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Physics 101
Second Semester
2012/2013

Suggested Problems from Chapter 4

1. A motorist drives south at 20.0 m/s for 3.00 min, then turns west and travels at 25.0 m/s for 2.00 min, and finally travels northwest at 30.0 m/s for 1.00 min. For this 6.00-min trip, find (a) the total vector displacement, (b) the average speed, and (c) the average velocity. Let the positive *x* axis point east.

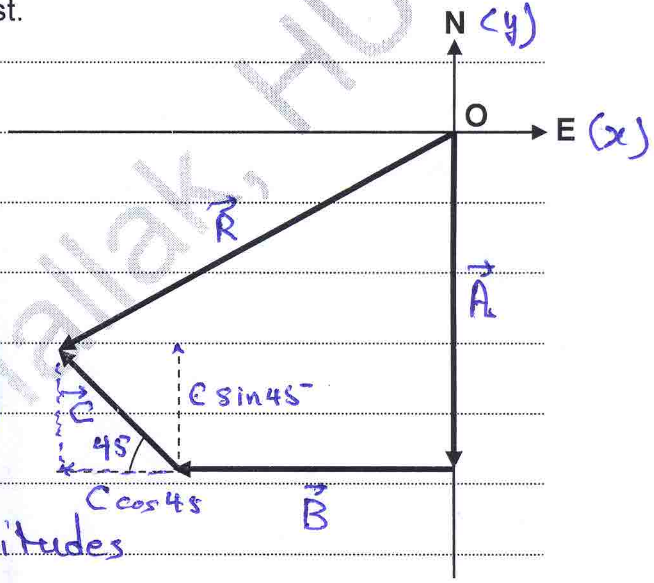
a) Driving south $\Rightarrow \vec{A}$

Driving west $\Rightarrow \vec{B}$

Driving northwest $\Rightarrow \vec{C}$

The resultant $\vec{R}$ is

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$ (Vector Sum)



① First we find the magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{C}$. i.e: A, B, C.

$A = 20 \times 3 \times 60 = 3600$ m

$B = 25 \times 2 \times 60 = 3000$ m.

$C = 30 \times 1 \times 60 = 1800$ m

② Writing $\vec{A}, \vec{B}, \vec{C}$ as vectors and using unit vectors

$\vec{A} = -A\hat{j} = -3600\hat{j}$

$\vec{B} = -B\hat{i} = -3000\hat{i}$

$\vec{C} = -1800\cos 45\,\hat{i} + 1800\cos 45\,\hat{j} = -1272.8\hat{i} + 1272.8\hat{j}$

③ Find $\vec{R} = \vec{A} + \vec{B} + \vec{C} = -3600\hat{j} - 3000\hat{i} - 1272.8\hat{i} + 1272.8\hat{j}$

$\vec{R} = -4272.8\hat{i} - 2327.2\hat{j}$ (في الربع الثالث)

$\Rightarrow R = \sqrt{(-4272.8)^2 + (-2327.2)^2} = 4865.5$ m.

$\tan\theta = \dfrac{-2327.2}{-4272.8} = 0.545 \Rightarrow \theta = 28.6$ or 208.6

b) Average speed $= \dfrac{\text{Total distance}}{\text{Total time} \leftarrow \Delta t}$

$= \dfrac{3600 + 3000 + 1800}{3\times 60 + 2\times 60 + 1\times 60}$

$= \dfrac{8400}{360} = 23.3 \text{ m/s}$

c) Average velocity $= \dfrac{\Delta \vec{r}}{\Delta t}$, $\Delta \vec{r} = \vec{R}$, $\Delta t = 360 \text{ s}$

$\Rightarrow \quad = \dfrac{-4272.8\hat{i} - 2327.2\hat{j}}{360} = -11.87\hat{i} - 6.46\hat{j}$

Magnitude of $\vec{v} = \sqrt{(-11.87)^2 + (-6.46)^2} = 13.5 \text{ m/s}$

Direction of $\vec{v} \Rightarrow \tan\theta = 0.545 \Rightarrow \theta = 208.6^\circ$ relative to $+x$

5. The vector position of a particle varies in time according to the expression $\vec{r} = 3.00\,\hat{i} - 6.00t^2\hat{j}$, where $\vec{r}$ is in meters and t is in seconds. (a) Find an expression for the velocity of the particle as a function of time. (b) Determine the acceleration of the particle as a function of time. (c) Calculate the particle's position and velocity at t = 1.00 s. $\vec{r} = (3\hat{i} - 6t^2\hat{j})$ m

a) $\vec{v} = \dfrac{d\vec{r}}{dt} = 0 - 6(2t)\hat{j}$ m/s $= \boxed{-12t\,\hat{j}}$ m/s

b) $\vec{a} = \dfrac{d\vec{v}}{dt} = \boxed{-12\hat{j}}$ m/s^2

c) Since $\vec{a}$ = constant $\Rightarrow$ Motion under constant acceleration

To find $\vec{r}_f$, we need first to find $\vec{r}_i$ and $\vec{v}_i$.

$\vec{r}_i(t=0) = (3\hat{i} + 0\hat{j})$ m $= 3\hat{i}$ m.

$v_i(t=0) = -12(0)\hat{j} = 0$

$\Rightarrow \vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$ (at t = 1 s)

$= 3\hat{i} + 0 + \frac{1}{2}(-12)(1)^2\hat{j} = \boxed{(3\hat{i} - 6\hat{j})\text{ m}}$

$\Rightarrow v_f = \vec{v}_i + \vec{a}t = 0 + (-12(1))\hat{j}$

$= (-12\hat{j})$ m/s^2

7. A fish swimming in a horizontal plane has velocity $\vec{v}_i = (4.00\,\hat{i} + 1.00\,\hat{j})$ m/s at a point in the ocean where the position relative to a certain rock is $\vec{r}_i = (10.0\,\hat{i} - 4.00\,\hat{j})$ m. After the fish swims with constant acceleration for 20.0 s, its velocity is $\vec{v}_f = (20.0\,\hat{i} - 5.00\,\hat{j})$ m/s. (a) What are the components of the acceleration of the fish? (b) What is the direction of its acceleration with respect to unit vector $\hat{i}$? (c) If the fish maintains constant acceleration, where is it at t = 25.0 s and in what direction is it moving?

a) At $t=0 \Rightarrow \vec{v}_i = (4\hat{i} + 1\hat{j})$ m/s $\Rightarrow v_{xi} = 4$ m/s , $v_{yi} = 1$ m/s

At $t = 20\,s \Rightarrow v_f = (20\hat{i} - 5\hat{j})$ m/s $\Rightarrow v_{xf} = 20$ m/s , $v_{yf} = -5$ m/s

Since $\vec{a}$ = constant $\Rightarrow \vec{a} = \bar{a}$

$$\Rightarrow a_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{20 - 4}{20 - 0} = 0.8 \text{ m/s}^2$$

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{v_{yf} - v_{yi}}{t_f - t_i} = \frac{-5 - 1}{20 - 0} = -0.3 \text{ m/s}^2$$

b) $\tan\theta = \frac{a_y}{a_x} = \frac{-0.3}{0.8} = -0.375$ (في الربع الرابع)

$\Rightarrow$

$$\theta = \tan^{-1}(-0.375) = -20.6 = (360 - 20.6) = 339.4^\circ$$

c) At $t = 25$ s , $\vec{r}_f = ?$

Since $\vec{r}_i = 10\hat{i} - 4\hat{j} \Rightarrow x_i = 10$ m , $y_i = -4$ m

From a) $v_{xi} = 4$ m/s , $v_{yi} = 1$ m/s , $a_x = 0.8$ m/s^2

$a_y = -0.3$ m/s^2

$\Rightarrow \vec{r}_f = x_f\,\hat{i} + y_f\,\hat{j}$ where

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 = 10 + 4(25) + \tfrac{1}{2}(0.8)(25)^2$$

$x_f = 360$ m

$$y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2 = -4 + 1(25) + \tfrac{1}{2}(-0.3)(25)^2$$

$= -72.7$ m

$$\Rightarrow \vec{r}_f = (360\,\hat{i} - 72.7\,\hat{j})\text{ m} \Rightarrow r_f = \sqrt{(360)^2 + (-72.7)^2} = 367.3 \text{ m}$$

$$\tan\theta_f = \frac{-72.7}{360} = -0.202 \Rightarrow \theta_f = -11.4^\circ$$

9. A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?

Maximum height $h = y_{max} = \dfrac{v_i^2 \sin^2\theta_i}{2g}$

Range $R = x_{max} = \dfrac{v_i^2 \sin(2\theta_i)}{g}$

Here we have $3h = R$.

$\Rightarrow$

$$\frac{3\cancel{v_i^2}\sin^2\theta_i}{2\cancel{g}} = \frac{\cancel{v_i^2}\sin(2\theta_i)}{\cancel{g}}$$

or

$$\frac{\sin^2\theta_i}{\sin(2\theta_i)} = \frac{2}{3}$$

But $\sin(2\theta_i) = 2\sin\theta_i\cos\theta_i$

$\Rightarrow$

$$\frac{\sin^{\cancel{2}}\theta_i}{2\cancel{\sin\theta_i}\cos\theta_i} = \frac{2}{3}$$

$\Rightarrow \tan\theta_i = \dfrac{4}{3}$

$\Rightarrow$

$$\theta_i = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\theta_i = 53.1^\circ$$

15. A firefighter, a distance d from a burning building, directs a stream of water from a fire hose at angle u*i* above the horizontal as shown in Figure P4.15. If the initial speed of the stream is v_i, at what height h does the water strike the building?

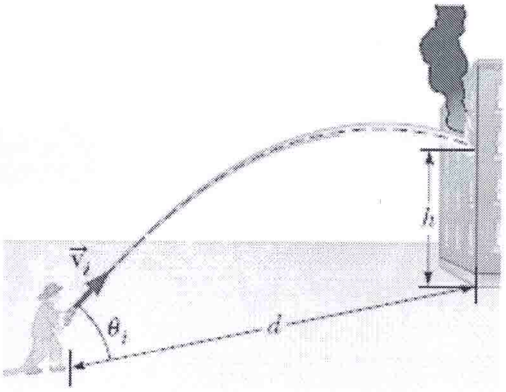


Figure P4.15

In projectile motion

$a_x = 0 \quad , \quad a_y = -g$

$\Rightarrow$

$\vec{a} = -g\hat{j}$

Therefore the x-motion is carried under $a_x = 0$

$\Rightarrow$ in the equation

$x_f = x_i + v_{x_i} t + \frac{1}{2} a_x t^2$, if $x_i = 0 \Rightarrow$

$x_f = 0 + v_{x_i} t + 0 = v_{x_i} t = v_i \cos\theta_i \, t$

In our case $x_f = d = v_i \cos\theta_i \, t$

$\Rightarrow$

$t = \left(\frac{d}{v_i \cos\theta_i}\right)$ (1), where t = time needed for water to reach the building

But $y_f = y_i + v_{y_i} t - \frac{1}{2} g t^2 \qquad (y_i = 0)$.

If t is as given in equation (1) then $y_f = h$.

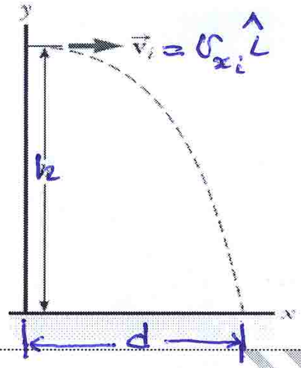
$\Rightarrow$

$h = 0 + v_i \sin\theta_i \left(\frac{d}{v_i \cos\theta_i}\right) - \frac{1}{2} g \left(\frac{d}{v_i \cos\theta_i}\right)^2$

$$\boxed{h = d \tan\theta_i - \left(\frac{g d^2}{2 v_i^2 \cos^2\theta_i}\right)}$$

18. A landscape architect is planning an artificial waterfall in a city park. Water flowing at 1.70 m/s will leave the end of a horizontal channel at the top of a vertical wall h=2.35 m high, and from there it will fall into a pool (Fig. P4.18). (a) Will the space behind the waterfall be wide enough for a pedestrian walkway? (b) To sell her plan to the city council, the architect wants to build a model to standard scale, which is one-twelfth actual size. How fast should the water flow in the channel in the model?

Figure 4.18 b shows a side view (منظر جانبي) of the waterfall. This is the case of a projectile fired from maximum height h where $\vec{v}_i = v_{x_i}\hat{i} + (0)\hat{j}$ Fig. 4.18 b



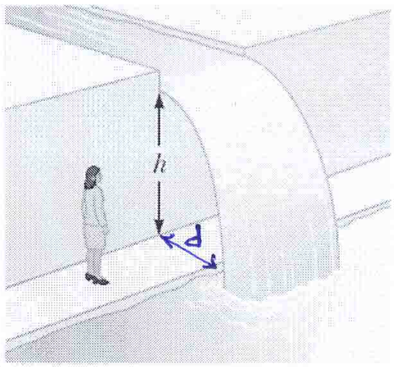


Figure P4.18

$\therefore$ we have $a_x = 0$, $a_y = -g$, $v_{x_i} = v_i$, $v_{y_i} = 0$, $y_i = h$, $y_f = 0$

Using x-motion equations $\Rightarrow$ $x_i = 0$, $x_f = d$

$$x_f = \cancel{x_i} + v_{x_i} t + \cancel{\tfrac{1}{2}a_x t^2} \quad \text{. Put } x_f = d$$

$\Rightarrow$

$$d = v_i t \quad (1)$$

. We need to find t to answer the question.

Using y-motion equations $\Rightarrow$

$$y_f = y_i + v_{y_i} t - \tfrac{1}{2} g t^2 \Rightarrow 0 = h + 0 - \tfrac{1}{2} g t^2$$

$$\Rightarrow t = \sqrt{\frac{2h}{g}}$$

Substitute in (1) $\Rightarrow$

$$d = v_i \sqrt{\frac{2h}{g}} = 1.7 \sqrt{\frac{2 \times 2.35}{9.8}} = 1.177 \text{ m}$$

Also

$$v_i = d\sqrt{\frac{g}{2h}}$$

. In the proposed model, the scale is $\frac{1}{12}$ actual size $\Rightarrow d_{model} = \frac{1.177}{12} = 0.098$ m

$$\Rightarrow h_{model} = \frac{2.35}{12} = 0.196 \text{ m}$$

$$\Rightarrow v_{i\,(model)} = 0.098\sqrt{\frac{9.8}{2 \times 0.196}} = 0.5 \text{ m/s}$$

32. Figure P4.32 represents the total acceleration of a particle moving clockwise in a circle of radius 2.50 m at a certain instant of time. For that instant, find (a) the radial acceleration of the particle, (b) the speed of the particle, and (c) its tangential acceleration.

$r = 2.5\ m \quad , \quad a = 15\ m/s^2$

$\vec{a} = \vec{a}_r + \vec{a}_t$

$a_r = a \cos 30°$

$a_t = a \sin 30°$

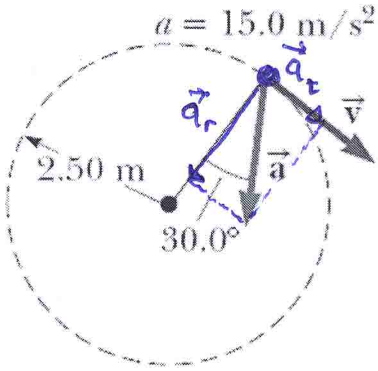


Figure P4.32

a) $a_r = 15 \cos 30°$

$= 15 \times 0.866$

$= 13.0\ m/s^2$

b) $a_r = \dfrac{v^2}{r} \Rightarrow v = \sqrt{r\, a_r}$

$\Rightarrow$

$v = \sqrt{2.5 \times 13.0} = 5.7\ m/s$

c) $a_t = 15 \sin 30°$

$= 15 \times 0.50$

$= 7.5\ m/s^2$

or

$a^2 = a_t^2 + a_r^2$

$(15)^2 = a_t^2 + (13)^2$

$\Rightarrow$

$a_t = \sqrt{(15)^2 - (13)^2} = 7.5\ m/s^2$

50. A basketball player is standing on the floor 10.0 m from the basket as in Figure P4.50. The height of the basket is 3.05 m, and he shoots the ball at a 40.0° angle with the horizontal from a height of 2.00 m. (a) What is the acceleration of the basketball at the highest point in its trajectory? (b) At what speed must the player throw the basketball so that the ball goes through the hoop without striking the backboard?

a) At the highest point and everywhere else

$\vec{a} = -g\hat{j}$

$\Rightarrow a_x = 0 \ , \ a_y = -g$

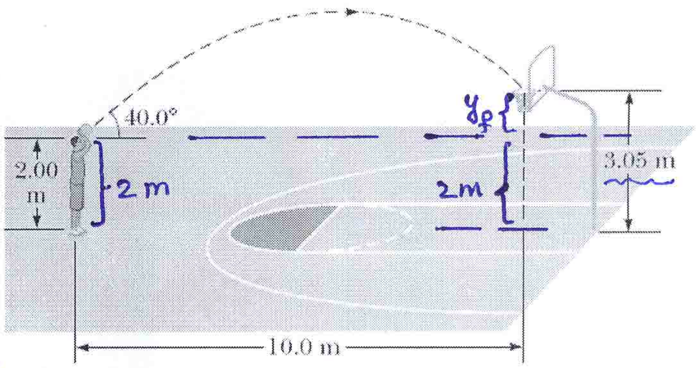


Figure P4.50

b) The ball was thrown at a point 2 m from ground $\Rightarrow \quad 3.05 = y_f + 2 \Rightarrow y_f = 1.05$ m

For x-motion, $x = 10$ m.

$x = v_{x_i} t \ , \ v_{xi} = v_i \cos 40°$

$\Rightarrow t = \dfrac{x}{v_i \cos 40°} = \dfrac{10}{v_i \cos 40°}$

For y-motion, $y_f = 1.05 \ , \ y_i = 0 \ , \ v_{y_i} = v_i \sin 40°$

$\Rightarrow y_f = y_i + v_{y_i} t - \frac{1}{2} g t^2$

Substitute for $t \Rightarrow$

$$1.05 = 0 + \cancel{v_i} \sin 40° \left(\frac{10}{\cancel{v_i} \cos 40°} \right) - \frac{1}{2} \times 9.8 \left(\frac{10}{v_i \cos 40°} \right)^2$$

$$1.05 = 10 \tan 40° - \frac{490}{v_i^2 \cos^2 40°}$$

$\Rightarrow v_i = 10.7$ m/s