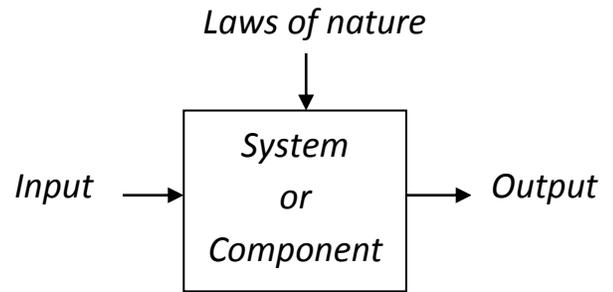


# CH 1: Introduction

Science explains what is; Engineering creates what never was.

	To find	From
Science	Laws	System, Input, Output
Engineering	System	Input, Output, Laws
Analysis	Output	System, Input, Laws
Inverse Analysis	Input	System, Output, Laws



The designed product must be: functional, safe, reliable, usable, competitive, manufacturable, and marketable.

## Codes and Standards

Codes and standards are made to organize and unify the engineering work.

*Imagine; what if there was no standard for bolts?*

- A code is a set of specifications for the analysis, design, manufacture, and construction of something.
- A standard is a set of specifications for parts, materials, or processes intended to achieve uniformity, efficiency and specific quality.

Examples of organizations that establish standards and design codes: AISI, AGMA, SAE, ASTM, ISO.

## Stress and Strength

When designing a component, the designer needs to make sure that the maximum stress is less than the strength of the material used for making that component.

- Stress ( $\sigma$ ) depends on geometry and load.
- Strength ( $S$ ) is a material property; strength is the stress level at which something occurs (such as yield strength or ultimate strength).



Using the stress notation, the design factor is defined as:

$$n_d = \frac{\text{Strength}}{\text{Stress}}$$

However when the load is not linearly related to stress, the design factor can be defined as:

$$n_d = \left( \frac{\text{Strength}}{\text{Stress}} \right)^2 \quad \text{for cylinders in contact}$$

$$n_d = \left( \frac{\text{Strength}}{\text{Stress}} \right)^3 \quad \text{for spheres in contact}$$

The factor of safety ( $n_s$ ) has the same definition as the design factor, but it is numerically different due to the rounding (usually up) caused by using standard sizes.

### **Reliability**

Reliability of a component is the probability that the component will not fail during use (*i.e., the probability of survival*).

The reliability ( $R$ ) can be expressed numerically in the range:  $0 \leq R < 1$

➤ If  $R = 0.9$ , it means that there is a chance of 90% that the component will perform its function without failure.

➤ If 4 parts fail out of each 1000, then the reliability is:  $R = 1 - \frac{4}{1000} = 0.996$

In the reliability method of design, the distributions of stresses and strengths need to be known. Then the designer makes the design decisions in order to meet a reliability goal.

## **Statistical Considerations ( 9<sup>th</sup> Ed. - CH 20)**

Statistics provide the designer with a method for dealing with characteristics whose values are variable (such as life of a product).

### **Random Variables**

The *total number of elements* (or the entire group of anything) is called the population, while a sample is a part of the population.

When studying the mechanical properties and the characteristics of mechanical elements we *can not study the entire population*, thus we have to take a sample.

A sample chosen *randomly* out of a large population is called a random sample.

Consider that we choose a random sample of steel pieces (say 20), out of a truck load of pieces, and use those to make tensile-test specimens and test them for ultimate strength. Such experiment is called a random experiment and the outcome of the random experiment (the ultimate strength in this case) is called a random variable.

There are two types of random variables:

- Discrete random variable (can take discrete values only, such as when tossing a die)
- Continuous random variable (can take any value, such as the ultimate strength of a specimen)

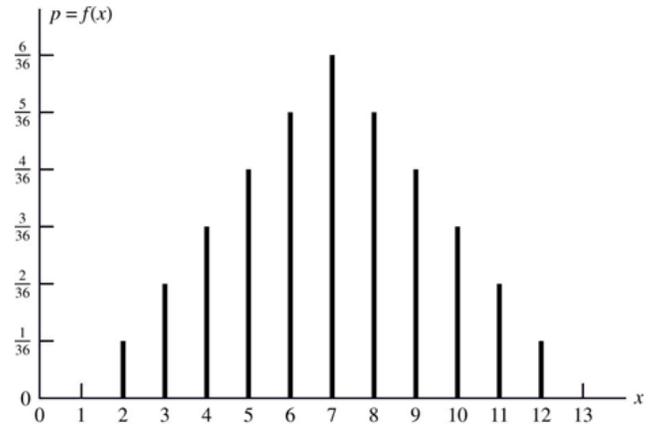
When two dice are tossed (knowing that each can display any number from 1 to 6), there are 36 possible outcomes. Thus the probability of each outcome is  $1/36$ .

If we define a random variable  $x$  as the sum of the numbers obtained on the two dice, the probability of any value of  $x$ , termed as  $p = f(x)$ , is obtained by summing the probabilities of individual events.

For example  $x = 4$  can be obtained from: (2, 2), (1, 3) & (3, 1); therefore  $f(4) = 3/36$

The function  $f(x)$  is usually called the probability or frequency function.

If we plot the probability function  $p = f(x)$  against  $x$  we obtain the frequency distribution diagram which is usually called the histogram.



The probability that  $x$  is less than or equal to a certain value  $x_i$  is called the cumulative probability and it is obtained by summing the probabilities of all  $x$ 's up to and including  $x_i$ .

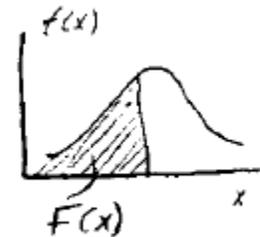
$$F(x_i) = \sum_{x_j \leq x_i} f(x_j)$$

*Example:*

The probability that  $x \leq 4$  is:  $F(4) = f(2) + f(3) + f(4) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \frac{6}{36}$

For continuous random variables, the probability of obtaining an observation less than or equal to a certain value is found as:

$$F(x) = \int_{-\infty}^x f(x) dx$$



### Median, Mean, Variance and Standard Deviation

The Median is the middle element among a group of elements listed in an increasing (or decreasing) order.

*Example:* 10, 13, 14, 18, 20, 25, 26

*Median = 18*

Note: median is not affected by *out-layers*.

The Mean (average) of a sample consisting of  $N$  elements is defined as:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_N}{N} = \frac{1}{N} \sum_{i=1}^N x_i$$

Note: - Median and mean should have reasonably close values (unless the data has some irregular distribution).

- The mean is sensitive to *out-layers*.

Besides the mean we need another kind of measure to tell us about the spread of the data.

The variation of an observation  $x_i$  from the mean is  $(x_i - \bar{x})$ ; however, if we sum the variations of all observations we get zero. Thus, we square them.

This is defined as the Sample Variance

$$S_x^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_N - \bar{x})^2}{N - 1} = \frac{1}{N - 1} \sum_{i=1}^N (x_i - \bar{x})^2$$

The square root of the variance is called the Standard Deviation

$$S_x = \sqrt{\frac{1}{N - 1} \sum_{i=1}^N (x_i - \bar{x})^2} = \sqrt{\frac{(\sum_{i=1}^N x_i^2) - N\bar{x}^2}{N - 1}}$$

Easier for calculation

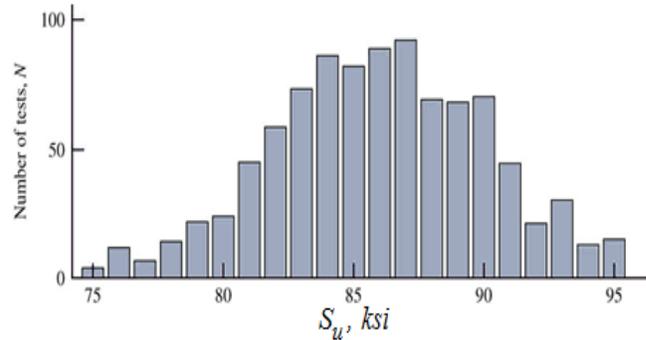
Note: -  $(N - 1)$  is used rather than  $(N)$  because it gives better estimate of the variance especially when the sample size is small.

- When dealing with the entire population, the mean and standard deviation are found using the same equation, but  $(N)$  is used instead of  $(N - 1)$  in the standard deviation equation. Also,  $\bar{x}$  and  $S_x$  are replaced with  $\mu_x$  and  $\hat{\sigma}_x$ .

For continuous variables the distribution is obtained by plotting a discrete frequency histogram.

- The data is divided into ranges or classes and plotted against the frequency (the number of occurrences within each class)

For example the ultimate strengths of a number of samples are plotted as:



**Example:** Nine rods were chosen randomly out of a truck load of 1030 hot-rolled steel rods. The rods were machined into standard tensile-test specimens and tested for ultimate strength. The ultimate strength values obtained from the tests are given in the table in an increasing order.

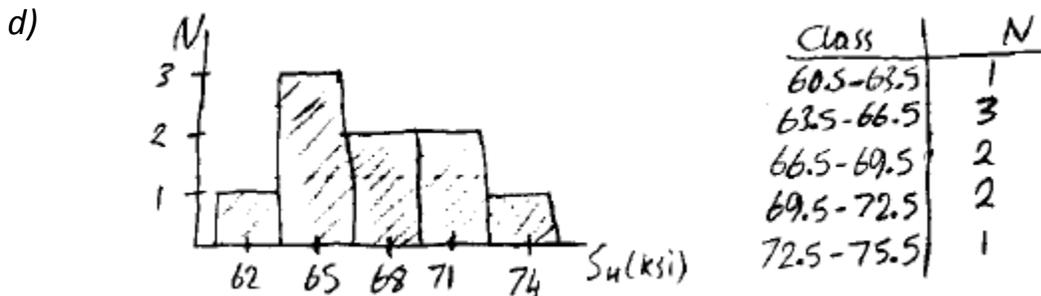
$S_u$ (ksi)	
$x$	$x^2$
62.8	3943.8
64.4	4174.4
65.8	4329.6
66.3	4395.7
68.1	4637.6
69.1	4774.8
69.8	4872.1
71.5	5112.3
74.8	5476
$\Sigma$ 611.8	41689

- Median
- Mean
- Standard deviation
- Plot the histogram of the data with a class width of 3 and the first class centered at 62 ksi

**Solution:**

- Median = 68.1 ksi
- Mean:  $\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i = \frac{1}{9} (611.8) = 67.98$  ksi
- Standard deviation:

$$S_x = \sqrt{\frac{(\sum_{i=1}^N x_i^2) - N\bar{x}^2}{N-1}} = \sqrt{\frac{41689 - 9(67.98)^2}{8}} = 3.54 \text{ ksi}$$



## Probability Distributions

There are a number of standard discrete and continuous probability distributions that are applicable to engineering problems.

The most important continuous distributions are:

- The Gaussian (or normal) distribution
- The Lognormal distribution
- The Uniform distribution
- The Weibull distribution

### The Gaussian (Normal) Distribution

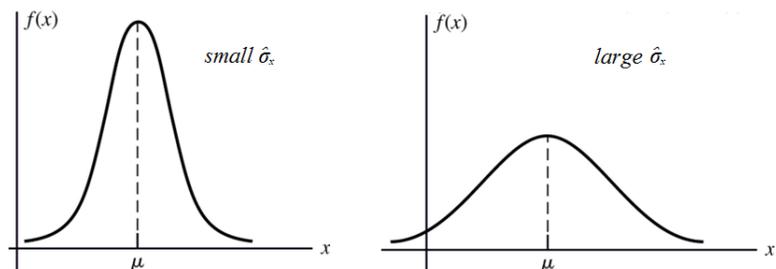
The normal distribution is symmetrical, bell-shaped, and is centered at the mean ( $\mu_x$ ).

The probability density function is expressed in terms of the mean ( $\mu_x$ ) and standard deviation ( $\hat{\sigma}_x$ ) as:

$$f(x) = \frac{1}{\hat{\sigma}_x \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x - \mu_x}{\hat{\sigma}_x} \right)^2 \right]$$

Same as for all distribution curves, the total area under the curve (*total probability*) is equal to one.

The curve gets taller and narrower for small values of  $\hat{\sigma}_x$ , and gets shorter and wider for large  $\hat{\sigma}_x$



For a normally distributed random variable:

- \* 68% of the data is bounded between  $\mu_x - \hat{\sigma}_x$  and  $\mu_x + \hat{\sigma}_x$
- \* 95% of the data is bounded between  $\mu_x - 2\hat{\sigma}_x$  and  $\mu_x + 2\hat{\sigma}_x$
- \* 99.7% of the data is bounded between  $\mu_x - 3\hat{\sigma}_x$  and  $\mu_x + 3\hat{\sigma}_x$

The normal distribution is usually applicable to sizes and dimensions.

### The Lognormal Distribution

If a random variable have the following two characteristics;

- The distribution is asymmetrical about the mean
- The variables have only positive values

Then, the distribution could be lognormal.

The distribution is called lognormal because the logarithms of the variance have a normal distribution.

The lognormal distribution is applicable to things as fatigue life.

### The Uniform Distribution

The uniform distribution is a closed interval distribution where the chance of an observation is the same as that of any other observation.

The probability density function is given as:

$$f(x) = \begin{cases} 1/(b - a) & a \leq x \leq b \\ 0 & x < a, x > b \end{cases}$$



This distribution is applicable to the dimensions of parts manufactured in mass-quantities using an automated operation, where the size gradually increases due to tool wear.

### The Weibull Distribution

The Weibull distribution is a non-classical distribution and it is especially applicable to experimental results (life of rolling-contact bearings for example).

The Weibull is a three parameter distribution that can take many different shapes.

The probability density function is expressed as:

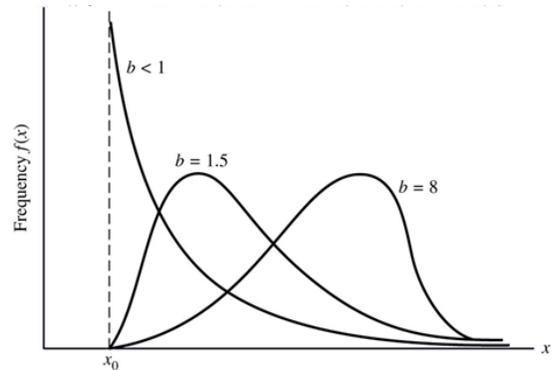
$$f(x) = \begin{cases} \frac{b}{\theta-x_0} \left(\frac{x-x_0}{\theta-x_0}\right)^{b-1} \exp\left[-\left(\frac{x-x_0}{\theta-x_0}\right)^b\right] & x \geq x_0 \geq 0 \\ 0 & x < x_0 \end{cases}$$

The three parameters are:

$x_0$ : minimum guaranteed value of  $x$

$b$ : shape parameter ( $b > 1$ )

$\theta$ : scale value ( $\theta \geq x_0$ ) \*63.2% of the observations are smaller than  $\theta$  \*



To find the Reliability of any given  $x$  value; knowing that the reliability is the complement of the cumulative probability  $R(x) = 1 - F(x)$

Thus, 
$$R(x) = \exp\left[-\left(\frac{x-x_0}{\theta-x_0}\right)^b\right] \quad x \geq x_0 \geq 0$$
 The reliability at any given value of  $x$

- If a specific required reliability is given, solving for  $x$  gives:

$$x = x_0 + (\theta - x_0) \left[\ln \frac{1}{R}\right]^{1/b}$$
Value of  $x$  at certain reliability

The mean and standard deviation for the Weibull distribution are given by:

$$\mu_x = x_0 + (\theta - x_0) \Gamma\left(1 + \frac{1}{b}\right)$$

$$\hat{\sigma}_x = (\theta - x_0) \sqrt{\Gamma\left(1 + \frac{2}{b}\right) - \Gamma^2\left(1 + \frac{1}{b}\right)}$$

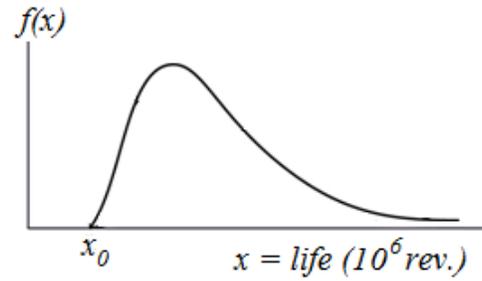
Where  $\Gamma$  is the Gama function and its value is found from Table A-34

- Table A-25 gives the Weibull parameters for yield & ultimate strengths for some metallic alloys (in kPsi units).

**Example:** If the life of rolling-contact bearings (under a fixed radial load) follows a Weibull distribution having the parameters ( $x_0 = 0.02$ ,  $\theta = 4.459$ ,  $b = 1.483$ ) where the unit of  $x$  (life) is one million revolution.

Find:

- Life at a reliability of  $R = 0.9$
- Life at a reliability of  $R = 0.1$
- The median life
- The average (mean) life
- The reliability that a bearing will survive five million revolutions



Solution:

a)  $L = x = x_0 + (\theta - x_0) \left[ \ln \frac{1}{R} \right]^{1/b}$

$L(R = 0.9) = 0.02 + (4.459 - 0.02) \left[ \ln \frac{1}{0.9} \right]^{1/1.483} = 1 \times 10^6 \text{ rev.}$

90% of bearings will live this long

b)  $L(R = 0.1) = 0.02 + (4.459 - 0.02) \left[ \ln \frac{1}{0.1} \right]^{1/1.483} = 7.81 \times 10^6 \text{ rev.}$

10% of bearings will live this long

c) Median life means  $R = 0.5$

$L(R = 0.5) = 0.02 + (4.459 - 0.02) \left[ \ln \frac{1}{0.5} \right]^{1/1.483} = 3.49 \times 10^6 \text{ rev.}$

d)  $\mu_x = x_0 + (\theta - x_0) \Gamma \left( 1 + 1/b \right)$

$\rightarrow \mu_x = 0.02 + (4.459 - 0.02) \underbrace{\Gamma(1 + 1/1.483)}_{\text{from Table A-34}} = 4.03 \times 10^6 \text{ rev.}$

e)  $R(x) = \exp \left[ - \left( \frac{x - x_0}{\theta - x_0} \right)^b \right]$

$R(L = 5) = \exp \left[ - \left( \frac{5 - 0.02}{4.459 - 0.02} \right)^{1.483} \right] = 0.33$

## Dimensions and Tolerances (Sec. 1-14)

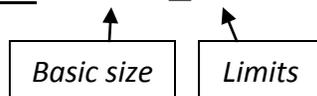
Tolerance is the maximum allowable variation in the size of a part.

- Nominal size: the size we use in speaking of an element.
- Basic size: the exact desired theoretical size.

There are two ways for specifying tolerance:

- ❖ Bilateral tolerance: the variation in both directions from the basic size.

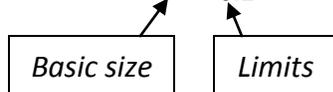
Example:  $100 \pm 0.2 \text{ mm}$



- Usually used for parts that fit besides each other

- ❖ Unilateral tolerance: the basic size is taken as one of the limits and the variation is only in one direction.

Example:  $40^{+0}_{-0.1} \text{ mm}$       or       $40^{+0.1}_{-0} \text{ mm}$



- Usually used for parts that fit inside each other

*Why do we specify a tolerance?*

Because parts can not be manufactured at exact sizes, so we specify the acceptable range of variation.

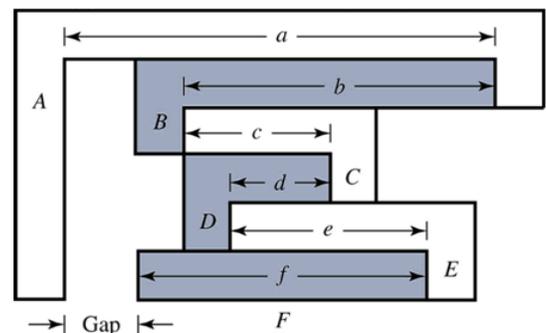
The tighter the tolerances, the higher the cost.

Variations in dimensions of manufactured parts occur at random (usually they follow a normal distribution).

When several parts are assembled, the gap (or interference) depends on both the dimensions and tolerances of the individual parts.

Consider the shown assembly:

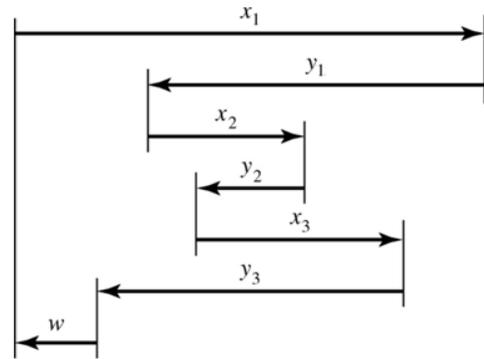
$$\text{Gap} = a - b + c - d + e - f$$



Using  $x$  for ( $\rightarrow$ ) and  $y$  for ( $\leftarrow$ ) we can write;

$$w = (x_1 + x_2 + \dots) - (y_1 + y_2 + \dots)$$

$$\rightarrow \boxed{w = \sum x_i - \sum y_j}$$



- The largest gap  $w_{max}$  occurs when the  $x$  values are the largest possible & the  $y$  values are the smallest possible.

If we call the bilateral tolerance as " $t$ " we get,

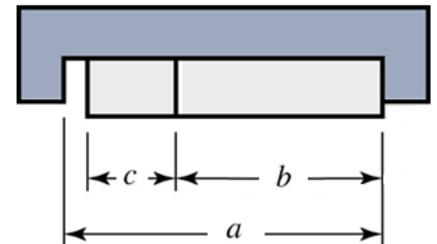
$$\rightarrow w_{max} = \sum(x_i + t_i) - \sum(y_j - t_j) = \boxed{\sum x_i - \sum y_j + \sum_{all} t}$$

- The smallest gap  $w_{min}$  occurs when the  $x$  values are the smallest possible & the  $y$  values are the largest possible.

$$\rightarrow w_{min} = \sum(x_i - t_i) - \sum(y_j + t_j) = \boxed{\sum x_i - \sum y_j - \sum_{all} t}$$

**Example:** Find the maximum and minimum gap then give the size of the gap using bilateral tolerance.

$$\begin{aligned} a &= 500 \pm 1 \text{ mm} \\ b &= 200 \pm 0.4 \text{ mm} \\ c &= 150 \pm 0.1 \text{ mm} \end{aligned}$$



Solution:

$$w_{max} = 500 - 200 - 150 + (1 + 0.4 + 0.1) = 151.5 \text{ mm}$$

$$w_{min} = 500 - 200 - 150 - (1 + 0.4 + 0.1) = 148.5 \text{ mm}$$

$$w = 150 \pm 1.5 \text{ mm}$$

## Limits & Fits (Sec. 7-8)

When a shaft or pin needs to be inserted into a hole, the type of fit is decided based on the intended application.

- For example if the shaft and the hole are to form a pin-joint or a journal bearing , then a clearance fit is needed to allow the shaft to rotate inside the hole.
- Another example when the shaft is inserted into a rolling-contact bearing, an interference fit is used to prevent any slipping between the shaft and the inner-ring of the bearing.

Limits and fits are standardized. Here we will use the *ANSI metric standard*.

- Capital letters refer to the hole and lower case letters refer to the shaft.
- The Basic size (or nominal size) is the size to which limits are assigned and it is the same for both members of the fit.

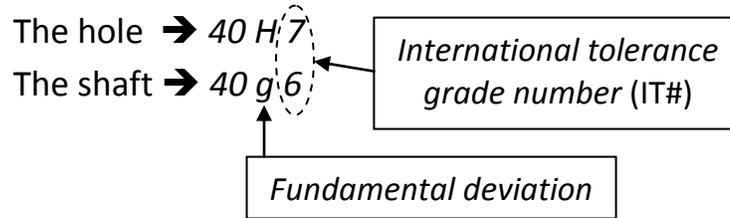
The *ANSI* preferred fits are:

Type of Fit	Sub-type	ISO symbol (Hole Basis)
Clearance	Lose running fit	H11/c11
	Free running fit	H9/d9
	Close running fit	H8/f7
	Sliding fit	H7/g6
	Locational clearance fit	H7/h6
Transition	Locational transition fit	H7/k6, H7/n6
Interference	Locational interference fit	H7/p6
	Medium drive fit	H7/s6
	Force fit	H7/u6

*Full descriptions for each type are given in Table 7-9*

Once the fit is decided according to the intended use, the fit is described by the basic size and the fit symbol.

Example: A joint with basic size of 40 mm having a sliding fit → 40 H7/g6



The maximum and minimum diameters for the hole and shaft are found as follows:

Hole:  $D_{max} = D + \Delta D$       &       $D_{min} = D$

Shaft:

- For clearance fits: *c, d, f, g, & h*

$$d_{max} = d + \delta_F \quad \& \quad d_{min} = d + \delta_F - \Delta d$$

- For interference fits: *k, n, p, s, & u*

$$d_{max} = d + \delta_F + \Delta d \quad \& \quad d_{min} = d + \delta_F$$

Where,

$D$  : basic size of the hole → the same

$d$  : basic size of the shaft → the same

$\delta_F$ : fundamental deviation → found from Table A-12 according to the fundamental deviation letter

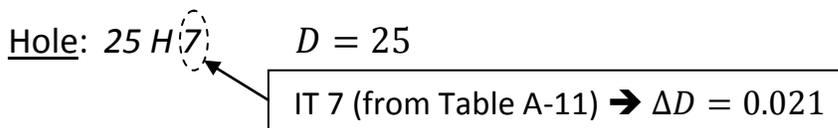
$\Delta D$ : tolerance grade for the hole → found from Table A-11 according to the IT#

$\Delta d$ : tolerance grade for the shaft → found from Table A-11 according to the IT#

**Example**: Find the shaft and hole dimensions for a sliding fit with 25mm basic size then specify the dimensions of both elements using unilateral tolerance.

Solution:

ISO symbol: 25 H7/g6



$$D_{max} = D + \Delta D = 25.021 \text{ mm}$$

$$D_{min} = D = 25 \text{ mm}$$

Shaft:  $25 \begin{matrix} g \\ 6 \end{matrix}$

$$d = 25$$

IT 6: (from Table A-11)  $\rightarrow \Delta d = 0.013$

(from Table A-12)  $\rightarrow \delta_F = -0.007$

$$\text{Sliding fit: } g \rightarrow \begin{cases} d_{max} = d + \delta_F = 25 + (-0.007) = 24.993 \text{ mm} \\ d_{min} = d + \delta_F - \Delta d = 25 + (-0.007) - 0.013 = 24.98 \text{ mm} \end{cases}$$

Hole:  $25_{-0}^{+0.021} \text{ mm}$

Shaft:  $24.993_{-0.013}^{+0} \text{ mm}$

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