Chapter 3
Vectors
by
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Vectors

Physics Department-The Hashemite University

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Vectors

Vector quantities

- Physical quantities that have both numerical and directional properties

Mathematical operations of vectors in this chapter

- Addition
- Subtraction

Coordinate Systems

Used to describe the position of a point in space

Common coordinate systems are:

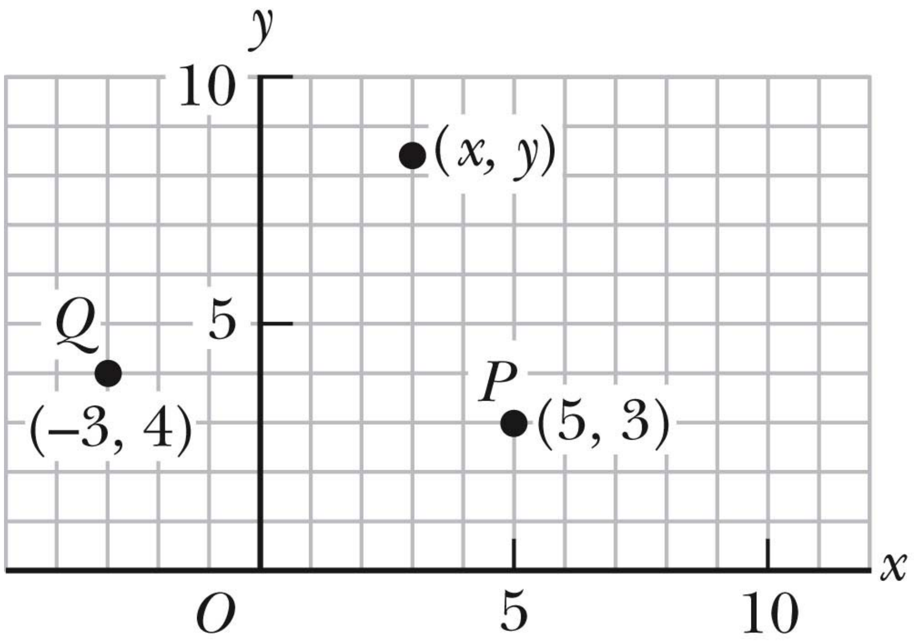
- Cartesian
- Polar

Cartesian Coordinate System

Also called rectangular coordinate system

x- and *y*- axes intersect at the origin

Points are labeled (*x*,*y*)



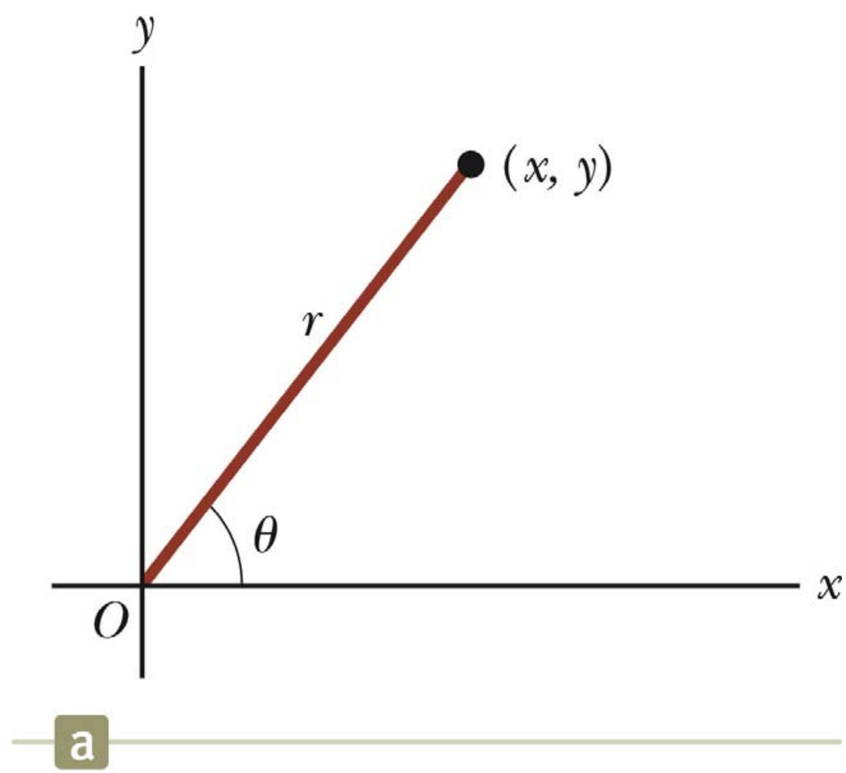
Polar Coordinate System

Origin and reference line are noted

Point is distance *r* from the origin in the direction of angle θ, ccw from reference line

- The reference line is often the x-axis.

Points are labeled (r, θ)



Polar to Cartesian Coordinates

Based on forming a right triangle from r and θ

$x = r\cos\theta$

$y = r\sin\theta$

If the Cartesian coordinates are known:

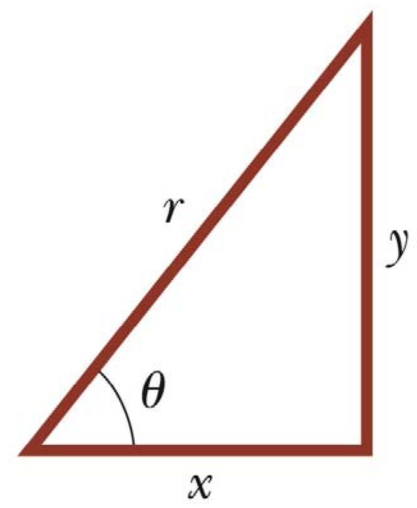
$$\tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



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Example 3.1

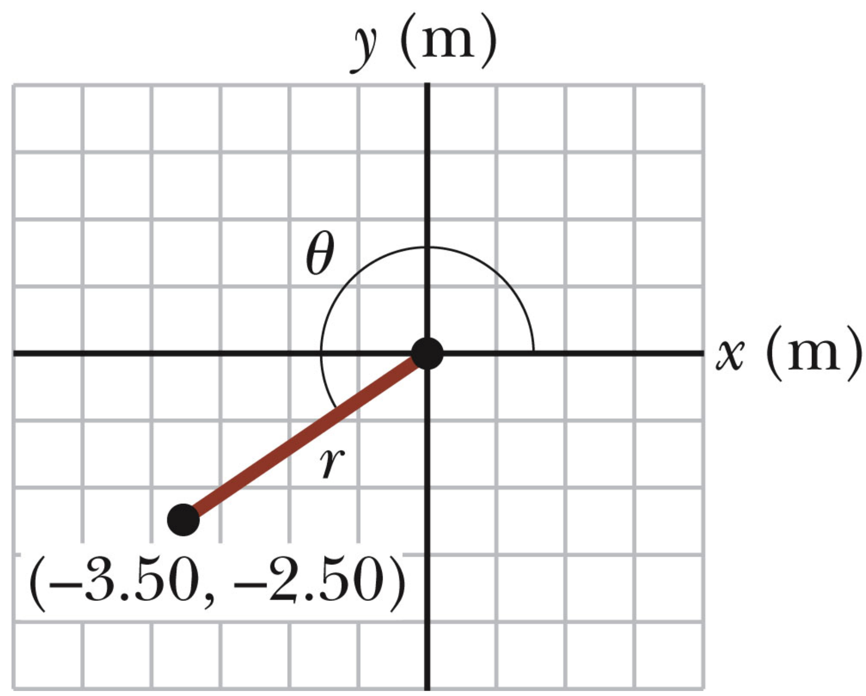
The Cartesian coordinates of a point in the *xy* plane are (*x*,*y*) = (-3.50, -2.50) m, as shown in the figure. Find the polar coordinates of this point.

Solution: From Equation 3.4,

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2}$$
$$= 4.30\text{ m}$$

and from Equation 3.3,

$$\tan\theta = \frac{y}{x} = \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$
$$\theta = 216^\circ \quad \text{(signs give quadrant)}$$



Vectors and Scalars

A ***scalar quantity*** is completely specified by a single value with an appropriate unit and has no direction.

- Many are always positive
- Some may be positive or negative
- Rules for ordinary arithmetic are used to manipulate scalar quantities.

A ***vector quantity*** is completely described by a number and appropriate units plus a direction.

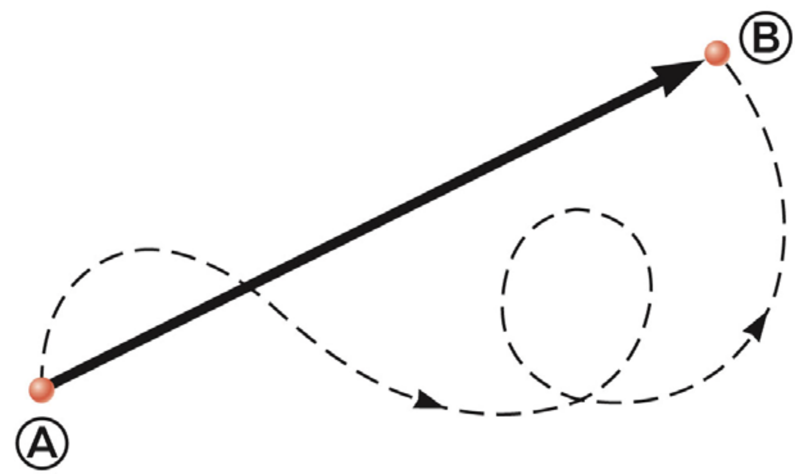
Vector Example

A particle travels from A to B along the path shown by the broken line.

- This is the ***distance*** traveled and is a scalar.

The ***displacement*** is the solid line from A to B

- The displacement is independent of the path taken between the two points.
- Displacement is a vector.



Vector Notation

Text uses bold with arrow to denote a vector: $\vec{\mathbf{A}}$

Also used for printing is simple bold print: **A**

When dealing with just the magnitude of a vector in print, an italic letter will be used: A or $|\vec{\mathbf{A}}|$

- The magnitude of the vector has physical units.
- The magnitude of a vector is always a positive number.

When handwritten, use an arrow: $\vec{A}$

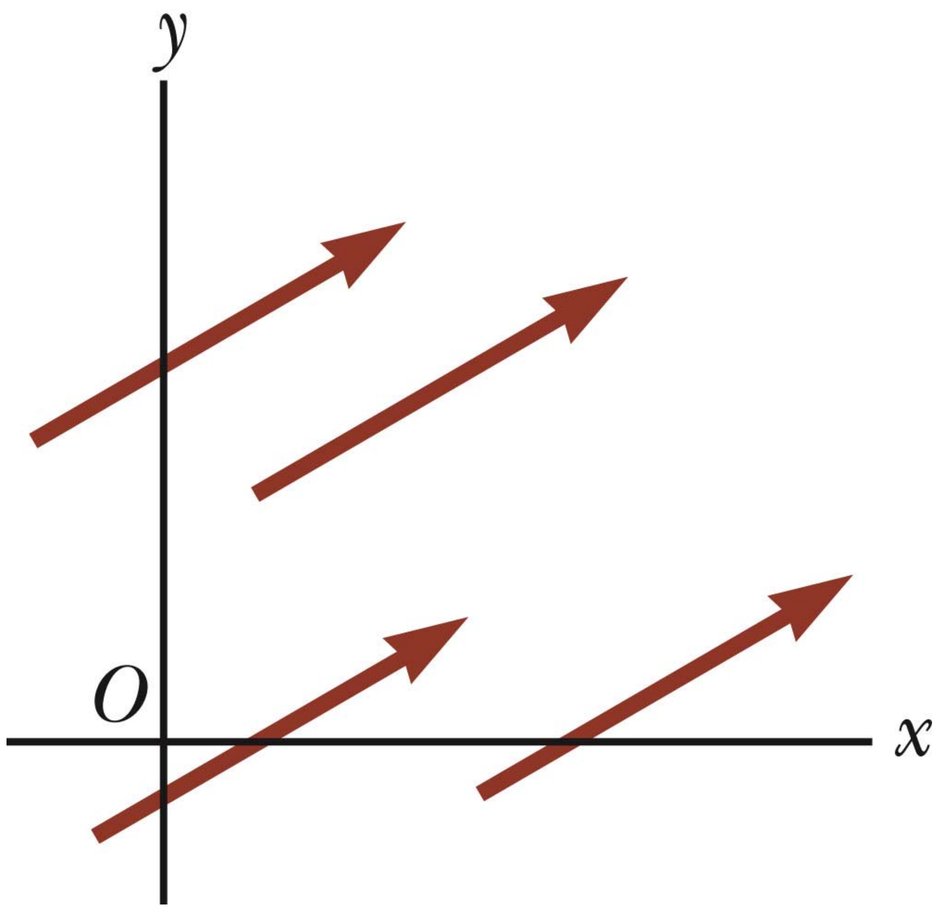
Equality of Two Vectors

Two vectors are ***equal*** if they have the same magnitude and the same direction.

$\vec{\mathbf{A}} = \vec{\mathbf{B}}$ if $A = B$ and they point along parallel lines

All of the vectors shown are equal.

Allows a vector to be moved to a position parallel to itself



Adding Vectors

Vector addition is very different from adding scalar quantities.

When adding vectors, their directions must be taken into account.

Units must be the same

Graphical Methods

- Use scale drawings

Algebraic Methods

- More convenient

Adding Vectors Graphically

Choose a scale.

Draw the first vector, $\vec{\mathbf{A}}$, with the appropriate length and in the direction specified, with respect to a coordinate system.

Draw the next vector with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector $\vec{\mathbf{A}}$ and parallel to the coordinate system used for $\vec{\mathbf{A}}$.

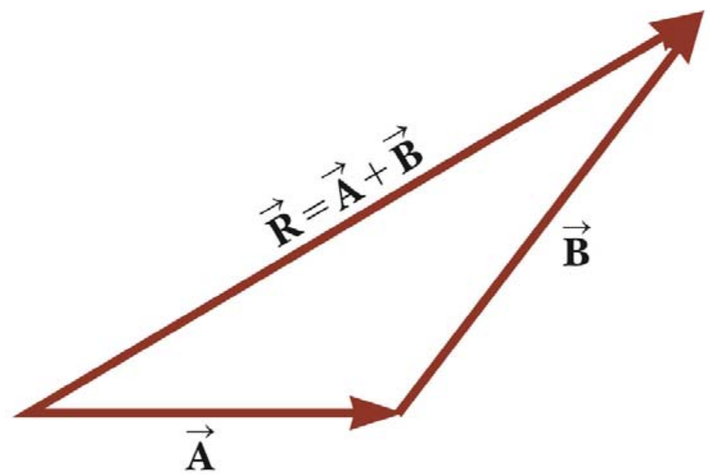
Adding Vectors Graphically, cont.

Continue drawing the vectors "tip-to-tail" or "head-to-tail".

The resultant is drawn from the origin of the first vector to the end of the last vector.

Measure the length of the resultant and its angle.

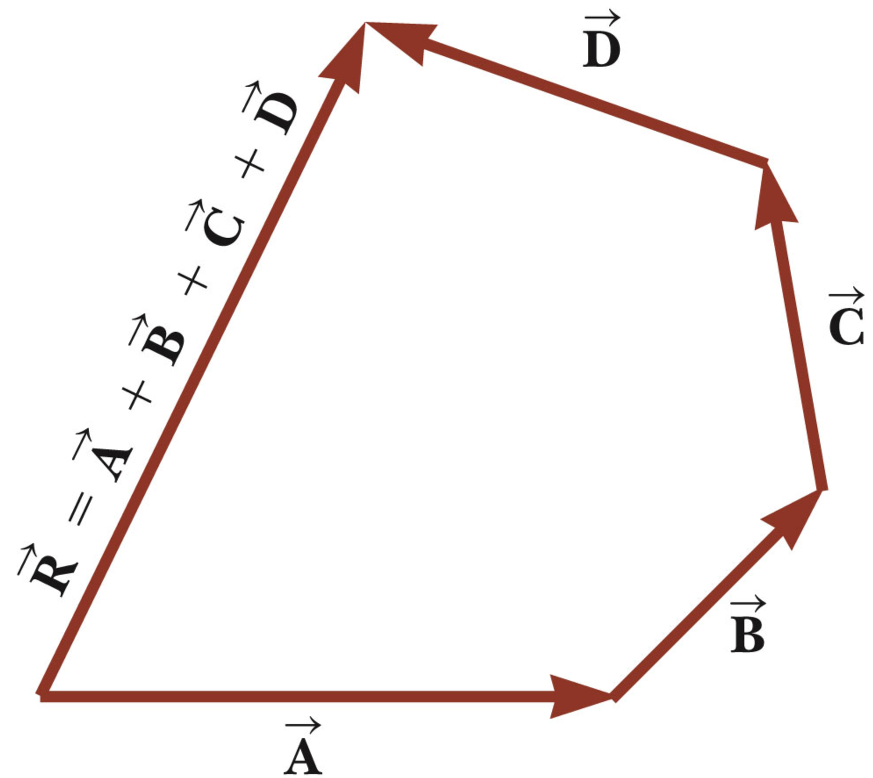
- Use the scale factor to convert length to actual magnitude.



Adding Vectors Graphically, final

When you have many vectors, just keep repeating the process until all are included.

The resultant is still drawn from the tail of the first vector to the tip of the last vector.

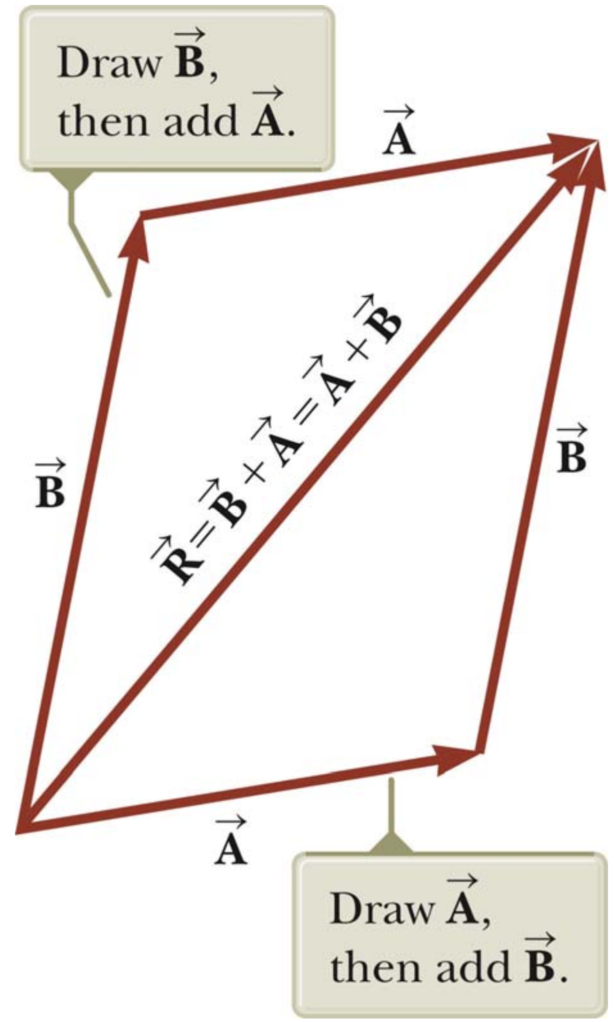


Adding Vectors, Rules

When two vectors are added, the sum is independent of the order of the addition.

- This is the ***Commutative Law of Addition.***

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$$



Adding Vectors, Rules cont.

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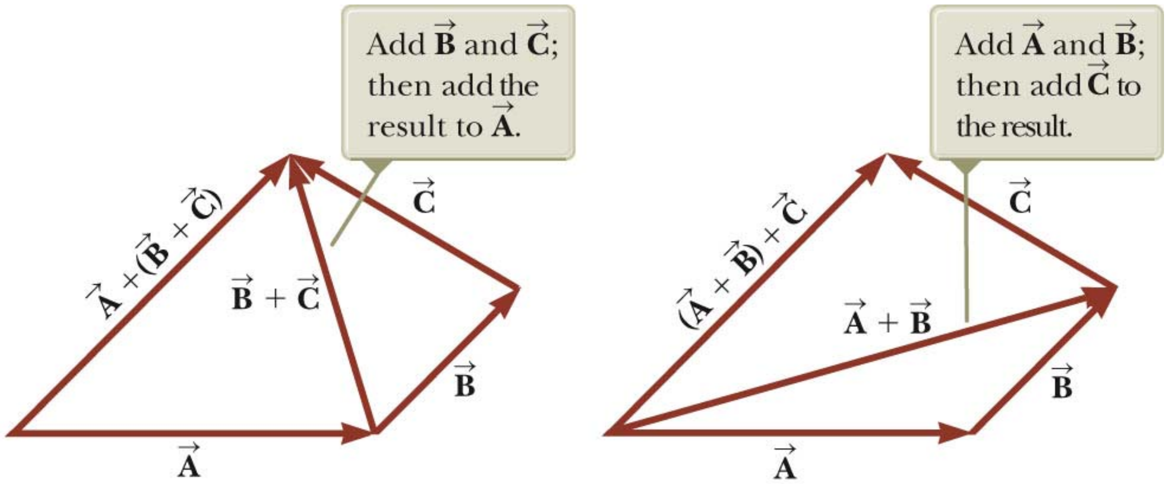
When adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped.

- This is called the ***Associative Property of Addition.***

$$\vec{\mathbf{A}} + \left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right) = \left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right) + \vec{\mathbf{C}}$$



Adding Vectors, Rules final

When adding vectors, all of the vectors must have the same units.

All of the vectors must be of the same type of quantity.

- For example, you cannot add a displacement to a velocity.

Negative of a Vector

The negative of a vector is defined as the vector that, when added to the original vector, gives a resultant of zero.

- Represented as $-\vec{\mathbf{A}}$
- $\vec{\mathbf{A}} + \left(-\vec{\mathbf{A}}\right) = 0$

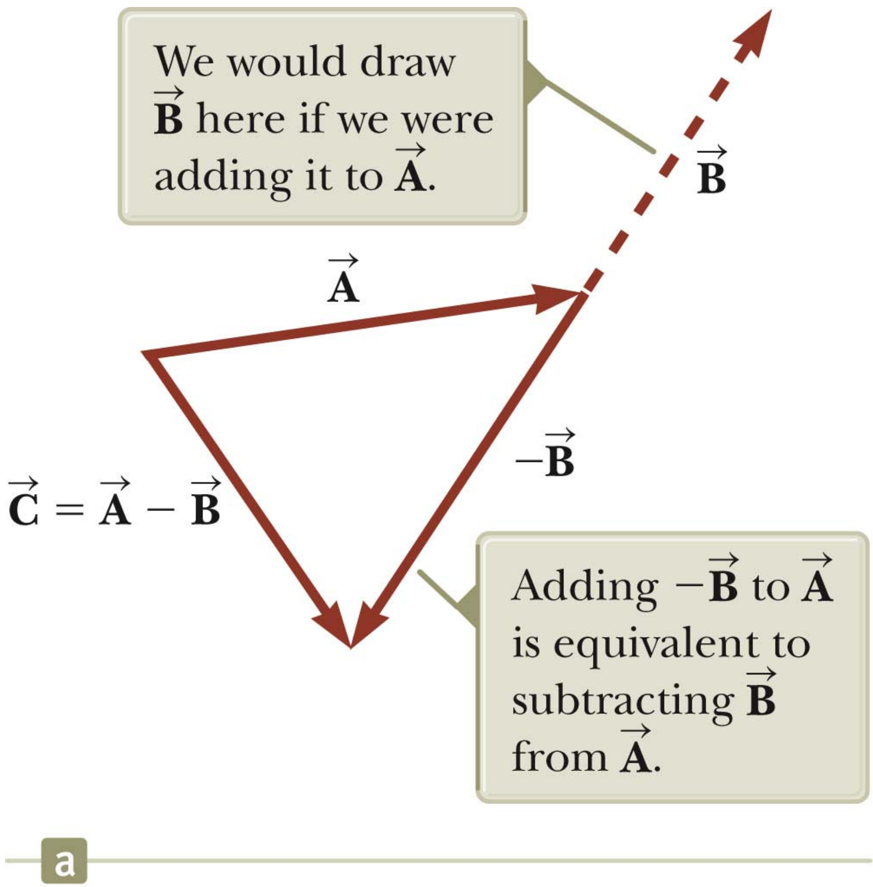
The negative of the vector will have the same magnitude, but point in the opposite direction.

Subtracting Vectors

Special case of vector addition:

If $\vec{\mathbf{A}} - \vec{\mathbf{B}}$, then use $\vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$

Continue with standard vector addition procedure.

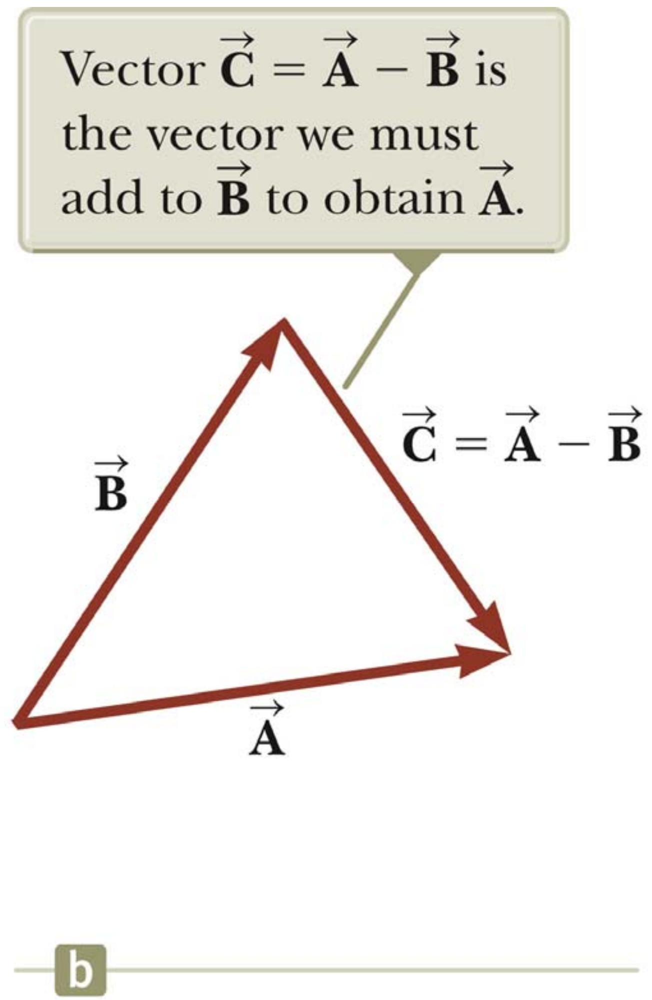


Subtracting Vectors, Method 2

Another way to look at subtraction is to find the vector that, added to the second vector gives you the first vector.

$$\vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right) = \vec{\mathbf{C}}$$

- As shown, the resultant vector points from the tip of the second to the tip of the first.



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Multiplying or Dividing a Vector by a Scalar

The result of the multiplication or division of a vector by a scalar is a vector.

The magnitude of the vector is multiplied or divided by the scalar.

If the scalar is positive, the direction of the result is the same as of the original vector.

If the scalar is negative, the direction of the result is opposite that of the original vector.

Component Method of Adding Vectors

Graphical addition is not recommended when:

- High accuracy is required
- If you have a three-dimensional problem

Component method is an alternative method

- It uses projections of vectors along coordinate axes

Components of a Vector, Introduction

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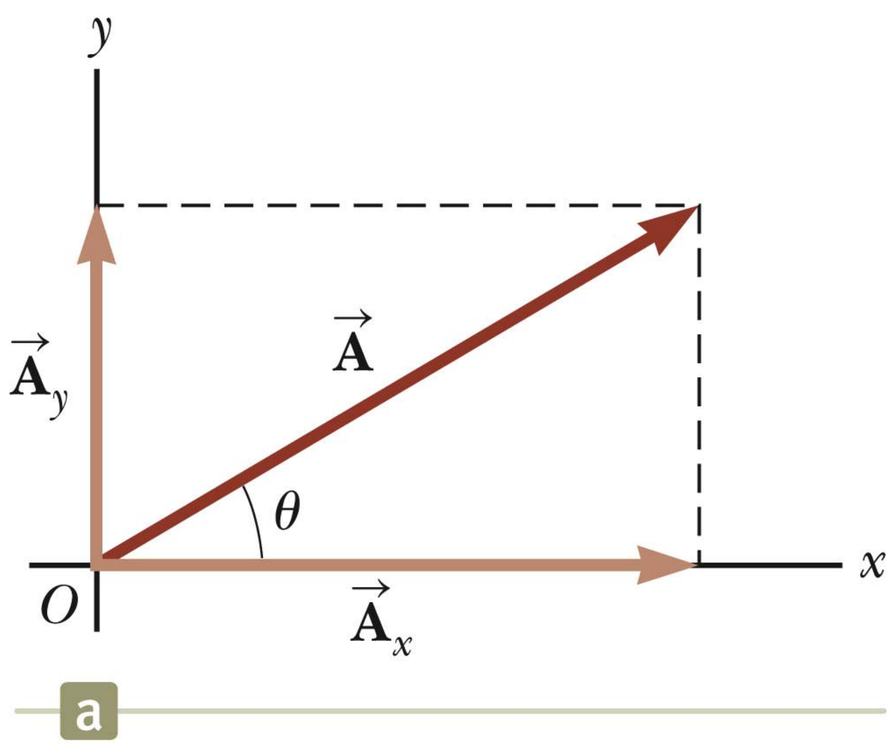
A **component** is a projection of a vector along an axis.

- Any vector can be completely described by its components.

It is useful to use **rectangular components.**

- These are the projections of the vector along the x- and y-axes.



Vector Component Terminology

$\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ are the ***component vectors*** of $\vec{\mathbf{A}}$.

- They are vectors and follow all the rules for vectors.

A_x and A_y are scalars, and will be referred to as the ***components*** of $\vec{\mathbf{A}}$.

Components of a Vector

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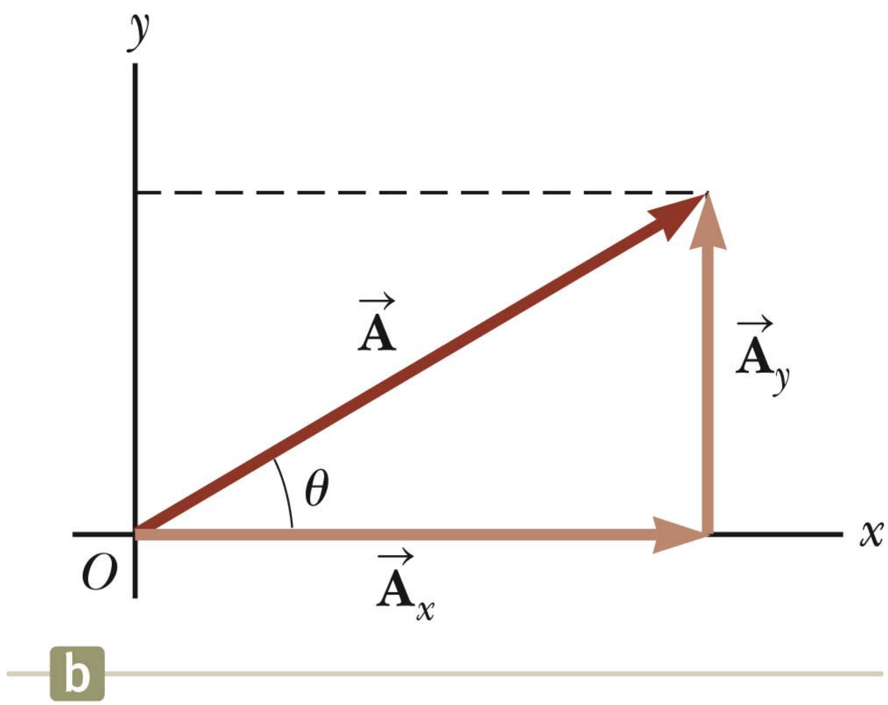
Assume you are given a vector $\vec{\mathbf{A}}$

It can be expressed in terms of two other vectors, $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$$

These three vectors form a right triangle.

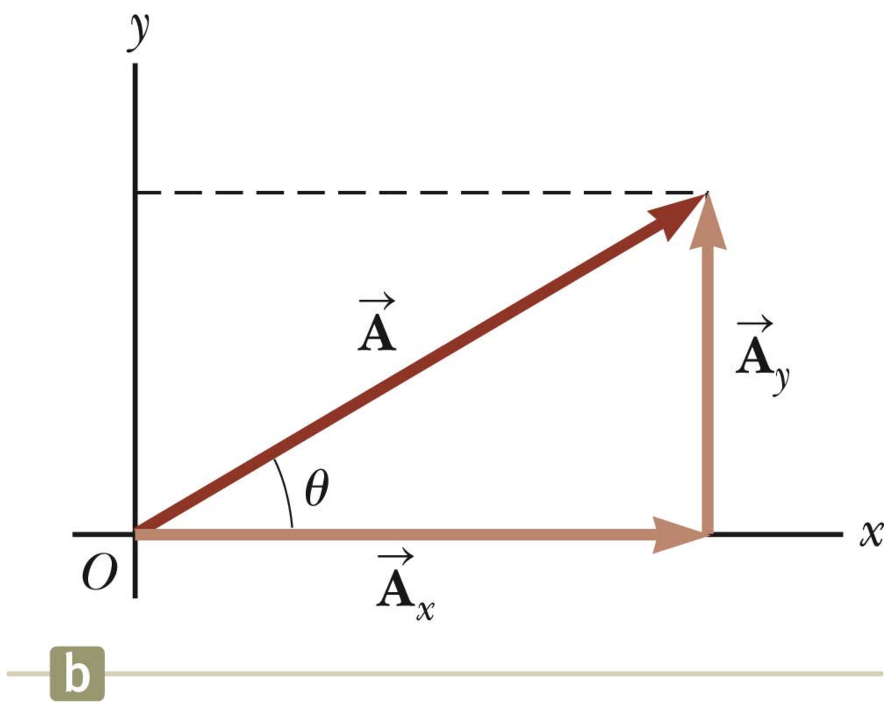


Components of a Vector, 2

The *y*-component is moved to the end of the *x*-component.

This is due to the fact that any vector can be moved parallel to itself without being affected.

- This completes the triangle.



Components of a Vector, 3

The x-component of a vector is the projection along the x-axis.

$$A_x = A\cos\theta$$

The y-component of a vector is the projection along the y-axis.

$$A_y = A\sin\theta$$

This assumes the angle θ is measured with respect to the x-axis.

- If not, do not use these equations, use the sides of the triangle directly.

Components of a Vector, 4

The components are the legs of the right triangle whose hypotenuse is the length of *A.*

- $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1} \frac{A_y}{A_x}$
- May still have to find θ with respect to the positive *x*-axis

In a problem, a vector may be specified by its components or its magnitude and direction.

Components of a Vector, final

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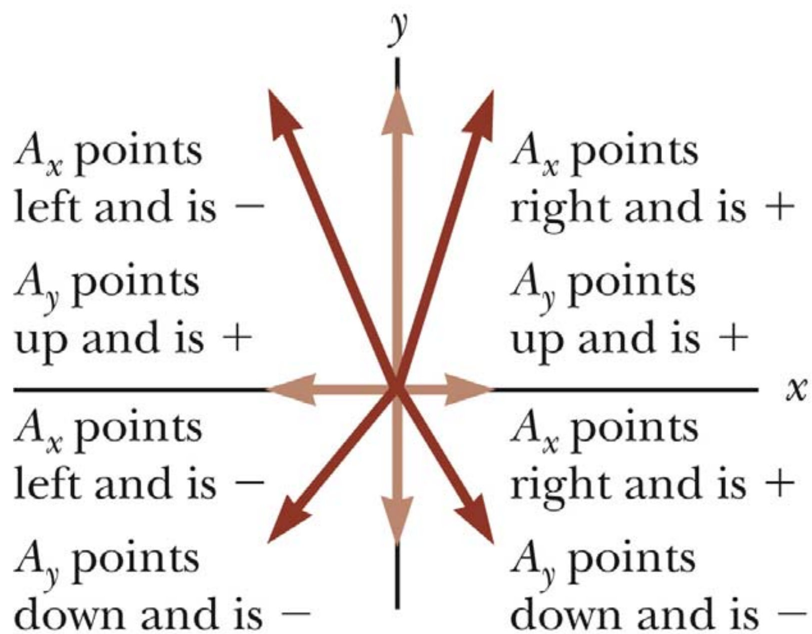
The components can be positive or negative and will have the same units as the original vector.

The signs of the components will depend on the angle.



Unit Vectors

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A ***unit vector*** is a dimensionless vector with a magnitude of exactly 1.

Unit vectors are used to specify a direction and have no other physical significance.

Unit Vectors, cont.

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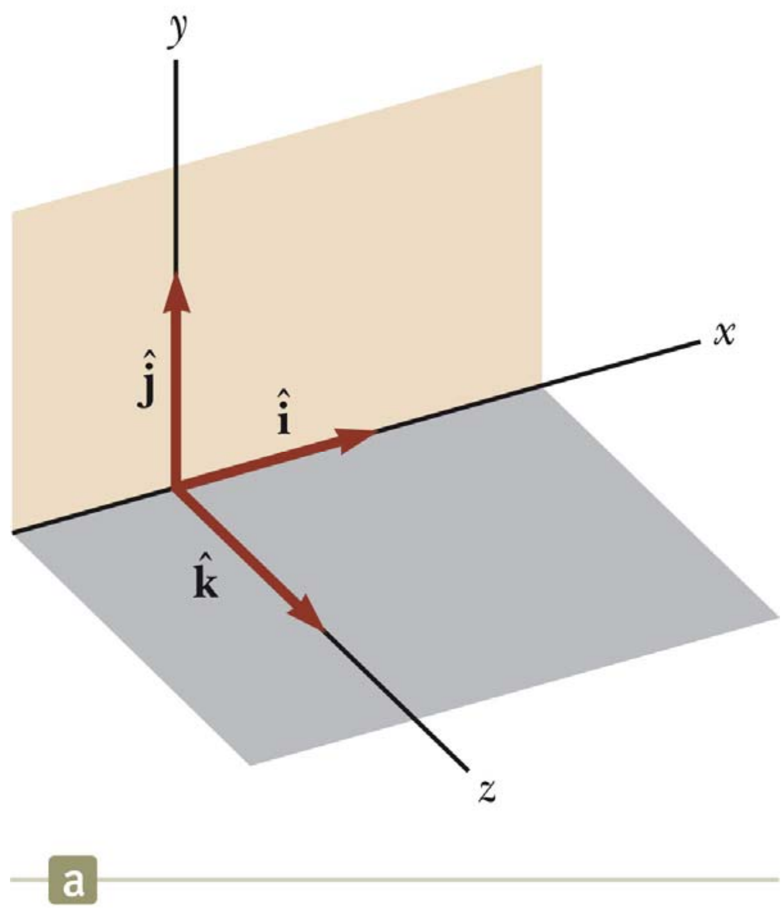
The symbols $\hat{\mathrm{i}}, \hat{\mathrm{j}}$, and $\hat{\mathrm{k}}$

represent unit vectors

They form a set of mutually perpendicular vectors in a right-handed coordinate system

The magnitude of each unit vector is 1

$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$



Unit Vectors in Vector Notation

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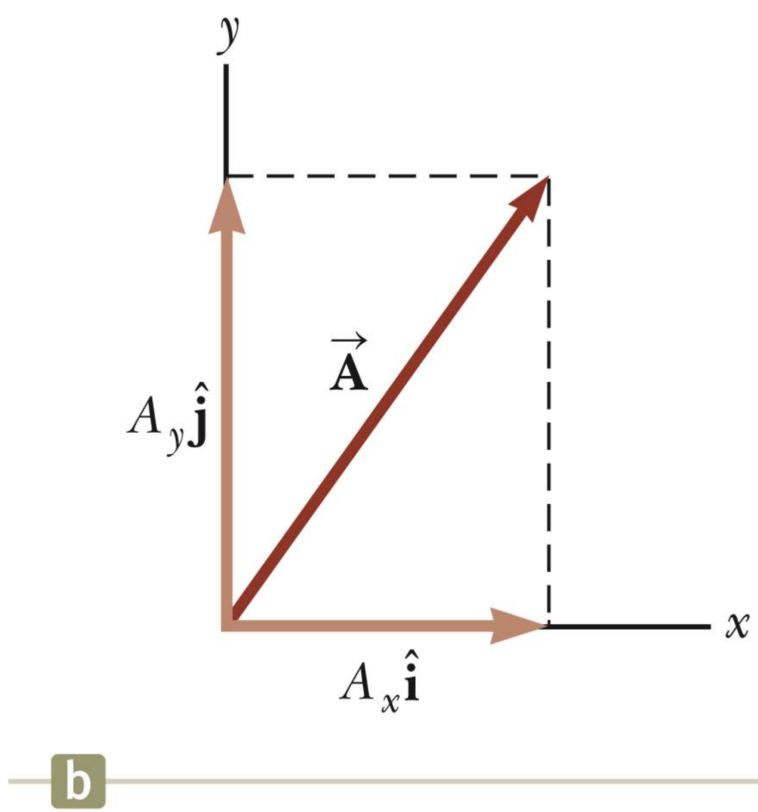
$\mathbf{A_x}$ is the same as $A_x\hat{\mathbf{i}}$ and $\mathbf{A_y}$ is the same as $A_y\,\hat{\mathbf{j}}$ etc.

The complete vector can be expressed as:

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$$



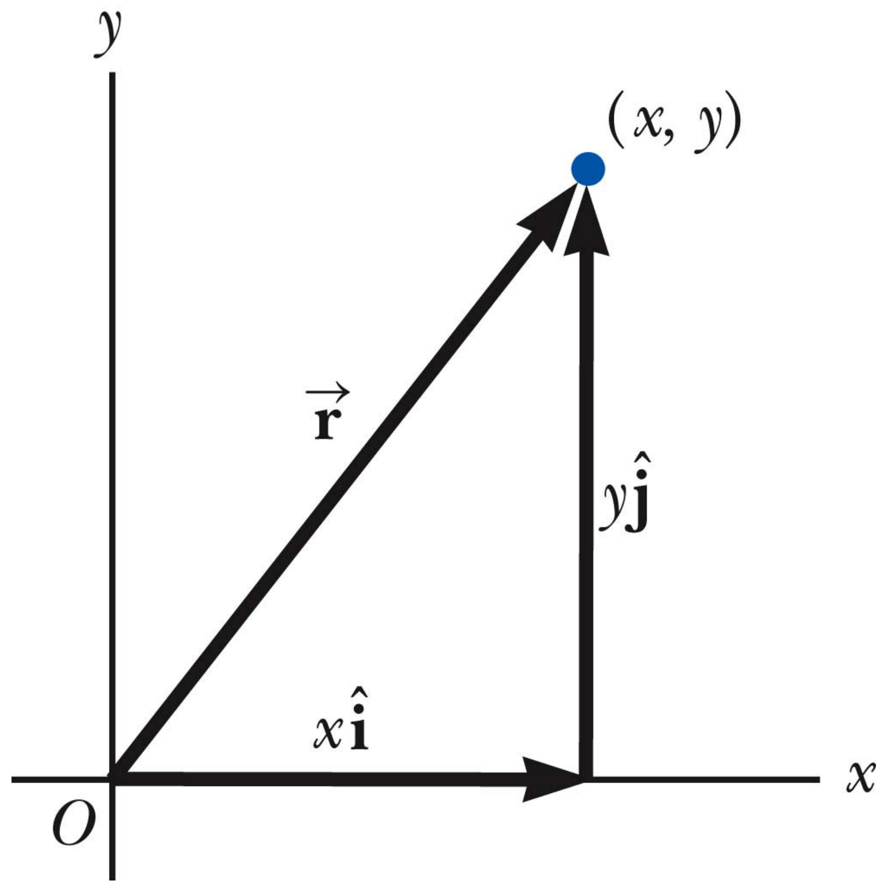
Position Vector, Example

A point lies in the *xy* plane and has Cartesian coordinates of (x, y).

The point can be specified by the position vector.

$$\hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

This gives the components of the vector and its coordinates.



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Adding Vectors Using Unit Vectors

Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$

Then

$$\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$$

So $R_x = A_x + B_x$ and $R_y = A_y + B_y$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

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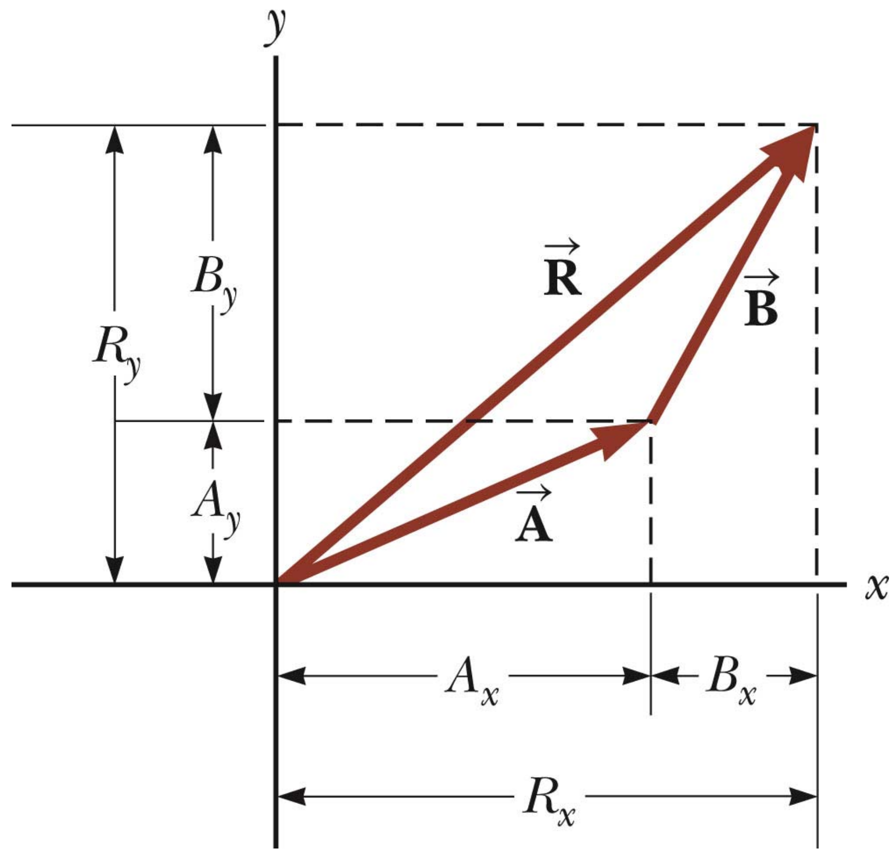
Adding Vectors with Unit Vectors

Note the relationships among the components of the resultant and the components of the original vectors.

$R_x = A_x + B_x$

$R_y = A_y + B_y$



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Three-Dimensional Extension

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Using $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$

Then

$$\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}} + \left(A_z + B_z\right)\hat{\mathbf{k}}$$

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} + R_z\hat{\mathbf{k}}$$

So $R_x = A_x + B_x$, $R_y = A_y + B_y$, and $R_z = A_z + B_z$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \qquad \theta_x = \cos^{-1}\frac{R_x}{R}, \textit{ etc.}$$

Adding Three or More Vectors

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The same method can be extended to adding three or more vectors.

Assume

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$$

And

$$\vec{\mathbf{R}} = \left(A_x + B_x + C_x\right)\hat{\mathbf{i}} + \left(A_y + B_y + C_y\right)\hat{\mathbf{j}}$$
$$+ \left(A_z + B_z + C_z\right)\hat{\mathbf{k}}$$

Example 3.5 – Taking a Hike

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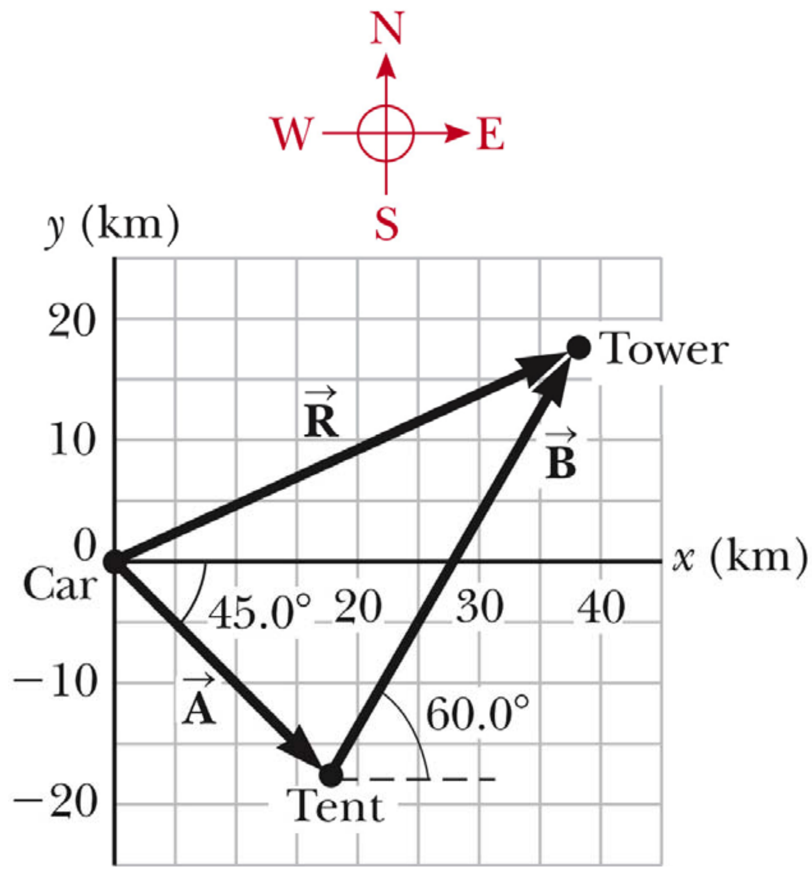
A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

Example 3.5 – Solution, Conceptualize and Categorize

- *Conceptualize* the problem by drawing a sketch as in the figure.
- Denote the displacement vectors on the first and second days by $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ respectively.
- Use the car as the origin of coordinates.
- The vectors are shown in the figure.
- Drawing the resultant $\vec{\mathbf{R}}$, we can now *categorize* this problem as an addition of two vectors.



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Example 3.5 – Solution, Analysis

Analyze this problem by using our new knowledge of vector components.

The first displacement has a magnitude of 25.0 km and is directed 45.0° below the positive *x* axis.

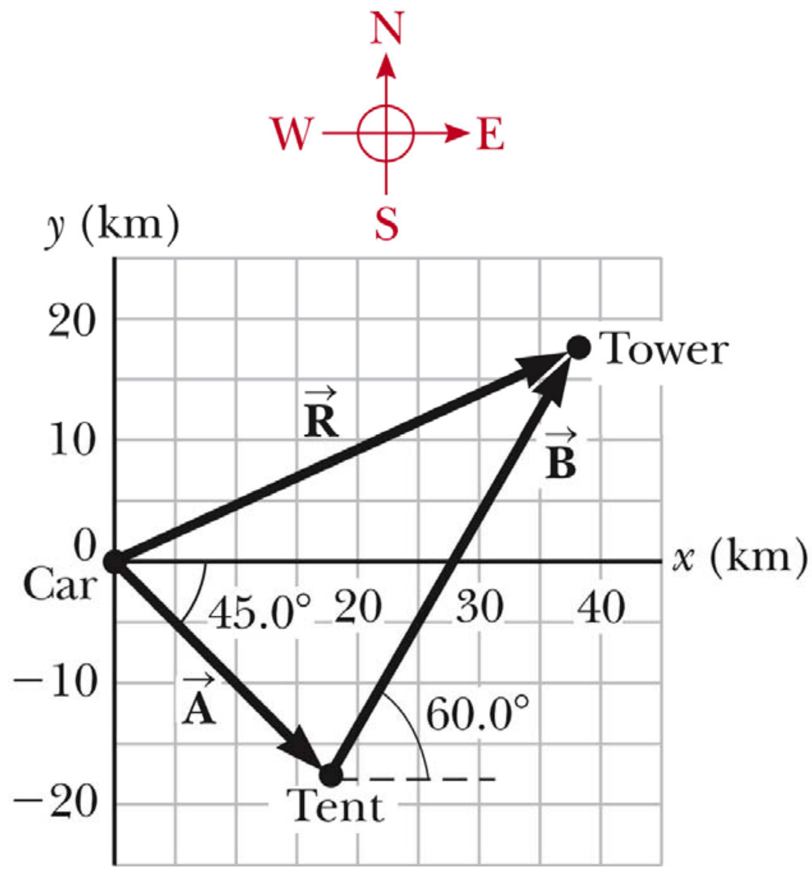
Its components are:

$A_x = A\cos(-45.0°) =$

$(25.0 \text{ km})(0.707) = 17.7 \text{ km}$

$A_y = A\sin(-45.0°)$

$= (25.0 \text{ km})(-0.707) = -17.7 \text{ km}$



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Example 3.5 – Solution, Analysis 2

The second displacement has a magnitude of 40.0 km and is 60.0° north of east.

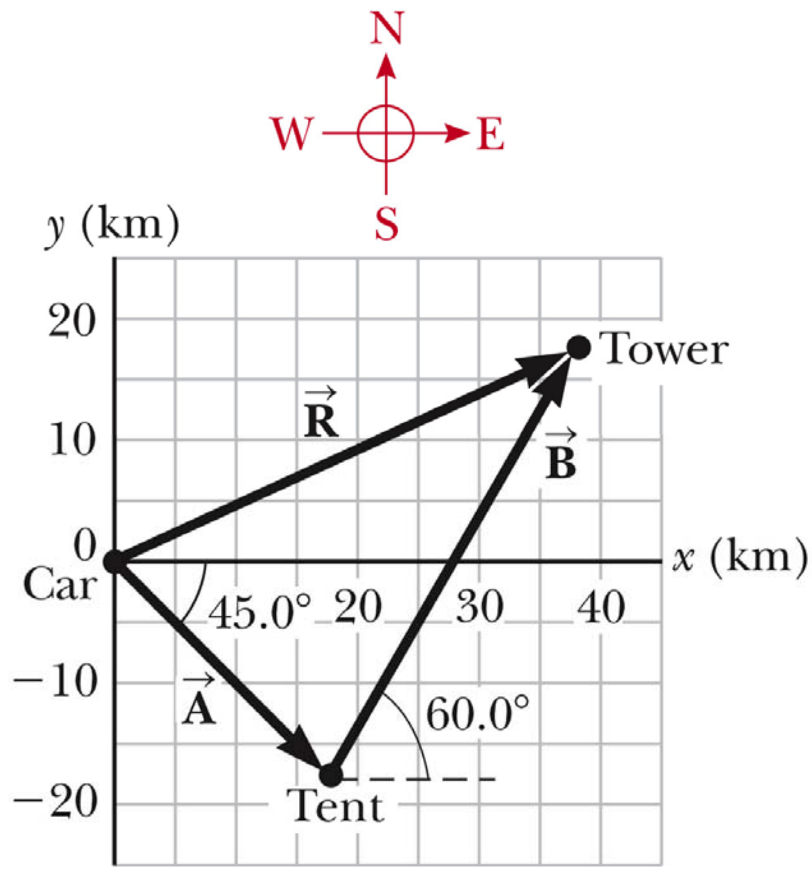
Its components are:

$B_x = B\cos 60.0° =$

$(40.0\text{ km})(0.500) = 20.0\text{ km}$

$B_y = B\sin 60.0°$

$= (40.0\text{ km})(0.866) = 34.6\text{ km}$



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Example 3.5 – Solution, Analysis 3

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The negative value of A_y indicates that the hiker walks in the negative y direction on the first day.

The signs of A_x and A_y also are evident from the figure.

The signs of the components of B are also confirmed by the diagram.

Example 3.5 – Analysis, 4

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Determine the components of the hiker's resultant displacement for the trip.

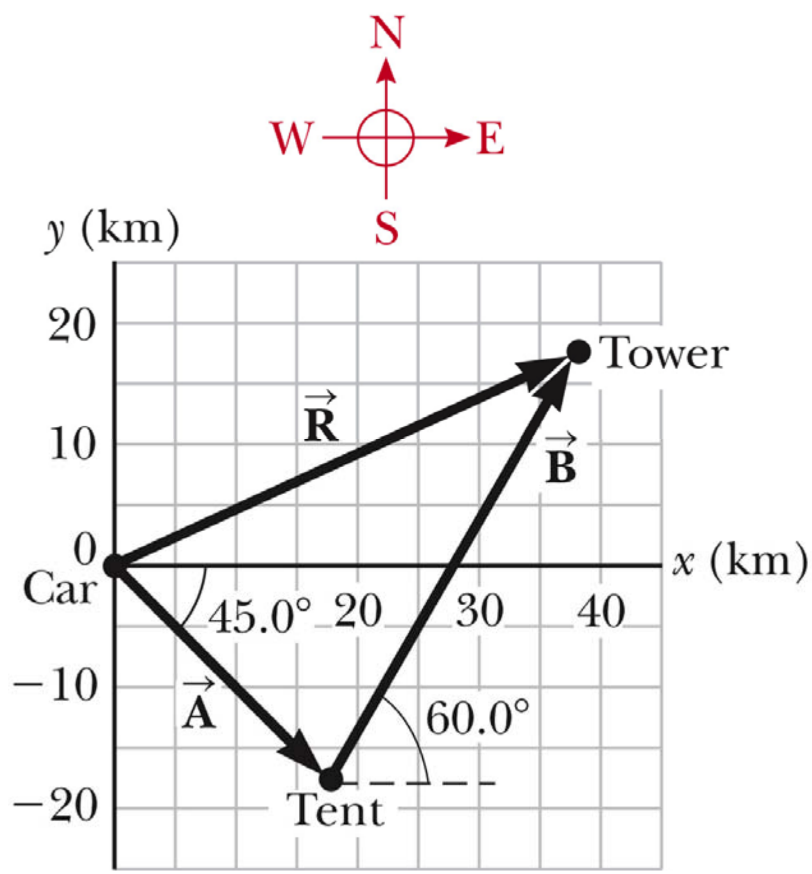
- Find an expression for the resultant in terms of unit vectors.

The resultant displacement for the trip has components given by

- $R_x = A_x + B_x$ = 17.7 km + 20.0 km = 37.7 km
- $R_y = A_y + B_y$ = -17.7 km + 34.6 km = 16.9 km

In unit vector form

$\vec{R} = (37.7\hat{\mathbf{i}} + 16.9\hat{\mathbf{j}})$ km



Example 3.5 – Solution, Finalize

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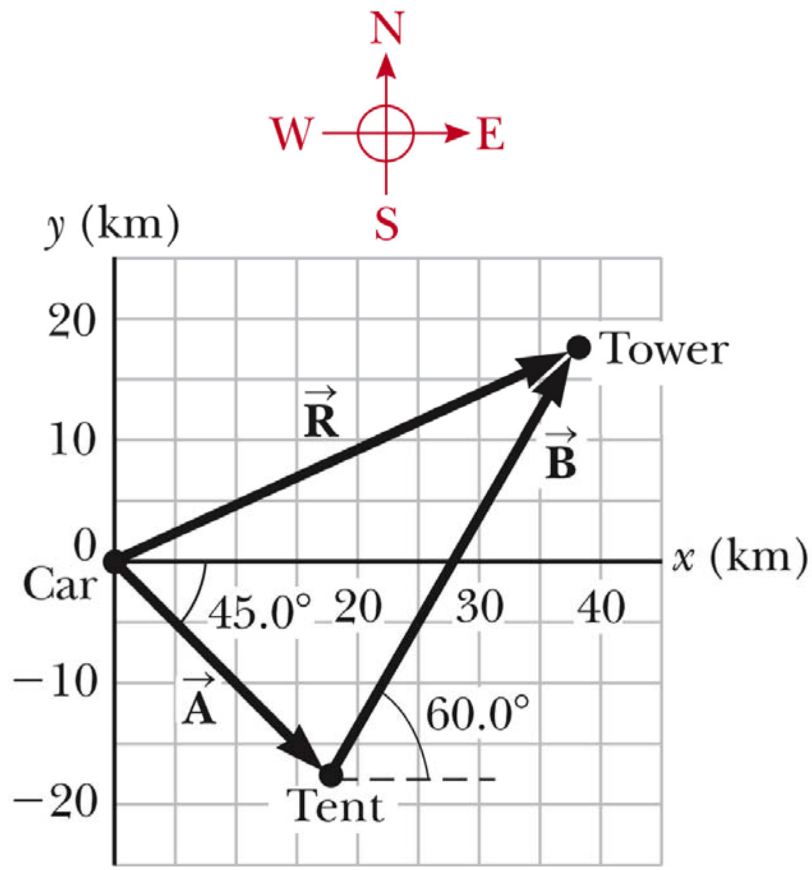
The resultant vector has a magnitude of 41.3 km and is directed 24.1° north of east.

The units of $\vec{R}$ are km, which is reasonable for a displacement.

From the graphical representation, estimate that the final position of the hiker is at about (38 km, 17 km) which is consistent with the components of the resultant.



Example 3.5 – Solution, Finalize, cont.

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Both components of the resultant are positive, putting the final position in the first quadrant of the coordinate system.

- This is also consistent with the figure.

Scalar Product of Two Vectors

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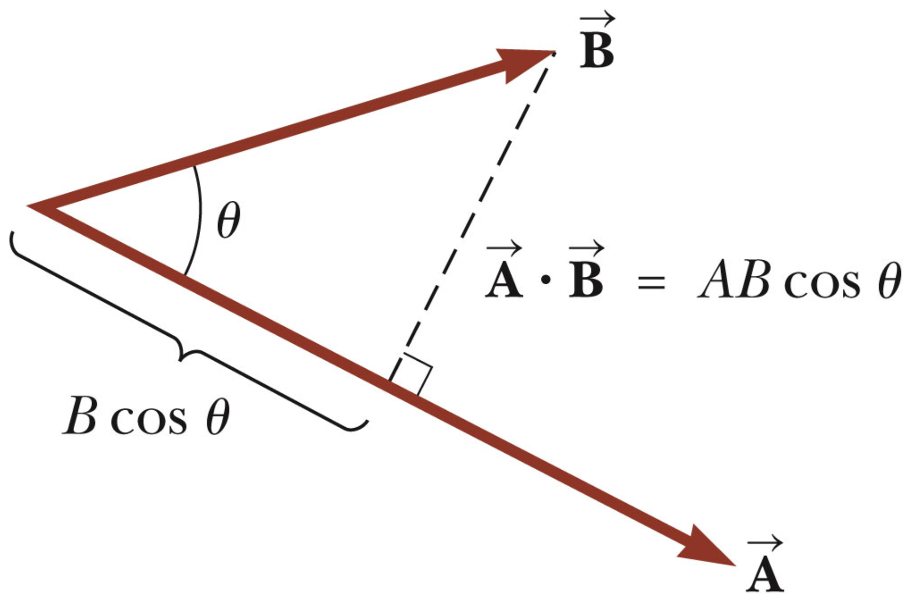
The scalar product of two vectors is written as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$.

- It is also called the dot product.

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv A\,B\cos\theta$$

- θ is the angle *between* A and B



Scalar Product, cont

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The scalar product is commutative.

- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$

The scalar product obeys the distributive law of multiplication.

- $\vec{\mathbf{A}} \cdot \left(\vec{\mathbf{B}} + \vec{\mathbf{C}} \right) = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$

Dot Products of Unit Vectors

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$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$$

Using component form with vectors:

$$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z$$

In the special case where

$$\vec{\mathbf{A}} = \vec{\mathbf{B}};$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{A}} = A_x^2 + A_y^2 + A_z^2 = A^2$$

The Vector Product Defined

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Given two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$

The vector (cross) product of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is defined as a *third vector*, $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$.

- **C** is read as "**A** cross **B**".

The magnitude of vector C is *AB* sin θ.

- θ is the angle between $\vec{\mathbf{A}}$ and. $\vec{\mathbf{B}}$

More About the Vector Product

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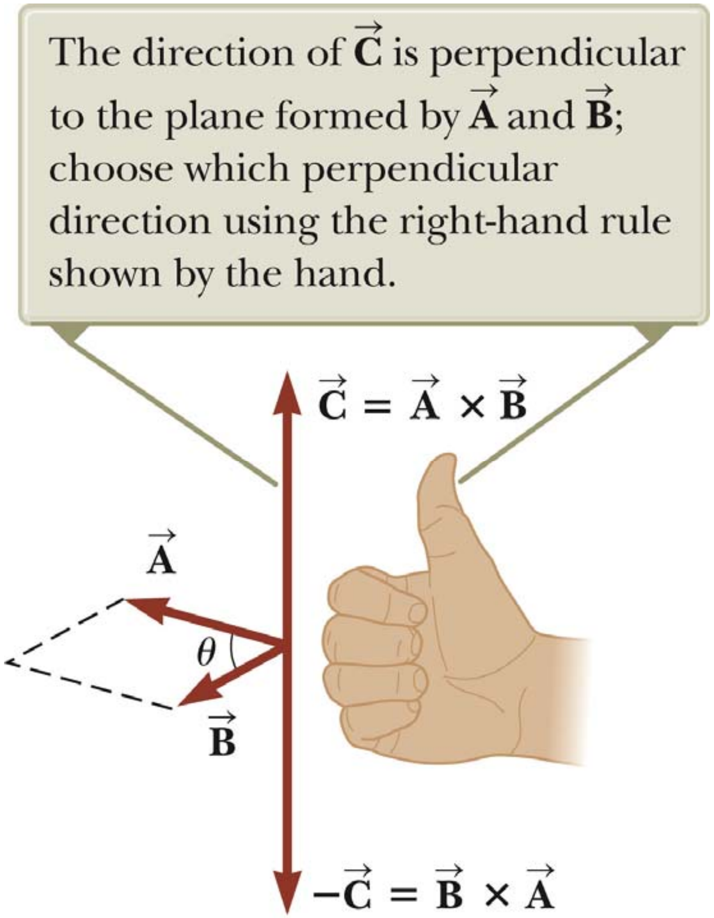
The quantity $AB \sin \theta$ is equal to the area of the parallelogram formed by $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

The direction of $\vec{\mathbf{C}}$ is perpendicular to the plane formed by $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

The best way to determine this direction is to use the right-hand rule.



Properties of the Vector Product

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The vector product is *not* commutative. The order in which the vectors are multiplied is important.

- To account for order, remember $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = -\vec{\mathbf{B}} \times \vec{\mathbf{A}}$

If $\vec{\mathbf{A}}$ is parallel to $\vec{\mathbf{B}}$ (θ = 0° or 180°), then $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = 0$

- Therefore $\vec{\mathbf{A}} \times \vec{\mathbf{A}} = 0$

If $\vec{\mathbf{A}}$ is perpendicular to $\vec{\mathbf{B}}$, then $\left|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right| = AB$

The vector product obeys the distributive law.

$$\vec{\mathbf{A}} \text{ x } (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = \vec{\mathbf{A}} \text{ x } \vec{\mathbf{B}} + \vec{\mathbf{A}} \text{ x } \vec{\mathbf{C}}$$

Final Properties of the Vector Product

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The derivative of the cross product with respect to some variable such as *t* is

$$\frac{d}{dt}\left(\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right)=\frac{d\vec{\mathbf{A}}}{dt}\times\vec{\mathbf{B}}+\vec{\mathbf{A}}\times\frac{d\vec{\mathbf{B}}}{dt}$$

where it is important to preserve the multiplicative order of the vectors.

Vector Products of Unit Vectors

Professor Wa'el Salah

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

Signs in Cross Products

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Signs are interchangeable in cross products

- $\vec{\mathbf{A}} \times \left(-\vec{\mathbf{B}} \right) = -\vec{\mathbf{A}} \times \vec{\mathbf{B}}$

- and $\hat{\mathbf{i}} \times \left(-\hat{\mathbf{j}} \right) = -\hat{\mathbf{i}} \times \hat{\mathbf{j}}$

Using Determinants

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The cross product can be expressed as

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{\mathbf{i}} + \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{\mathbf{k}}$$

Expanding the determinants gives

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \left(A_y B_z - A_z B_y \right) \hat{\mathbf{i}} + \left(A_x B_z - A_z B_x \right) \hat{\mathbf{j}} + \left(A_x B_y - A_y B_x \right) \hat{\mathbf{k}}$$

Vector Product Example

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Given $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}; \; \vec{\mathbf{B}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

Find $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$

Result

$$\begin{aligned}
\vec{\mathbf{A}} \times \vec{\mathbf{B}} &= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \\
&= 2\hat{\mathbf{i}} \times (-\hat{\mathbf{i}}) + 2\hat{\mathbf{i}} \times 2\hat{\mathbf{j}} + 3\hat{\mathbf{j}} \times (-\hat{\mathbf{i}}) + 3\hat{\mathbf{j}} \times 2\hat{\mathbf{j}} \\
&= 0 + 4\hat{\mathbf{k}} + 3\hat{\mathbf{k}} + 0 = 7\hat{\mathbf{k}}
\end{aligned}$$

Do the same example using the determinant method

Torque Vector Example

Professor Wa'el Salah

Given the force and location

$$\vec{\mathbf{F}} = (2.00\,\hat{\mathbf{i}} + 3.00\,\hat{\mathbf{j}})\ N$$

$$\vec{\mathbf{r}} = (4.00\,\hat{\mathbf{i}} + 5.00\,\hat{\mathbf{j}})\ m$$

Find the torque produced

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = [(4.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})\text{N}] \times [(2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})\text{m}]$$

$$= [(4.00)(2.00)\hat{\mathbf{i}} \times \hat{\mathbf{i}} + (4.00)(3.00)\hat{\mathbf{i}} \times \hat{\mathbf{j}}$$

$$+(5.00)(2.00)\hat{\mathbf{j}} \times \hat{\mathbf{i}} + (5.00)(3.00)\hat{\mathbf{i}} \times \hat{\mathbf{j}}$$

$$= 2.0\,\hat{\mathbf{k}}\ \text{N} \cdot \text{m}$$

Do the same example using the determinant method