

# Vectors

*Prepared By*

Prof. Rashad Badran

# Coordinate Systems: Cartesian Coordinates vs. Polar coordinates

$x$   $y$   $z$  coordinate system is called *Cartesian coordinate system*.

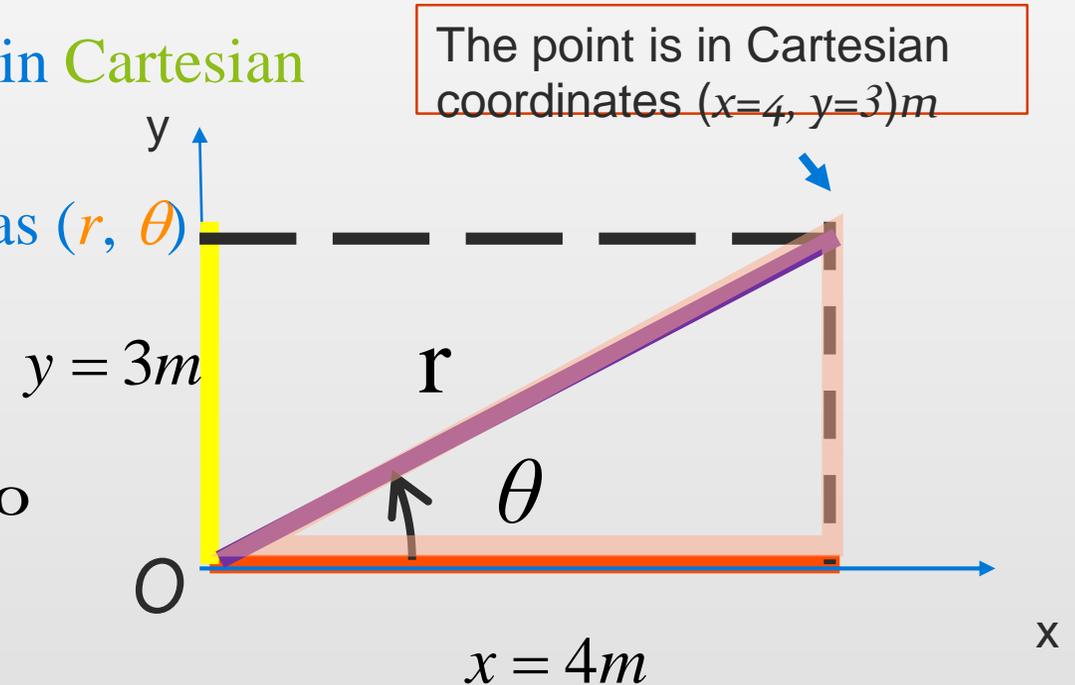
In  $xy$  plane, a point given as  $x = 4\text{ m}$  and  $y = 3\text{ m}$  is in Cartesian coordinates.

This point can be transformed into polar coordinates as  $(r, \theta)$

$$\tan \theta = \frac{\text{Opposite side } y}{\text{Adjacent side } x}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{3}{4}\right) \Rightarrow \theta = 37^\circ$$

$$\Rightarrow r = \sqrt{x^2 + y^2} = \sqrt{4^2 + 3^2} \Rightarrow r = 5\text{ m}$$



**Notes:** The point expressed in Cartesian coordinate system ( $x = 4, y = 3$ )  $m$ , is transformed into polar coordinate system ( $r = 5\text{ m}, \theta = 37^\circ$ )

# Coordinate Systems: Cartesian Coordinates *vs.* Polar coordinates

**Given:**  $r = 5m$ ,  $\theta = 37^\circ$  in polar coordinates.

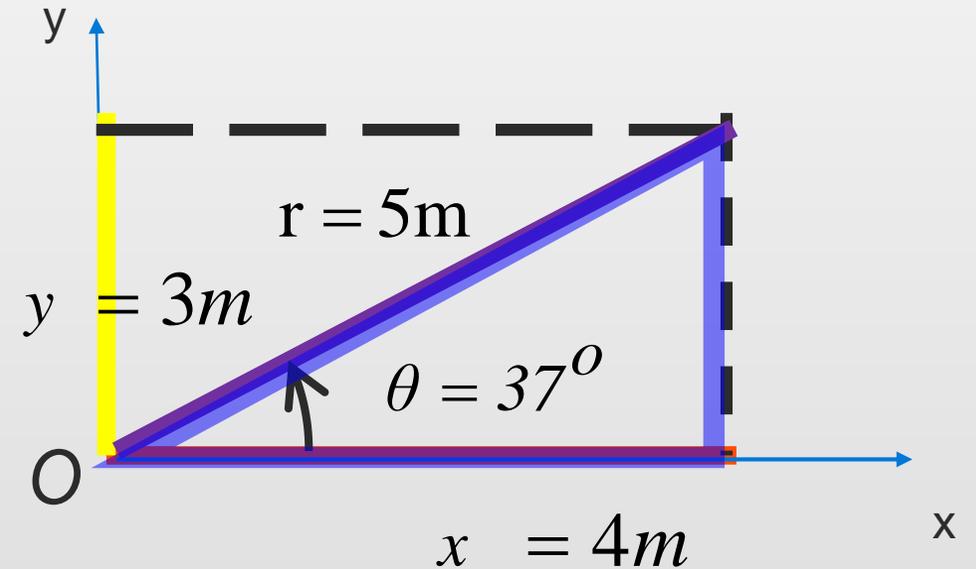
The point is in polar coordinates  $(r, \theta)$  and can be transformed into Cartesian Coordinates  $(x, y)$

$$\cos \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse side}} = \frac{x}{r}$$

$$\Rightarrow x = r \cos \theta = (5) \cos 37^\circ = 4m$$

$$\sin \theta = \frac{\text{Opposite side}}{\text{Hypotenuse side}} = \frac{y}{r}$$

$$\Rightarrow y = r \sin \theta = (5) \sin 37^\circ = 3m$$



**Notes:** The point given in polar coordinate system ( $r = 5m$ ,  $\theta = 37^\circ$ ) is transformed into Cartesian (or Rectangular) coordinate system ( $x = 4$ ,  $y = 3$ )  $m$ .

# Coordinate Systems: Cartesian Coordinates *vs.* Polar coordinates

## Problem

The polar coordinates of a point are  $r = 5.5 \text{ m}$  and  $\theta = 240^\circ$ . What are the Cartesian coordinates of this point?

## Solution

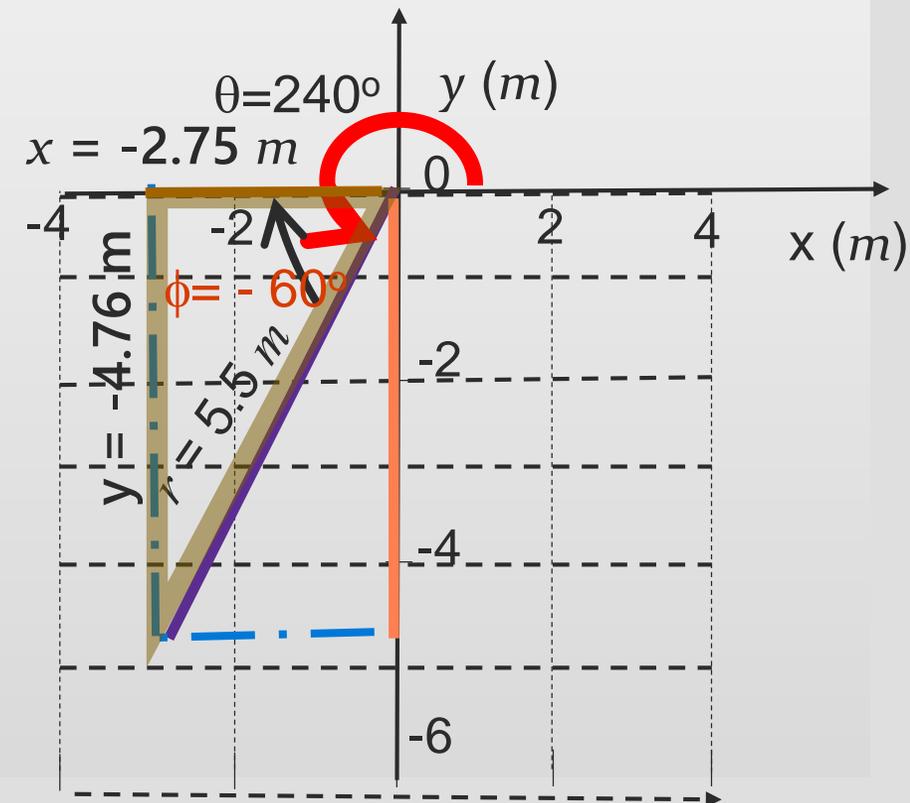
$$x = r \cos \theta = (5.5\text{m})(\cos 240)$$

$$\Rightarrow x = (5.5 \text{ m})(-0.5) = -2.75 \text{ m}$$

$$y = r \sin \theta = (5.5 \text{ m})(\sin 240)$$

$$\Rightarrow y = (5.5\text{m})(-0.86) = -4.76 \text{ m}$$

$$(r = 5.5 \text{ m}, \theta = 240^\circ) \longrightarrow (x = -2.75 \text{ m}, y = -4.76 \text{ m})$$



# Coordinate Systems: Cartesian Coordinates *vs.* Polar coordinates

## Problem

The Cartesian (or Rectangular) coordinates of a point are given  $(2, y)$ , and its polar coordinates are  $(r, 30^\circ)$ . Determine (a) the value of  $y$  and (b) the value of  $r$ .

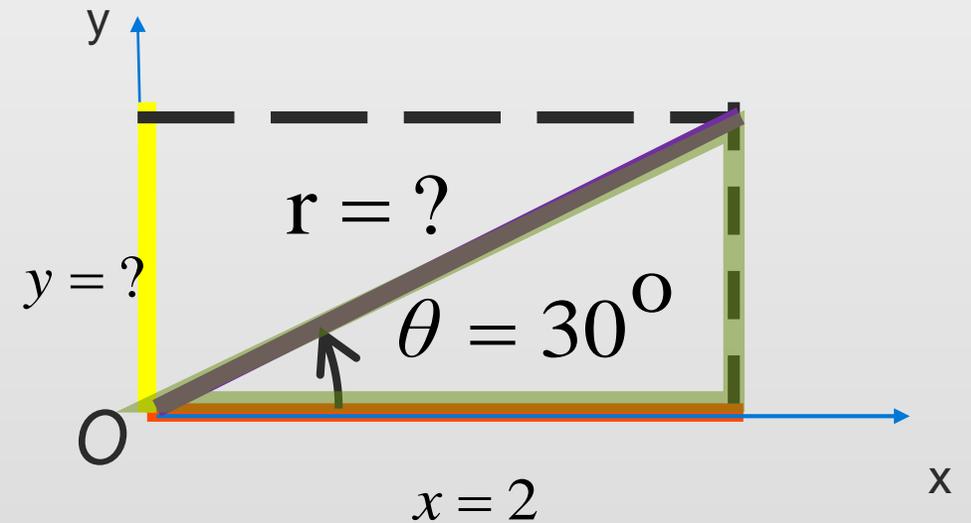
## Solution

$$(a) \quad \tan \theta = \frac{y}{x}$$

$$\Rightarrow y = x \tan \theta = (2)(\tan 30^\circ) = 1.15$$

$$(b) \quad \cos \theta = \frac{x}{r}$$

$$\Rightarrow r = \frac{x}{\cos \theta} = \frac{2}{\cos 30} = \frac{2}{0.86} = 2.31$$



# Physical Quantities: Scalars and Vectors

## Scalar quantity:

A physical quantity that can be described by a *magnitude* (single number) with a suitable unit.

Examples: Distance, average speed, mass, time, energy, temperature, ...etc.

# Scalar Quantities: **Example**

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Mass

160 kg !!!

**Note:** 160 is the number that represents the magnitude. Here **kg** is the unit of mass in the international system (SI)

# Scalar Quantities:

## Example



## Temperature

What is your temperature?

40° C

40 is the magnitude of temperature while ° C refers to the SI unit (degree Celsius or centigrade)

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## Vector quantity:

A physical quantity that has both *magnitude* *and* *direction*.

Examples: Displacement, velocity, acceleration, force, momentum,.....etc.

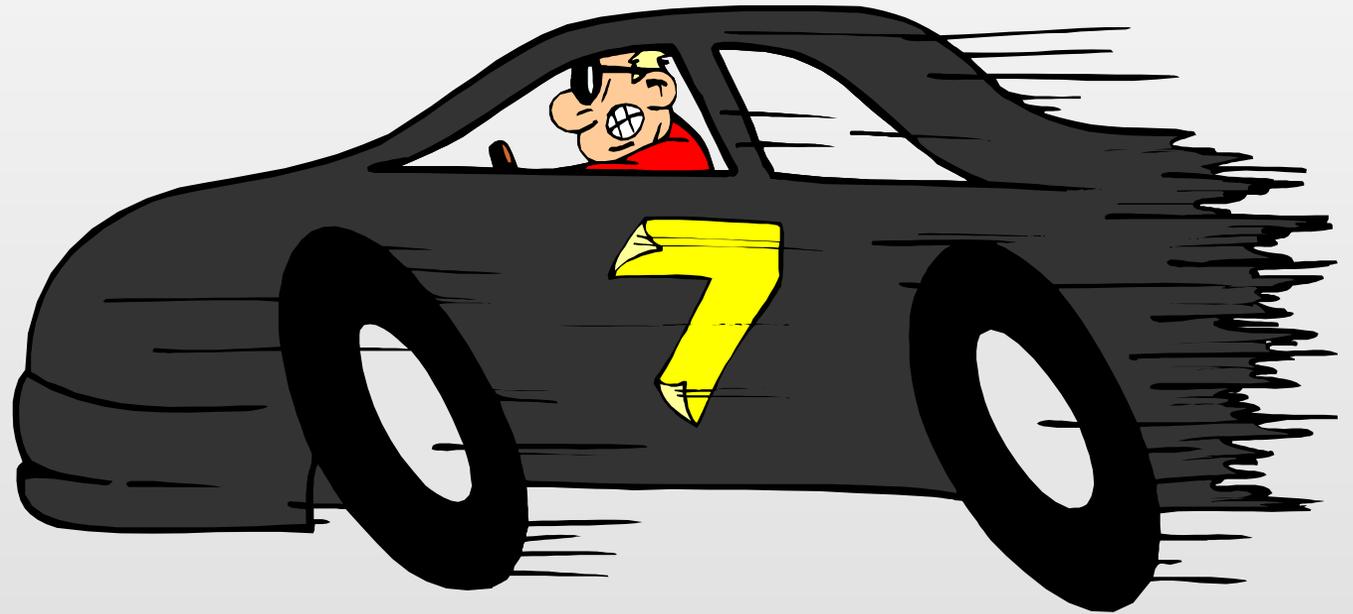
# Vector Quantities: **Example**

## Velocity

160 km/h (magnitude)



West (**direction**)



# Vector Quantities: **Example**

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Force

1000 N (**magnitude**)

Upward (**direction**)

# Vector Notation and Representation

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$\vec{A}$  is a vector notation where the arrow at the top of letter A indicates that this vector must have both magnitude and direction.

- A vector is represented by an arrow which connects an initial point with a terminal point.
- Magnitude of the vector  $\vec{A}$  is written as  $A$  or  $|\vec{A}|$
- The direction of the vector is specified with respect to an axis of reference (e.g. positive x-axis).

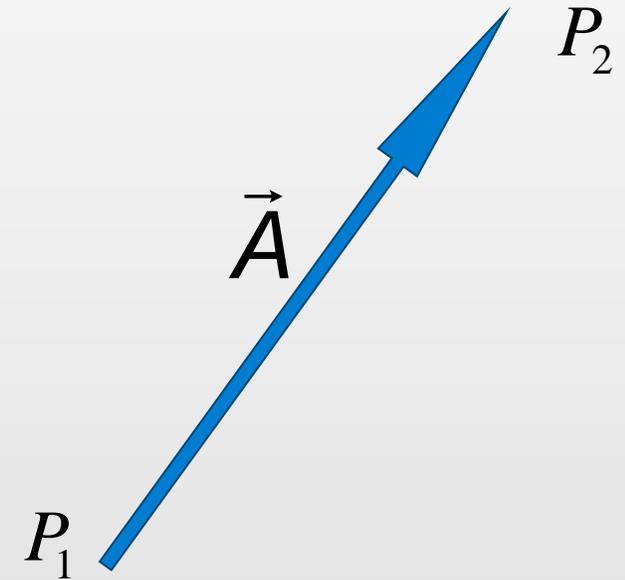
**Notes:** (1) the magnitude of the vector is always positive and (2) it is represented by the length of the vector.

# Vector Quantities: **Example**

□ **Displacement:** the change in position of a point.

➤ Represented by a line directed from the initial point to the final point

*e.g.* Vector  $\vec{A}$  represents the displacement from point  $P_1$  to point  $P_2$



- The **length** of this vector represents its **magnitude**
- The **head** of the vector refers to its **direction**

# Conceptual Questions

## Conceptual Question

Can the magnitude of a vector quantity have a negative value ?

**Answer:**

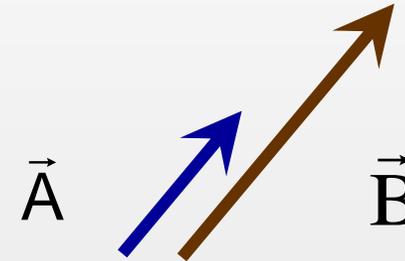
No it cannot be negative. The magnitude of a vector quantity is always positive.

# Some Properties of Vectors

## 1) Parallel and Equal Vectors

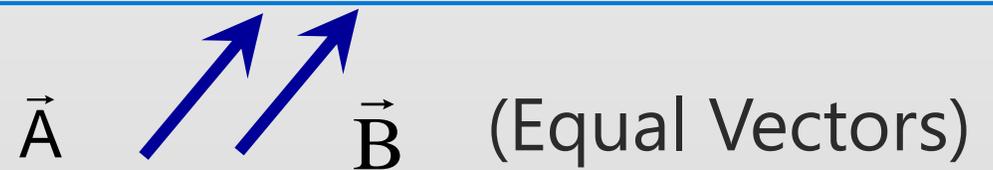
- If two vectors have the same **direction**, they are parallel.

- If they have the same **direction** and the same **magnitude**, they are equal.



Parallel Vectors:  $\vec{B} \neq \vec{A}$

Vectors have different magnitudes although they are in same direction



Equal Vectors:  $\vec{B} = \vec{A}$

Both vectors have same magnitude and direction

# Some Properties of Vectors

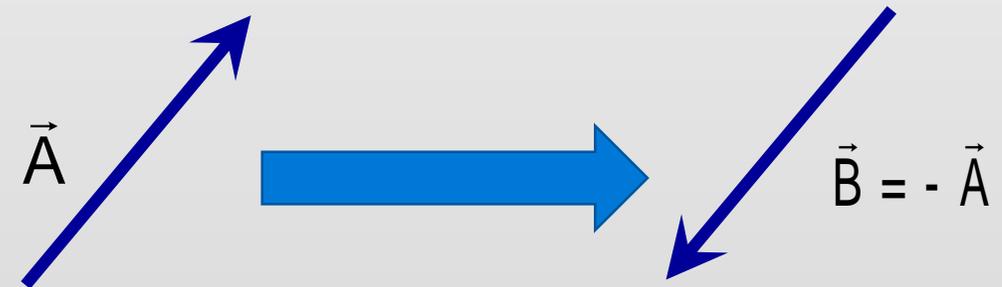
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## 2) Negative of a Vector

- Negative of a vector is a vector with the same **magnitude** but opposite direction

**Note:**  $\vec{A}$  and  $\vec{B}$  are antiparallel vectors. They have opposite directions

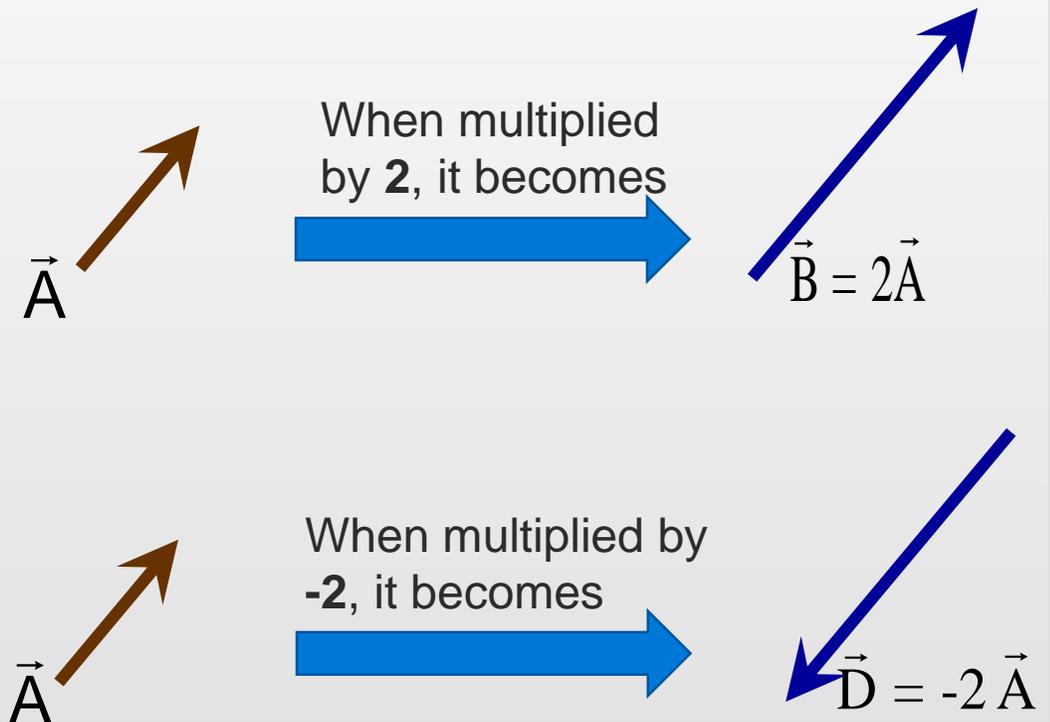
A vector  $\vec{A}$  which has certain **magnitude** and **direction becomes a different vector when multiplied by a negative sign**



# Some Properties of Vectors

## 3) Multiplying a Vector by a Scalar

- When a vector  $\vec{A}$  is multiplied by a positive scalar quantity, say, 2 then the new vector (call it  $\vec{B}$ ) has a magnitude twice that of the original vector with same direction.
- But when  $\vec{A}$  is multiplied by, say, -2 ( a negative scalar quantity) then the new vector (call it  $\vec{D}$ ) is  $-2 \vec{A}$



**Conclusions:** (1) the vector changes when either its direction or magnitude changes. Obviously  
(2) the vector also changes when both magnitude and direction change.

# Some Properties of Vectors

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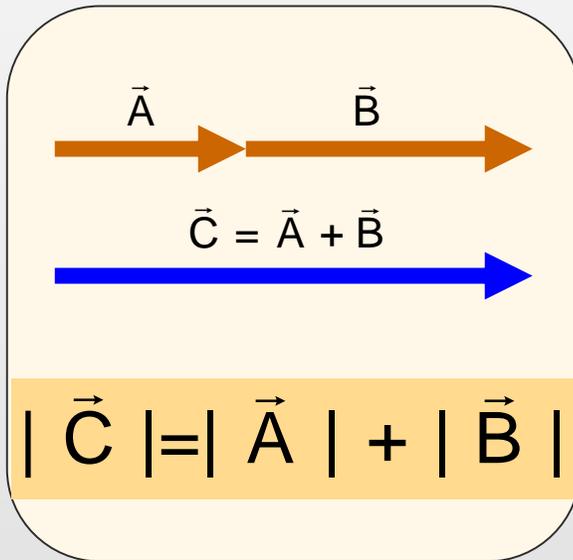
## 4) Addition of Vectors

Vector addition is different from number addition. The addition of two vectors  $\vec{A}$  and  $\vec{B}$  gives a third vector (call it  $\vec{C}$ ) which represents the sum or resultant of them, namely: 
$$\vec{C} = \vec{A} + \vec{B}$$

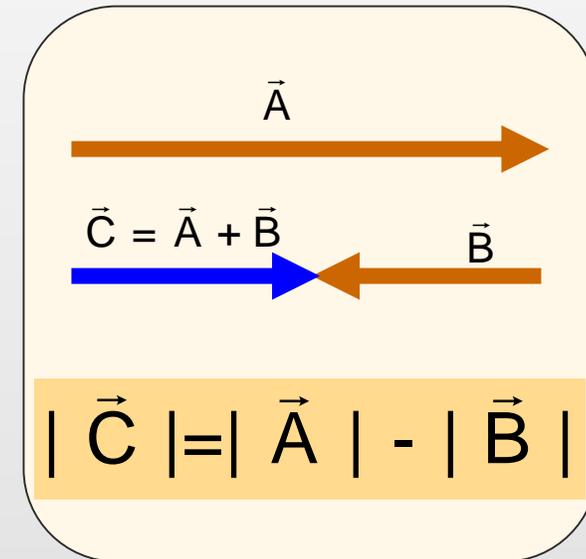
When  $\vec{B}$  is multiplied by a negative sign (or expressed as  $-\vec{B}$ ) then the addition of vectors may be given by  $\vec{D} = \vec{A} + (-\vec{B})$ , where this operation represents the subtraction of the two vectors  $\vec{A}$  and  $\vec{B}$  which is written as: 
$$\vec{D} = \vec{A} - \vec{B}$$

# Some Properties of Vectors

## Addition of Vectors: A) Vectors in one dimension



When vectors are in same direction (e.g. towards east) then magnitudes will be added and the sum of the vectors (or their resultant) will be in same direction (east)



When vectors are in opposite directions ( $\vec{A}$  towards east) and ( $\vec{B}$  towards west) then the magnitude the sum of the vectors will be subtracted and this sum (or resultant) will be in direction of largest vector (east)

# Conceptual Questions

## Conceptual Question

Is it possible to add a vector quantity to a scalar quantity ?

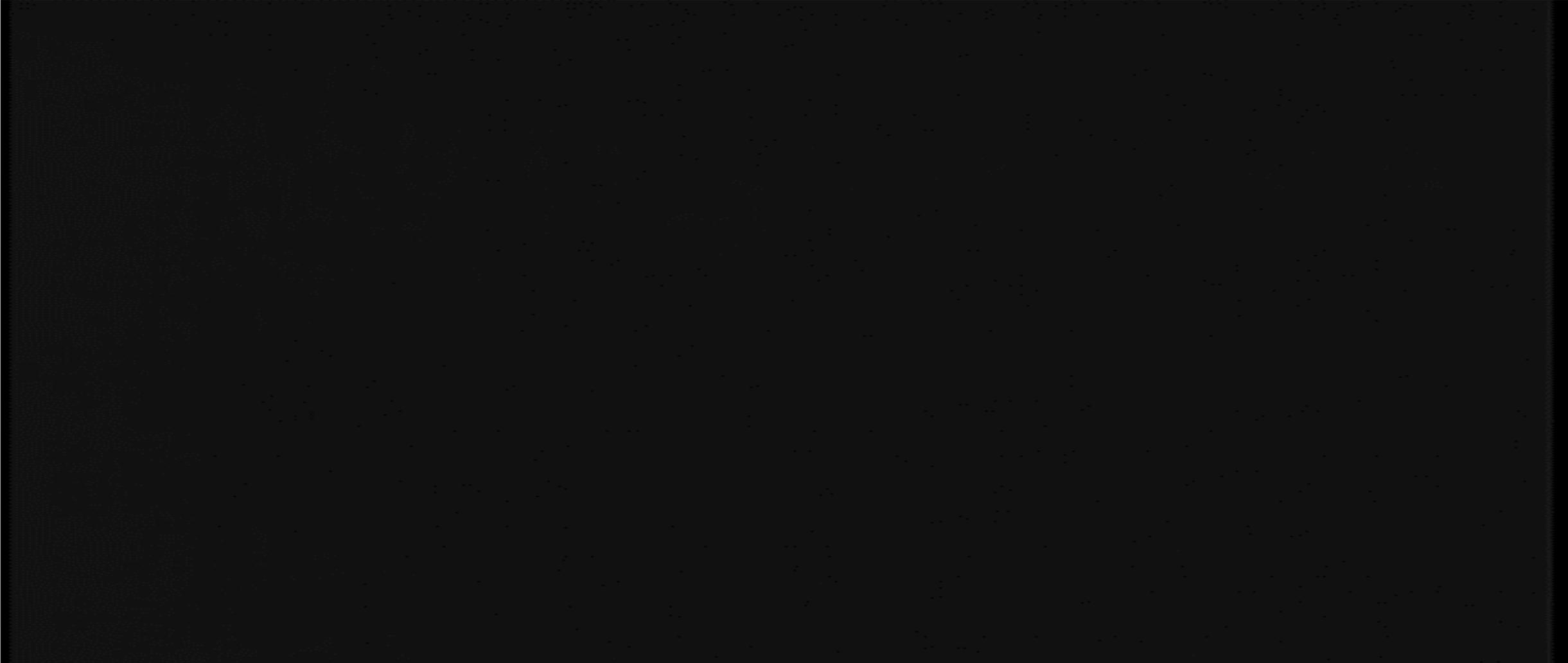
**Answer:**

No it is not possible because the addition of vector quantities has rules different from that of scalar quantities

# Some Properties of Vectors

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Addition of Vectors: B) Vectors in two dimensions



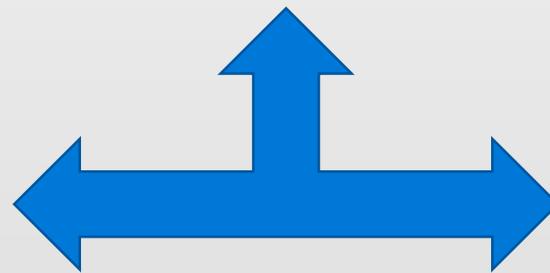
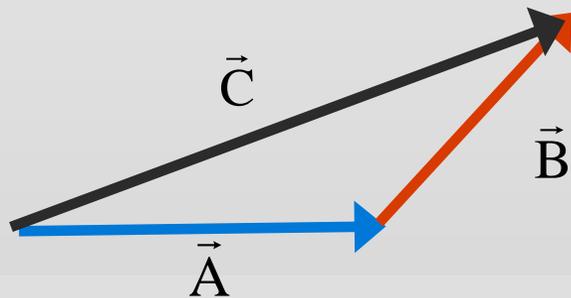
# Some Properties of Vectors

## Addition of Vectors: B) Vectors in two dimensions

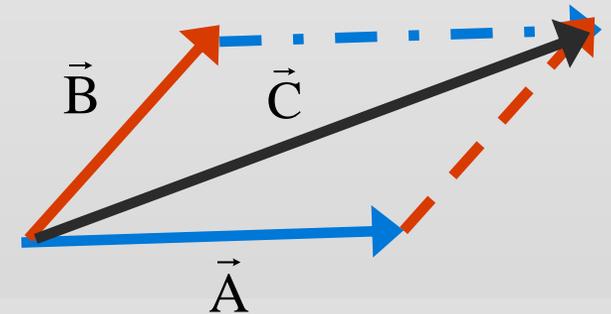
- There are two methods to add vectors in 2-D: The sum (or resultant) can be obtained by:

### 1) Geometric (or Graphical) Method:

1-a) Triangle Method  
(Vectors are head-to-tail)



1-b) Parallelogram Method  
(Vectors are tail-to-tail)



# 1-a) Triangle Method

## Example

Two vectors  $\vec{A}$  and  $\vec{B}$  are head - to - tail vectors. Vector  $\vec{A}$  has a magnitude of 8 units and directed towards east and vector  $\vec{B}$  has a magnitude of 5 units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$ . Use the triangle method to determine geometrically (a) the magnitude of the sum  $\vec{R}$  of the two vectors and, (b) the direction of  $\vec{R}$  with respect to that of vector  $\vec{A}$ .

# 1-a) Triangle Method

**Solution:**

(a)  $|\vec{A}| = 8$  units and directed due east

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$

To draw the head-to-tail vectors geometrically, we start with the first vector



# 1-a) Triangle Method

**Solution:**

(a)  $|\vec{A}| = 8$  units and directed due east

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$

To draw the second vector geometrically, we must specify its angle using the protractor

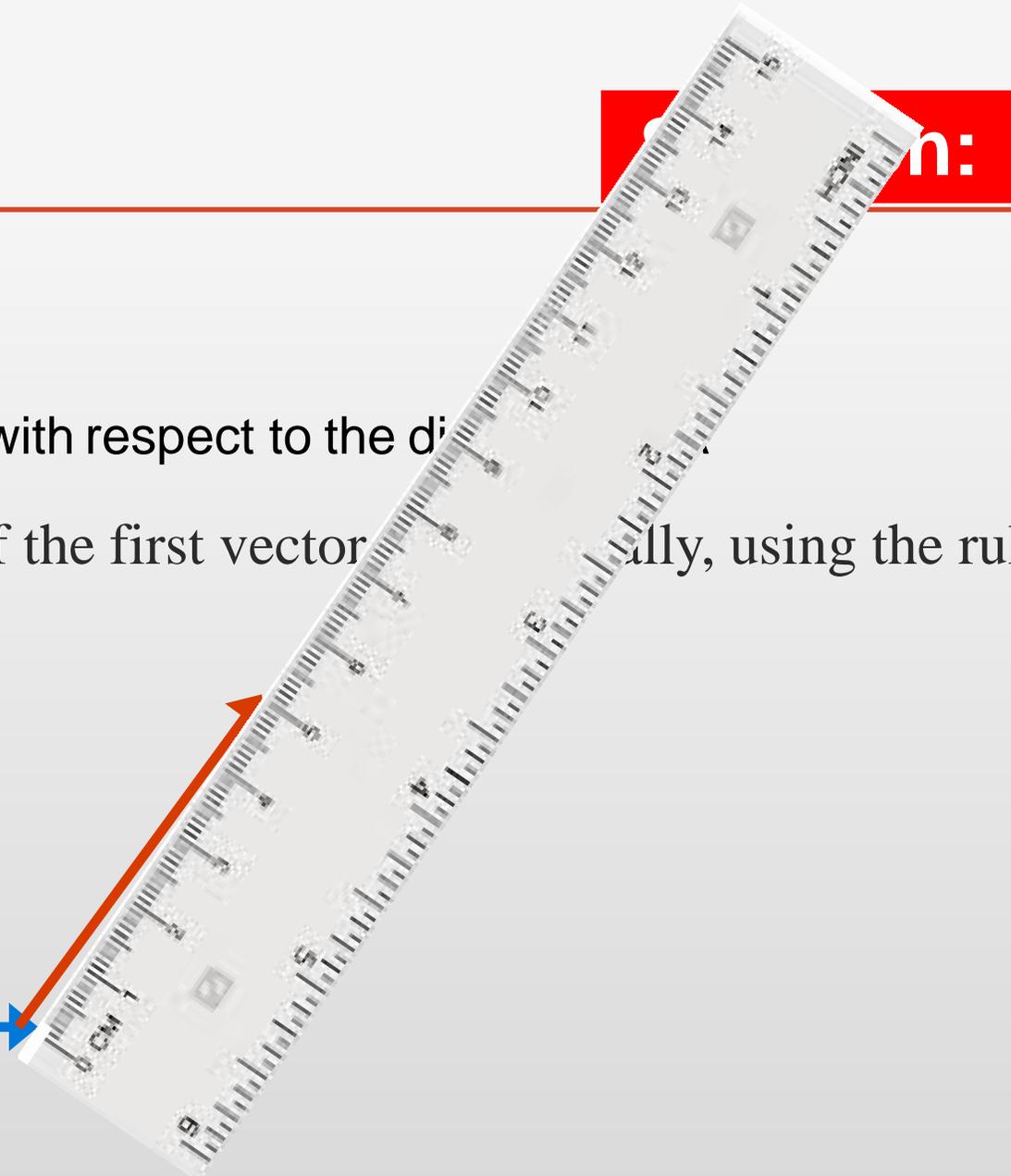


# 1-a) Triangle Method

(a)  $|\vec{A}| = 8$  units and directed due east

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the di

The second vector will be drawn from the head of the first vector  $60^\circ$  using the ruler



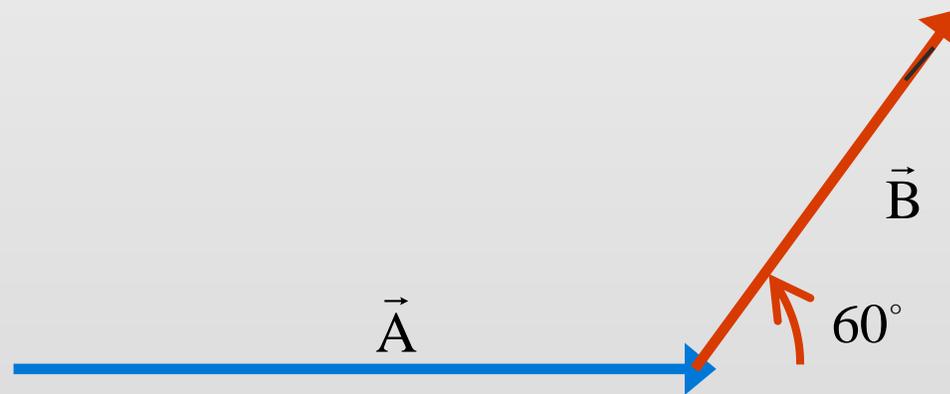
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The second vector is drawn from the head of first vector



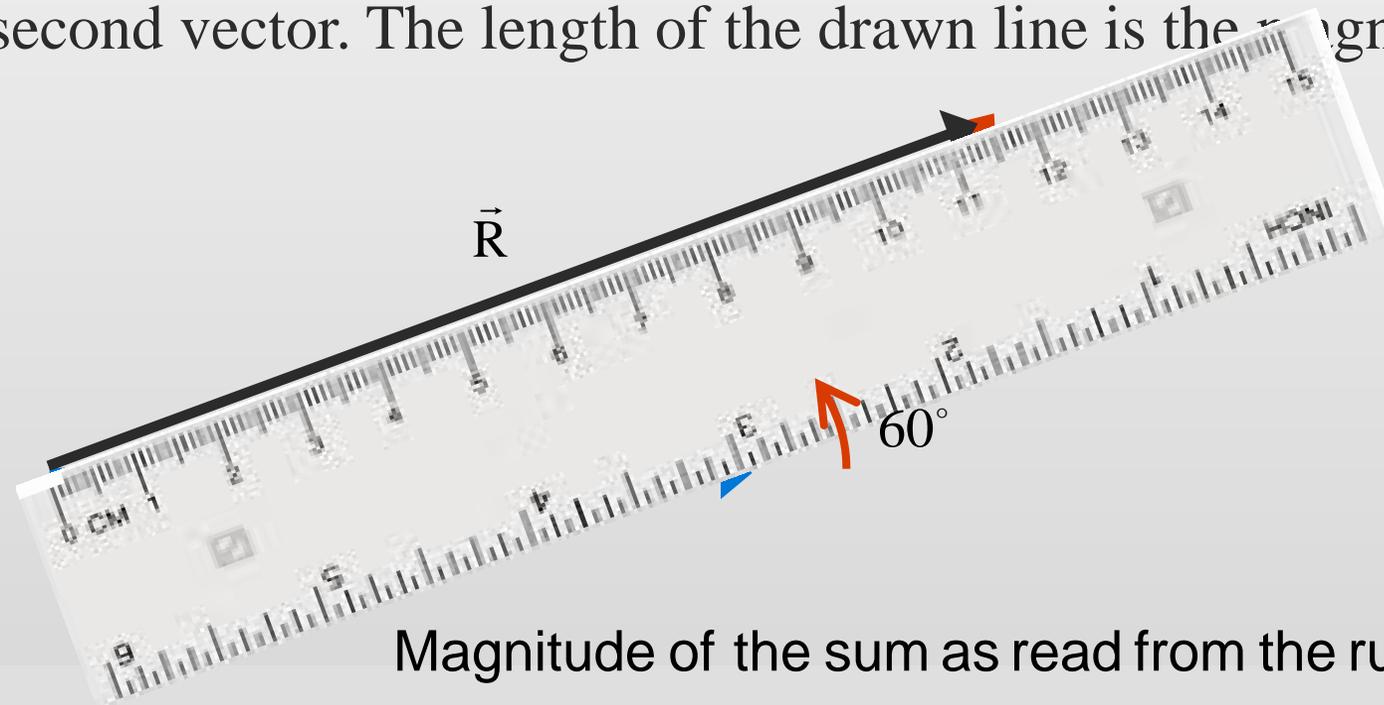
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**Solution:**

(a)  $|\vec{A}| = 8$  units and directed due east

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$

To draw the vector sum geometrically, use the ruler to draw a line from the tail of first vector up to the head of the second vector. The length of the drawn line is the magnitude of the sum



Magnitude of the sum as read from the ruler is  $|\vec{R}| = 11.35$  units

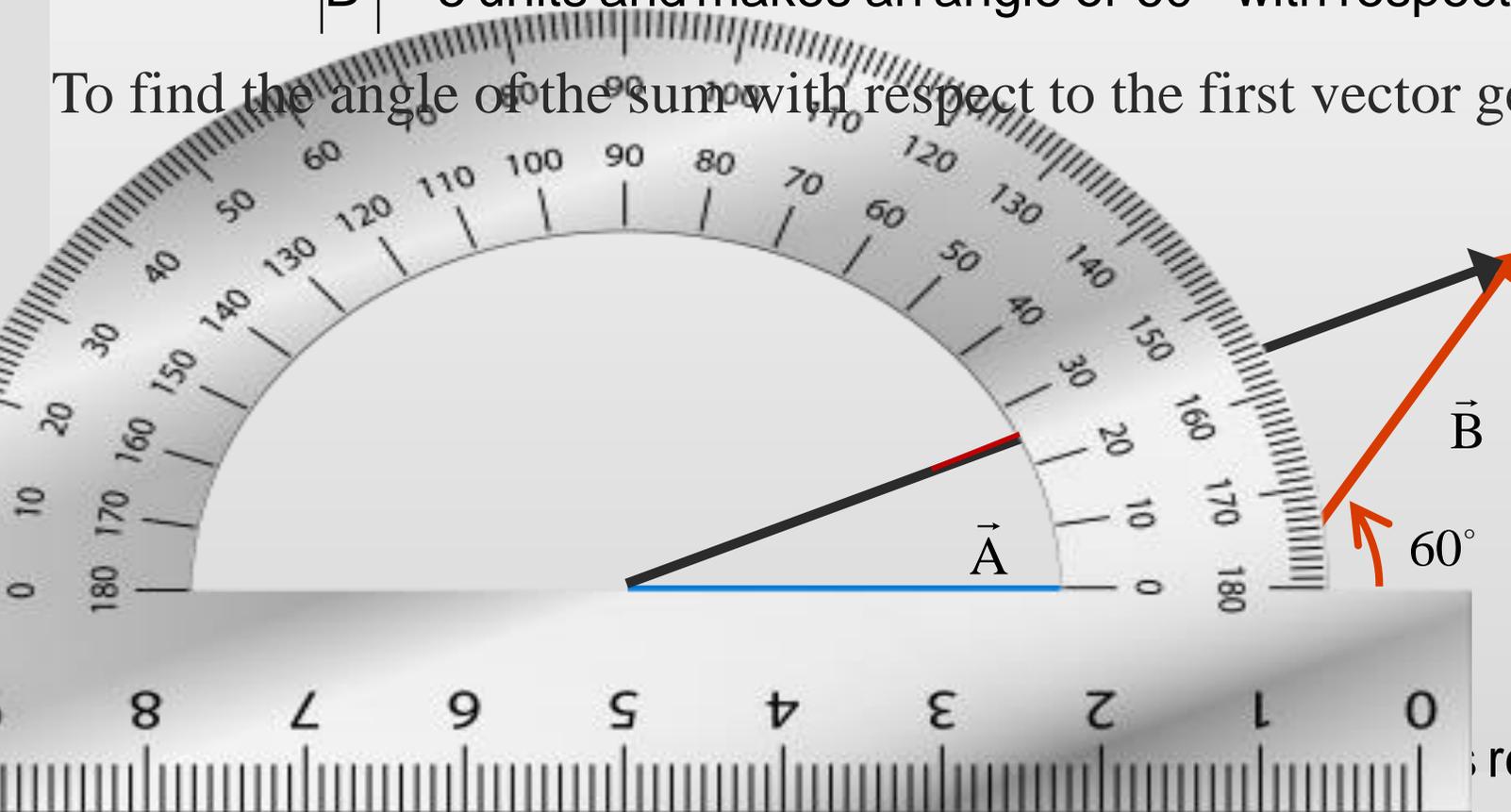
# 1-a) Triangle Method

**Solution:**

(b)  $|\vec{A}| = 8$  units and directed due east

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$

To find the angle of the sum with respect to the first vector geometrically, we use the protractor



read from the ruler is  $|\vec{R}| = 11.35$  units

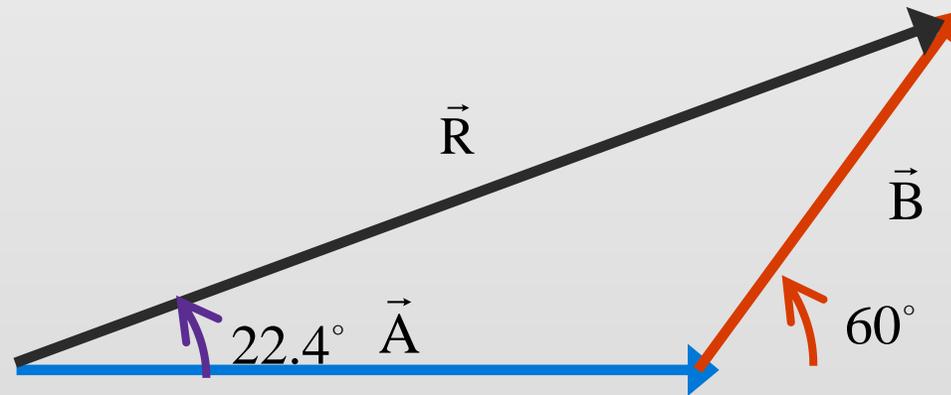
# 1-a) Triangle Method

**Solution:**

(a)  $|\vec{A}| = 8$  units and directed due east

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$

The reading of the protractor (direction of  $\vec{R}$ ) =  $22.4^\circ$



Magnitude of the sum as read from the ruler is  $|\vec{R}| = 11.35$  units

# 1-b) Parallelogram Method

## Example

Two vectors  $\vec{A}$  and  $\vec{B}$  are tail - to - tail vectors. The magnitudes of vectors  $\vec{A}$  and  $\vec{B}$  are 8 units and 5 units, respectively, and the angle between the two vectors is  $60^\circ$ . Use the parallelogram method to determine geometrically, (a) the magnitude of the sum  $\vec{R}$  of the two vectors and, (b) the direction of  $\vec{R}$  with respect to that of vector  $\vec{A}$ .

[Hint : Assume vector  $\vec{A}$  along the positive x - axis]

# 1-b) Parallelogram Method

**Solution:**

(a)  $|\vec{A}| = 8$  units and directed towards the positive x - axis

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$ .

To draw the tail-to-tail vectors geometrically. Use the ruler to draw the first vector along x-axis



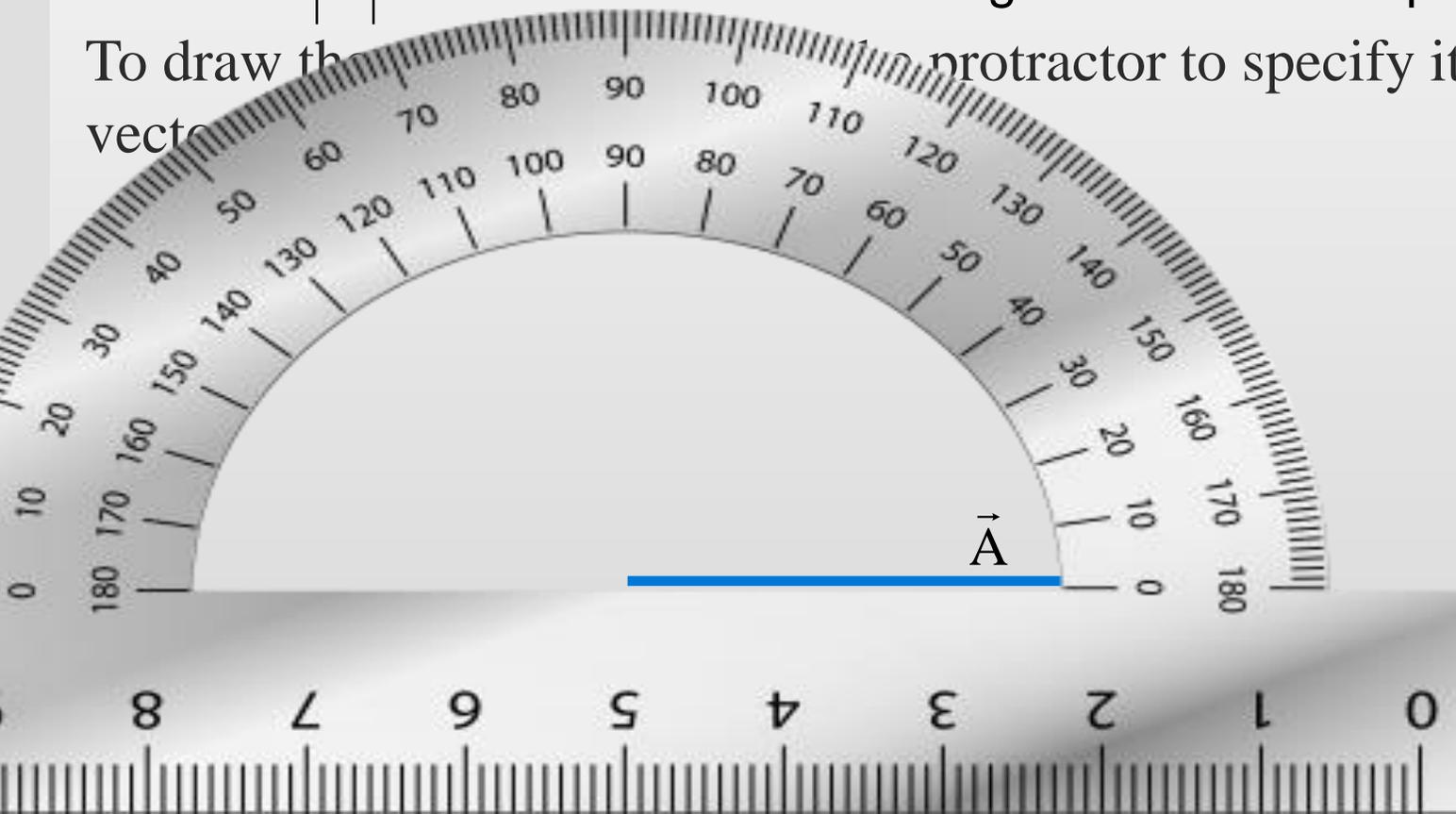
# 1-b) Parallelogram Method

**Solution:**

(a)  $|\vec{A}| = 8$  units and directed towards the positive x - axis

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$ .

To draw the vector  $\vec{B}$ , use a protractor to specify its angle of  $60^\circ$  with respect to first vector  $\vec{A}$ .



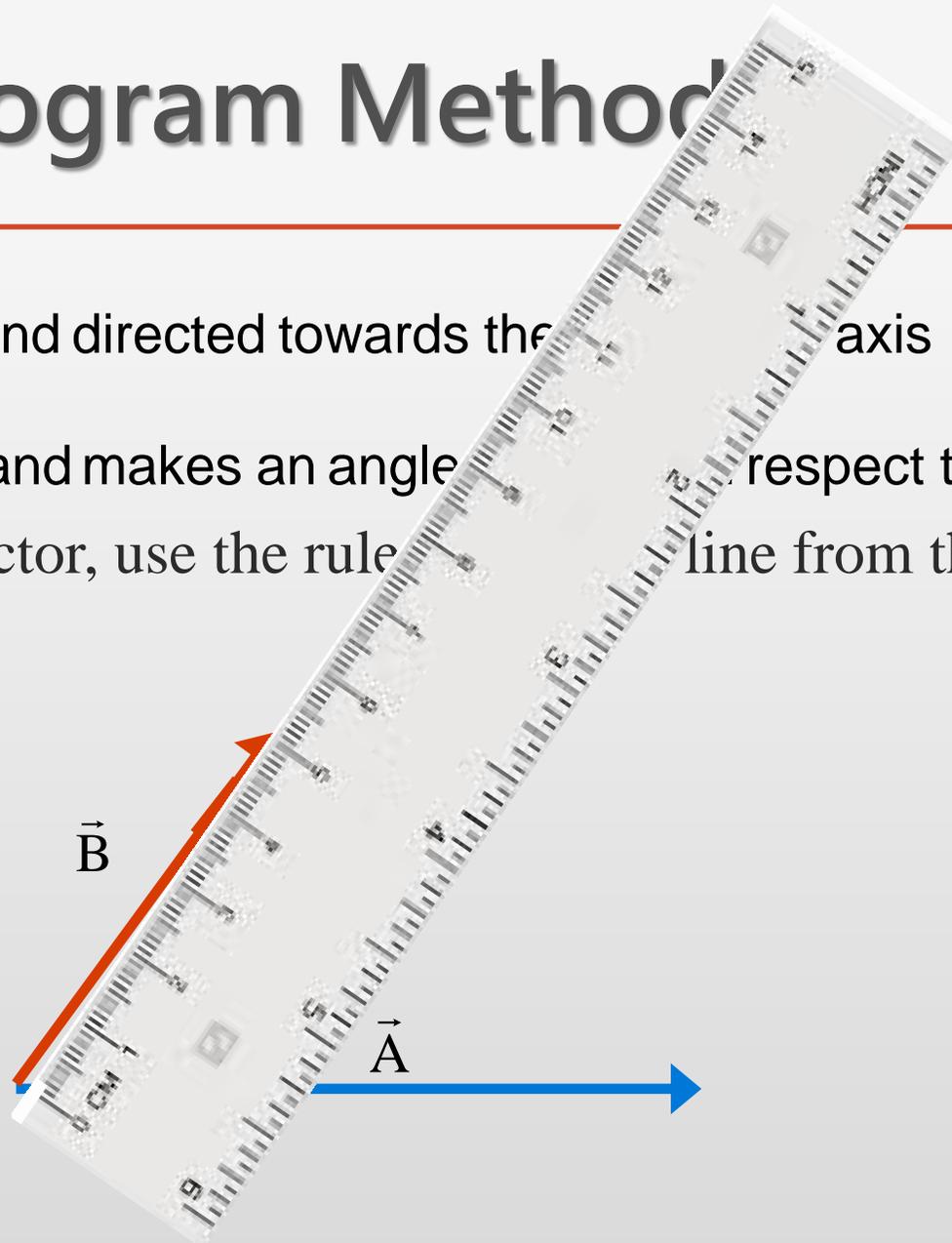
# 1-b) Parallelogram Method

**Solution:**

(a)  $|\vec{A}| = 8$  units and directed towards the positive x-axis

$|\vec{B}| = 5$  units and makes an angle of  $30^\circ$  with respect to the direction of  $\vec{A}$ .

To draw the second vector, use the ruler to draw a line from the tail of the first vector



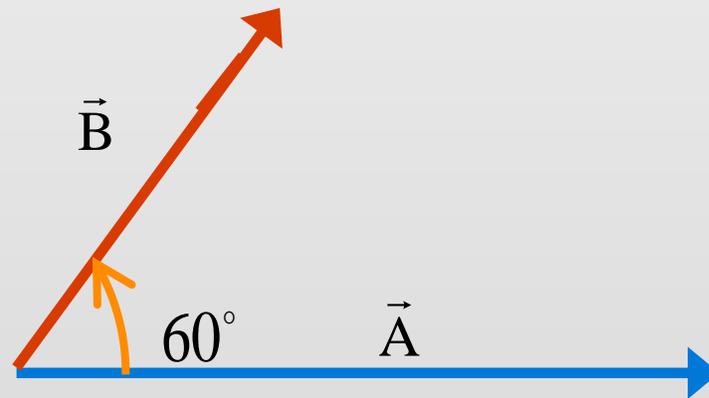
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The two vectors are drawn tail-to-tail and they form two sides of a parallelogram



# 1-b) Parallelogram Method

**Solution:**

(a)  $|\vec{A}| = 8$  units and directed towards the positive x - axis

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$ .

To draw the image of the second vector specify the angle of  $60^\circ$  with respect to the first vector in order to complete the parallelogram

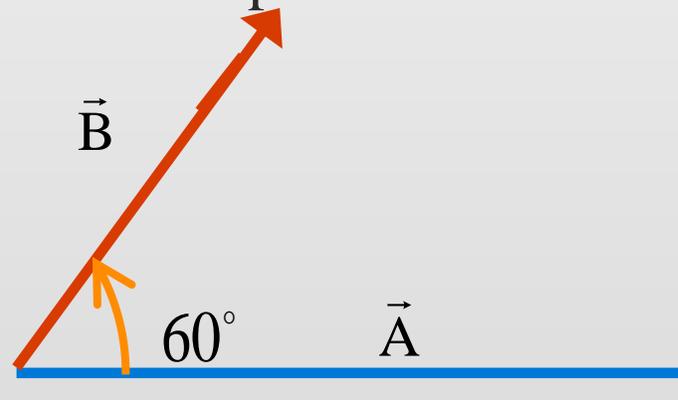


# 1-b) Parallelogram Method

(a)  $|\vec{A}| = 8$  units and directed towards the positive x - axis

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the dir

To draw the image of the second vector, use ruler to draw a line from the tip of the first vector which has the same length as the second vector with same direction thus the other side of the parallelogram will be complete



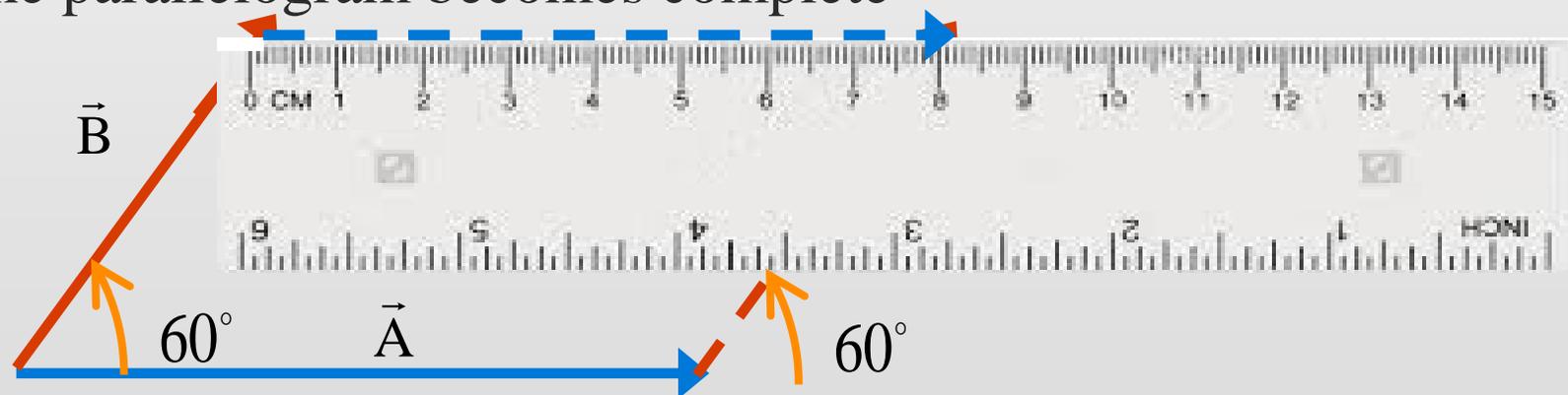
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**Solution:**

(a)  $|\vec{A}| = 8$  units and directed towards the positive x - axis

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$ .

To draw the image of the first vector, use ruler to draw a line from the head of the second vector to the head of its image vector which has the same length as the first vector and parallel to it and thus the parallelogram becomes complete



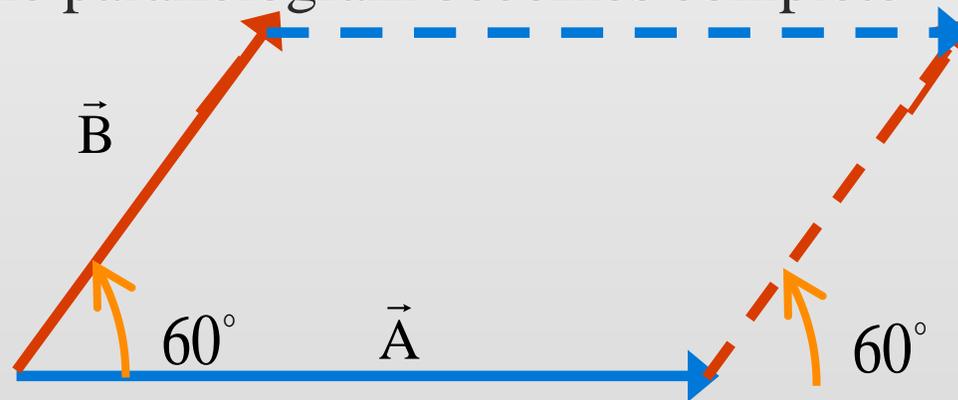
# 1-b) Parallelogram Method

**Solution:**

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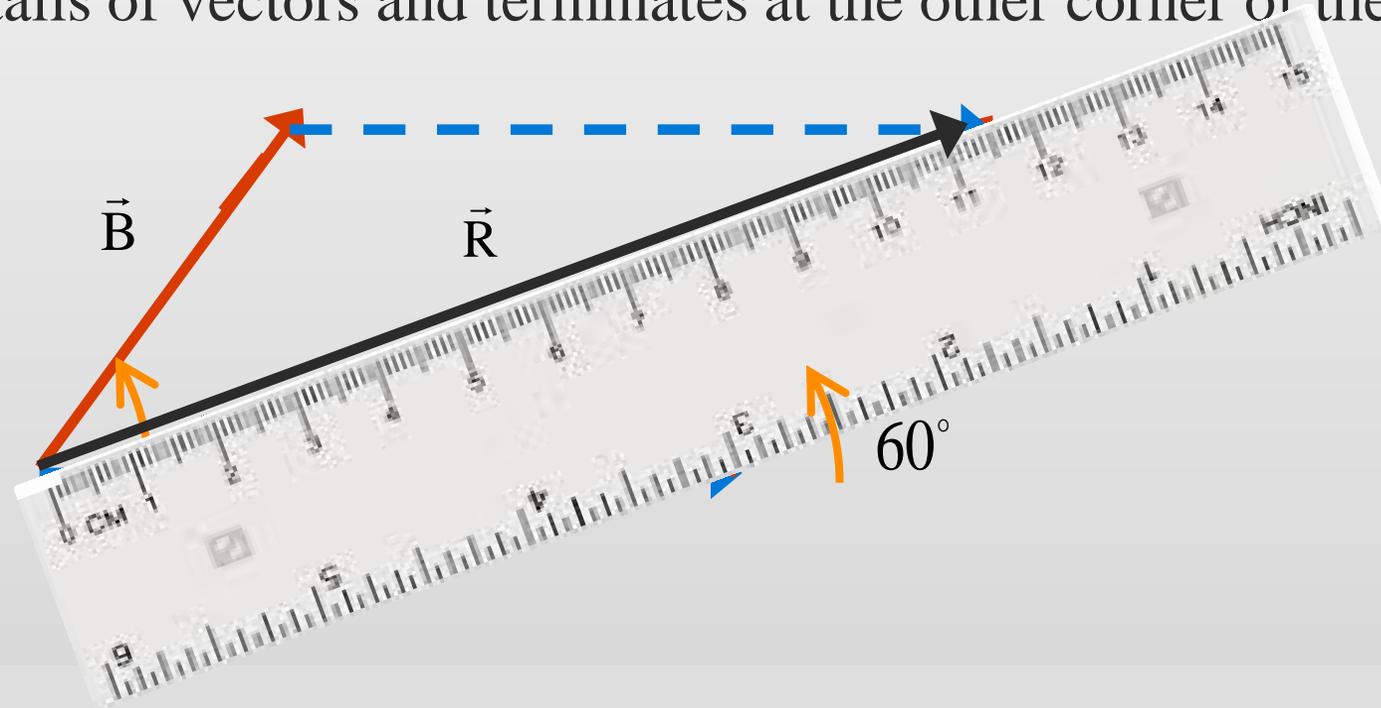
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(a)  $|\vec{A}| = 8$  units and directed towards the positive x - axis

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$ .

Use ruler to draw the diagonal of the parallelogram which starts from the point (or the corner) of the meeting of the tails of vectors and terminates at the other corner of the parallelogram



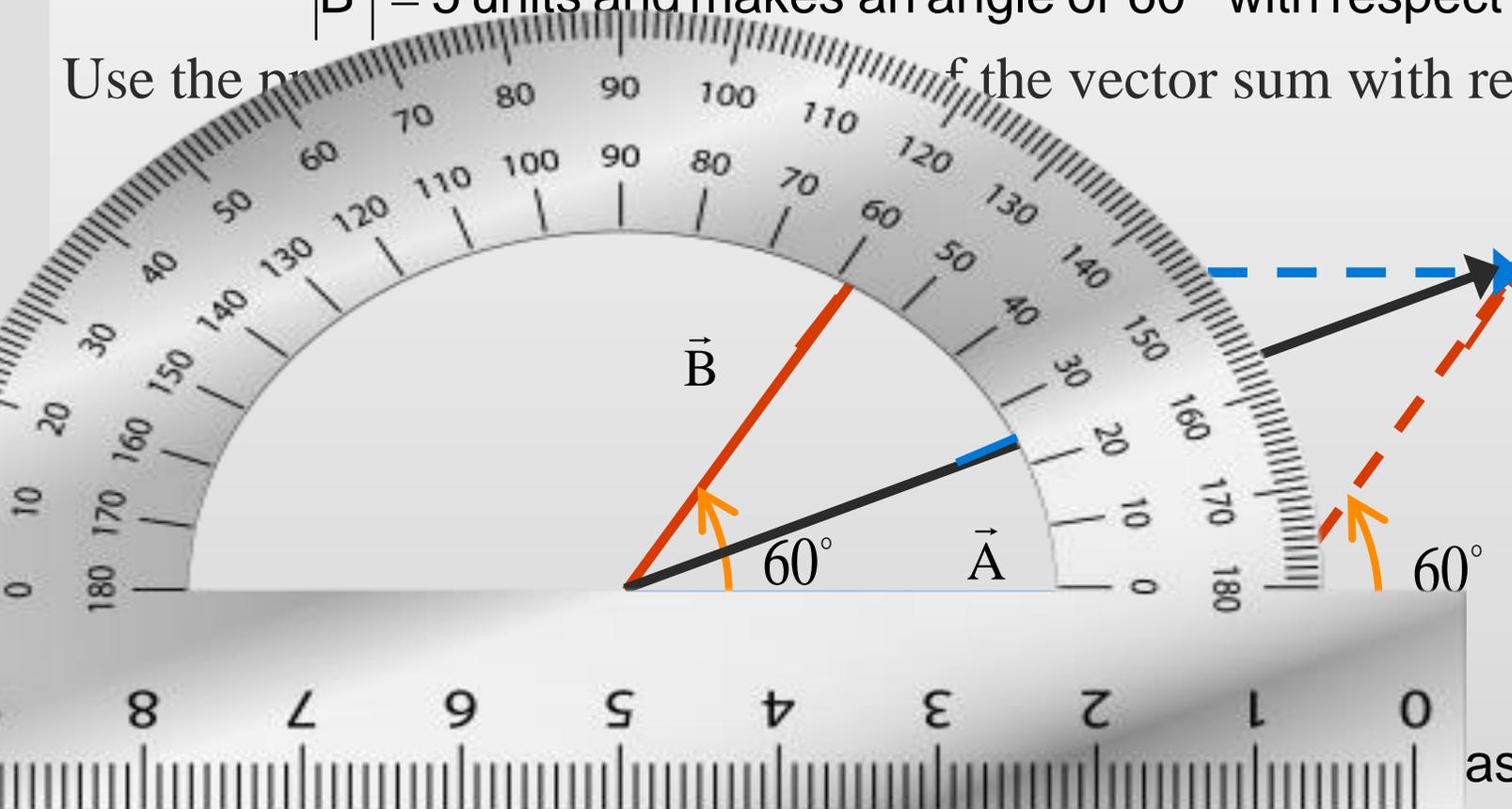
# 1-b) Parallelogram Method

**Solution:**

(b)  $|\vec{A}| = 8$  units and directed towards the positive x - axis

$|\vec{B}| = 5$  units and makes an angle of  $60^\circ$  with respect to the direction of  $\vec{A}$ .

Use the parallelogram method to find the vector sum with respect to the first vector



as read from the ruler is  $|\vec{R}| = 11.35$  units

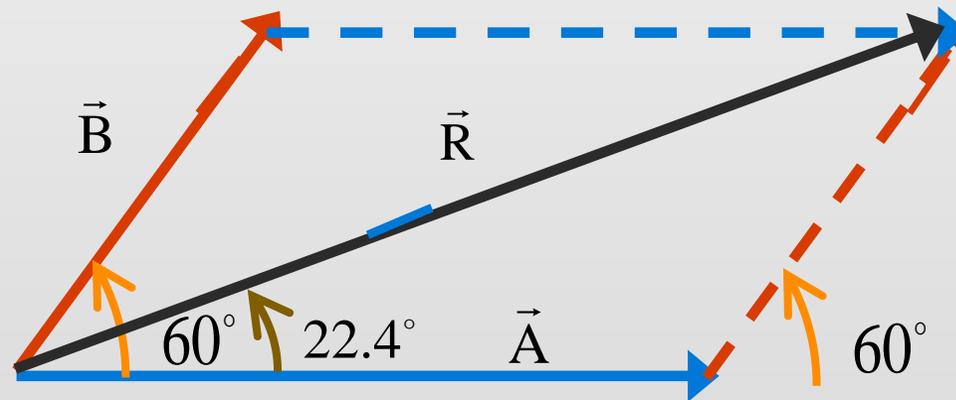
# 1-b) Parallelogram Method

**Solution:**

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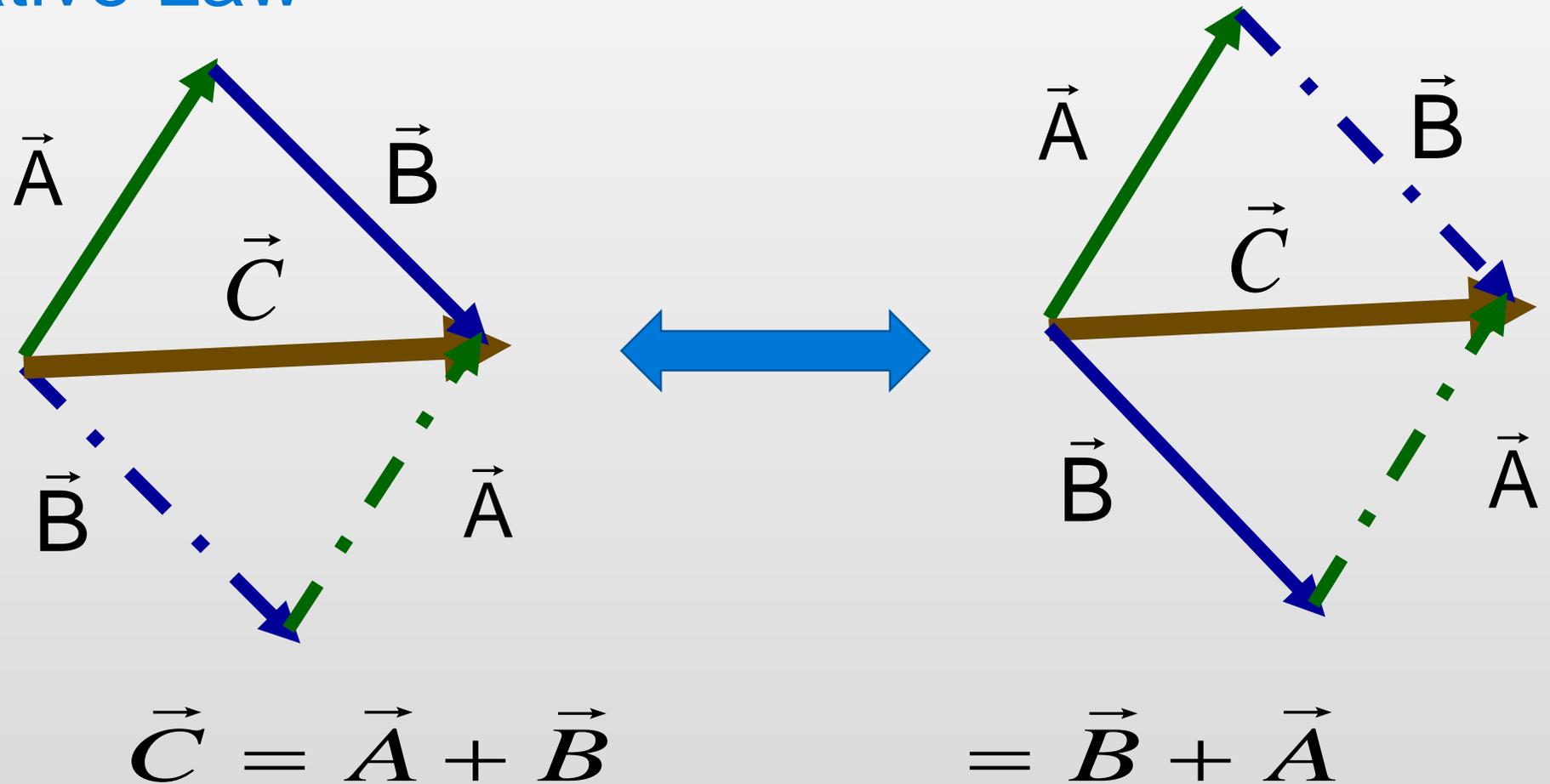
The reading of the protractor (direction of  $\vec{R}$ ) =  $22.4^\circ$



Magnitude of the sum as read from the ruler is  $|\vec{R}| = 11.35$  units

# Some Properties of Vectors

## □ Commutative Law



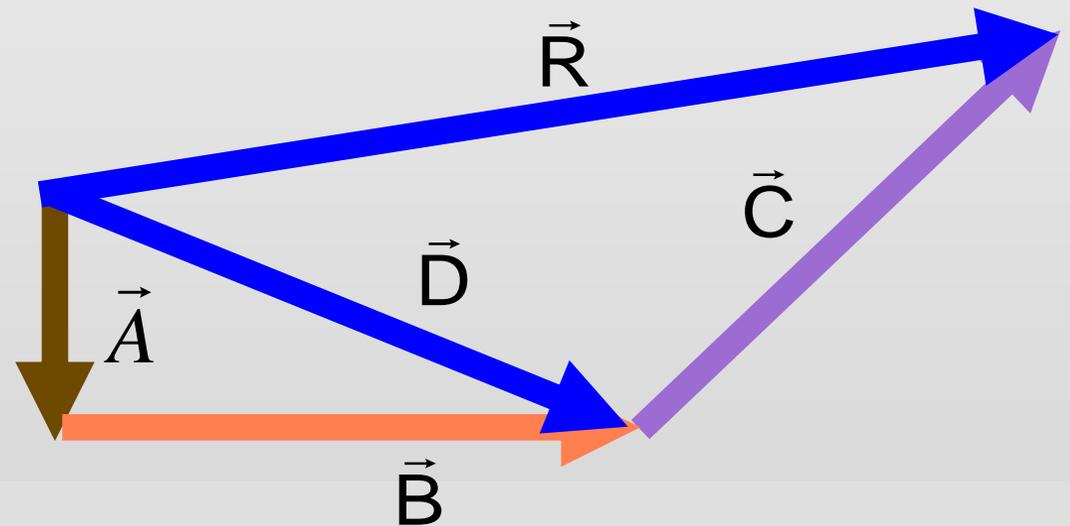
# Some Properties of Vectors

## □ Associative Law

The resultant (or sum)  $\vec{R}$  of more than two vectors can be obtained as follows:

1) The resultant of the three vectors shown in the diagram can be obtained by adding  $\vec{A}$  and  $\vec{B}$  first and then their resultant will be added to  $\vec{C}$

$$\begin{aligned}\vec{R} &= (\vec{A} + \vec{B}) + \vec{C} \\ &= \vec{D} + \vec{C}\end{aligned}$$



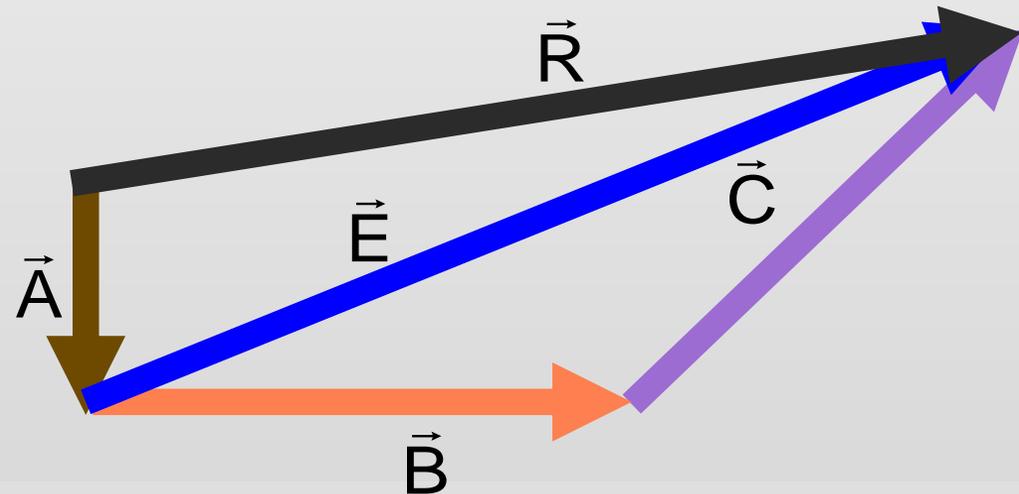
# Some Properties of Vectors

## □ Associative Law

The resultant (or sum)  $\vec{R}$  of more than two vectors can also be obtained as follows:

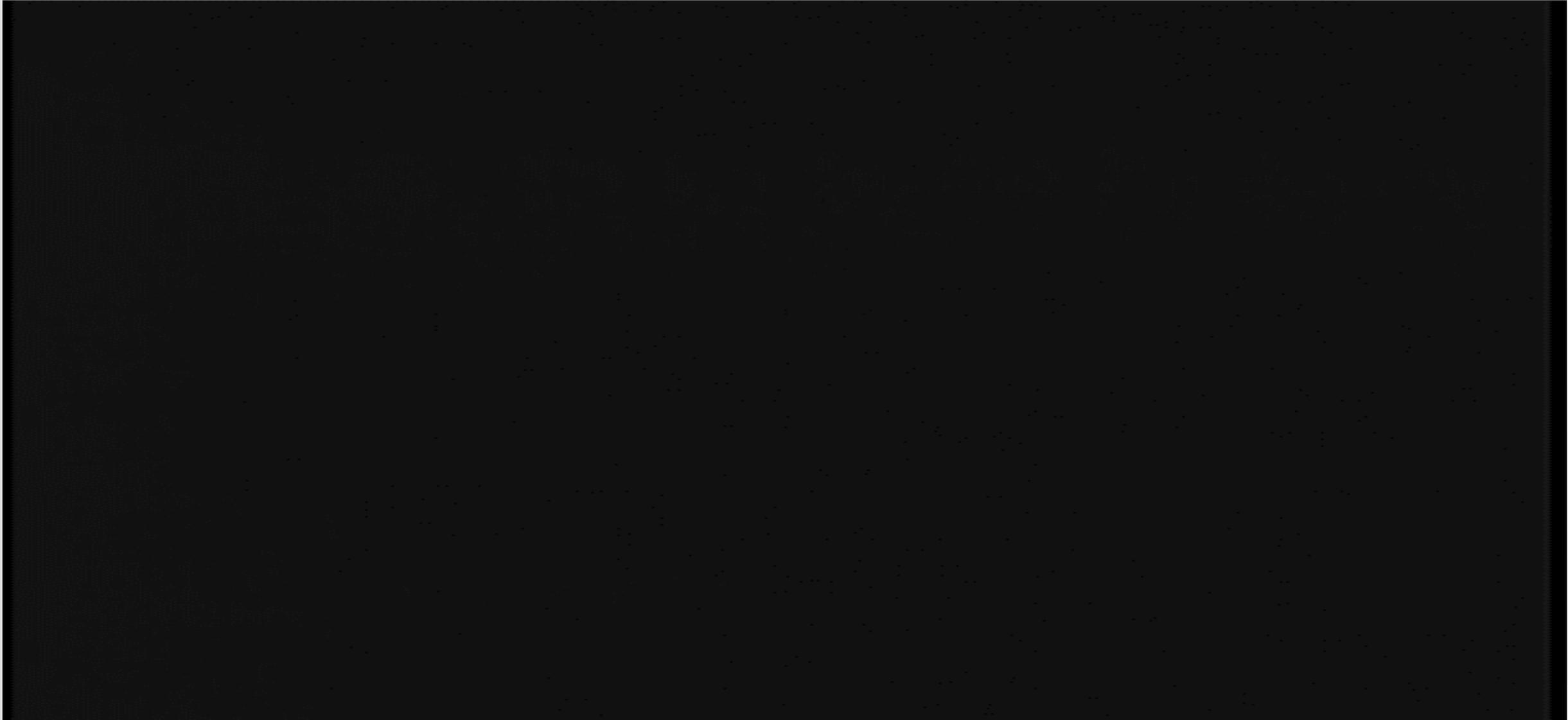
2) The resultant of the three shown vectors in the diagram can also be obtained by adding  $\vec{B}$  and  $\vec{C}$  first and then their resultant will be added to  $\vec{A}$

$$\begin{aligned}\vec{R} &= \vec{A} + (\vec{B} + \vec{C}) \\ &= \vec{A} + \vec{E}\end{aligned}$$



# Head-to-Tail Addition of Vectors

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# Tail-to-Tail Addition of Vectors

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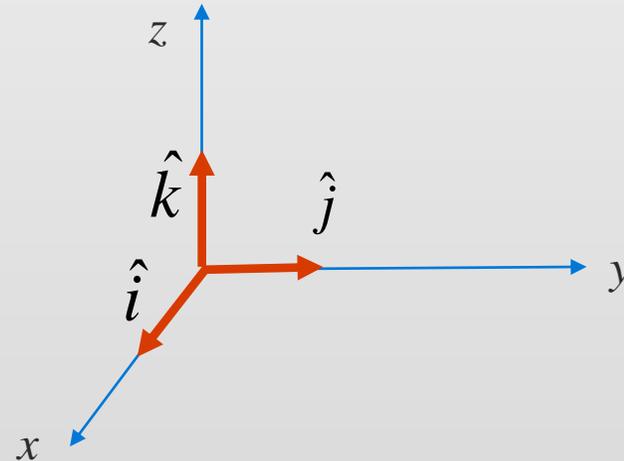
# Components of a Vector and Unit Vectors

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## Unit Vectors

- A **unit vector** is a vector which has a magnitude of **one** with **no** units.
- In Cartesian coordinates there are three unit vectors  $\hat{i}$ ,  $\hat{j}$  and  $\hat{k}$  along the  $x$ ,  $y$  and  $z$  axes, respectively.

**Notes:**(1) The three units vectors are perpendicular to each other and(2) their magnitudes are equal to each other, namely,  $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

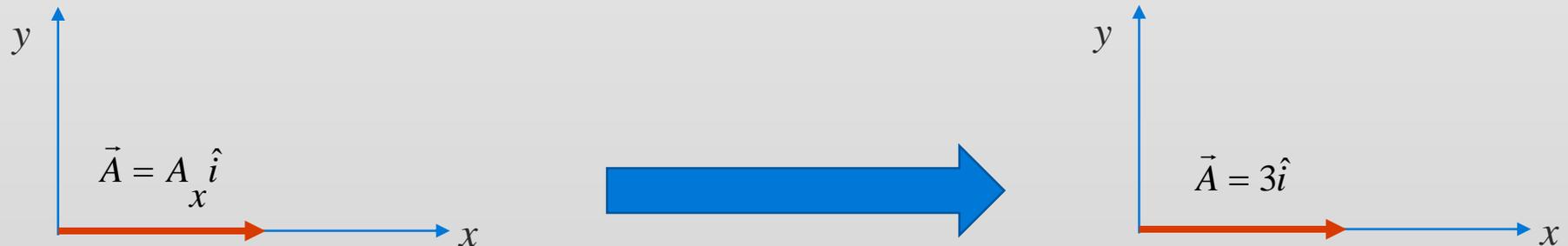


# Components of a Vector: 1-D Vector

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A vector along  $x$ -axis is written in terms of unit vector  $\hat{i}$  as follows:

$\vec{A} = A_x \hat{i}$  where  $A_x$  is the  $x$ -component of this vector which has zero  $y$ -component. However, the magnitude of this vector equals the magnitude of its component along  $x$ -axis. Positive  $\hat{i}$  refers to the direction of the vector along positive  $x$ -axis. *e.g.* The vector  $\vec{A} = 3\hat{i}$  has a magnitude of 3 units and a direction towards east.



# Components of a Vector: 1-D Vector

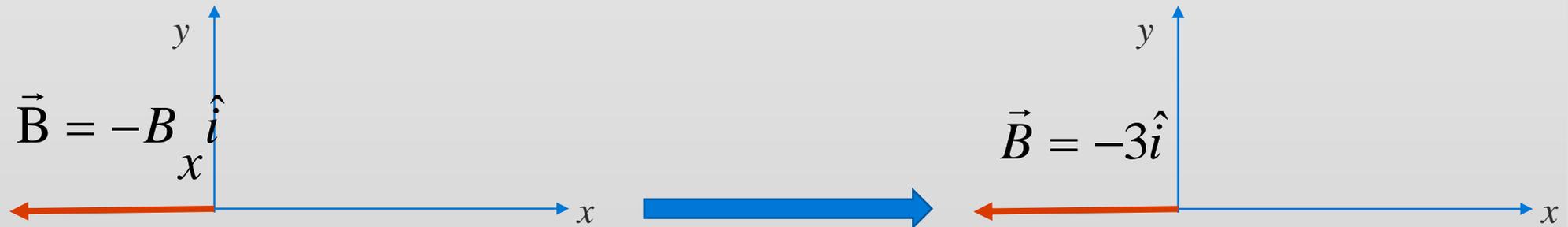
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A vector along  $x$ -axis is written in terms of unit vector  $\hat{i}$  as follows:

$\vec{B} = B_x \hat{i}$  where  $B_x$  is the  $x$ -component of this vector which has zero  $y$ -component. However, the magnitude of this vector equals the magnitude of its component along  $x$ -axis.

*e.g.* The vector  $\vec{B} = -3\hat{i}$  has a magnitude of 3 units and a direction towards west.

Negative  $\hat{i}$  refers to the direction of the vector along negative  $x$ -axis



# Components of a Vector: 1-D Vector

A vector along  $y$ -axis is written in terms of unit vector  $\hat{j}$  as follows:

$\vec{D} = D_y \hat{j}$  where  $D_y$  is the  $y$ -component of this vector which has zero  $x$ -component. However, the magnitude of this vector equals the magnitude of its component along  $y$ -axis.

*e.g.* The vector  $\vec{D} = 3\hat{j}$  has a magnitude of 3 units and a direction towards north. Positive  $\hat{j}$  refers to the direction of the vector along positive  $y$ -axis



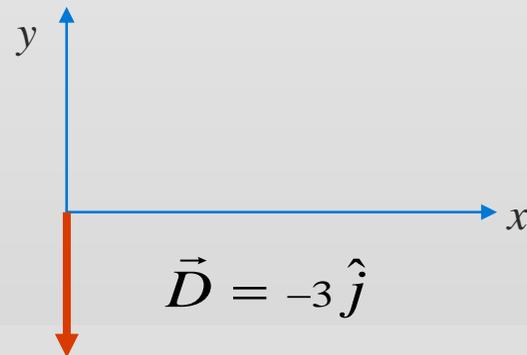
# Components of a Vector: 1-D Vector

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A vector along  $y$ -axis is written in terms of unit vector  $\hat{j}$  as follows:

$\vec{D} = D_y \hat{j}$  where  $D_y$  is the  $y$ -component of this vector which has zero  $x$ -component. However, the magnitude of this vector equals the magnitude of its component along  $y$ -axis.

*e.g.* The vector  $\vec{D} = -3\hat{j}$  has a magnitude of 3 units and a direction towards south. Negative  $\hat{j}$  refers to the direction of the vector along Negative  $y$ -axis.

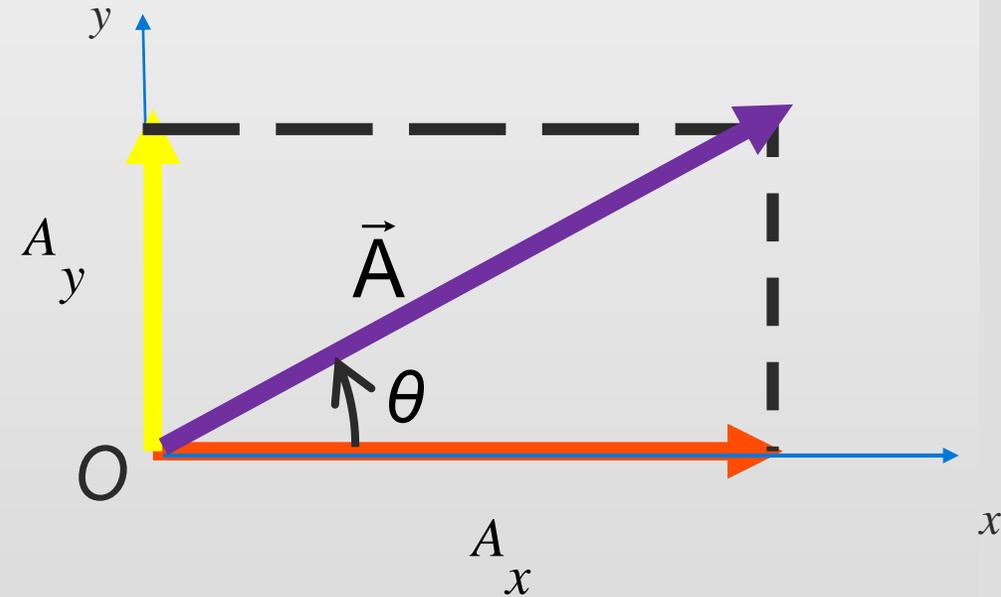


# Components of a Vector: 2-D Vector

A vector, in  $x$ - $y$  plane, is written in terms of unit vector  $\hat{i}$  and  $\hat{j}$  as follows:

$$\vec{A} = A_x \hat{i} + A_y \hat{j} \quad (\text{The vector expressed in unit vector notation})$$

$A_x$  and  $A_y$  are the  $x$ - and  $y$ - components of the vector. The angle  $\theta$  gives the direction of the vector  $\vec{A}$  with respect to the positive  $x$ -axis (this angle is always taken in counterclockwise direction from positive  $x$ -axis). The vector is chosen in the first quadrant.

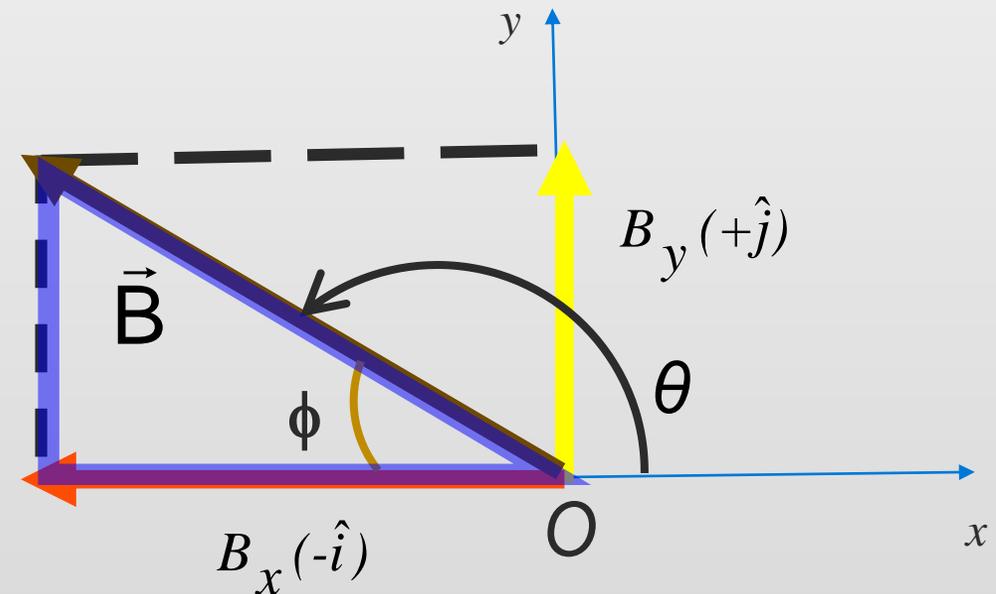


# Components of a Vector

When the vector, in  $x$ - $y$  plane, is in the second quadrant

$$\vec{B} = -B_x \hat{i} + B_y \hat{j} \quad (\text{The vector expressed in unit vector notation})$$

$y$ -component is along positive  $y$ -axis while  $x$ -component is along negative  $x$ -axis

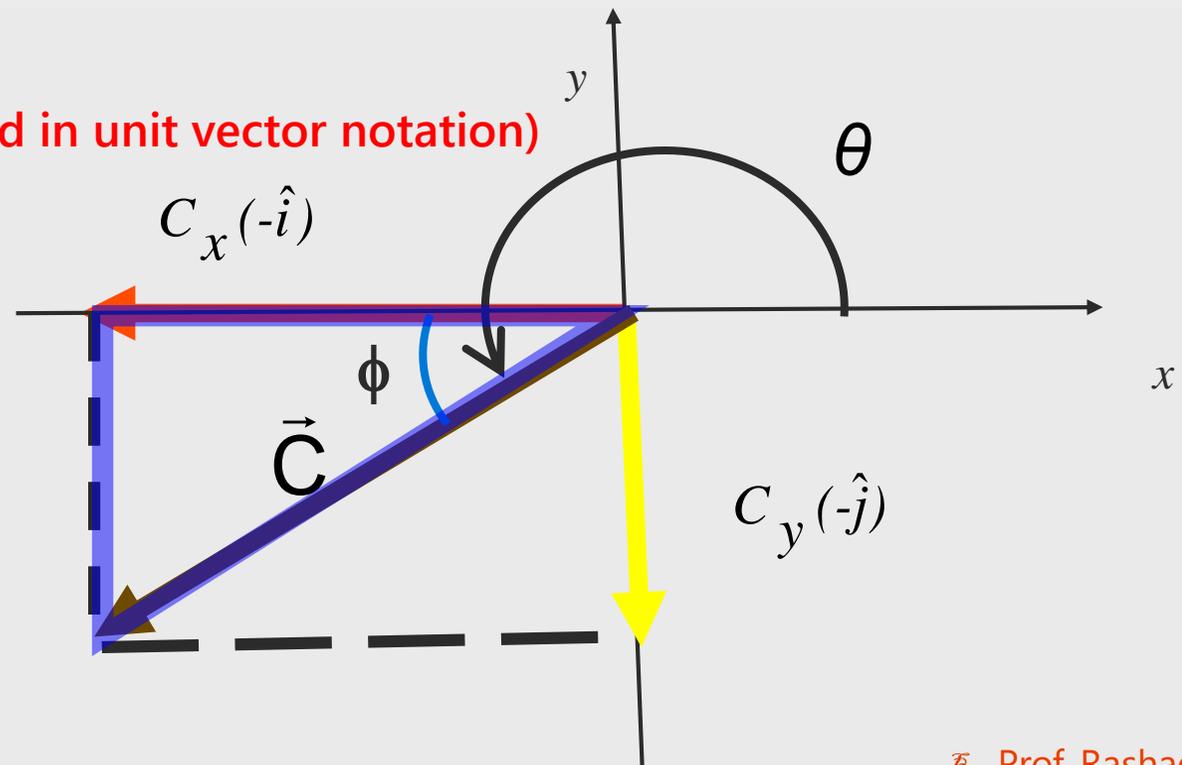


# Components of a Vector

When the vector in  $x$ - $y$  plane is in the third quadrant

$$\vec{C} = -C_x \hat{i} - C_y \hat{j} \quad (\text{The vector expressed in unit vector notation})$$

**Note:** Both components of the vector are in the negative directions



## Finding the components of a Vector:

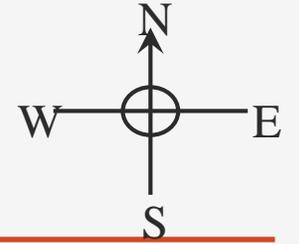
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### Example

( a ) What are the  $x$ - and  $y$ - components of a vector of magnitude 5.00 m and angle  $37^\circ$  from the +ve  $x$ -axis rotating counterclockwise direction?

(b) What are the  $x$ - and  $y$ - components of a vector of magnitude 5.00 m and angle  $37.0^\circ$  from the + ve  $x$ -axis rotating in clockwise direction?

# Finding the components of a Vector



## Solution

$$(a) A = 5 \text{ m}, \theta = 37^\circ$$

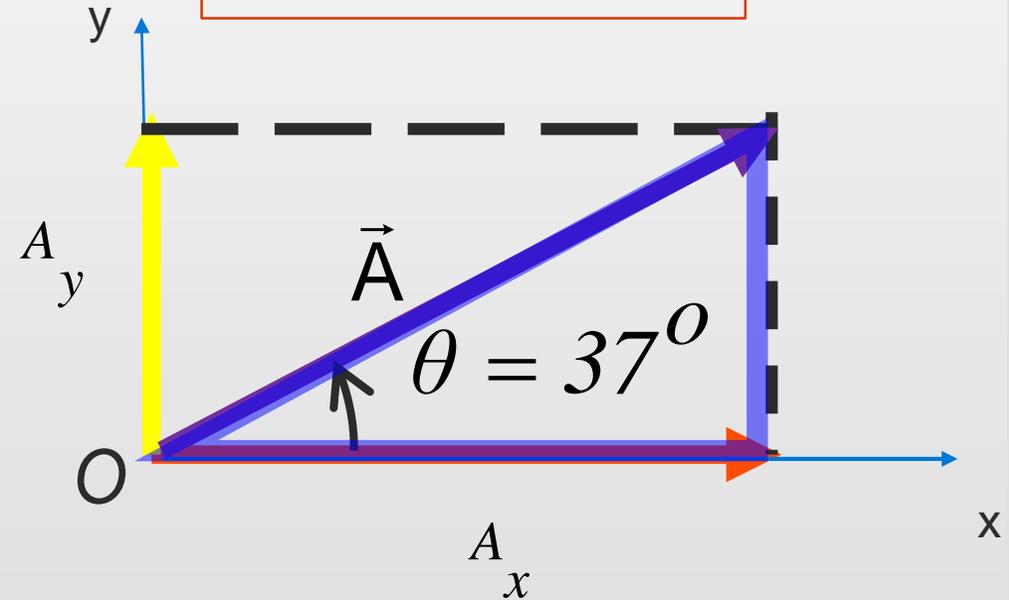
The vector is  $37^\circ$  north of east

$$\cos \theta = \frac{\text{Adjacent side } A_x}{\text{Hypotenuse side } A}$$

$$\rightarrow A_x = A \cos \theta = (5) \cos 37^\circ = 4\text{m}$$

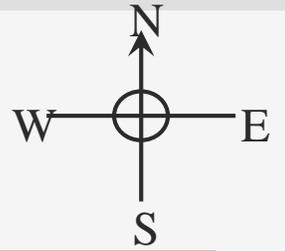
$$\sin \theta = \frac{\text{Opposite side } A_y}{\text{Hypotenuse side } A}$$

$$\rightarrow A_y = A \sin \theta = (5) \sin 37^\circ = 3\text{m}$$



**Notes:** Since the vector is in the first quadrant and started from +ve  $x$ -axis revolving in counterclockwise direction towards +ve  $y$ -axis then both components are directed in +ve  $x$ - and  $y$ - axes as expected. The vector can be expressed by  $\vec{A} = 4\hat{i} + 3\hat{j}$  or  $\vec{A} = (4,3)$

# Finding the components of a Vector



## Solution

$$(b) \quad D = 5m, \quad \phi = -37^\circ \quad \text{or} \quad \theta = 360 - \phi = 323^\circ$$

The vector is  $37^\circ$  south of east.

$$\cos \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse side}} = \frac{D_x}{D}$$

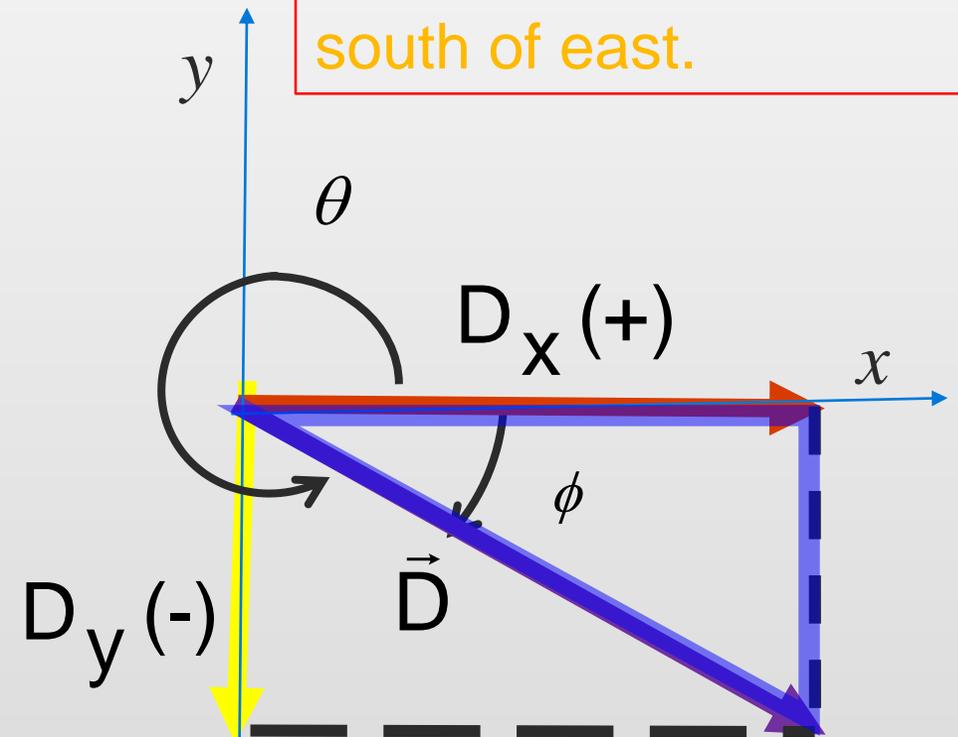
$$\rightarrow D_x = D \cos \theta = (5) \cos 323^\circ$$

$$\text{or} = (5) \cos \phi = (5) \cos (-37^\circ) = 4 \text{ m}$$

$$\sin \theta = \frac{\text{Opposite side}}{\text{Hypotenuse side}} = \frac{D_y}{D}$$

$$\rightarrow D_y = D \sin \theta = (5) \sin 323^\circ$$

$$\text{or} = (5) \sin \phi = (5) \sin (-37^\circ) = -3 \text{ m}$$



**Notes:**  $D_x$  is along +ve  $x$ - axis while  $D_y$  is along -ve  $y$ -axis as long as the vector is in the fourth quadrant. The vector can be expressed by  $\vec{D} = 4\hat{i} - 3\hat{j}$  or  $\vec{D} = (4, -3)$

# Components of a vector

## Problem

A vector has an  $x$ - component of  $-25$  units and a  $y$ - component of  $40$  units. Find the magnitude and direction of this vector.

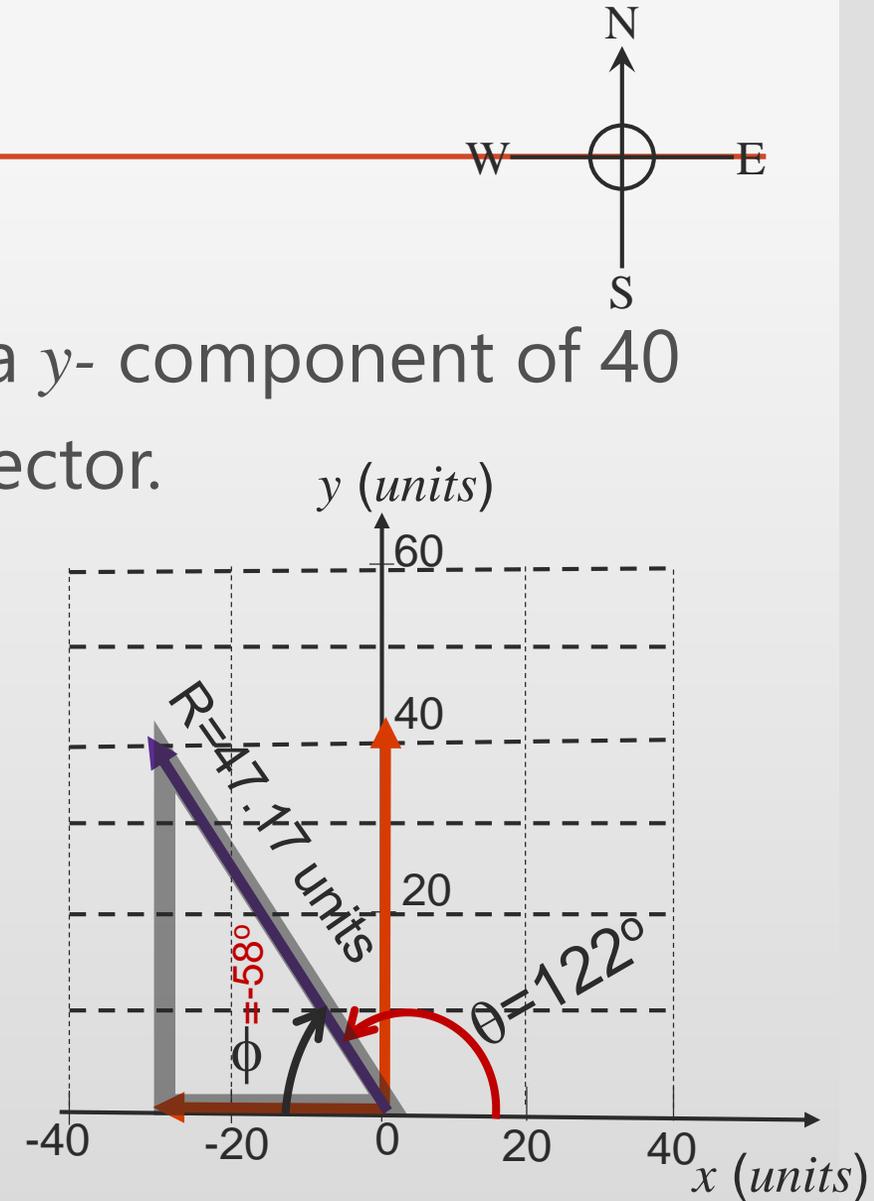
## Solution

$$\vec{r} = x\hat{i} + y\hat{j} = -25\hat{i} + 40\hat{j}$$

$$\Rightarrow r = \sqrt{x^2 + y^2} = \sqrt{(-25)^2 + (40)^2} = 47.17 \text{ units}$$

$$\tan \phi = \frac{y}{x} = \frac{40}{-25} \Rightarrow \phi = \tan^{-1}\left(\frac{40}{-25}\right) = -58^\circ$$

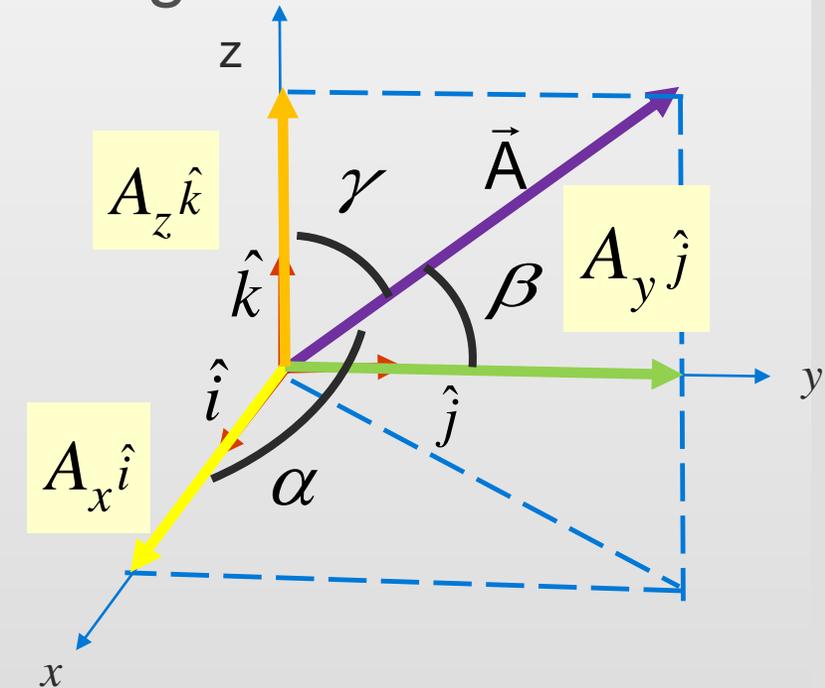
$$\theta = 180 - \phi = 180^\circ - 58^\circ = 122^\circ \text{ Counterclockwise}$$



# Components of a Vector: 3-D Vector

A vector in  $xyz$  coordinate system is expressed in terms of three unit vectors  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$  as follows and is shown in the diagram:

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$



**Note:** The shown vector has angles  $\alpha$ ,  $\beta$  and  $\gamma$  with the  $x$ -,  $y$ -, and  $z$ -axes.

## Exercise

A vector is given by  $\vec{R} = 2\hat{i} + \hat{j} + 3\hat{k}$ . Find (a) the magnitude of the x, y, and z components, (b) the magnitude of  $\vec{R}$ , and (c) the angles between  $\vec{R}$  and the x, y, and z axes.

**Answer:** (a)  $x = 2$ ,  $y = 1$  and  $z = 3$

(b)  $R = 3.74$

(c)  $\theta_x = 57.7^\circ$ ,  $\theta_y = 74.5^\circ$  and  $\theta_z = 36.7^\circ$

# Objective Questions

## Objective Question

What is the y component of the vector  $(3\hat{i} - 10\hat{k})$  m/s ?

**Answer:**

(a) 3 m/s

(b) -10 m/s

(c) 0

(d) 10 m/s

(e) None of those answers

# Objective Questions

## Objective Question

What is the magnitude of the vector  $\vec{D} = (10\hat{i} - 10\hat{k})$  m/s ?

**Answer:**

- (a) 0
- (b) 10 m/s
- (c) -10 m/s
- (d) 10
- (e) 14.1 m/s

# Change in Components of a Vector

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