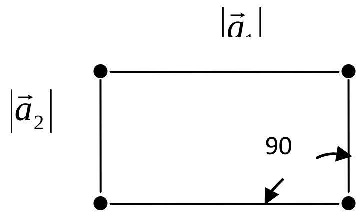
1) The diagram shows the conventional unit cell of a rectangular lattice (sides a_1 and a_2).

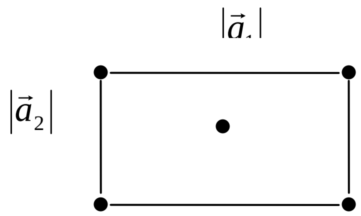


Rectangular unit cell

i) Write down the fractional co-ordinates of all the lattice points.
ii) Write down suitable primitive lattice vectors.
iii) Calculate the area of a primitive unit cell.
iv) Sketch the Wigner Seitz cell for this lattice.
v) What is the maximum packing fraction?

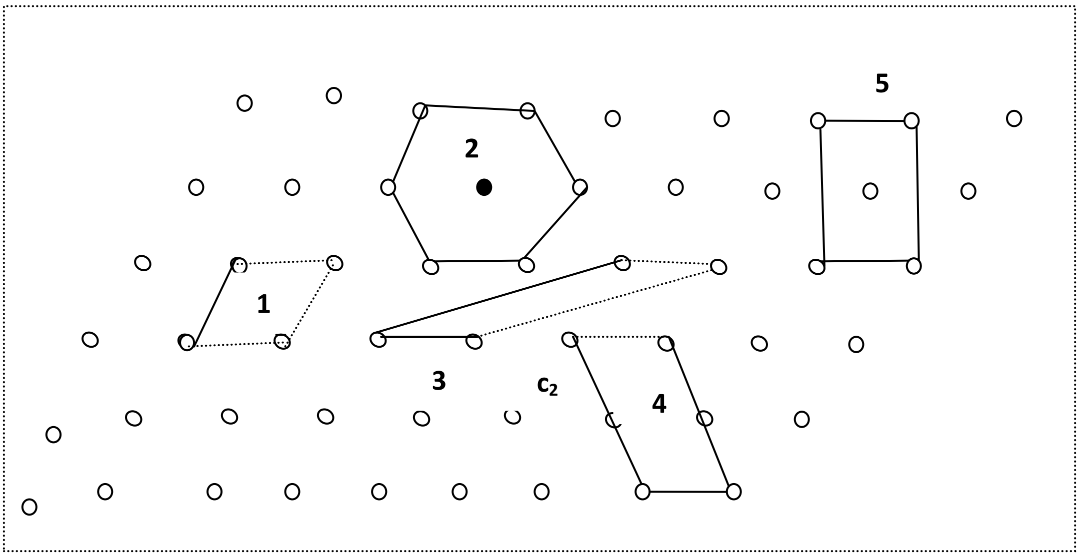
[Hint for (v): first work out the maximum radius of circles which just touch one another when placed at nearest neighbour sites; then compare the area of such a circle with that of the primitive unit cell].

2) Repeat question (1) for a centered rectangular lattice.



[N.B. the primitive lattice vectors are not the same and still there must be only one lattice point per primitive unit cell].

3) The diagram shows a portion of a 2D lattice. Which of the outlines represent
(i) primitive unit cells,
(ii) non-primitive unit cells;
(iii) not unit cells at all



4) Prove that the ideal ratio of *c*/*a* for a hexagonal close packed structure is 1.633.

5) What is the coordination number for a bcc lattice with a one-atom basis? Show that the packing fraction for identical hard spheres arranged in a bcc lattice is 0.680.

6) For the fcc lattice with a one atom basis:
  (i) what is the co-ordination number?
  (ii) how many second-nearest-neighbours does each atom have?
  (iii) what is the distance to these, in units of the conventional cubic lattice side *a*?
  (iv) what is the separation between close-packed planes? Answer: Close packed planes are (111) with spacing $a/\sqrt{3}$.

7) At 1190 K, iron has a cubic-F (fcc) lattice with cube edge 0.3647 nm, while at 1670 K it has a cubic-I (bcc) lattice with cube edge 0.2932 nm. In both cases there is a one-atom basis. Calculate the ratio of the densities of iron at these two temperatures. Answer :Ratio of densities is 0.962.

8) A 2D direct lattice has primitive lattice vectors: $\vec{a}_1 = a_1\hat{x}$, $\vec{a}_2 = a_2((\cos\theta)\hat{x} + (\sin\theta)\hat{y})$
[Note: these are just vectors of magnitudes a_1 and a_2 with angle θ between them].

a) Sketch a portion of this lattice, indicating the cartesian axes and the primitive lattice vectors
b) Show that the reciprocal lattice has primitive vectors: $\vec{b}_1 = \frac{2\pi}{a_1}(\hat{x} - \frac{\cos\theta}{\sin\theta}\hat{y})$,

$\vec{b}_2 = \frac{2\pi}{a_2}(\frac{1}{\sin\theta}\hat{y})$.

[Note: the 2D reciprocal lattice can be determined in two ways:
EITHER: write $\vec{b}_1$ and $\vec{b}_2$ as general vectors in 2D and then determine the values of their *x* and *y* components that allow them to satisfy the Laue condition;
OR: use the formulae relating 3D reciprocal and direct lattice vectors, but use $\vec{a}_3 = a_3\hat{z}$ for the third direct lattice vector and allow a_3 to tend to infinity. The third reciprocal lattice vector will then be of magnitude zero, and can simply be ignored.]

c) Sketch the reciprocal lattice indicating its orientation with respect to the direct lattice. [Hint:you should find that $\vec{b}_1$ is perpendicular to $\vec{a}_2$ and $\vec{b}_2$ is perpendicular to $\vec{a}_1$]
d) Sketch both the direct and reciprocal lattices in the case $a_1 = 0.1$ nm, $a_2 = 0.2$ nm, θ= 60°. [Hint: they should be rectangular.]
e) On your sketch of the rectangular reciprocal lattice above, construct the first and second Brillouin zones. [Note: you know how to construct the first one. The second Brillouin zone is the region of *k*-space (not necessarily a single region) reached from the chosen reciprocal lattice point, by crossing one, and only one, perpendicular bisector – it should have the same 'volume' as the first Brillouin zone.