CH 17: Flexible Mechanical Elements

Flexible mechanical elements (belts, chains, ropes) are used in conveying systems and to transmit power over long distances (*instead of using shafts and gears*).

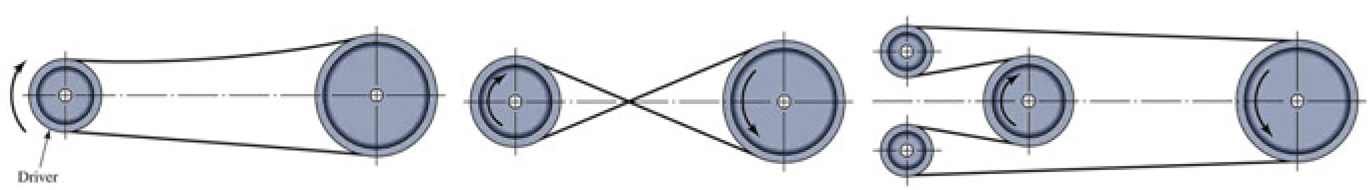
- The use of flexible elements simplifies the design and reduces cost.
- Also, since these elements are elastic and usually long, they play a role in absorbing shock loads and reducing vibrations.
- Disadvantage, they have shorter life than gears, shafts, etc.

Belts

- There are four basic types of belts (*Table 17-1*):
  - Flat belts ~ *crowned pulleys*.
  - Round belts ~ *grooved pulleys*.
  - V-belts ~ *grooved pulleys*.
  - Timing belts ~ *toothed pulleys*.

- Characteristics of belt drives:
  - Pulley axis must be separated by certain minimum distance. *Why?*
  - Can be used for long centers distance.
  - Except for timing belts, there is some slipping between belt and pulley, thus angular velocity ratio is not constant or equal to the ratio of pulley diameters.
  - A tension pulley can be used to maintain tension in the belt.

- There are two main configurations for belt drives; open and crossed (*Fig 17-1*) where the direction of rotation will be reversed for the crossed belt drive.
- The figure shows reversing and non-reversing belt drives, always there is one *loose side* depending on the driver pulley and the direction of rotation.



- ❖ *Fig. (17-3)* shows flat belt drive for out of-plane pulleys.
- ❖ *Fig. (17-4)* shows how clutching action can be obtained by shifting the belt from loose to a tight pulley.
- ❖ *Fig. (17-5)* shows two types of variable-speed belt drives.

Flat and Round Belt Drives

Flat belt drivers produce very little noise and they absorb more vibration from the system than V-belts.

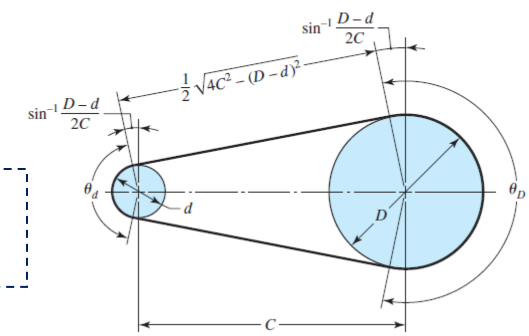
Also, flat belts drives have high efficiency of about *98 %* (*same as for gears*) compared to *70-96 %* for V-belts.

- For open belt drives, the contact angles are:

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

Larger contact-angle for the large pulley



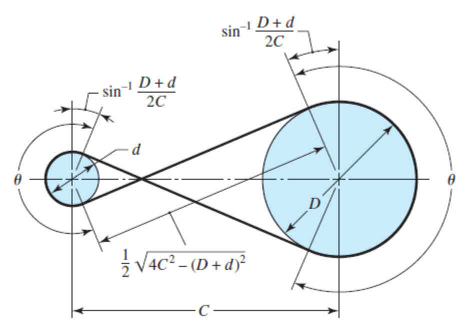
where: D : diameter of larger pulley
d : diameter of smaller pulley
C : centers distance

  - And the length of the belt is:

$$L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(D\theta_D + d\theta_d)$$

- For crossed belt drives, the contact angle is the same for both pulleys:

$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C}$$



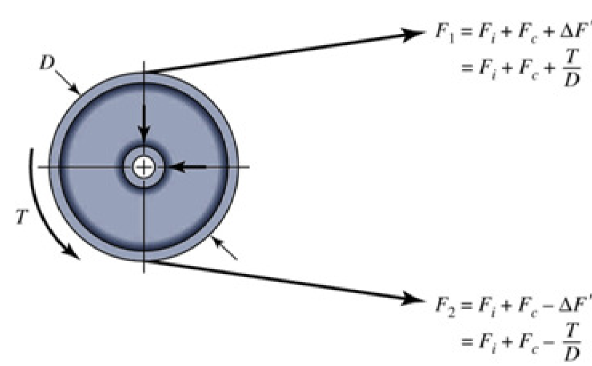
  - And the belt length is:

$$L = \sqrt{4C^2 - (D+d)^2} + \frac{1}{2}(D+d)\theta$$

- *Force Analysis*:

  - Tight side tension:

$$F_1 = F_i + F_c + \Delta F' = F_i + F_c + T/D$$



- Loose side tension:

$$F_2 = F_i + F_c - \Delta F' \\ = F_i + F_c - T/D$$

Note that "D" refers to the diameter of the driver pulley

where F_i : initial tension, F_c : hoop tension due to centrifugal force, and $\Delta F'$: tension due to transmitted torque.

- The total transmitted force is the difference between F_1 & F_2

$$F_1 - F_2 = \frac{2T}{D}$$

- The centrifugal tension F_C can be found as:

$$F_c = mr^2\omega^2$$

where ω is the angular velocity & m is the mass per unit length.

It also can be written as:

$$F_c = \frac{w}{g}V^2$$

where g = *9.81 m/s*^{2}, w : is weight per unit length, $V=\pi Dn$

- The initial tension can be expressed as:

$$F_i = \frac{F_1 + F_2}{2} - F_c \qquad (1)$$

- The belting equation relates the possible belt tension values with the coefficient of friction and it is defined as:

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{f\phi}$$

Note that ϕ is the <u>smallest</u> value of the contact angle

where *f*: coefficient of friction, ϕ: contact angle.

- Substituting in *eqn.(1)* we find the relation between F_i and T

$$F_i = \frac{T}{D}\frac{e^{f\phi}+1}{e^{f\phi}-1}$$

Minimum value of F_i needed to transmit a certain value of torque without slipping

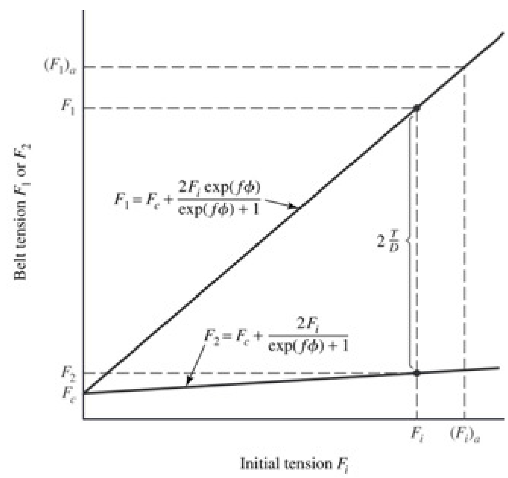
  - This equation shows that if F_i is zero; then T is zero (*i.e. there is no transmitted torque*).

- Substituting in F_1 & F_2 equations we get:

$$F_1 = F_c + F_i \frac{2\, e^{f\phi}}{e^{f\phi}+1}$$

$$F_2 = F_c + F_i \frac{2}{e^{f\phi}+1}$$

Used to find the F_1 & F_2 values when the belt is on the verge of slipping ***or*** to find F_1 & F_2 for small F_i values where slipping is occurring (*note that the kinetic coefficient of friction should be used in such case*)



- Plotting F_1 & F_2 vs. F_i we can see that the initial tension needs to be sufficient so that the difference between F_1 & F_2 curves is $2T/D$.

❖ *Table 17-2* gives the manufacturers specifications for the allowable tension for each type of belts.

- When a belt is selected, the tension in the tight side is set to be equal to the max allowable tension for that belt type.
  - However, severity of flexing at the pulley, and the belt speed affect the belt life, thus they need to be accounted for.

- Therefore the max allowable tension is found as:

$$(F_1)_a = bF_a C_P C_V$$

where:

F_a : allowable tension per unit width for a specific belt material (kN/m) (*Table 17-2*)

b : belt width (m)

C_P : pulley correction factor (*for the severity of flexing*), it is found from (*Table 17-4*) for the small pulley diameter.

Use C_P=1 for urethane belts

C_V : velocity correction factor. (*For velocities other than 3 m/s*), it is found from *Fig. 17-9* for leather belts.

For polyamide or urethane belts use C_V=1

- The transmitted horsepower can be found as:

$$H = (F_1 - F_2)V = Tn$$

- However, when designing a belt drive, a <u>design factor</u> n_d needs to be included to account for unquantifiable effects. Also another <u>correction factor</u> K_S is included to account for load deviations from the nominal value (*i.e., over loads*).
  - Thus the design horsepower is:

$$H_d = H_{nom} K_S n_d$$

- <u>Steps for analyzing flat belts include</u>:

1. Find ϕ for the <u>smallest</u> pulley from geometry (*find* $e^{f\phi}$ *if needed*).
2. From belt material and speed find F_C. $F_C = \frac{w}{g} V^2$
3. Find the transmitted torque.

$$T = H_d / n = (H_{nom} K_S n_d)/n$$

4. From torque T, find the transmitted load.

$$(F_1)_a - F_2 = 2T/D$$

5. From belt material, drive geometry & speed, find $(F_1)_a$.

$$(F_1)_a = bF_a C_P C_v$$

6. Find F_2

$$F_2 = (F_1)_a - ((F_1)_a - F_2)$$

Note that F_2 must be larger than zero

7. From $(F_1)_a$, F_2 & F_C find F_i.

$$F_i = \frac{(F_1)_a + F_2}{2} - F_C$$

8. Check if the friction of the belt material is sufficient to transmit the torque.

$$f > \grave{f}$$

where $$f' = \frac{1}{\phi}\ln\frac{(F_1)_a - F_C}{F_2 - F_C}$$

Minimum friction needed to transmit the load without slipping

Alternatively, the comparison can be made between the calculated F_i and the minimum required value of F_i

9. Find the factor of safety $n_{fs} = H_a / H_{nom} K_S$

*See **Example 17-1** from text*

V-Belts

The cross sectional dimensions of V-belts are standardized. Each letter designates a certain cross section (see *Table 17-9*).

- A V-belt can be specified by the cross section letter followed by the inside circumference length.
  - ❖ *Table 17-10* gives the standard lengths for V-belts.
  - ▪ However, calculations involving the belt length are usually based on pitch length for standard belts.
  - ❖ Table 17-11 gives the quantity to be added to the inside length.

    Example: Pitch length of *C-1500* belt is: *1500 + 72 = 1572 mm*.

- The standard angle for the V-belts cross section is *40˚*; however the sheave angle is slightly smaller causing the belt to wedge itself inside the sheave to increase friction.
- The operating speed for V-belts needs to be high and the recommended speed range is from *5* to *25 m/s*. Best performance is obtained at speed of *20 m/s*.

- For V-belts, the pitch length L_P, and center-to-center distance are found as:

$$L_P = 2C + \pi(D+d)/2 + (D-d)^2/(4C)$$

and

$$C = 0.25\left\{\left[L_P - \frac{\pi}{2}(D+d)\right] + \sqrt{\left[L_P - \frac{\pi}{2}(D+d)\right]^2 - 2(D-d)^2}\right\}$$

- While there are no limitations on the center-to-center distance for flat belts, for V-belts the center-to-center distance <u>should not exceed</u> "$3(D+d)$" because the excessive vibrations of the loose side will shorten the belt life. *<u>why</u>?*
- Also the centers distance should not be less than D.

- Horsepower:

❖ *<u>Table 17-12</u>* gives the <u>horsepower rating</u> for each belt cross-section (*according to sheave pitch diameter and belt speed*).

- The <u>allowable horsepower</u> per-belt, H_a is found as:

$$\boxed{H_a = K_1 K_2 H_{tab}}$$

Power that can be transmitted by each belt

H_{tab}: *from <u>Table 17-12</u>*

where,

K_1: contact angle correction factor (*<u>Table 17-13</u>*).

<u>Note</u>: the contact angles for V-belts are found using the <u>same</u> equations used for flat belts.

K_2: belt length correction factor (*<u>Table 17-14</u>*).

- The <u>design horsepower</u> is found as:

$$\boxed{H_d = H_{nom} K_S n_d}$$

Power that needs to be transmitted from the power source to the driven machine

where,

H_{nom}: nominal horsepower of the power source.
K_S: service factor for overloads (*<u>Table 17-15</u>*).
n_d: design factor of safety.

- The <u>number of belts</u> needed to transmit the design horsepower is found as:

$$\boxed{N_b \geq \frac{H_d}{H_a}}$$

where N_b is an integer

- The belting equation for V-belts is the <u>same equation</u> used for flat belts. The effective coefficient of friction for *Gates Rubber Company* belts is *0.5123*

Thus, $$\frac{F_1 - F_c}{F_2 - F_c} = e^{0.5123\phi}$$

  - Where the centrifugal tension F_c is found as:

$$F_c = K_C\left(\frac{V}{2.4}\right)^2$$

K_C: accounts for mass of the belt (<u>*Table 17-16*</u>).

- The transmitted force <u>per belt</u> $(F_1 - F_2)$ is found as:

$$F_1 - F_2 = \frac{H_d / N_b}{n(d/2)}$$

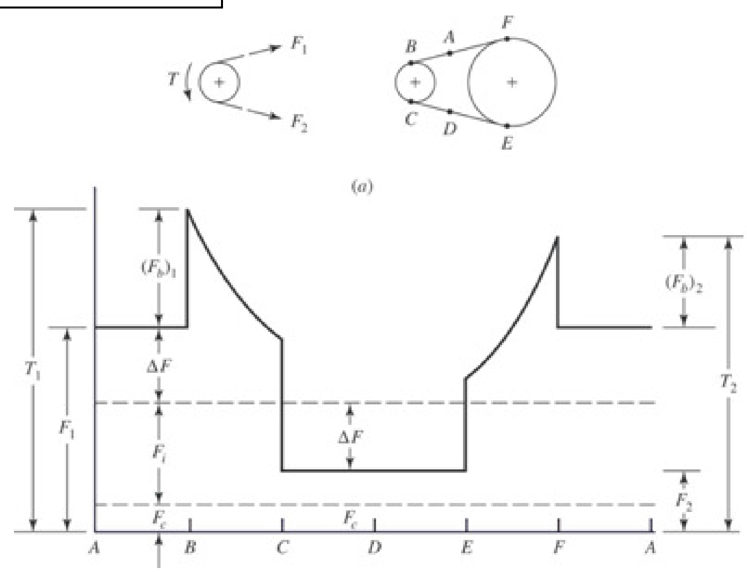
where n (rad/s) & d are for the <u>driver</u> pulley.

  - Thus, F_1 can be found as: $$F_1 = F_c + (F_1 - F_2)\frac{e^{f\phi}}{e^{f\phi} - 1}$$

  - Then F_2 can be found from: $$F_2 = F_1 - (F_1 - F_2)$$

  - And F_i is found as: $$F_i = \frac{F_1 + F_2}{2} - F_c$$

- In flat-belt force analysis, the tension induced from bending the belt was not accounted for explicitly (*since belt thickness is not that large*), however, in V-belts the effect of <u>flexural stress</u> is more pronounced, and thus it affects the durability (*life*) of the belt. The figure shows the <u>two tension peaks</u> T_1 & T_2 resulting from belt flexure.



  - The values of tension peaks are found as:

$$T_1 = F_1 + (F_b)_1 = F_1 + \frac{K_b}{d}$$

$$T_2 = F_1 + (F_b)_2 = F_1 + \frac{K_b}{D}$$

Where,

$(F_b)_1$ & $(F_b)_2$ are the <u>added components</u> of tension due to the flexure of the belt on the smaller and larger pulleys.

K_b is used to account for belt flexure and it is found from <u>*Table 17-16*</u>.

- The life of V-belts is defined as the <u>number of passes</u> the belt can do (N_P), and it is found as:

$$N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1}$$

❖ where K & b are found from <u>*Table 17-17*</u>.

Then, <u>life time</u> (*in hours*) is found as:

$$t = \frac{N_P L_P}{3600V}$$

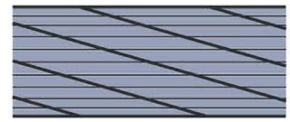
<u>*Note*</u>*:* K & b values given in *Table 17-17* are <u>valid only</u> for the indicated range. Thus, if N_P is found to be larger than 10^9 it is reported as $N_P=10^9$ and life time in hours "t" is found using $N_P=10^9$. Also, if it is found to be less than 10^8, the belt life is considered to be short and inappropriate.

- <u>Steps for analyzing V-belts include</u>:
  - Find V, L_P, C, ϕ and $e^{0.5123\phi}$
  - Find H_d, H_a then the number of belts N_b
  - Find F_c, ΔF, F_1, F_2 & F_i
  - Find T_1, T_2 and then belt life N_P & t

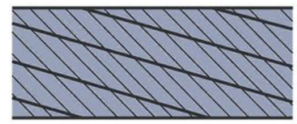
*See **Example 17-4** from text*

Wire Ropes

Wire ropes are made out of steel wires and are used in many applications (such as hoisting, haulage, aircraft, etc...).

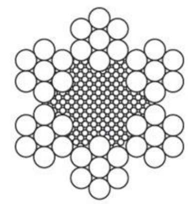


(*a*) Regular lay



(*b*) Lang lay

- There are two basic ways of winding of wire ropes:
  - *Regular lay*: wires and strands are twisted in opposite directions (*do not kink or untwist*).
  - *Lang lay*: wires and strands are twisted in the same direction (*more resistance to wear and fatigue*).



(*c*) Section of 6×7 rope

- Ropes are designated by size and configuration, for example: *25-mm 6x7* haulage rope means: diameter is *25 mm* and has *6* strands each having *7* wires.
  - ❖ *Table 17-24* lists some of the standard ropes along with their properties. Also see *Table 17-27*.

- When a rope passes around a sheave, bending stress develops (*especially in the outer wires*) due to flexing.
  - Using mechanics principles, the stress in one of the wires of the rope can be found as:

$$\sigma = E_r \frac{d_w}{D}$$

where,

E_r: modules of elasticity of the rope

d_w: diameter of the wire

D: sheave diameter

✓ This equation shows the importance of using large diameter sheaves (where it reduces the stress developed in the outer wires).

✓ The recommended D/d_w ratio is *400* and up.

  - Tension in the rope causing the same stress caused by bending is called "*the equivalent bending load*", F_b, and it is found as:

$$\boxed{F_b = \sigma A_m = \frac{E_r d_w A_m}{D}}$$

where,

A_m: is the area of the metal in the rope, and it is usually $A_m = 0.38d^2$ (or from *Table 17-27*)

d_w: diameter of the wire & D: sheave diameter

- Wire ropes are selected according to two considerations:
  - Static considerations: the ability of the rope to carry the loads.
  - Wear life (*fatigue*) considerations: the ability of the rope to live for a certain number of loading cycles.

- *Static considerations*:
  - First step is to determine the tension caused in the rope by the loads (*this includes the dead weight and tension caused by acceleration and shock loads*)
    - For example, the tension caused in a hoisting rope due to load and acceleration /deceleration is:

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right)$$

where, W: total weight of the load
m: number of ropes supporting the load
w: weight per unit length of the rope
l: suspended length of the rope
a: maximum acceleration/deceleration experienced
g: gravity acceleration

  - The tension due to loads is then compared to the ultimate tensile load of the rope to find the static factor of safety.

$$\boxed{F_u = \textit{strength of the rope} \times \textit{nominal area of the rope}}$$

Maximum load that can be supported

Thus, the static factor of safety is:

$$n_S = \frac{F_u}{F_t}$$

    - However, the ultimate tensile load F_u must be reduced due to the increased tension caused by flexing the rope over the sheave and thus the factor of safety can be found as:

$$\boxed{n_s = \frac{F_u - F_b}{F_t}}$$

more accurate

❖ *Table 17-25* gives the minimum rope factors of safety for different applications.

- *Fatigue considerations*:
  - The amount of wear that occurs in ropes depends on the bearing pressure on the rope caused by the sheave and by the number of bends (*number of the passes of the rope over the sheave*) of the rope during operation.
  - The allowable fatigue tension (*fatigue strength*) for a rope is found as:

$$F_f = \frac{(P/S_u)S_u dD}{2}$$

Maximum tension that can be supported under certain bearing pressure (P) for a certain number of bends.

Where,

(P/S_u): bearing pressure to ultimate strength ratio. It is found according to the specified life from *Fig 17-21*.

S_u: ultimate tensile strength of the wires.

d: diameter of the rope & D: diameter of the sheave.

- It should be noted that S_u is the ultimate strength of the wires not the strength of the rope. (*it usually not listed in the tables but it can be determined from a hardness test*).
  - Approximate ranges of the ultimate strength of wires S_u for different wire materials are listed below:

Improved plow steel (monitor)	$1655 < S_u < 1930$ MPa
Plow steel	$1448 < S_u < 1665$ MPa
Mild plow steel	$1241 < S_u < 1448$ MPa

- Thus, the fatigue factor of safety can be found as:

$$n_f = \frac{F_f - F_b}{F_t}$$

- It should be understood that the fatigue failure in wire ropes is not sudden, as in solid bodies, but rather progressive. It shows as breaking of the outside wires (*since they are subjected to the highest stress*). Therefore, it can be detected by periodic inspection.

*See **Example 17-6** from text*