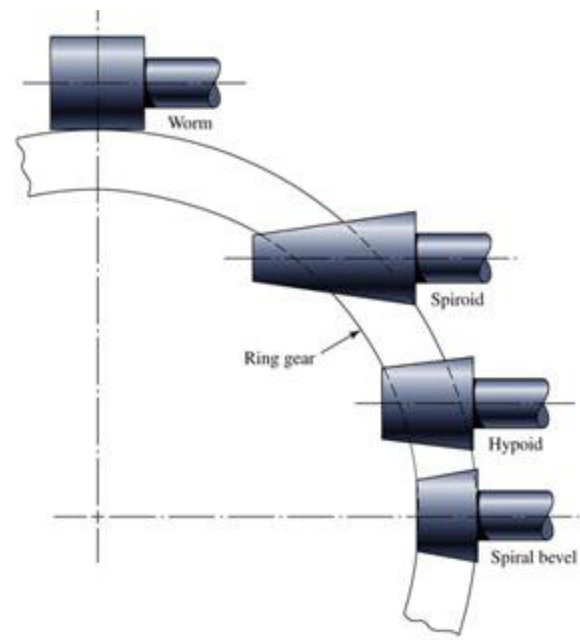
CH 15: Bevel and Worm Gears

Bevel Gears - General

Bevel gears are classified as follows:

- *Straight Bevel Gears*: (Fig. 13-35) used for pitch line velocities up to *5m/s*, noise level is high.
- *Spiral Bevel Gears*: (Fig. 15-1) used for higher speeds, noise level is lower than straight gears because of gradual engagement of teeth (*similar to helical gears*). Spiral angle; see Fig. 15-2.
- *Zerol Bevel Gears*: it has curved teeth but zero spiral angle. Usually used instead of straight bevel gears because of lower noise level. It has smaller thrust component than spiral gears (because of zero spiral angle).
- *Hypoid Bevel Gears*: (Fig. 15-3) used for offseted shafts. Teeth action is combination of rolling and sliding (*more friction*).
- *Spiroid Bevel Gears*: used for shafts with large offset (the *pinion is similar to a worm*).



Bevel-Gears Stresses and Strengths *(Straight Bevel Gears)*

There are two major difficulties that arise for bevel gears:

- Shaft deflection: typically one of the gears has to be mounted outboard of the bearing which makes the shaft deflection more pronounced and thus have greater effect on the nature of tooth contact.
- Teeth deflection: since teeth are tapered, more deflection occurs at the smaller section and that causes non-uniform line contact (to obtain a perfect line contact the larger section needs to deflect more). To overcome this difficulty, the face width needs to be kept fairly "small".

- AGMA Stress equations:

  - Bending stress:

$$\begin{cases} S_t = \sigma = \dfrac{W^t}{F} P_d K_o K_\upsilon \dfrac{K_s K_m}{K_x J} & \textit{US units} \\[2ex] \sigma_F = \dfrac{1000W^t}{b} \dfrac{K_A K_\upsilon}{m_{et}} \dfrac{Y_X K_{H\beta}}{Y_{\beta J}} & \textit{(SI units)} \end{cases}$$

  - Contact stress:

$$\begin{cases} S_C = \sigma_C = C_P \left(\dfrac{W^t}{F d_P I} K_o K_\upsilon K_m C_S C_{XC}\right)^{1/2} & \textit{US units} \\[2ex] \sigma_H = Z_E \left(\dfrac{1000W^t}{b d Z_1} K_A K_\upsilon K_{H\beta} Z_X Z_{XC}\right)^{1/2} & \textit{(SI units)} \end{cases}$$

- AGMA Strength (*allowable stress*) equations:

  - Bending strength:

$$\begin{cases} S_{WT} = \sigma_{all} = \dfrac{S_{at} K_L}{S_F K_T K_R} & \textit{US units} \\[2ex] \sigma_{FP} = \dfrac{\sigma_{F\lim} Y_{NT}}{S_F K_\theta Y_Z} & \textit{(SI units)} \end{cases}$$

  - Contact strength:

$$\begin{cases} S_{WC} = \sigma_{c,all} = \dfrac{S_{ac} C_L C_H}{S_H K_T C_R} & \textit{US units} \\[2ex] \sigma_{HP} = \dfrac{\sigma_{H\lim} Z_{NT} Z_W}{S_H K_\theta Z_Z} & \textit{(SI units)} \end{cases}$$

AGMA Equations Factors

- *Overload factor*, K_o (K_A): it is used to account for external loads exceeding the normal tangential load W^t.
  - ❖ See *Table 15-2* for K_o values.

- *Dynamic factor*, K_v : it is used to account for deviations from the uniform angular speed due to inaccuracies in manufacturing and meshing of gears. Transmission accuracy-level number Q_v is used to indicate the manufacturing precision.
  - ❖ The value of K_υ is found from *Fig. 15-5* as a function of Q_v and pitch-line velocity v_t where:

$$v_t = \pi d_P n_P / 12 \qquad \textit{US units}$$

$$v_{et} = 5.236\,(10^{-5})d_1 n_1 \qquad \textit{(SI units)}$$

  - ✓ Also there are curve fit equations given in text.

- *Bending stress geometry factor*, J (Y_J): it is used to account for geometry of the tooth and location of the load W^t.
  - ❖ The value for J is found from *Fig. 15-7*.

- *Contact stress geometry factor*, I (Z_I): it is used to account for the geometry of teeth surfaces (*i.e., instantaneous radii of curvature*) and thus the contact area of the two surfaces.
  - ❖ The value of I is found from *Fig. 15-6*.

- *Load distribution factor*, K_m $(K_{H\beta})$: it is used to account for non-uniform load distribution along the line of contact.

  K_m is found as :

$$K_m = K_{mb} + 0.0036\,F^2 \qquad \textit{US units}$$

$$K_{H\beta} = K_{mb} + 5.6(10^{-6})b^2 \qquad \textit{(SI units)}$$

  where

$$K_{mb} = \begin{cases} 1.00 & \textit{both members straddle} - \textit{mounted} \\ 1.10 & \textit{one member straddle} - \textit{mounted} \\ 1.25 & \textit{neither member straddle} - \textit{mounted} \end{cases}$$

- *Size factor for bending*, $K_s\ (Y_x)$:

$$K_s = \begin{cases} 0.4867 + \frac{0.2132}{P_d} & 0.5 \le P_d \le 16 \quad in^{-1} \\ 0.5 & P_d > 16 \quad in^{-1} \end{cases} \qquad \textit{US units}$$

$$Y_x = \begin{cases} 0.5 & m_{et} < 1.6 \ mm \\ 0.4867 + 0.008339\, m_{et} & 1.6 \le m_{et} \le 50 \ mm \end{cases} \qquad \textit{(SI units)}$$

- *Size factor for contact stress*, $C_s\ (Z_x)$:

$$C_s = \begin{cases} 0.5 & \mathrm{F} < 0.5 \ in \\ 0.125F + 0.4375 & 0.5 \le \mathrm{F} \le 4.5 \ in \\ 1 & \mathrm{F} > 4.5 \ in \end{cases} \qquad \textit{US units}$$

$$Z_x = \begin{cases} 0.5 & b < 12.7 \ mm \\ 0.00492b + 0.4375 & 12.7 \le b \le 114.3 \ mm \\ 1 & b > 114.3 \ mm \end{cases} \qquad \textit{(SI units)}$$

- *Crowing factor for contact stress*, $C_{xc}\ (Z_{xc})$:

  The teeth of most bevel gears are crowned to accommodate the shaft deflections.

$$C_{xc} = Z_{xc} = \begin{cases} 1.5 & crowned\ teeth \\ 2 & non-crowned\ teeth \end{cases}$$

- *Lengthwise curvature factor for bending stress*, $K_x\ (Y_\beta)$:

$$K_x = Y_\beta = 1 \qquad \textit{for straight bevel gears}$$

- *Elastic coefficient for contact stress*, C_p : it combines the elastic properties of the gear and pinion

$$C_p = \sqrt{\frac{1}{\pi((1-\nu_P^2)/E_P + 1 - \nu_G^2)/E_G))}} \qquad \sqrt{psi}\ (\sqrt{MPa})$$

❖ Or can be found from *Table 14-8*.

- *Bending strength*, S_{at} $(\sigma_{F\ all})$: material property under tensile cyclic loading conditions (*tensile fatigue strength*).

  - ❖ S_{at} values are found from *Tables 15-6, 15-7* and *Fig. 15-13*.
    - ➢ S_{at} values are based on *10^7* cycles and *0.99* reliability.
    - ➢ For reversed loading such as in idler gears, *AGMA* recommends using 70% of S_{at} value.

- *Contact strength*, S_{ac} $(\sigma_{H\ all})$: material property under compressive cyclic loading conditions (*compressive fatigue strength*).

  - ❖ S_{ac} values are found from *Tables 15-4, 15-5* and *Fig. 15-12*.
    - ➢ ***Note:*** S_{ac} values are based on *10^9 cycles* and *0.99* reliability.

- *Stress-cycle factor for bending strength*, $K_L(Y_{NT})$: it accounts for lives other than *10^7* cycles.

  - ❖ The value of $K_L(Y_{NT})$ is found from *Fig. 15-9*.

- *Stress-cycle factor for contact strength*, C_L (Z_{NT}): it accounts for lives other than *10^9* cycles.

  - ❖ The value of C_L (Z_{NT}) is found from *Fig 15-8*.

- *Hardness ratio factor*, C_H (Z_W) : it is used to account for the difference of the hardness between gear and pinion.

  - ❖ For through-hardened gear and pinion , use *Fig. 15-10*.
  - ❖ For surface-hardened pinion mating with through-hardened gear, use *Fig. 15-11*.

- *Temperature factor*, K_T (K_θ): it accounts for the change in material strength at increased temperature.

$$K_T = \begin{cases} 1 & 32°\text{F} \le T \le 250\ °\text{F} \\ (460+T)\ /710 & T > 250\ °\text{F} \end{cases}$$

$$K_\theta = \begin{cases} 1 & 0\ ℃ \le T \le 120\ ℃ \\ (273+T)\ /393 & T > 120\ ℃ \end{cases}$$

- *Reliability factors*, K_R and C_R : used to account for reliabilities other than *0.99*

  K_R: reliability factor for bending strength
  C_R: reliability factor for contact strength
  where $C_R = \sqrt{K_R}$

  ❖ Values of K_R and C_R are found from *Table 15-3*.

- *Safety factors* S_F and S_H: used to account for unquantifiable elements affecting the stresses.
  - When designing the safety factor become a design factor.
  - When analyzing, the safety factor is the ratio of strength-to-stress.
  - when comparing bending and contact factors of safety, we compare S_F with $(S_H)^2$

Summary: Figures 15-14 & 15-15 give "roadmaps" for bending and contact equations for straight bevel gears.

Straight Bevel Gear Analysis

*See **Example 15-1** from text*

Design of straight Bevel Gears

The decision set can be classified in two groups:

Priori decisions
$$\begin{cases} Function \\ Design\ factor \\ Tooth\ system \\ Quality\ number \\ Gear\ ratio\ and\ tooth\ count \end{cases}$$

Design decisions
$$\begin{cases} Diametral\ pitch\ and\ face\ width \\ Gear\ material, core\ and\ case\ hardness \\ Pinion\ material, core\ and\ case\ hardness \end{cases}$$

Since there are limitations on the face width of bevel gears (due to teeth deflection), it is suggested that the face width is chosen such that:

$$F = \min(0.3A_o, 10/P_d)$$

Where A_o is the cone distance

$$A_o = \frac{d_p}{2sin\gamma} = \frac{d_G}{2sin\Gamma}$$

*See **Example 15-2** from text*