<u>CH 6: Fatigue Failure Resulting from Variable Loading</u>

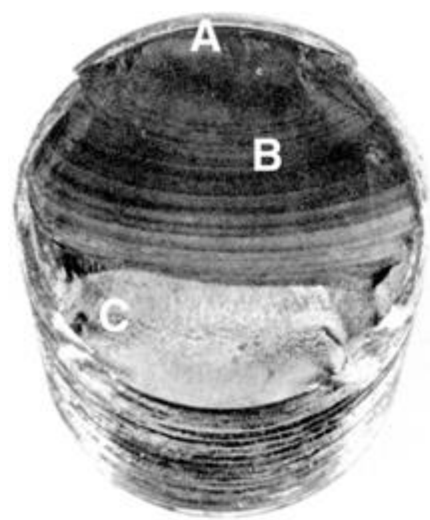
Some machine elements are subjected to static loads and for such elements static failure theories are used to predict failure (yielding or fracture). However, most machine elements are subjected to <u>varying or fluctuating stresses</u> (*due to the movement*) such as shafts, gears, bearings, cams & followers,etc.

Fluctuating stresses (repeated over long period of time) will cause a part to <u>fail</u> (*fracture*) at a stress level <u>much smaller than the ultimate strength</u> (or even the yield strength in some casses).

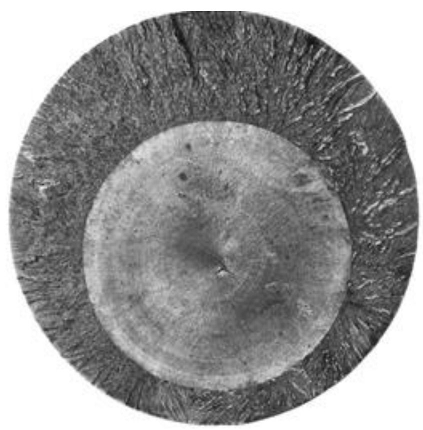
Unlike static loading where failure usualy can be detected before it happens (due to the large deflections associated with plastic deformation), fatigue failures are usualy <u>sudden</u> and therefore dangerous.

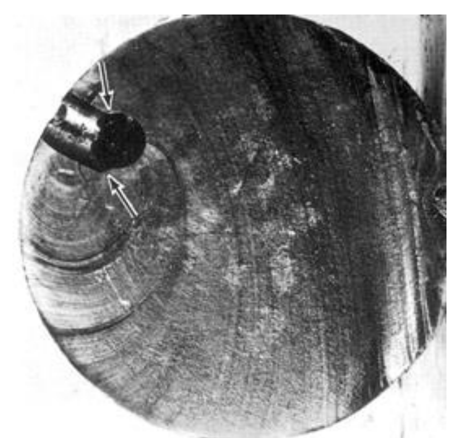
Fatigue failure is somehow similar to brittle fracture where the fracture surfaces are <u>prependicular</u> to the load axis.

- According to Linear-Elastic Fracture Mechanics (LEFM), fatigue failure develops in <u>three stages</u>:
  - *<u>Stage1</u>*: development of one or more <u>micro cracks</u> (the size of two to five grains) due to the cyclic local plastic deformation.
  - *<u>Stage2</u>*: the cracks progress from micro cracks to larger cracks (macro cracks) and <u>keep growing</u> making a smooth plateau-like fracture surfaces with <u>beach marks</u>.
  - *<u>Stage3</u>*: occurs during the <u>final stress cycle</u> where the remaining material cannot support the load, thus resulting in a <u>sudden fracture</u> (*can be brittle or ductile fracture*).



- Fatigue failure is due to crack formation and propagation. Fatigue cracks usually <u>initiate at locations with high stresses</u> such as discontinuities (*hole, notch, scratch, sharp corner, crack, inclusions, etc.*).





- Fatigue cracks can also initiate at surfaces having rough surface finish or due to the presence of tensile residual stresses. Thus all parts subjected to fatigue loading are heat treated and polished in order to increase the fatigue life.

Fatigue Life Methods

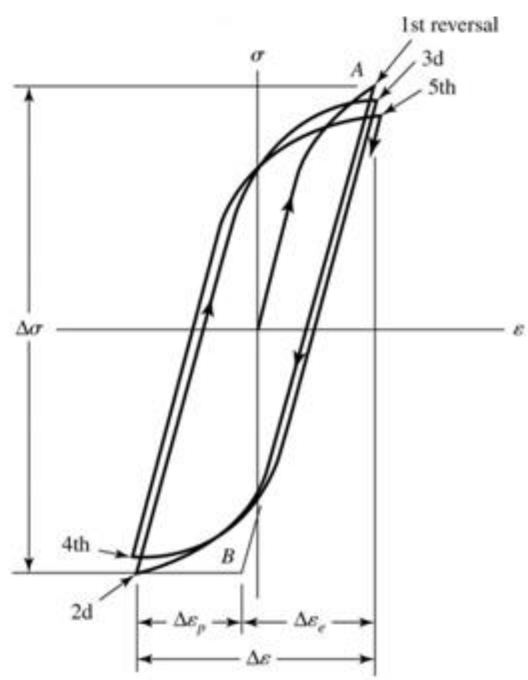
Fatigue failure is a much more complicated phenomenon than static failure where much complicating factors are involved. Also, testing materials for fatigue properties is more complicated and much more time consuming than static testing.

Fatigue life methods are aimed to determine the life (*number of loading cycles*) of an element until failure.

- There are three major fatigue life methods where each is more accurate for some types of loading or for some materials. The three methods are: the stress-life method, the strain-life method, the linear-elastic fracture mechanics method.

- The fatigue life is usually classified according to the number of loading cycles into:
  - *Low cycle fatigue* ($1 \leq N \leq 1000$) and for this low number of cycles, designers sometimes ignore fatigue effects and just use static failure analysis.
  - *High cycle fatigue* ($N>10^3$):
    - *Finite life*: from $10^3 \rightarrow 10^6$ cycles
    - *Infinite life*: more than 10^6 cycles

The Strain-Life Method



This method relates the fatigue life to the amount of plastic strain suffered by the part during the repeated loading cycles.

- When the stress in the material exceeds the yield strength and the material is plastically deformed, the material will be strain hardened and the yield strength will increase if the part is reloaded again. However, if the stress direction is reversed (from tension to compression), the yield strength in the reversed direction will be smaller than its initial value which means that the material has been softened in the reverse loading direction (*this is referred to as Bauschinger Effect*). Each time the stress is reversed, the yield strength in the other direction is decreased and the material gets softer and undergoes more plastic deformation until fracture occurs.

- The strain-life method is applicable to Low-cycle fatigue.

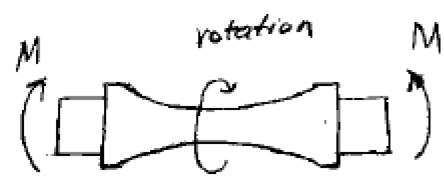
The Linear Elastic Fracture Mechanics Method

This method assumes that a crack initiates in the material and it keeps growing until failure occurs (*the three stages described above*).

- The LEFM approach assumes that a small crack already exists in the material, and it calculates the number of loading cycles required for the crack to grow to be large enough to cause the remaining material to fracture completely.
- This method is more applicable to High-cycle fatigue.

The Stress-Life Method

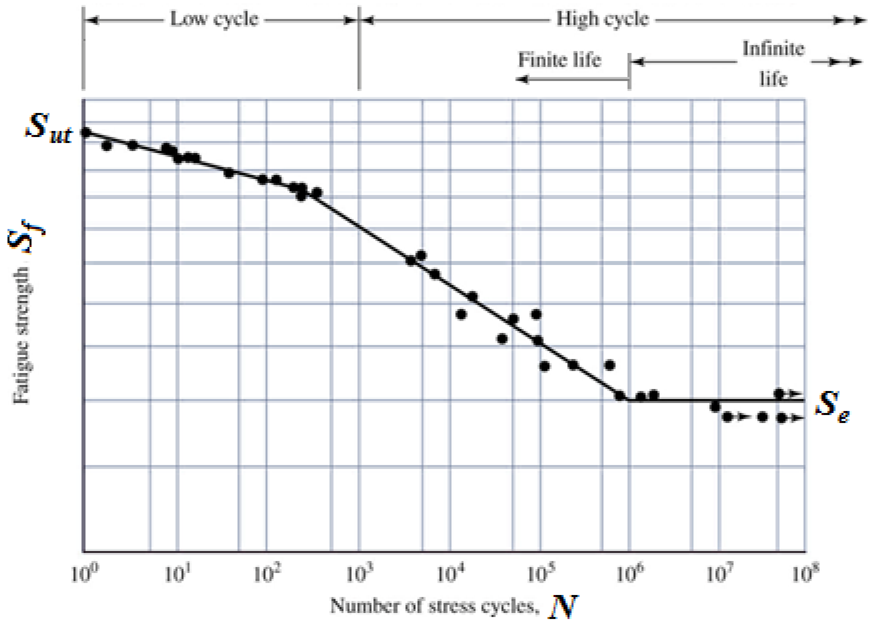
This method relates the fatigue life to the alternating stress level causing failure but it does not give any explanation to why fatigue failure happens.



- The stress-life relation is obtained experimentally using *Moore* high-speed rotating beam test.
  - The test is conducted by subjecting the rotating beam to a pure bending moment (of a fixed known magnitude) until failure occurs. (*Due to rotation, the specimen is subjected to an alternating bending stress*)
  - The data obtained from the tests is used to generate the fatigue strength *vs.* fatigue life diagram which is known as the *S-N diagram*.
  - The first point is the ultimate strength which corresponds to failure in half a cycle.
  - The alternating stress amplitude is reduced below the ultimate strength and the test is run until failure. The stress level and the number of cycles until failure give a data point on the chart.
  - The testing continues and each time the stress amplitude is reduced (*such that the specimen will live longer*) and new point is obtained.

- For steel alloys the low-cycle fatigue and the high-cycle fatigue (*finite and infinite*) can be recognized as having different slopes. (*they are straight lines but keep in mind it is a log-log curve*)
- For steels if we keep reducing the stress amplitude (for each test) we will reach to a stress level for which the specimen will never fail, and this value of stress is known as the *Endurance Limit* (S_e).

Typical S-N Diagram for Steel (log-log scale)



- The number of stress cycles associated with the Endurance Limit defines the boundary between *Finite-life* and *Infinite-life*, and it is usually between 10^6 to 10^7 cycles.

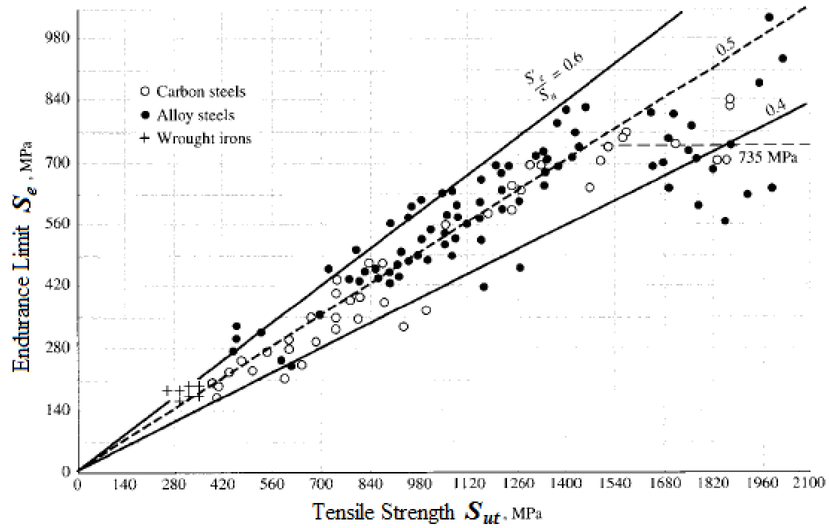
- Steel and Titanium alloys have a clear endurance limit, but this is not true for all materials.
  - For instance, Aluminum alloys do not have an endurance limit and for such materials the fatigue strength is reported at $5(10^8)$ cycles.
  - Also, most polymers do not have an endurance limit.

The Endurance Limit

The determination of the endurance limit is important for designing machine elements that are subjected to High-cycle fatigue. The common practice when designing such elements is to make sure that the fatigue stress level in the element is below the endurance limit of the material being used.

- Finding the Endurance Limit using the rotating beam experiment is time consuming where it requires testing many samples and the time for each test is relatively long. Therefore they try to relate the endurance limit to other mechanical properties which are easier to find (*such as the ultimate tensile strength*).

- The figure shows a plot of the *<u>Endurance Limits</u>* <u>versus</u> *<u>Tensile Strengths</u>* for a large number of <u>steel and iron</u> specimens.
  - The graph shows a <u>correlation between the ultimate strength and endurance limit</u> for ultimate strengths <u>up to</u> *1400* MPa then the endurance limit seems to have a <u>constant value</u>.



  - The relationship between the endurance limit and ultimate strength for <u>steels</u> is given as:

$$\boxed{S_e{}' = \begin{cases} 0.5\, S_{ut} & for \quad S_{ut} \le 1400\ MPa \\ 700\ MPa & for \quad S_{ut} > \ 1400\ MPa \end{cases}}$$

  - The *prime* (‘) is used to denote that this is the endurance limit value obtained for the test specimen (*modifications are still needed*).

<u>Endurance Limit Modifications Factors</u>

Endurance limit is obtained from the rotating beam test. The test is conducted under closely <u>controlled conditions</u> (*polished specimen of small size at a constant known temperature, etc.*). It is not realistic to expect a <u>machine element</u> to have the exact same endurance limit value as that obtained from the rotating beam test because it has <u>different conditions</u> (*size, surface finish, manufacturing process, environment, etc.*)

- Thus some <u>modification factors</u> are used to correlate the endurance limit for a given mechanical element to the value obtained from tests:

Size *Temp.* *Misc.*

$$S_e = k_a k_b k_c k_d k_e k_f S_e{}'$$

Finish *Load* *Reliability*

Where,

S_e: The endurance limit at the critical location of a machine element with the geometry and conditions of use.

S_e': The endurance limit obtained from the rotating beam test.

k_a … k_f: Modification factors (*obtained experimentally*).

Surface Condition Factor (k_a)

The rotating-beam test specimens are highly polished. A rough surface finish will reduce the endurance limit because there will be a higher potential for crack initiation.

- The surface condition modification factor depends on the surface finish of the part (*ground, machined, as forged, etc.*) and on the tensile strength of the material. It is given as:

$$\boxed{k_a = a\, S_{ut}{}^{b}}$$

❖ Constants a & b depend on surface condition and are given in *Table 6-2*.

Size Factor (k_b)

The rotating-beam specimens have a specific (*small*) diameter (*7.6mm*). Parts of larger size are more likely to contain flaws and to have more non-homogeneities.

- The size factor is given as:

For bending and torsion

$$\boxed{k_b = \begin{cases} 1.24\, d^{-0.107} & 2.79 \le d \le 51\ mm \\ 1.51\, d^{-0.157} & 51 < d \le 254\ mm \end{cases}}$$

where d is the diameter,

and:

$$\boxed{k_b = 1} \qquad \textit{for axial loading}$$

▪ When a member with circular cross-section is *not rotating*, we use an *effective diameter* value instead of the actual diameter, where:

$$\boxed{d_e = 0.37\, d}$$

❖ For *other cross-sections*, d_e is found using *Table 6-3* (*obtain $A_{0.95\sigma}$ from table then solve eqn. 6-23 for the "equivalent diameter" d and finally find d_e using the equation above "eqn. 6-24"*).

Loading Factor (k_c)

The rotating-beam specimen is loaded in bending. Other types of loading will have a different effect.

- The load factor for the different types of loading is:

$$\boxed{k_c = \begin{cases} 1 & bending \\ 0.85 & axial \\ 0.59 & torsion \end{cases}}$$

Temperature Factor (k_d)

When the operating temperature is below room temperature, the material becomes more brittle. When the temperature is high the yield strength decreases and the material becomes more ductile (and creep may occur).

- For steels, the tensile strength , and thus the endurance limit, slightly increases as temperature rises, then it starts to drop. Thus, the temperature factor is given as:

$$k_d = 0.9877 + 0.6507(10^{-3})T_c - 0.3414(10^{-5})T_c^2 + 0.5621(10^{-8})T_c^3 - 0.6246(10^{-11})T_c^4$$

For $40 \le T_c \le 540\ °C$

❖ The same values calculated by the equation are also given in *Table 6-4* where: $k_d = \left(\frac{S_T}{S_{RT}}\right)$

Reliability Factor (k_e)

The endurance limit obtained from testing is usually reported at mean value (*it has a normal distribution with* $\hat{\sigma} = 8\%$).

❖ For other values of reliability, k_e is found from *Table 6-5*.

Miscellaneous-Effects Factor (k_f)

It is used to account for the reduction of endurance limit due to all other effects (*such as residual stress, corrosion, cyclic frequency, metal spraying, etc.*).

However, those effects are not fully characterized and usually not accounted for. Thus we use ($k_f = 1$).

Stress Concentration and Notch Sensitivity

Under fatigue loading conditions, crack initiation and growth usually starts in locations having high stress concentrations (such as grooves, holes, etc.). The presence of stress concentration reduces the fatigue life of an element (*and the endurance limit*) and it must be considered in fatigue failure analysis.

However, due to the difference in ductility, the effect of stress concentration on fatigue properties is not the same for different materials.

- For materials under fatigue loading, the maximum stress near a notch (hole, fillet, etc.) is:

$$\boxed{\sigma_{max} = K_f\ \sigma_o} \qquad \underline{or} \qquad \boxed{\tau_{max} = K_{fs}\ \tau_o}$$

Where,

σ_o : is the nominal stress

K_f : is the *fatigue stress concentration factor* which is a reduced value of the stress concentration factor (K_t) because of the difference in material sensitivity to the presence of notches.

and K_f is defined as:

$$K_f = \frac{max.\, stress\ in\ notched\ specimen}{stress\ in\ notch-free\ specimen}$$

- *Notch sensitivity* (q) is defined as:

$$q = \frac{K_f - 1}{K_t - 1} \qquad \underline{or} \qquad q_{shear} = \frac{K_{fs} - 1}{K_{ts} - 1}$$

The value of q ranges from *0* to *1*

$q = 0$ ➔ $K_f = 1$ (*material is not sensitive*)

$q = 1$ ➔ $K_f = K_t$ (*material is fully sensitive*)

- Thus,

$$\boxed{K_f = 1 + q(K_t - 1)} \qquad \underline{or} \qquad \boxed{K_{fs} = 1 + q_{shear}(K_{ts} - 1)}$$

❖ For Steels and Aluminum (*2024*) the *notch sensitivity* for *Bending and Axial* loading can be found from *Figure 6-20* and for *Torsion* is found from *Figure 6-21*.

- Alternatively, instead of using the figures, the fatigue stress concentration factor K_f, can be found as:

$$\boxed{K_f = 1 + \frac{K_t - 1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}}$$ *The Neuber equation*

where, r : radius

$\sqrt{a}$: is a material constant known as the Neuber constant.

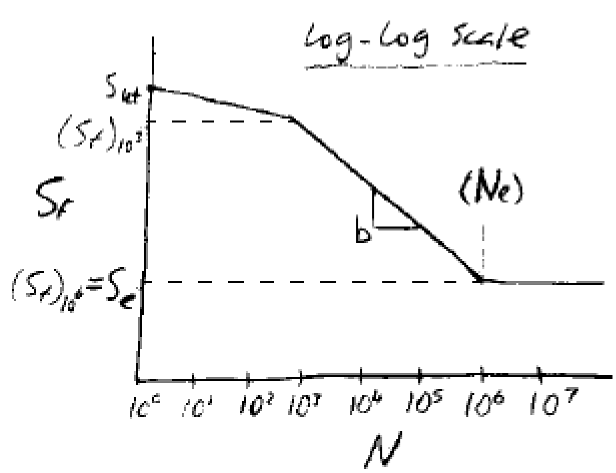
❖ For steels, $\sqrt{a}$ can be found using *Eqns. 6-35a & 6-35b* given in the text (*note that S_{ut} needs to be in "$kpsi$" and $\sqrt{a}$ will be given in "$\sqrt{in}$"*)

- For cast iron, the notch sensitivity is very low from *0* to *0.2,* but to be conservative it is recommended to use $q = 0.2$

- For simple loading, K_f can be multiplied by the stress value, or the endurance limit can be reduced by dividing it by K_f. However, for combined loading each type of stress has to be multiplied by its corresponding K_f value.

Fatigue Strength

In some design applications the number of load cycles the element is subjected to, is limited (*less than 10^6*) and therefore there is no need to design for infinite life using the endurance limit.



- In such cases we need to find the *Fatigue Strength* associated with the desired life.

- For the High-cycle fatigue (*$10^3 \rightarrow 10^6$*), the line equation is $S_f = aN^b$ where the constants "a" (*y intercept*) and "b" (*slope*) are determined from the end points $(S_f)_{10^3}$ and $(S_f)_{10^6}$ as:

$$a = \frac{(S_f)^2_{10^3}}{S_e} \qquad and \qquad b = -\frac{log(\sigma'_f/S_e)}{log(2N_e)}$$

S_e is the modified Endurance Limit

Where σ'_f is the *True Stress at Fracture* and for steels with $H_B \leq 500$, it is approximated as:

$$\sigma'_f = S_{ut} + 345\ Mpa$$

- $(S_f)_{10^3}$ can be related to S_{ut} as:

fraction of

$$(S_f)_{10^3} = fS_{ut}$$

where f is found as:

$$f = \frac{\sigma'_f}{S_{ut}}(2 \times 10^3)^b$$

❖ Using the above equations, the value of f is found as a function of S_{ut}(*using* $N_e = 10^6$) and it is presented in graphical form in *Figure 6-18*.

For S_{ut} values less than 490 MPa, use $f = 0.9$ to be conservative

- If the value of (f) is known, the constant b can be directly found as:

$$\boxed{b = -\frac{1}{3}log\left(\frac{fS_{ut}}{S_e}\right)}$$

and a can be rewritten as:

$$\boxed{a = \frac{(fS_{ut})^2}{S_e}}$$

- Thus for $10^3 \leq N \leq 10^6$, the fatigue strength associated with a given life (N) is:

$$\boxed{(S_f)_N = aN^b}$$

and the fatigue life (N) at a given fatigue stress (σ) is found as:

$$\boxed{N = \left(\frac{\sigma}{a}\right)^{\frac{1}{b}}}$$

- Studies show that for ductile materials, the Fatigue Stress Concentration Factor (K_f) reduces for $N < 10^6$, however the conservative approach is to use K_f as is.

Example: For a <u>rotating-beam specimen</u> made of *1045 CD* steel, find:

a) The endurance limit ($N_e = 10^6$)
b) The fatigue strength corresponding to (5×10^4) cycles to failure
c) The expected life under a completely reversed stress of $400\ MPa$

<u>Solution</u>:

From *<u>Table A-20</u>* $S_{ut} = 630\ MPa$

a) $S_e{}' = 0.5(S_{ut}) = 315\ MPa$

Note that no modifications are needed since it is a specimen: $S_e = S_e{}'$

b) $\sigma_f' = S_{ut} + 345 = 975\ MPa$

$$b = -\frac{log(\sigma_f'/S_e)}{log(2N_e)} = -\frac{log(975/315)}{log(2 \times 10^6)} = -0.0779$$

$$f = \frac{\sigma_f'}{S_{ut}}(2 \times 10^3)^b = \frac{975}{630}(2 \times 10^3)^{-0.0779} = 0.856$$

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{(0.856 \times 630)^2}{315} = 923.4\ MPa$$

<u>OR</u>, easier, from Figure 6-18: $f \cong 0.857$

Then,

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{(0.857 \times 630)^2}{315} = 925.4\ MPa$$

$$b = -\frac{1}{3}log\left(\frac{fS_{ut}}{S_e}\right) = -\frac{1}{3}log\left(\frac{0.857 \times 630}{315}\right) = -0.078$$

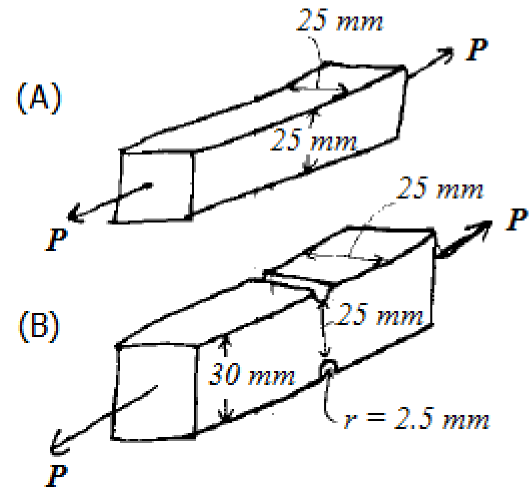
$$(S_f)_N = aN^b \quad \Rightarrow \quad (S_f)_{5\times10^4} = 923.4(5 \times 10^4)^{-0.0779}$$

$$\Rightarrow \quad (S_f)_{5\times10^4} = \boxed{397.5\ MPa}$$

c)

$$N = \left(\frac{\sigma}{a}\right)^{\frac{1}{b}} = \left(\frac{400}{923.4}\right)^{\frac{1}{-0.0779}} = \boxed{46.14 \times 10^3 cycles}$$

Example: The two axially loaded bars shown are made of *1050 HR* steel and have machined surfaces. The two bars are subjected to a <u>completely reversed</u> load P.



a) Estimate the maximum value of the load P for each of the two bars such that they will have infinite life (*ignore buckling*).
b) Find the static and fatigue factors of safety n_s & n_f for bar (***B***) if it is to be subjected to a completely reversed load of $P = 50\ kN$.
c) Estimate the fatigue life of bar (***B***) under reversed load of $P = 150\ kN$ (*use* $f = 0.9$)

<u>*Solution*</u>:

From <u>*Table A-20*</u> $S_{ut} = 620\ MPa$ & $S_y = 340\ MPa$

a) $S_e{}' = 0.5(S_{ut}) = 310\ MPa$

<u>*Modifying factors*</u>:

- *Surface factor*: $k_a = a\, S_{ut}{}^b$, from <u>*Table 6-2*</u>: $a = 4.51, b = -0.265$
  ➔ $k_a = 4.51(620)^{-0.265} = 0.821$
- *Size factor*: $k_b = 1$ *since the loading is axial*
- *Loading factor*: $k_c = 0.85$ (*for axial loading*)
- *Other factors*: $k_d = k_e = k_f = 1$

<u>*Stress concentration*</u> (*for bar **B***):

From <u>*Figure A-15-3*</u> with $w/d = 1.2$ & $r/d = 0.1$ ➔ $K_t \approx 2.38$

The fatigue stress concentration factor: $K_f = 1 + q(K_t - 1)$

From <u>*Figure 6-20*</u> for steel: $q \approx 0.81$

➔ $K_f = 1 + 0.81(2.38 - 1) = 2.12$

Thus the maximum stress for each should not exceed,

<u>*Bar (**A**)*</u>: $S_e = k_a k_c S_e' = (0.821)(0.85)(310) = 216.3\ MPa$

<u>*Bar (**B**)*</u>: $(S_e)_{mod} = \frac{S_e}{K_f} = \frac{216.3}{2.12} = 102.03\ MPa$

And the maximum load P for each is,

<u>*Bar (**A**)*</u>: $P_{max} = 216.3 \times (25 \times 25) = 135187.5\ N$

<u>*Bar (**B**)*</u>: $P_{max} = 102.03\ \times (25 \times 25) = 63767.7\ N$

- *Note that the maximum load for bar (**B**) is <u>smaller</u> than that of bar (**A**) because of the notch.*

b) Static factor of safety n_s:

From <u>*Table A-20*</u>: $\epsilon_f = 0.15$ ➔ *Ductile material, thus stress concentration is not applicable*.

$$\sigma_o = \frac{P}{A_{net}} = \frac{50 \times 10^3}{25 \times 25} = 80\ MPa$$

$$➔ \quad n_s = \frac{S_y}{\sigma_o} = \frac{340}{80} = \boxed{4.25}$$

Fatigue factor of safety n_f:

$$n_f = \frac{(S_e)_{mod}}{\sigma_o} \qquad or \qquad n_f = \frac{S_e}{(K_f \sigma_o)} = \frac{216.3}{(2.12)(80)} = \boxed{1.28}$$

c) If we calculate the fatigue factor of safety with $P = 150\ kN$ we will find it to be <u>less than one</u> and thus the bar will not have infinite life.

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{(0.9 \times 620)^2}{216.3} = 1439.5\ MPa$$

$$b = -\frac{1}{3} log\left(\frac{f S_{ut}}{S_e}\right) = -\frac{1}{3} log\left(\frac{0.9 \times 620}{216.3}\right) = -0.137$$

$$\sigma_o = \frac{P}{A_{net}} = \frac{150 \times 10^3}{25 \times 25} = 240\ MPa$$

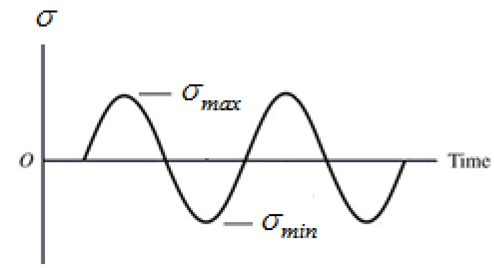
$$\sigma = K_f \sigma_o = 2.12 \times 240 = 508.8\ MPa$$

$$➔ \quad N = \left(\frac{\sigma}{a}\right)^{1/b} = \left(\frac{508.8}{1439.5}\right)^{1/-0.137} = \boxed{1.98 \times 10^3\ cycles}$$

- This gives more conservative results than dividing (S_e) by K_f, and using σ_o as is.

<u>Characterizing Fluctuating Stress</u>

In the rotating-beam test, the specimen is subjected to <u>completely reversed stress</u> cycles ($\sigma_{max} = |\sigma_{min}|$)



In the case of the <u>rotating shaft</u> subjected to <u>both radial and axial loads</u> (*such as with helical gears*) the fluctuating stress pattern will be different since there will be a component of stress that is always present (*due to the axial load*).

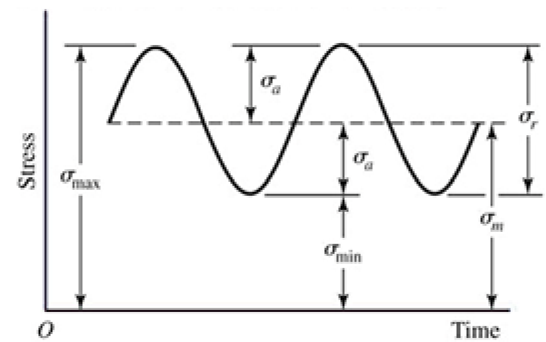
- The following stress components can be defined for distinguishing different states of fluctuating stress:

  σ_m: Mean or average stress, $\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2}$

  σ_r: Stress range, $\sigma_r = |\sigma_{max} - \sigma_{min}|$

  σ_a: Stress amplitude (*half of the stress range*),

  $$\sigma_a = \left|\frac{\sigma_{max}-\sigma_{min}}{2}\right|$$

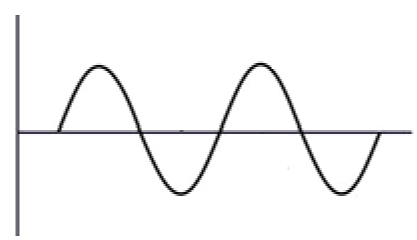


- For uniform periodic fluctuating stress, σ_m & σ_a are used to characterize the stress pattern.

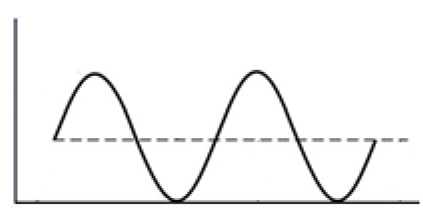
- We also define:
  - Stress ratio: $R = \sigma_{min}/\sigma_{max}$
  - Amplitude ratio: $A = \sigma_a/\sigma_m$

- Some common types of fluctuating stress:

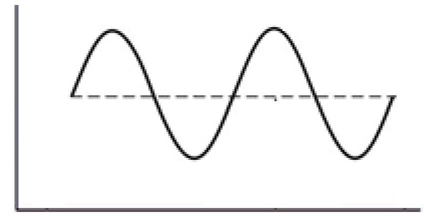
Completely reversed stress:
$\sigma_m = 0$
$\sigma_a = \sigma_{max} = |\sigma_{min}|$



Repeated stress:
Tension $\sigma_a = \sigma_m = \sigma_{max}/2$
Compression $\sigma_m = \sigma_{min}/2$



General fluctuating stress:
(non-zero mean)
$\sigma_a \neq \sigma_m \neq 0$

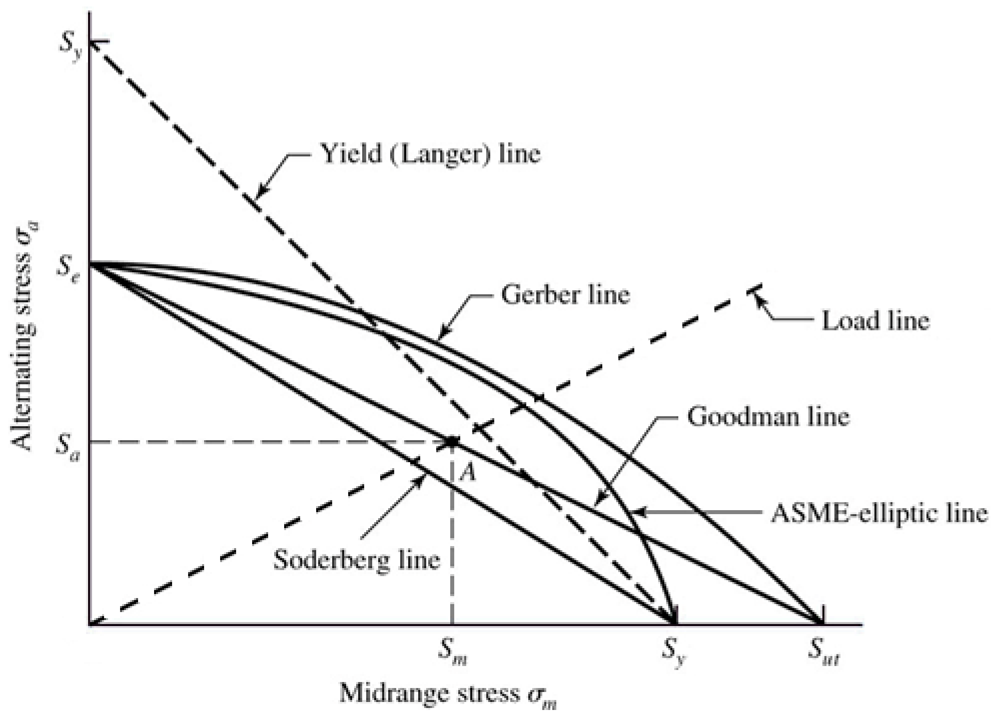


Fatigue Failure Criteria for Fluctuating Stress

When a machine element is subjected to completely reversed stress (*zero mean*, $\sigma_m = 0$) the endurance limit is obtained from the rotating-beam test (*after applying the necessary modifying factors*).

However, when the mean (*or midrange*) is non-zero the situation is different and a fatigue failure criteria is needed.

- If we plot the alternating stress component (σ_a) *vs.* the mean stress component (σ_m), this will help in distinguishing the different fluctuating stress scenarios.



  - When $\sigma_m = 0$ & $\sigma_a \neq 0$, this will be a completely reversed fluctuating stress.
  - When $\sigma_a = 0$ & $\sigma_m \neq 0$, this will be a static stress.
  - Any combination of σ_m & σ_a will fall between the two extremes (*completely reversed & static*).

- Different theories are proposed to predict failure in such cases:

  Yield (Langer) line: It connects S_y on the σ_a axis with S_y on σ_m axis. *But it is not realistic because S_y is usually larger than S_e.*

  Soderberg line: The most conservative, it connects S_e on σ_a axis with S_y on σ_m axis.

$$\boxed{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}}$$

Where (n) is the design factor

  ASME-elliptic line: Same as *Soderberg* but it uses an ellipse instead of the straight line.

$$\boxed{\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1}$$

It fits experimental data better (see fig 6-25)

  Goodman line: It considers failure due to static loading to be at S_{ut} rather than S_y, thus it connects S_e on σ_a axis with S_{ut} on σ_m axis using a straight line.

$$\boxed{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}}$$

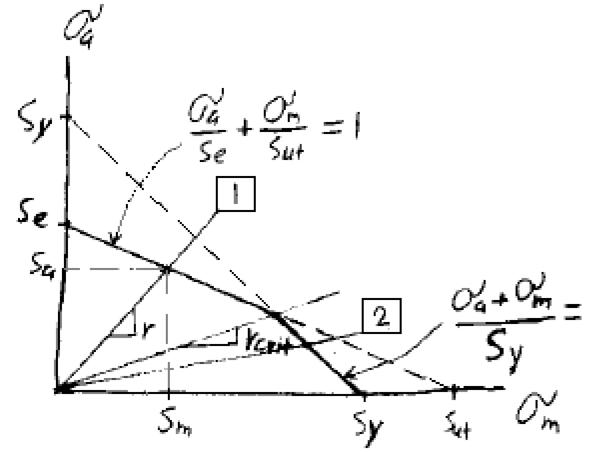
  Gerber line: Same as *Goodman* but it uses a parabola instead of the straight line.

$$\boxed{\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1}$$

  - *The factor of safety is found as:* $$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

- It should be noted that S_e is the modified endurance limit.
- The fatigue stress concentration factor (K_f) should be multiplied with both σ_a & σ_m for conservative results.
- The load line represents any combination of σ_a and σ_m , the intersection of the load line with any of the failure lines gives the limiting values S_a and S_m according to the line it intercepts.

Modified Goodman (Goodman and Langer)



It combines the *Goodman* and *Langer* lines.

- The slope of the loading line passing through the intersection point of the two lines is called the critical slope and it is found as:

  $$r_{crit} = S_a / S_m$$

  $$where \quad S_m = \frac{(S_y - S_e)S_{ut}}{S_{ut} - S_e} \quad \& \quad S_a = S_y - S_m$$

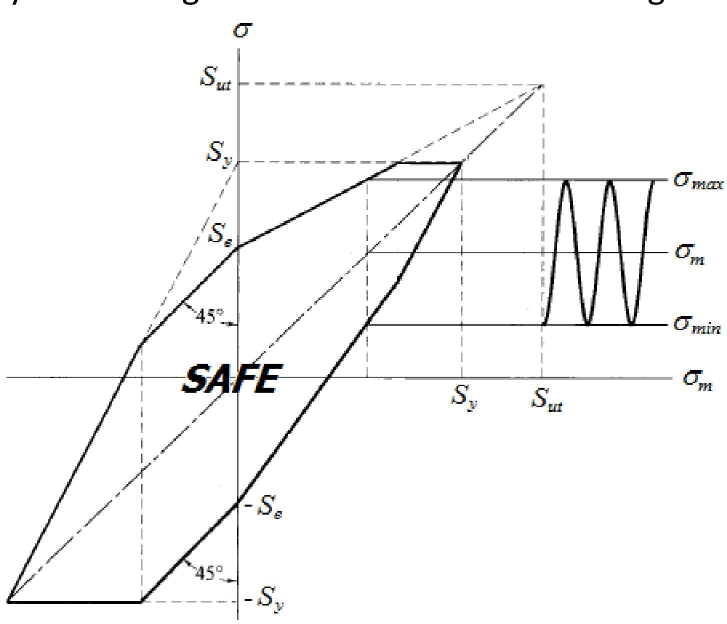
- According to the slope of the load line ($r = \sigma_a/\sigma_m$), it could intersect any of the two lines:

$$r > r_{crit} \quad \Rightarrow \quad \boxed{1} \; S_a = \frac{rS_eS_{ut}}{rS_{ut} + S_e} \quad \& \quad S_m = \frac{S_a}{r}, \quad n_f = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m} = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

$$r < r_{crit} \quad \Rightarrow \quad \boxed{2} \; S_a = \frac{rS_y}{1 + r} \quad \& \quad S_m = \frac{S_y}{1 + r}, \quad n_s = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m} = \frac{S_y}{\sigma_a + \sigma_m}$$

  - Where case $\boxed{2}$ is considered to be a static *yielding* failure.

- If we plot the *Modified Goodman* on stress (σ) *vs.* mean stress (σ_m) axes we obtain the complete *Modified Goodman* diagram where it defines a failure envelope such that any alternating stress that falls inside the diagram will not cause failure.



The "Complete" Modified Goodman Diagram

- Also, there are other modified criteria:
  - *Gerber-Langer* (*see Table 6-7*)
  - *ASME-elliptic-Langer* (*see Table 6-8*)

Torsional Fatigue Loading

For shafts that are subjected to fluctuating shear stress with non-zero mean (*due to pulsating torque*), a fatigue criterion (*ASME elliptic, Gerber, etc.*) needs to be used.

- It should be noted that the endurance limit S_e already accounts for the torsional loading since $k_c = 0.59$ is used in such case.
- Similarly, the yield or ultimate strengths need to be corrected where the "shear yield strength" S_{ys} or the "shear ultimate strength" S_{us} need to be used and those are found as:

$$\boxed{S_{ys} = 0.577 S_y} \qquad \mathit{and} \qquad \boxed{S_{us} = 0.67 S_{ut}}$$

Combination of Loading Modes

The procedures presented earlier can be used for fatigue calculations for a component subjected to general fluctuating stress (*or fully reversed stress, easier*) under one of the three modes of loading; Axial, Bending or Torsion.

- For a component subjected to general fluctuating stress under combination of loading modes:
  - The stress corresponding to each mode of loading is split into its alternating (σ_a) and midrange (σ_m) components.
  - The fatigue stress concentration factor corresponding to each mode of loading is applied to the (σ_a & σ_m) of that mode.
  - An equivalent *von Mises* stress is calculated for the alternating and midrange components as:

$$\boxed{\sigma_a' = \sqrt{\left[(K_f)_{bending}(\sigma_a)_{bending} + (K_f)_{axial}\frac{(\sigma_a)_{axial}}{0.85}\right]^2 + 3\left[(K_{fs})_{torsion}(\tau_a)_{torsion}\right]^2}}$$

$$\boxed{\sigma'_m = \sqrt{\left[\left(K_f\right)_{bending}(\sigma_m)_{bending} + \left(K_f\right)_{axial}(\sigma_m)_{axial}\right]^2 + 3\left[\left(K_{fs}\right)_{torsion}(\tau_m)_{torsion}\right]^2}}$$

- ✓ Note that alternating component of the axial load is divided by 0.85 (i.e., k_c for axial loading).
- ✓ The torsional alternating stress is not divided by its corresponding k_c value (i.e., 0.59) since that effect is already accounted for in the *von Mises* stress.

- The endurance limit is calculated assuming the loading is bending (i.e., $k_c = 1$).
- Finally, a fatigue failure criterion (Gerber, Goodman, ASME-elliptic, etc.) is selected and applied as usual.

A road map summarizing all the important equations for the stress-life method is given in Sec. 16-7 page 338.

Example: A $40\ mm$ diameter bar has been machined from *AISI-1045 CD* bar. The bar will be subjected to a fluctuating tensile load varying from 0 to $100\ kN$. Because of the ends fillet radius, $K_f = 1.85$ is to be used.

Find the critical mean and alternating stress values S_a & S_m and the fatigue factor of safety n_f according to the *Modified Goodman* fatigue criterion.

Solution:

From *Table A-20* $\quad S_{ut} = 630\ MPa \quad \& \quad S_y = 530\ MPa$

$$S_e{}' = 0.5(S_{ut}) = 315\ MPa$$

Modifying factors:

- *Surface factor*: $\quad k_a = 4.51(630)^{-0.265} = 0.817 \quad$ (*Table 6-2*)
- *Size factor*: $\quad k_b = 1 \quad$ *since the loading is axial*
- *Loading factor*: $\quad k_c = 0.85 \quad$ (*for axial loading*)
- *Other factors*: $\quad k_d = k_e = k_f = 1$

$$\Rightarrow \quad S_e = k_a k_c S'_e = (0.817)(0.85)(315) = 218.8\ MPa$$

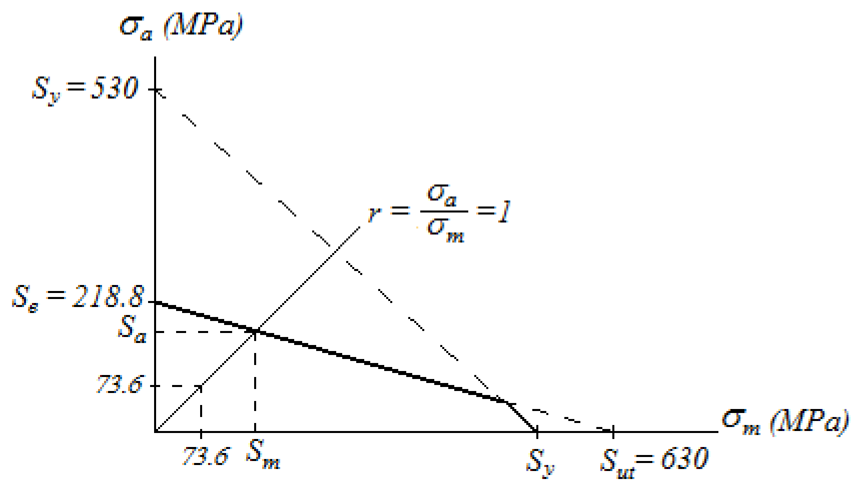
Fluctuating stress: $\quad \sigma = \frac{F}{A} \quad , \quad A = \frac{\pi}{4} d^2 = 1.257 \times 10^3\ mm^2$

$$\sigma_{max} = \frac{100 \times 10^3}{1.257 \times 10^3} = 79.6\ MPa \qquad \& \qquad \sigma_{min} = 0$$

$$\sigma_{m_o} = \frac{\sigma_{max} + \sigma_{min}}{2} = 39.8\ MPa \qquad \& \qquad \sigma_{a_o} = \frac{\sigma_{max} - \sigma_{min}}{2} = 39.8\ MPa$$

Applying K_f to <u>both components</u>: $\quad \sigma_m = K_f\, \sigma_{m_o} \quad \& \quad \sigma_a = K_f\, \sigma_{a_o}$

➔ $\sigma_m = \sigma_a = 1.85(39.8)$
$= 73.6\ MPa$



- The plot shows that the load line intersects the *<u>Goodman</u>* line:

$$S_a = \frac{rS_eS_{ut}}{rS_{ut} + S_e} = \frac{1(218.8)(620)}{1(620) + 218.8}$$

➔ $S_a = \boxed{162.4\ MPa} = S_m$

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} \qquad or \qquad n_f = \frac{S_a}{\sigma_a} = \frac{S_m}{\sigma_m}$$

➔ $n_f = \dfrac{162.4}{73.6} = \boxed{2.21}$

See ***Example 6-12*** *from text*