CH 12: Lubrication and Journal Bearings

The purpose of lubrication is to reduce friction, wear and heating of machine parts moving relative to each other.

In sleeve bearings, a shaft (or Journal) rotates within a sleeve (or bushing) and the relative motion is sliding, where in ball bearings the relative motion is rolling.

Journal bearings are more applicable for extreme operational conditions (high loads and rotational speeds). Also they are used for low demand applications (without external lubrication) because they are more cost effective than antifriction bearings.

Types of Lubrication

Five forms of lubrication can be identified:

1. Hydrodynamic
2. Hydrostatic
3. Elastohydrodynamic
4. Boundary
5. Solid film

- **Hydrodynamic (or full-film):** in this type, the surfaces of the bearing are separated by a relatively thick film of lubricant (to prevent metal to metal contact). The film pressure is created by the moving surface forcing the lubricant into a wedge-shaped zone, therefore creating a pressure that separates the sliding surfaces.

- **Hydrostatics:** in this type, the lubricant is forced into the bearing at a pressure high enough to separate the surfaces (relative motion of the surfaces is not required in this case).

- **Elastohydrodynamic:** in this type, the lubricant is introduced between surfaces that are in rolling contact (such as mating gears or rolling bearings).

- **Boundary:** this type is special case of hydrodynamic lubrication where the film thickness is reduced to be “very thin”. This may happen because of increased load, reduced lubricant supply, reduced rotational speed, reduced viscosity, etc.

- **Solid-film:** in this type, self-lubricating solid materials such as graphite are used in the bearing. This is used when bearings must operate at very high temperature.
Viscosity

- When we have a film of lubricant of thickness \( h \) trapped between two plates where one is moving with velocity \( U \) and the other is stationary, the lubricant film will be sheared such that the layer in contact with the moving plate will move at the same velocity of the plate and the layer in contact with the stationary plate will stay stationary.

- The intermediate layers will have the velocities proportional to their distance \( y \) from the stationary plate.

- *Newton viscous effect* states that the shear stress in the fluid is proportional to the velocity gradient. Thus we can write:

\[
\tau = \frac{F}{A} = \mu \frac{du}{dy}
\]

where \( \mu \), the dynamic (or absolute) viscosity, is the constant of proportionality.

- For *Newtonian fluids* the velocity gradient is constant; \( \frac{du}{dy} = \frac{U}{h} \) (i.e., linear increase)

Thus,

\[
\tau = \frac{F}{A} = \mu \frac{U}{h}
\]

- The unit for viscosity \( \mu \) in SI system is (Pa.s).
- In US system the unit is lb/s/in\(^2\) (psi.s) and it is called “reyn”.
  - The conversion factor is: 1 reyn (psi.s) = 6895 Pa.s

Petof’s Equation

The Petroff equation gives the coefficient of friction in journal bearings. It is based on the assumption that the shaft is concentric. Though the shaft is not concentric but the coefficient of friction predicted by this equation turns out to be quite good.

- Consider a shaft of radius \( r \) rotating inside a bearing with rotational speed \( N \), and the clearance between the shaft and sleeve \( c \) is filled with oil (leakage is negligible).
• From Newton’s viscosity equation we get:

\[ \tau = \mu \frac{U}{h} = \mu \frac{2\pi r N}{c} \]

• The force needed to shear the film is \( F = \tau A \) (where \( A = 2\pi rl \)) and the torque \( T = Fr = \tau Ar \).

Thus the torque can be written as:

\[ T = \frac{4\pi^2 r^3 \mu N}{c} \quad (1) \]

• The pressure on the projected area is \( P = \frac{W}{2rl} \rightarrow W = 2rlP \) and the torque created by the frictional force “\( fW \)” is:

\[ T = fW r = (f)(2rlP)(r) = 2r^2 f lP \quad (2) \]

• Equating 1 & 2 and solving for “\( f \)” gives:

\[ f = 2\pi^2 \frac{\mu N r}{P c} \quad \text{Pitroff’s equation} \]

\[ \begin{align*}
\text{The coefficient of friction} & \quad \rightarrow \\
& \quad f = 2\pi^2 \frac{\mu N r}{P c}
\end{align*} \]

\[ \text{\blacktriangleright} \quad \text{The quantities} \ \left( \frac{\mu N}{P} \right) \text{\&} \ \left( \frac{r}{c} \right) \text{\ are non-dimensional, and they are very important parameters in lubrication.} \]

\[ \quad \text{\blacktriangleright} \quad \text{They are used to give the “bearing characteristic number” or the “Sommerfeld number” (also non-dimensional) which is:} \]

\[ S = \left( \frac{r}{c} \right)^2 \frac{\mu N}{P} \quad \text{(N is in (rev/s))} \]

\[ \text{\blacktriangleright} \quad \text{The Sommerfeld number is very important in lubrication analysis because it contains many of the parameters specified by the designer.} \]

**Stable Lubrication**

The difference between boundary (thin film) and hydrodynamic (thick film) lubrication can be explained by the figure (which was obtained from testing).
- Suppose we are operating to the right of point “C” and something happens and increases the temperature of lubricant:
  - Temperature↑, viscosity “μ” ↓, friction “f” ↓, heat generated by shearing the lubricant ↓, temperature ↓
  - **Self-correcting** *stable lubrication*
- If we operate to the left of point “C” and the temperature increased
  - Temperature↑, viscosity “μ” ↓, friction “f” ↑, (metal to metal contact) more heat is generated which increases the temperature more.
  - **Damage** *unstable lubrication*
- To ensure thick-film lubrication we should have:
  \[ \frac{\mu N}{P} \geq 1.7 \times 10^{-6} \]

**Thick-Film Lubrication**

- Suppose the journal starts to rotate in clockwise direction while it is still dry, the journal will roll up the right side of the bearing, as seen in (a).
- Once the lubricant is introduced, the rotating journal will pump the lubricant around the bearing by forcing into a wedge-shaped space, and this forces the journal to move to the other side (left side) of the bearing, as seen in (b).
  - The “minimum film thickness”, \( h_0 \), occurs at the bottom half of the bearing but slightly to the left (for clockwise rotation), as seen in (b).
- The nomenclature of a journal bearing is:
  - *Radial clearance* “c” = \( r_b - r_j \)
  - *Eccentricity* “e”: the distance between the centers of the bushing and journal.
\[ e = c - h_o \]

- We also define an “eccentricity ratio”, \( \varepsilon \), as:
  \[ \varepsilon = \frac{e}{c} \]

- **Full bearing**: the bushing encloses the journal.
- **Partial bearing**: the angle \( \beta \) describes the angular length of the partial bushing.

**Hydrodynamic Theory**

The present hydrodynamic theory is based on the experiments conducted by Tower, which later Reynolds developed a mathematical model to explain it.

- **Reynolds’ work** was based on some assumptions which are:
  - The film thickness is very small compared with the bearing radius; therefore the curvature could be neglected.
  - The lubricant is Newtonian, incompressible and its viscosity is constant.
  - Forces due to inertia of the lubricant are neglected.
  - The film pressure is constant in the axial and vertical directions.
  - The bearing has infinite length in the axial direction (**no side leakage**).
  - The velocity of any particle in the lubricant depends only on the \( x \) and \( y \) coordinates.

- **Reynolds’ equation** for one-dimensional flow (**with negligible side leakage**) is: “See derivation in the text”
  \[
  \frac{d}{dx} \left( \frac{h^3 \frac{dp}{dx}}{\mu \frac{dx}{dx}} \right) = 6U \frac{dh}{dx}
  \]

- When side leakage is **not neglected**, the equation becomes:
  \[
  \frac{\partial}{\partial x} \left( \frac{h^3 \frac{dp}{dx}}{\mu \frac{dx}{dx}} \right) + \frac{\partial}{\partial z} \left( \frac{h^3 \frac{dp}{dz}}{\mu \frac{dz}{dz}} \right) = 6U \frac{\partial h}{\partial x}
  \]
• There is no general analytical solution for this equation; one of the important solutions (numerical) was introduced by Sommerfeld which is:

\[
\frac{r}{c} f = \varphi \left[ \left( \frac{r}{c} \right)^2 \frac{\mu N}{P} \right]
\]

\[\varphi \text{ := functional relationship}\]

_Sommerfeld number_

**Design Considerations**

The variables involved in the design of sliding bearings may be divided in two groups:

- **Independent (or design) variables:** might be controlled directly by the designer which include: viscosity “\(\mu\)”, load per unit projected area “\(P\)”, angular speed “\(N\)” and bearing dimensions \(r, c, l, \beta\) (though speed “\(N\)” and sometimes viscosity “\(\mu\)” may be forced on the designer).

- **Dependent variables:** may be controlled indirectly by changing one or more of the first group, which are: coefficient of friction “\(f\)”, temperature rise “\(\Delta T\)”, oil flow rate “\(Q\)”, and the minimum film thickness “\(h_o\)”. These variables tell about the performance of the bearing, and may be called the “**performance factors**” (**the designer may impose limitations on those variables to ensure satisfactory performance**).

**Significant angular speed:**
The rotational speed “\(N\)” that is used in the Sommerfeld number depends on the rotation of the journal, the bearing and the load. It can be found as:

\[N = |N_j + N_b - 2N_f|\]

where:

- \(N_j\): journal angular speed (\(\text{rev/s}\))
- \(N_b\): bearing (bushing) angular speed (\(\text{rev/s}\))
- \(N_f\): load vector angular speed (\(\text{rev/s}\))

**Fig. 12-11** shows some examples.
**Trumpler’s Design Criterion**

Based on his experience, *Trumpler* introduced some limitation for the design of journal bearings, which are:

- **Minimum film thickness** \( h_0 \)
  
  When bearing starts rotation some debris are generated because of metal to metal contact and they move with the lubricant. It is important that the minimum film thickness is kept thick enough such that the debris can pass and will not block the lubricant flow. Therefore *Trumpler* suggest that:
  
  \[
  h_0 \geq 0.00508 + 0.00004d \quad \text{mm}
  \]
  
  where \( d \) is the journal diameter in “mm”.

- **Maximum lubricant temperature**
  
  When temperature increases beyond a certain limit, lighter components of the lubricant starts to evaporate which increases viscosity and thus friction. For light oils, *Trumpler* suggests:
  
  \[
  T_{\text{max}} \leq 121 \quad \text{°C}
  \]

- **Starting load**
  
  Journal bearing usually consist of a steel journal and a bushing of softer material. If the starting load is high, the bushing will be worn-out quickly because of the metal to metal contact. Thus, it is suggested that the starting load divided by the projected area is:
  
  \[
  \frac{W_{\text{st}}}{ld} \leq 2068 \quad \text{kPa}
  \]

  **Note that starting load is usually smaller than running load**

- **Running load design factor**
  
  To account for external vibrations, a design factor is to be used:
  
  \[
  n_d \geq 2
  \]

  **For running load not starting load**
The Relation of the Variables

Raimondi and Boyd used numerical solution to solve the Reynolds’ equation. They presented their numerical results relating the different variables in the form of charts.

- The charts presented in the text are for full bearing ($\beta = 360^\circ$)

- Viscosity charts (figs. 12-12 to 12-14).

- Table 12-1 gives the viscosity vs. temperature in table from using curve fits.

- In their solution, Raimondi-Boyd, assumed the temperature of the lubricant to stay constant as it passes through the bearing. In reality, temperature increases because of the work done on the lubricant. Thus, when finding viscosity from the chart we should use an average temperature value which is:

$$T_{av} = T_1 + \frac{\Delta T}{2}$$

- When we know the oil inlet temperature and need to find the outlet temperature we have to use trial-and-error where we assume the temperature rise and find viscosity from chart then use it to compute a temperature rise. If it does not match the assumed $\Delta T$ then another value is tried and so on.

- The charts give the variables against the Sommerfeld number “S” for different $l/d$ ratios.

- The figure explains the notation used in the charts.

- Minimum film thickness and its angular position (figs. 12-16 and 12-17)
  - note that $h_0 = c - e \Rightarrow \frac{h_0}{c} = 1 - \varepsilon$

  See Example 12-1 from text

- The coefficient of friction variable (fig. 12-18)

  See Example 12-2 from text
Lubricant flow and side flow ratio (figs. 12-19 and 12-20)
- Why there is no \( l/d = \infty \) curve in fig. 12-20?

See Example 12-3 from text

Maximum film pressure and its angular position (figs. 12-21 and 12-22)

See Example 12-4 from text

- **Lubricant Temperature Rise**

The lubricant temperature will increase until a heat balance is reached (heat generated by shearing the lubricant = heat lost to the surroundings).

It can be shown that heat balance calculations (assuming side flow) will give the following equation: (see derivation in text)

\[
0.12 \Delta T_C = \frac{r/c f}{P_{(MPa)}} \left( 1 - \frac{1}{2} \left[ \frac{Q_s}{Q} \right] \right) \left( \frac{Q}{rCNl} \right)
\]

The RHS of this equation can be evaluated using charts in figs. 12-18, 19 & 20 or easier using fig. 12-24 where it combines the three charts together which makes the iterative approach to find \( \Delta T \) easier.

See the interpolation equation in the text (Eqn. 12-16)

**Steady-State Conditions in Self Contained Bearings**

In self-contained bearings the lubricant stays within the bearing housing and it is cooled within the housing by dissipating the heat to the surroundings. This type of bearings is also called *pillow-block* bearings. In this type, the sump is expanded peripherally in the top half of the bearing and the bushing covers the lower half.
(β = 180°). As the oil film exits the lower half of the bearing it mixes with sump contents, then heat is transferred to the surroundings.

- The heat lost from the housing to the surroundings can be estimated as:

\[ H_{loss} = h_{CR}A(T_b - T_\infty) \]

where:

- \( H_{loss} \): Dissipated heat, J/s or W
- \( h_{CR} \): Combined coefficient of radiation & convection, W/(m\(^2\).°C)
- \( A \): Bearing surface area, m\(^2\)
- \( T_b \) & \( T_\infty \): Housing surface temperature & ambient temp, °C

- Some representative values of \( h_{CR} \) are given as:

\[
\begin{align*}
    h_{CR} &= \begin{cases} 
        11.4 \text{ W/(m}^2\text{.°C) for still air} \\
        15.3 \text{ W/(m}^2\text{.°C) for shaft – stirred air} \\
        33.5 \text{ W/(m}^2\text{.°C) for air moving at 24.5 m/s}
    \end{cases}
\end{align*}
\]

- Similar expression can be written for the temperature difference between the lubricant film and housing surface. If we define \( T_f \) as the average film temperature between the inlet \( T_s \) and the outlet \((T_s + \Delta T)\), then the following proportionality can be observed:

\[
T_f - T_b = \alpha(T_b - T_\infty)
\]

where \( \alpha \) depends on lubrication system and housing geometry

- Table 12-2 gives representative values of \( \alpha \).

- Solving for \( T_b \) we get:

\[
T_b = T_f + \alpha T_\infty / (1 + \alpha)
\]

- Substituting in the heat loss equation, we get:

\[
H_{loss} = \frac{h_{CR}A}{1 + \alpha} (T_f - T_\infty)
\]

- Because of the shearing of lubricant film heat is generated. In steady-state condition, the heat generated in the lubricant film is equal to the heat dissipated from the housing to the surrounding.
• The heat generated can be found as:

\[ H_{gen} = T(2\pi N) \]

But,

\[ T = 4\pi^2 r^3 \mu N/c \]

Thus,

\[ H_{gen} = \frac{248\mu N^2 lr^3}{c} \]

Note that this torque equation is based on the assumption that the journal is concentric. The equation that should be used is (see the derivation of Petroff’s equation):

\[ T = fWr \]

\[ \Rightarrow H_{gen} = fWr(2\pi N) = 2\pi NC \left( \frac{fr}{c} \right) \]

• In steady-state analysis, the average film temperature \( \bar{T}_f \) is unknown and therefore the viscosity is unknown. Thus, a trial value of \( \bar{T}_f \) is used (the corresponding \( \mu \) is found) and both \( H_{loss} \) & \( H_{gen} \) are evaluated. Then, iterations continue until we get \( H_{loss} = H_{gen} \).

• Equating \( H_{loss} \) & \( H_{gen} \) and solving for \( \bar{T}_f \) we get,

\[ \bar{T}_f = T_\infty + 248(1 + \alpha) \frac{\mu N^2 lr^3}{h_{CR}Ac} \]

Note that:

\[ \bar{T}_f = T_s + \Delta T/2 \]

See Example 12-5 from text

Clearance

When designing journal bearings for thick film lubrication, the designer selects the lubricant and suitable values for the bearing parameters to give satisfactory performance. However, the clearance “c” is difficult to hold accurate during manufacturing. Also, clearance increases with time because of wear.

• The figure shows the effect of wide range of clearances on the performance of a bearing (for bearing parameters given in examples 12-1 to 12-4). It can be seen from the figure that lubricant flow increases with increased clearance and this decreases the generated heat and outlet temperature. The minimum
film thickness “$h_0$” increases with clearance then it starts to decrease.

- If the clearance is too small, dirt (debris) may block the oil flow and therefore cause overheating and failure.
- If the clearance is too high, the bearing becomes noisy and $h_0$ decreases.

Thus, the optimum range of clearances is shown by shaded area in the figure. If clearance value is within this range the performance of the bearing will improve with wear.

**Pressure-Fed Bearings**

The load carrying capacity of self-contained bearings is limited because of the limited heat-dissipating capability. To increase the heat-dissipation, an external pump is used to increase the lubricant flow through the bearing. The pump supplies the bearing with lubricant of high pressure therefore increasing the lubricant flow and heat dissipation. The lubricant sump may also be cooled with water to reduce temperature further.

- A circumferential groove at the center of the bearing, with an oil-supply hole located opposite to the load zone, is usually used to feed the lubricant (fig. 12-27). The oil flows from the groove in the middle towards the ends of the bearing.
  
  ➢ Note that in this type of bearings, the lubricant is supplied at high pressure (pressure is not created by the sliding of the journal surface).

- When determining the lubricant flow, eccentricity is first neglected, then a correction factor for eccentricity is applied. Also, rotation of the shaft is neglected.

- For lubricant supplied at pressure “$P_s$ ”, the average velocity of lubricant flowing towards the ends of the bearing can be found as (see derivation in text):

$$u_{av} = \frac{P_s}{12 \mu l'} (c - e \cos \theta)^2$$

where $l'$ is length of each half of the bearing (see fig. 12-28).

➢ Note that minimum film thinness “$h_0$” is assumed to be at the bottom of the bearing (fig. 12-30) because rotation is neglected.
Thus, lubricant flow out of both ends of the bearing is:

\[ Q_s = \frac{\pi P_s r c^3}{3 \mu l'} (1 + 1.5 \epsilon^2) \]

- In this type of bearings, the pressure per projected area for each half of the bearing is:

\[ P = \frac{w/2}{2rl'} = \frac{w}{4rl'} \]

Therefore, the Sommerfeld number is:

\[ s = \left( \frac{r}{c} \right)^2 \frac{\mu N}{p} = \left( \frac{r}{c} \right)^2 \frac{4rl' \mu N}{W} \]

- The heat generated and heat loss can be found as:

\[ H_{loss} = \rho C_p Q_s \Delta T \quad \text{where} \quad \rho: \text{density} \]
\[ H_{gen} = 2\pi WN c \frac{fr}{c} \quad C_p: \text{heat capacity} \]

- Equating \( H_{loss} \) & \( H_{gen} \), the temperature rise can be found as:

\[ \Delta T_c = \frac{978 (10^6) (fr/c) SW^2}{1 + 1.5 \epsilon^2} \frac{P_s r^4}{} \quad \text{for} \quad \Delta T \text{ in } ^\circ C \]

Or

\[ \Delta T_F = \frac{0.0123 (fr/c) SW^2}{(1 + 1.5 \epsilon^2) P_s r^4} \quad \text{for} \quad \Delta T \text{ in } ^\circ F \]

**Thrust Bearings**

Journal bearings are designed to take radial loads only, if there is a thrust component, a sliding thrust bearing can be used.

- The working principle of thrust bearings is similar to that of journal bearings. The figure shows a fixed-pad thrust bearing consisting of a runner sliding over a fixed pad. The lubricant is brought into the radial grooves and pumped into the wedge shaped spaces (see fig. 12-36).
• Hydrodynamic lubrication is obtained if the speed of the runner is continuous and sufficiently high and lubricant is available in sufficient quantity.

**Boundary-Lubricated Bearings**

When two surfaces slide relative to each other with only partial lubricant film between them, **boundary (or thin-film)** lubrication is said to exist.

• Boundary lubrication occurs in hydrodynamically lubricated bearings on starting or stopping condition, or when load increases or if lubricant supply or rotational speed decreases.  
  ➢ To reduce friction in boundary lubricated bearings, animal or vegetable oils can be mixed with mineral oils because they form a soap film that sticks to metallic surfaces.

• “Mixed-film lubrication” is said to exist if the bearing operates **partly** under hydrodynamic conditions and **partly** under boundary lubrication conditions. Mixed-film lubrication condition occurs if lubricant is supplied manually using drop or mechanical feed, viscosity is too low, bearing speed is too low, the bearing is overloaded, the clearance is too tight, or the journal is not properly aligned.

**Linear sliding wear:**

Considering a block with surface area “A” sliding over a fixed surface with contact pressure “P” where the coefficient of sliding friction “\( f_s \)” and define a linear wear measure “\( w \)”.

• The **work** done by the frictional force “\( f_sPA \)” during displacement “\( S \)” is “\( f_sPAS \)” or “\( f_sPAvt \)” (where “V” velocity and “t” time).

• The **volume of material** removed because of wear “\( wA \)” is proportional to the work (\( wA \propto f_sPAvt \)), or we can write:

\[
wA = KPAvt
\]

where “\( K \)” is the constant of proportionality and it is called the “**wear factor**” which includes the coefficient of friction “\( f_s \)”.

The unit of “\( K \)” in SI units is: \( m^3 \cdot s/(N \cdot m \cdot s) \)
Table 12-8 gives the values of the wear factor “K” for different bushing materials (they are determined from testing).

- Cancelling “A” from both sides, “w” can be expressed as:
  \[ w = KPVt \]
- Additional correction factors \( f_1 \) & \( f_2 \) can be included such that:
  \[ w = f_1f_2KPVt \]
  where, \( f_1 \) correctio factor for motion type; Table 12-10
  \( f_2 \) correctio factor for enviroment; Table 12-11

**Bushing wear:**
For the case of journal bearing of diameter “D” and Length “L” rotating at speed “N”, the wear of the bushing “w” can be found from the previous equation knowing that:

\[ P_{max} = \frac{4}{\pi} P = \frac{4F}{\pi DL} \quad \& \quad F = W = \text{radial load} \]

Thus, the wear is found to be:

\[
  w = f_1f_2K \frac{4F}{\pi DL} Vt = \frac{4f_1f_2KFNt}{L}
\]

Where
- \( V \) (m/s)
- \( N \) (rev/s)
- \( t \) (s)

See Example 12-7 from text