4+1} = 1.567 \text{ in}$$

$$r_n = \frac{1/2\,(1+1.4)(1.2)}{1 - 1.4 + [(1.4*2.2 - 1*1)/1.2]\ln\left(\frac{2.2}{1}\right)}$$

$r_i = 1$ in

$r_o = r_i + h = 2.2$ in

$$\Rightarrow r_n = 1.49 \text{ in}$$

$$\Rightarrow e = r_c - r_n = 0.077 \text{ in}$$

$$M = 3000(4 + 1.567) = 16700 \text{ lb.in}$$

* at the inner surface (due to bending) $C_i = r_n - r_i = 0.49$ in

$$\sigma_i = \frac{M c_i}{A e r_i} = \frac{16700(0.49)}{1.44(0.077)(1)} = 73.14 \text{ MPa}$$

* at the outer surface (due to bending) $C_o = r_o - r_n = 0.71$ in

$$\sigma_o = -\frac{M C_o}{A e r_o} = \frac{-16700(0.71)}{1.44(0.077)(2.2)} = -48.17 \text{ MPa}$$

* Stress due to normal force

$$\sigma = F/A = 3000/1.44 = 2.08 \text{ MPa}$$

Ans.

Total

* at inner surface

$$\sigma_i = 73.14 + 2.08 = 75.22 \text{ MPa}$$

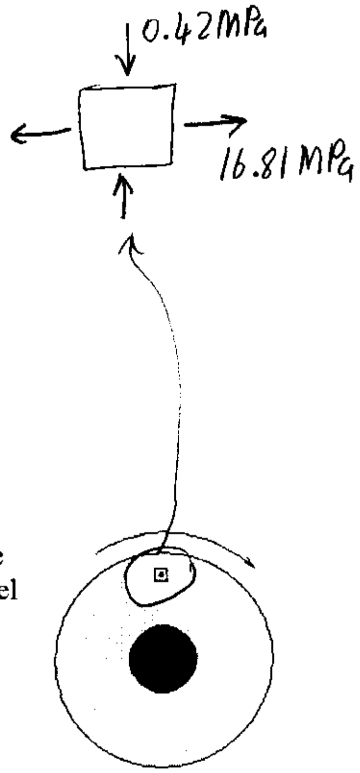
* at outer surface

$$\sigma_o = -48.17 + 2.08 = -46.09 \text{ MPa}$$

Cont. Problem 9

Total stress at $r = 120$ mm

$$\Rightarrow \boxed{\begin{aligned} \sigma_t &= 12.24 + 4.57 = 16.81 \text{ MPa} \\ \sigma_r &= -2.69 + 2.27 = -0.42 \text{ MPa} \end{aligned}} \quad \text{Ans.}$$



9. A 50 mm nominal diameter shaft is press fitted (Force fit "H7/u6") into the hole of a fly wheel. The outer diameter of the flywheel is 300 mm. The shaft/flywheel assembly rotates at an angular velocity of 300 rad/s. Both the shaft and the flywheel are made of steel (**E** = 200 GPa, **ν** = 0.32, **ρ** = 7850 Kg/m^3). For a maximum interference assembly find the state of stress at a point on the surface of the flywheel located 120 mm away from the center, and show it on a stress element.

50 H7/u6 fit

hole: 50 H7 : From table A-11, IT:7 $\Rightarrow \Delta D = 0.025$ mm $\Rightarrow D = 50^{+0.025}_{-0}$

Shaft: 50 u6 : From table A-11, IT:6 $\Rightarrow \Delta d = 0.016$ mm ; From table A-12, u $\Rightarrow \delta_F = +0.07$ mm $\Rightarrow d = 50.07^{+0.016}_{-0}$

maximum Interference $\Rightarrow 2\delta = d_{max} - D_{min} = 50.086 - 50 = 0.086$ mm

$$\Rightarrow \boxed{\delta = 0.043 \text{ mm}}$$

* Contact pressure due to interference

$$P = \frac{E\delta}{R}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{2R^2(r_o^2 - r_i^2)}\right] \qquad r_i = 0,\; R = 25 \text{ mm},\; r_o = 150 \text{ mm}$$

$$\Rightarrow P = \frac{200 \times 10^3 (0.043)}{25}\left[\frac{(150^2 - 25^2)(25^2 - 0)}{2(25)^2(150^2 - 0)}\right] = 167.2 \text{ MPa}$$

* Stress in the flywheel due to the force fit (internal pressure) $r_i = 25$ mm, $r_o = 150$ mm

at $r = 120$ mm

$$\sigma_t = \frac{r_i^2 P_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right) = \frac{25^2(167.2)}{150^2 - 25^2}\left(1 + \frac{150^2}{120^2}\right) \Rightarrow \boxed{\sigma_t = 12.24 \text{ MPa}}$$

$$\sigma_r = \frac{r_i^2 P_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = \frac{25^2(167.2)}{150^2 - 25^2}\left(1 - \frac{150^2}{120^2}\right) \Rightarrow \boxed{\sigma_r = -2.69 \text{ MPa}}$$

* Stress in the flywheel due to rotation

at $r = 120$ mm

$$\sigma_t = \rho \omega^2 \left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu} r^2\right)$$

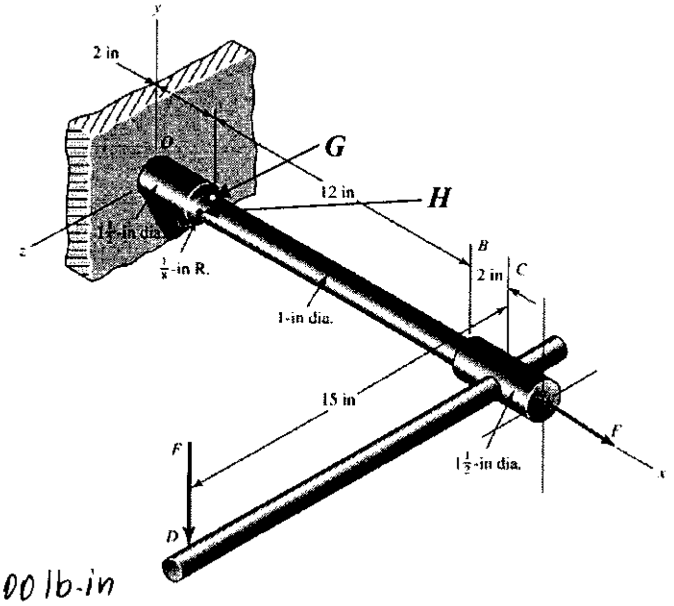
$$= 7850(300^2)\left(\frac{3+0.32}{8}\right)\left(0.025^2 + 0.15^2 + \frac{0.025^2 * 0.15^2}{0.12^2} - \frac{1+3(0.32)}{3+0.32}(0.12^2)\right)$$

$$\Rightarrow \boxed{\sigma_t = 4.57 \text{ MPa}}$$

$$\sigma_r = \rho \omega^2 \left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

$$= 7850(300)^2\left(\frac{3+0.32}{8}\right)\left(0.025^2 + 0.15^2 - \frac{0.025^2 * 0.15^2}{0.12^2} - 0.12^2\right) \Rightarrow \boxed{\sigma_r = 2.27 \text{ MPa}}$$

10. Considering the stress concentration and knowing that the value of the load $\boldsymbol{F} = 100$ lb (<u>*note that the load is applied at two locations*</u>), determine:
   a) The state of stress at point ***G*** (on the top surface and facing the positive *y* direction) and <u>show it on a stress element</u>.
   b) The state of stress at point ***H*** (on the side surface and facing the positive *z* direction) and <u>show it on a stress element</u>.



at Points G & H: axial force $= F = 100\,lb$

shear force $V = F = 100\,lb$

Moment $M_z = -F(14\,in) = -1400\ lb\cdot in$

Torque $T_x = F(15\,in) = 1500\ lb\cdot in$

* axial stress $\sigma_o = F/A = 100/\frac{\pi}{4}(1)^2 = 127.3\ Psi$; From Fig A-15-7, $D/d = 1.5$, $r/d = 0.125$ $\Rightarrow K_t \simeq 1.76$

$\Rightarrow \sigma = 127.3(1.76) = \underline{224\ Psi}$

* shear stress due to torsion

$\tau = \frac{Tr}{J}$ (on the outer surface), $J = \frac{\pi}{32} d^4 = 0.098\ in^4$

$\tau_o = \frac{1500(0.5)}{0.098} = 7639\ Psi$

From Fig A-15-8, $D/d = 1.5$, $r/d = 0.125$ $\Rightarrow K_{ts} \simeq 1.39$

$\Rightarrow \tau = 7639(1.39) = \underline{10619\ Psi}$

* Bending stress due to moment (at Point G)

$I = \frac{\pi}{64} d^4 = 0.049\ in^4$

$\sigma = -\frac{My}{I} = -\frac{(-1400)(0.5)}{0.049} = 14260\ Psi$

From Fig A-15-9, $D/d = 1.5$, $r/d = 0.125$ $\Rightarrow K_t \simeq 1.57$

$\Rightarrow \sigma = 14260(1.57) = \underline{22389\ Psi}$

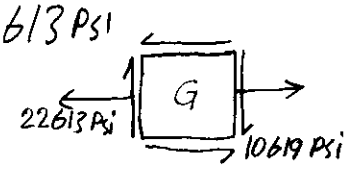
* Shear stress due to shear force (at point H)

$\tau_{max} = \frac{4V}{3A} = \frac{4(100)}{3(\frac{\pi}{4} 1^2)} = \underline{169.8\ Psi}$

<u>Point G</u>

$\sigma = 224 + 22389 = 22613\ Psi$

$\tau = 10619\ Psi$



<u>Point H</u> $\sigma = 224\ Psi$

$\tau = 10619 + 169.8$

$\Rightarrow \tau = 10789\ Psi$