General

Physics 101

Freely Falling Objects

Prepared By

Prof. Rashad Badran

One Dimensional Motion: An Application

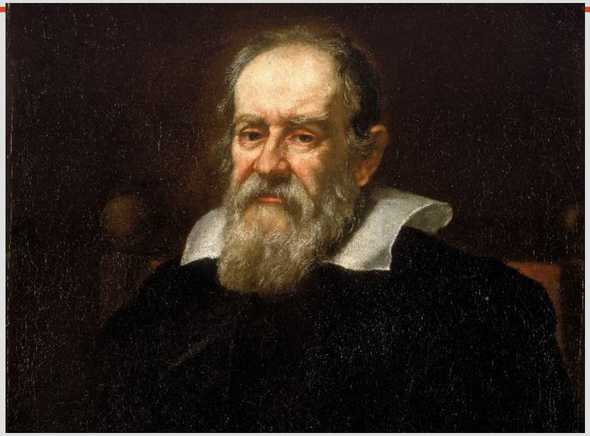
Freely Falling Objects

- A freely falling Object is any object falling freely under the influence of gravity, regardless of its initial motion.
- The ideal case of this motion occurs when the air resistance is negligible.
- Thus, only one force acts on the object, that is, the force of gravity.

By Prof. Rashad Badran

One Dimensional Motion: Freely Falling Objects

Galileo



Leaning Tower of Pisa

- Two different weights dropped simultaneously from Tower of Pisa hit the ground at approximately the same time!!
- Rate of fall does not depend on mass.

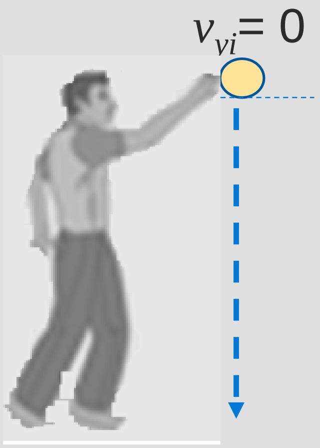
By Prof. Rashad Badran

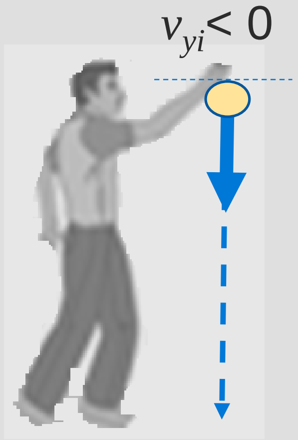
One Dimensional Motion:

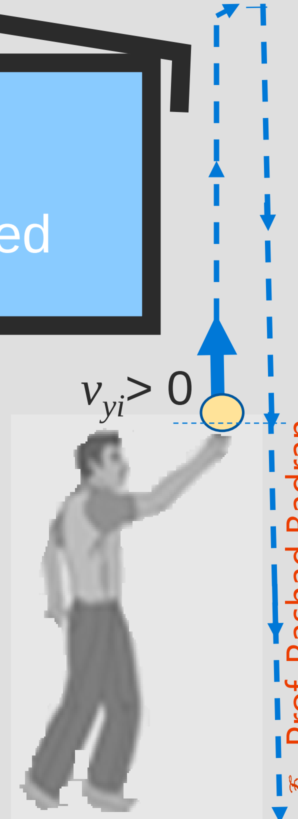
Freely Falling Objects

An object may be released under the effect of gravity only in three different cases:

(1) From rest	(2) Thrown downward with an initial speed	(3) Thrown upward with an initial speed







One Dimensional Motion:

Freely Falling Objects

In the three different cases, two assumptions can be adopted:

(1) At the Earth's surface, the free-fall acceleration (or acceleration due to gravity) does not change with altitude over short vertical distances: Freely falling object along y-axis is approximately equivalent to the motion of a particle under constant acceleration in 1-D.

$\Rightarrow a_y = -g$ where g is acceleration due to gravity which has a magnitude of $9.8\ m/s^2$.

(2) Air resistance is neglected.

One Dimensional Motion: Freely Falling Objects

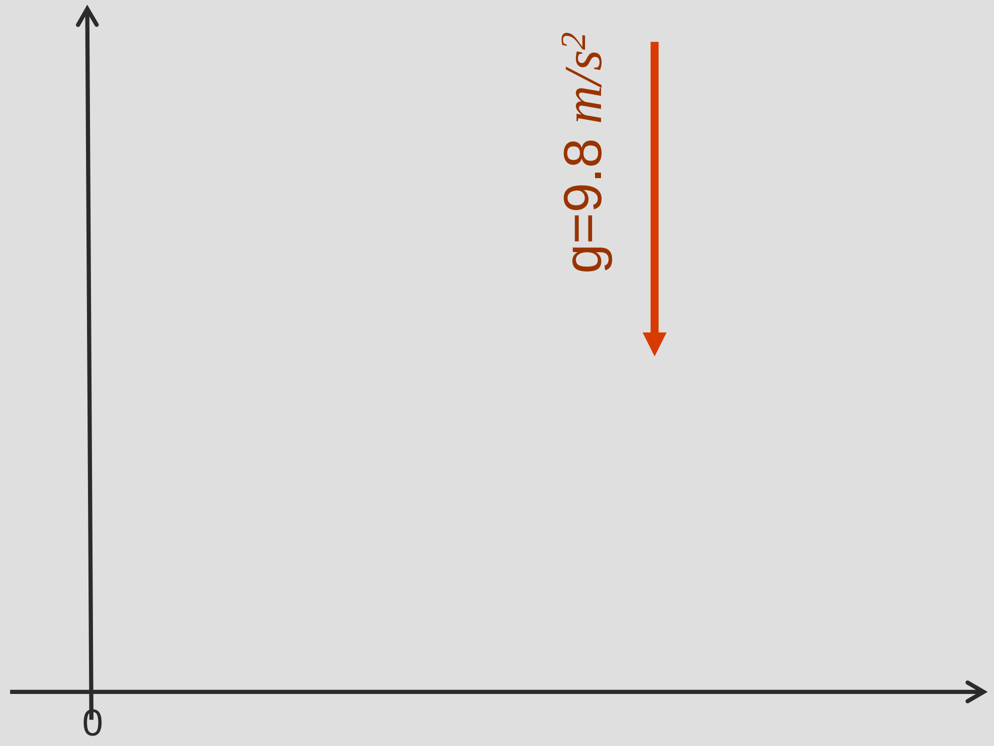
Recalling the equations of motion in one dimension by replacing x by y to indicate that the motion is along y-axis.

$$v_{yf} = v_{yi} + a_y t$$

$$y_f - y_i = \frac{1}{2}(v_{yi} + v_{yf})t$$

$$y_f - y_i = v_{yi} t + \frac{1}{2} a_y t^2$$

$$v_{yf}^2 = v_{yi}^2 + 2a_y (y_f - y_i)$$



One Dimensional Motion: Freely Falling Objects

Insert $a_y = -g$ into the equations of motion in 1-D to get:

$$v_{yf} = v_{yi} - gt$$

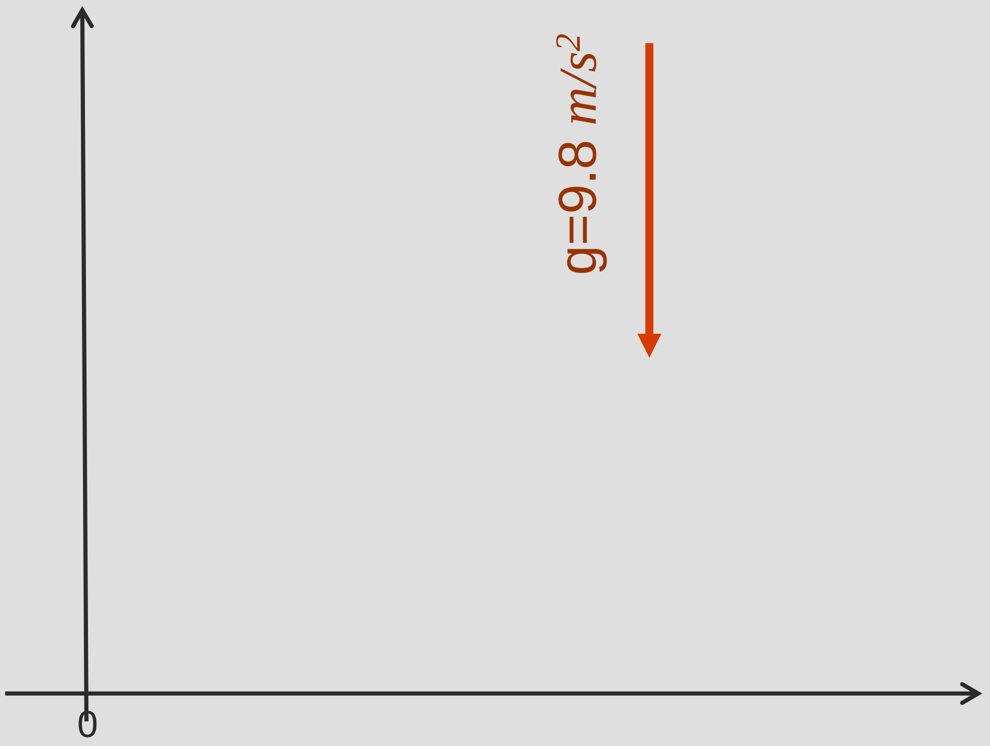
$$\Delta y = \frac{1}{2}(v_{yi} + v_{yf})t$$

$$\Delta y = v_{yi}t - \frac{1}{2}gt^2$$

$$v_{yf}^2 = v_{yi}^2 - 2g\Delta y$$

where $\Delta y = y_f - y_i$

The minus sign for the free-fall acceleration is already inserted into the equations



One Dimensional Motion: Freely Falling Objects

$$v_{yf} = v_{yi} - gt$$

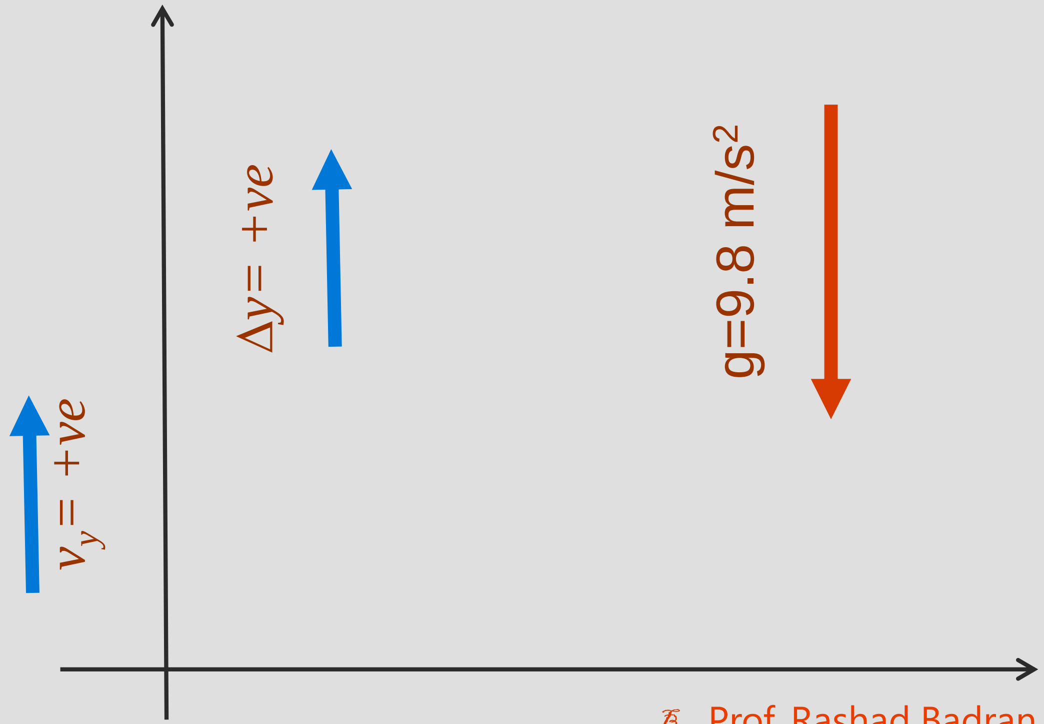
$$\Delta y = \frac{1}{2}(v_{yi} + v_{yf})t$$

$$\Delta y = v_{yi}t - \frac{1}{2}gt^2$$

$$v_{yf}^2 = v_{yi}^2 - 2g\Delta y$$

where $\Delta y = y_f - y_i$

If the object moves upward and is displaced upward then its velocity (v_y) and displacement (Δy) must have positive signs



By Prof. Rashad Badran

One Dimensional Motion: Freely Falling Objects

$$v_{yf} = v_{yi} - gt$$

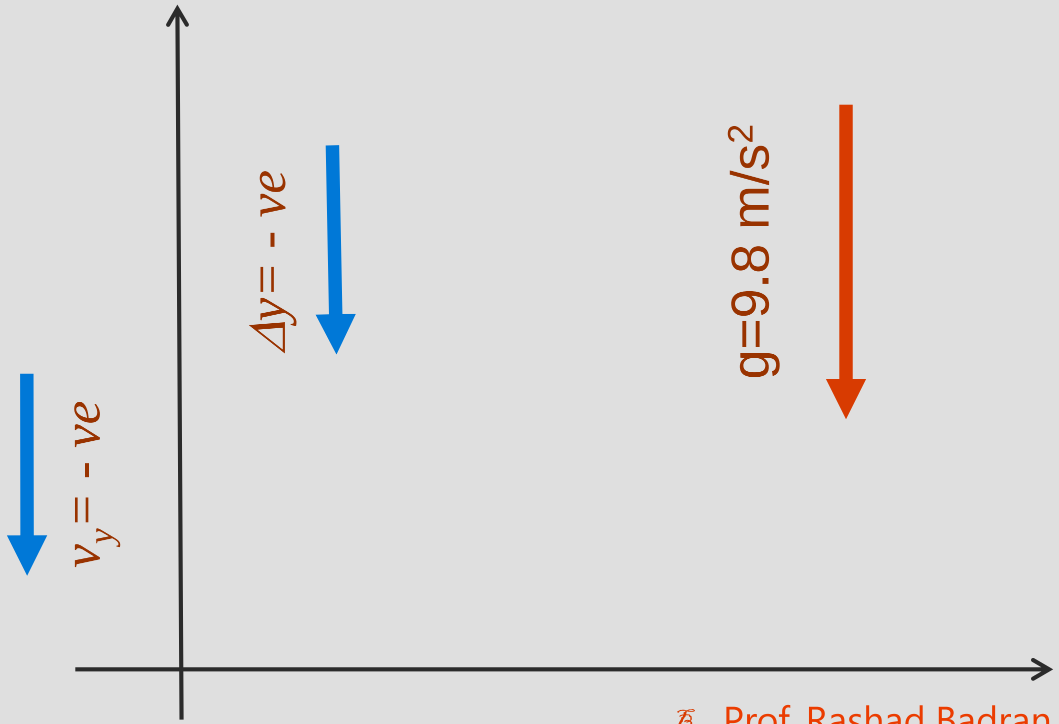
$$\Delta y = \frac{1}{2}(v_{yi} + v_{yf})t$$

$$\Delta y = v_{yi}t - \frac{1}{2}gt^2$$

$$v_{yf}^2 = v_{yi}^2 - 2g\Delta y$$

where $\Delta y = y_f - y_i$

If the object moves downward and is displaced downward then its velocity (v_y) and displacement (Δy) must have negative signs

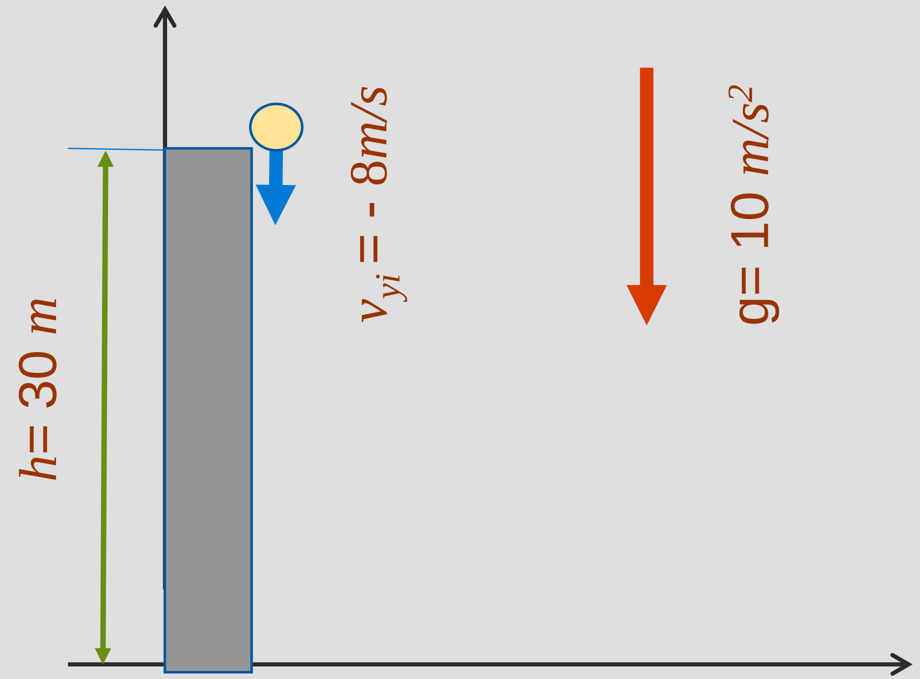


By Prof. Rashad Badran

One Dimensional Motion:

Example

A ball is thrown directly downward with an initial speed of 8 *m/s* from a height of 30 *m*. (a) after what time interval does it strike the ground?(b) what is its final speed just before hitting the ground? [Hint: Take g= 10 m/s^2]



One Dimensional Motion:

Solution:

(a) Here initial time is taken t_i = 0. Initial velocity has a negative sign (i.e. v_{yi} = - 8 m/s). Also the displacement must have a negative sign (i.e. Δy = - 30 m). Substitute g = 10 m/s^2. To find t_f use the equation:

$$\Delta y = v_{yi}t - \frac{1}{2}gt^2$$

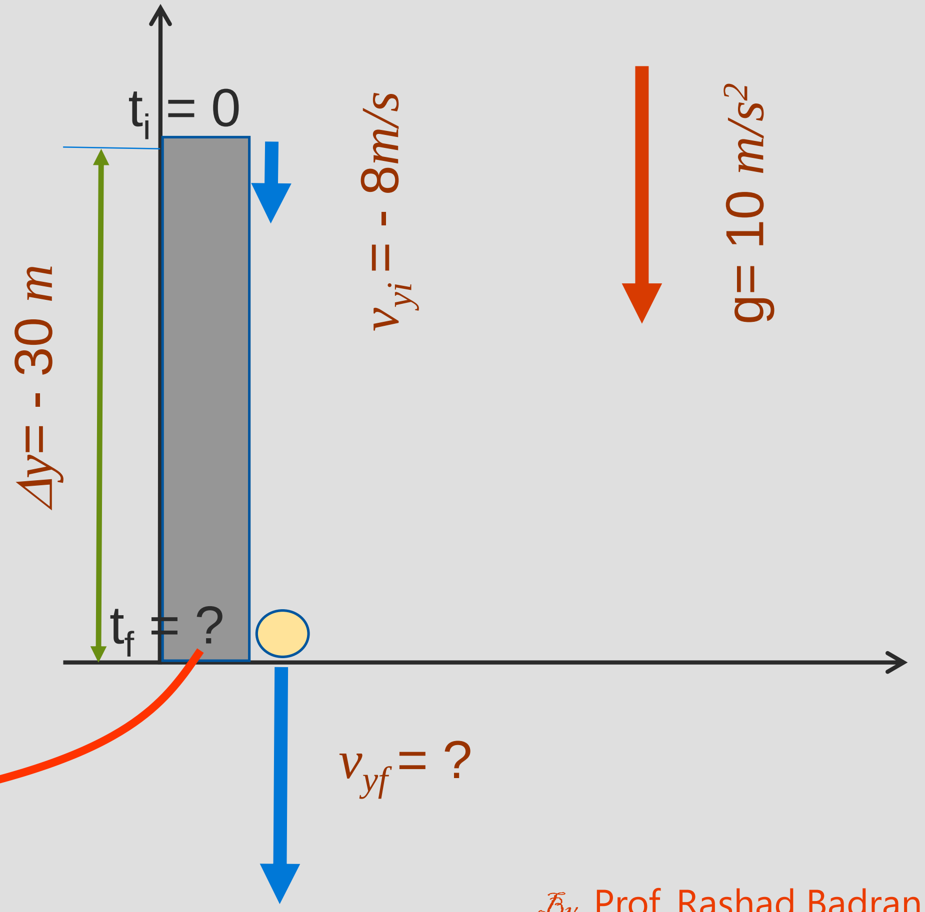
$$\Rightarrow -30 = (-8)t - (\frac{10}{2})t^2$$

$$5t^2 + 8t - 30 = 0 \quad \Longleftrightarrow \quad at^2 + bt + c = 0$$

$$t = \frac{-8 \pm \sqrt{(8)^2 - 4(5)(-30)}}{10} \quad \Longleftrightarrow \quad t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-8 + 25.77}{10} \quad or \quad t = \frac{-8 - 25.77}{10}$$

$$\Rightarrow t = 1.77s$$



One Dimensional Motion:

Solution:

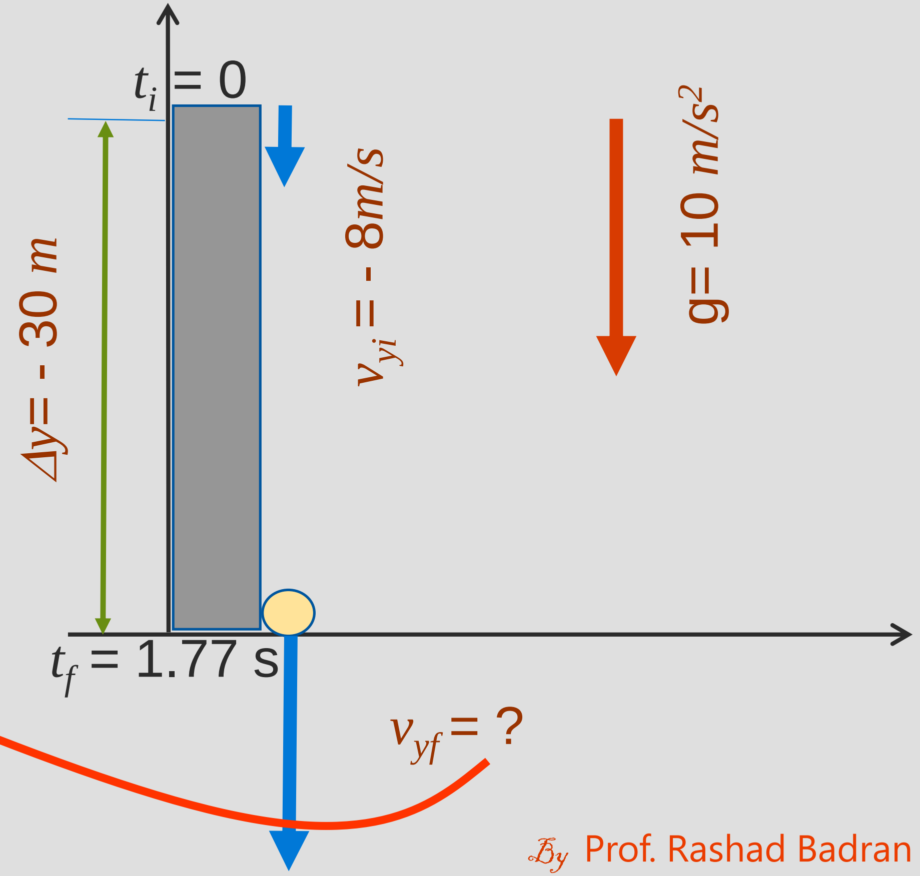
(b) To find the final speed use the information: [v_{yi} = - 8 m/s, Δy = - 30 m, and g= 10 m/s^2] and employ the equation:

$$v_{yf}^{2} = v_{yi}^{2} - 2g\Delta y$$

$$\Rightarrow v_{yf}^{2} = (-8)^{2} - 2(10)(-30)$$

$$\Rightarrow v_{yf} = -25.77\,m/s$$

Note: The final speed is 25.77 m/s while the negative sign refers to the direction of velocity (downward).

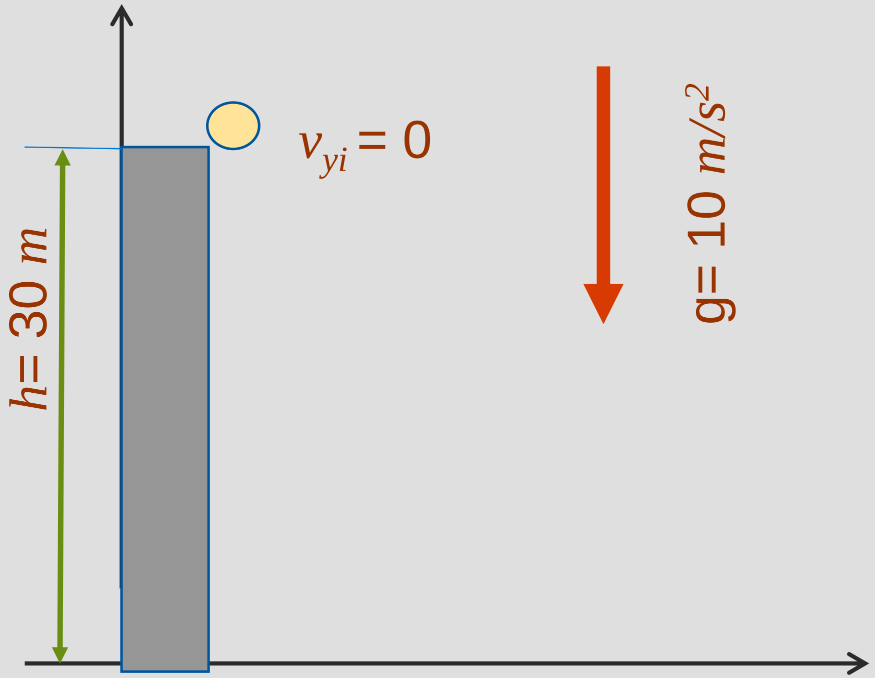


By Prof. Rashad Badran

One Dimensional Motion:

Example

A ball is released from rest from a height of 30 *m*. (a) after what time interval does it strike the ground?(b) what is its final speed just before hitting the ground? (c) where is the ball (with respect to the level of thrower (or the top of building)) after one second of its release? [Hint: Take g= 10 m/s^2].



One Dimensional Motion:

Solution:

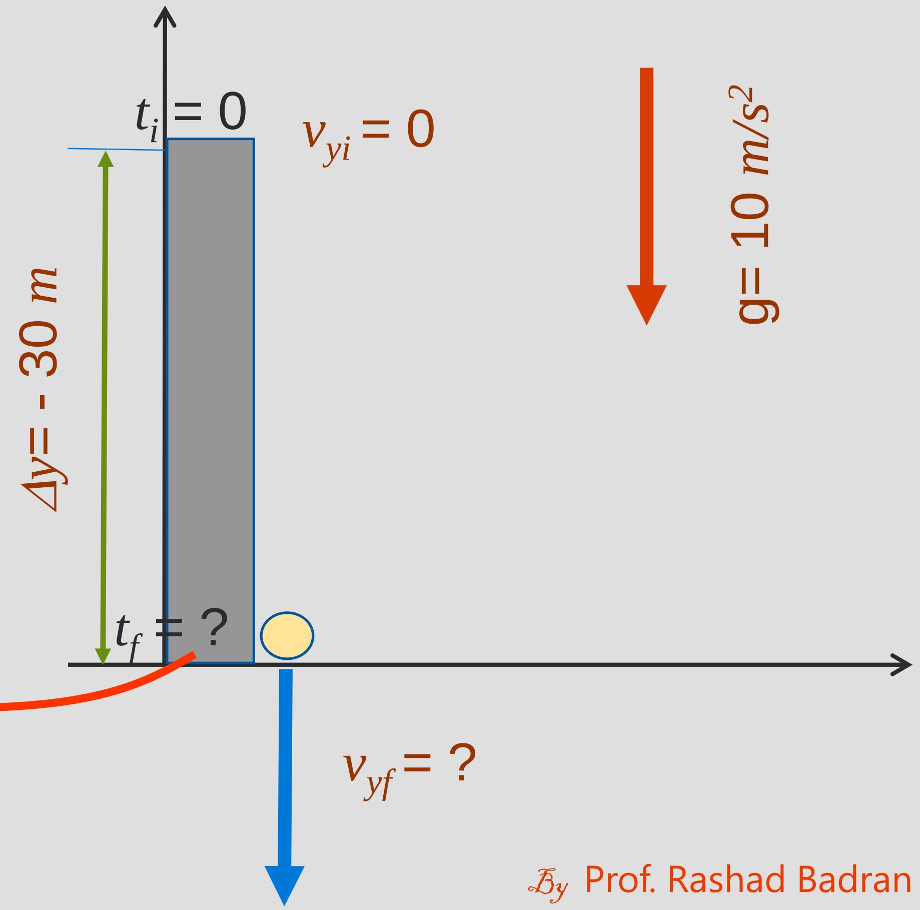
(a) Here initial time is taken $t_i = 0$. Initial velocity $v_{yi} = 0$. The displacement must have a negative sign (i.e. $\Delta y = -30\ m$). Substitute g = 10 m/s^2. To find t_f use the equation:

$$\Delta y = v_{yi}t - \frac{1}{2}gt^2$$

$$\Rightarrow -30 = 0 - (\frac{10}{2})t^2$$

$$\Rightarrow 5t^2 = 30$$

$$\Rightarrow t = 2.45s$$



The ball strikes the ground after the time interval 2.45 s

One Dimensional Motion:

Solution:

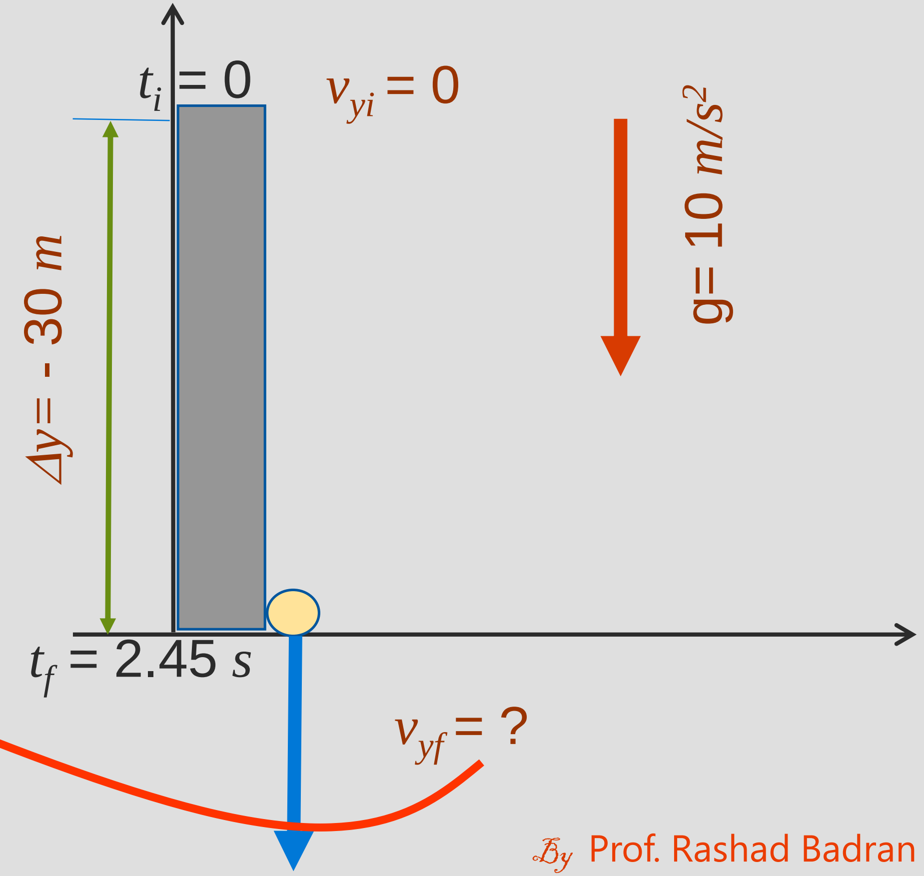
(b) To find the final speed use the information: [v_{yi} = 0, Δy = - 30 *m*, and g= 10 m/s^2] and employ the equation:

$$v_{yf}^2 = v_{yi}^2 - 2g\Delta y$$

$$\Rightarrow v_{yf}^2 = 0 - 2(10)(-30)$$

$$\Rightarrow v_{yf} = -24.5 m/s$$

Note: The final speed is 24.5 *m/s* while the negative sign refers to the direction of velocity (downward).



By Prof. Rashad Badran

One Dimensional Motion:

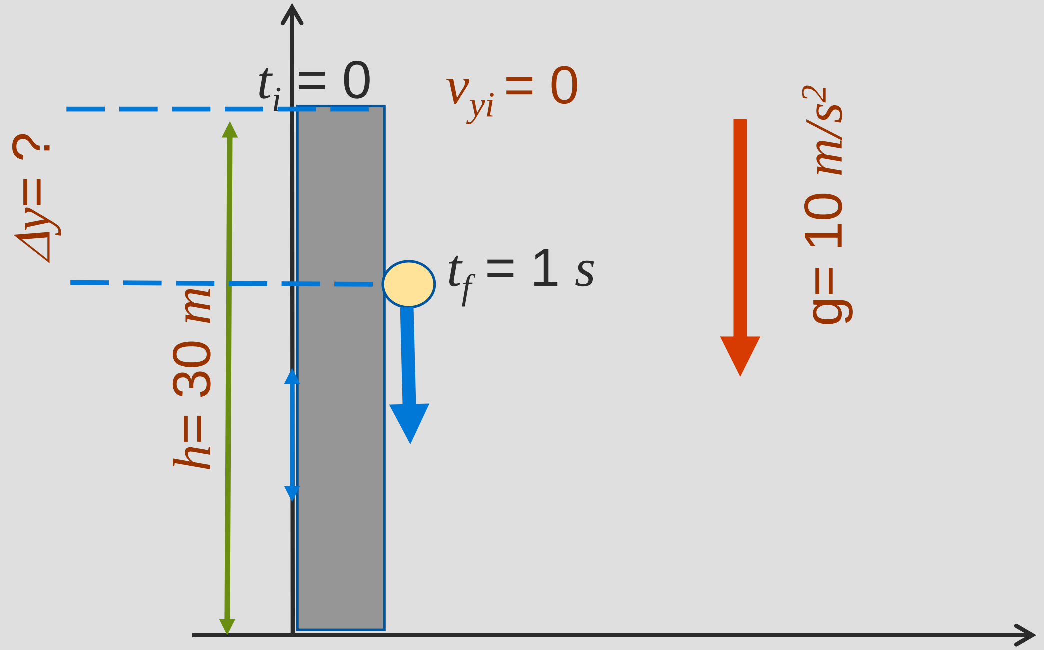
Solution:

(c) To find the location of ball after 1 *s* of its release: use the information: [$v_{yi} = 0$, and g= 10 m/s^2] and employ the equation:

$$\Delta y = v_{yi} t - \frac{1}{2} g t^2$$

$$\Rightarrow \Delta y = 0 - (\frac{10}{2})(1)^2$$

$$\Rightarrow \Delta y = -5m$$



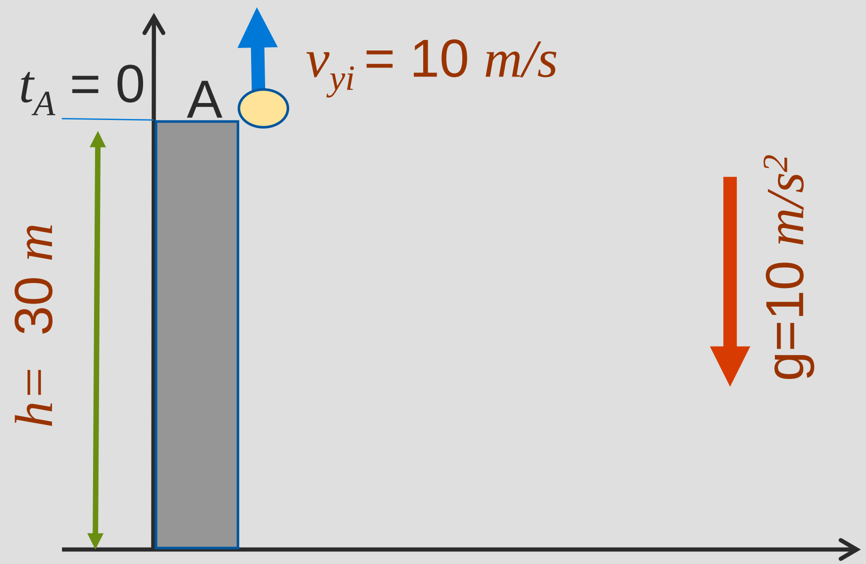
Note: The ball is 5 *m* below the top of the building (or 25 *m* from the ground).

By Prof. Rashad Badran

One Dimensional Motion:

Example

A ball is thrown vertically upward with an initial speed of 10 m/s from the top of a building of 30 m height. (a) Using $t_A = 0$ as the time the ball leaves the thrower's hand at point A, determine the time at which the ball reaches its maximum height.
(b) find the maximum height of the ball. (c) Determine the velocity of the ball when it returns to the height from which it was thrown. (d) Find the position and velocity of the ball at $t = 3$ s. (e) what is the ball's velocity just before it strikes the ground [Hint: Take g = 10 m/s^2]



One Dimensional Motion:

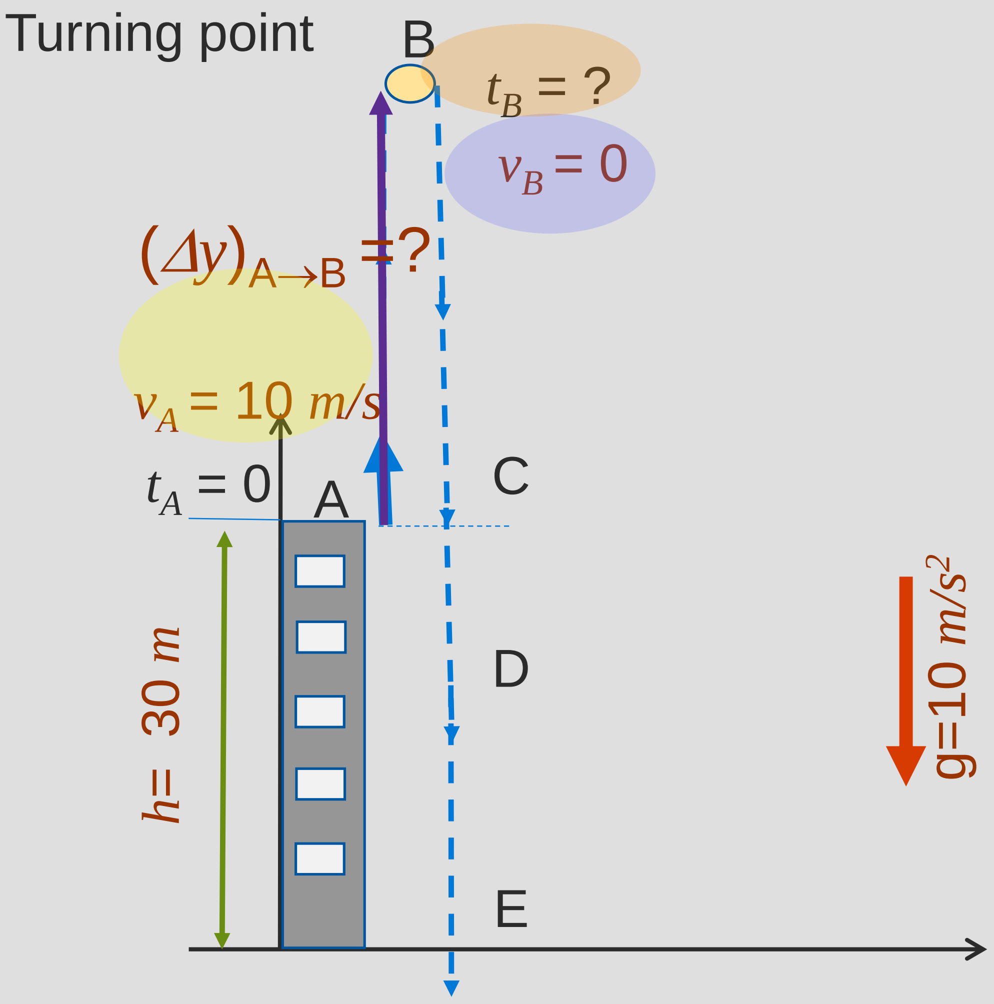
Solution:

(a)

$$v_{y_B} = v_{y_A} - gt_B$$

$$0 = 10 - 10t_B$$

$$\Rightarrow t_B = 1s$$



The time it takes the ball to reach its maximum height is 1s.

Note: It can be easily shown that the time it takes the ball to reach point C from A is 2s.

One Dimensional Motion:

Solution:

(b)

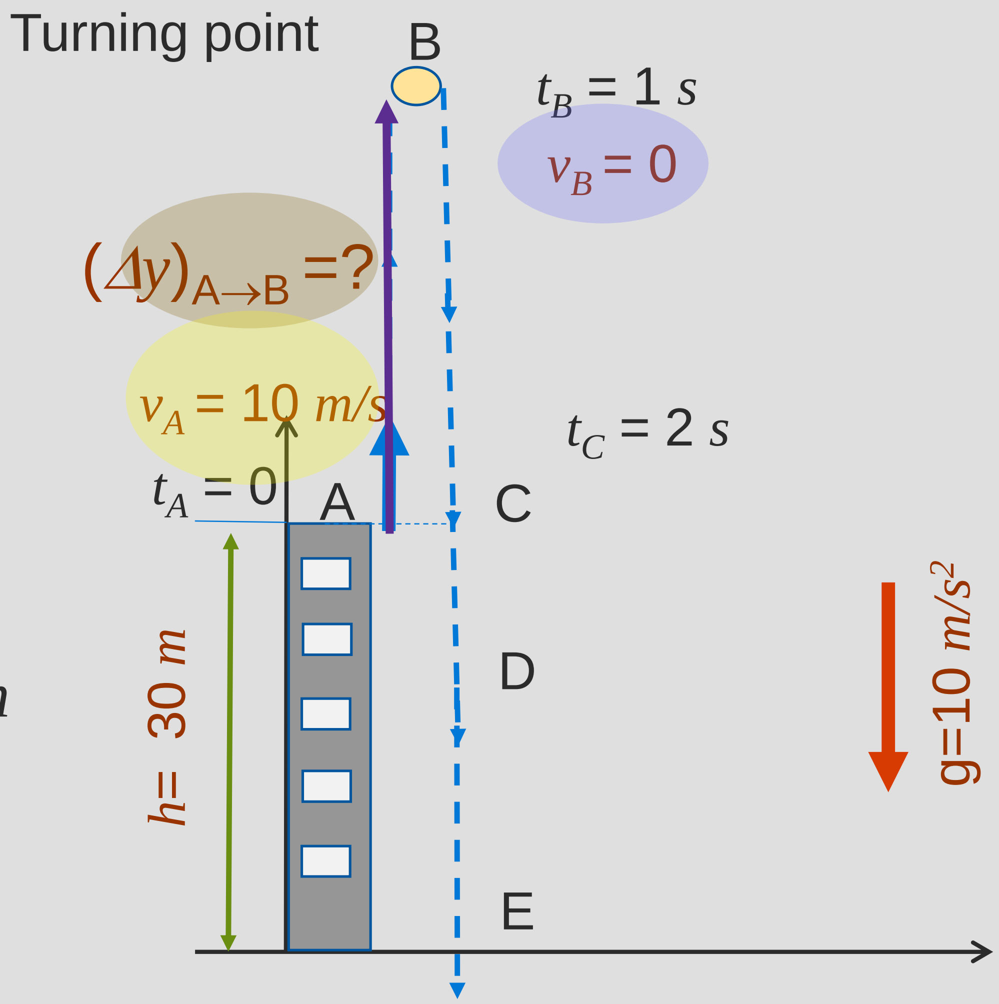
$$v_{y_B}^2 = v_{y_A}^2 - 2g(\Delta y)_{A \to B}$$

$$0 = (10)^2 - 2(10)(\Delta y)_{A \to B}$$

$$(\Delta y)_{A \to B} = 5m$$

The maximum height with respect to point A is 5 m

Note: The result shows that the displacement is positive as the ball moves from point A from B.

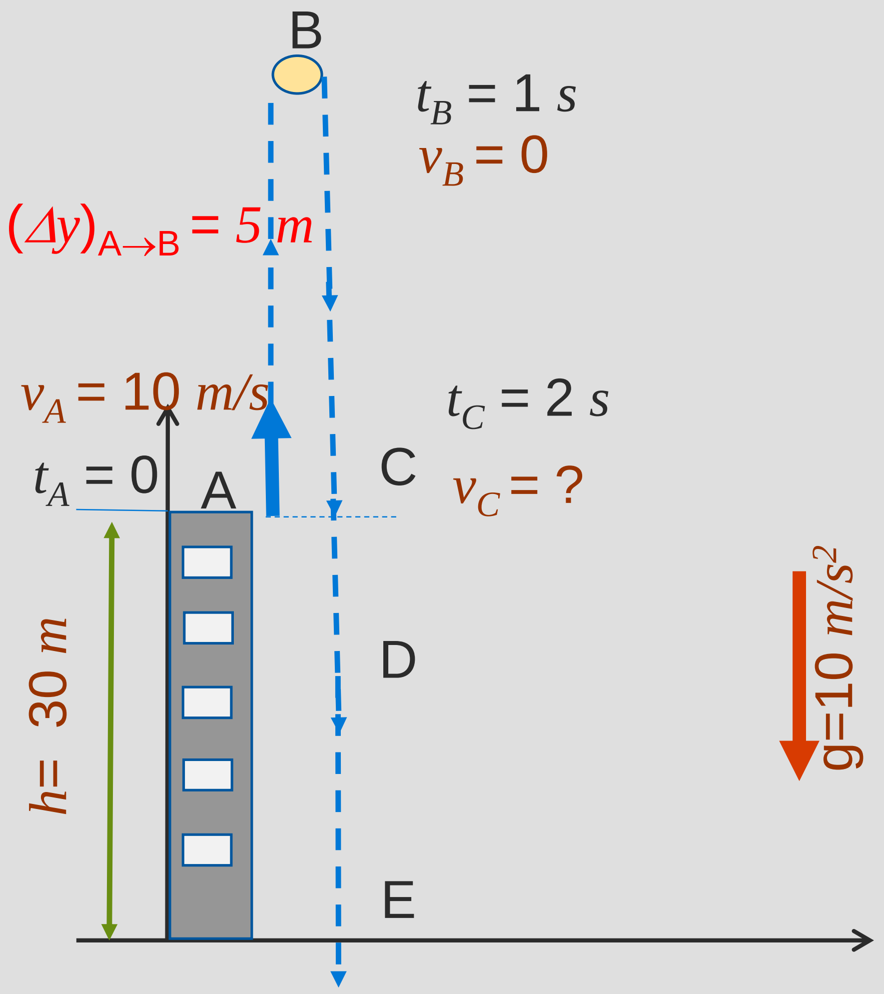


One Dimensional Motion:

Solution:

(c)

At the level of thrower's hand (point A), the ball returns to the same position



By Prof. Rashad Badran

One Dimensional Motion:

Solution:

(c)

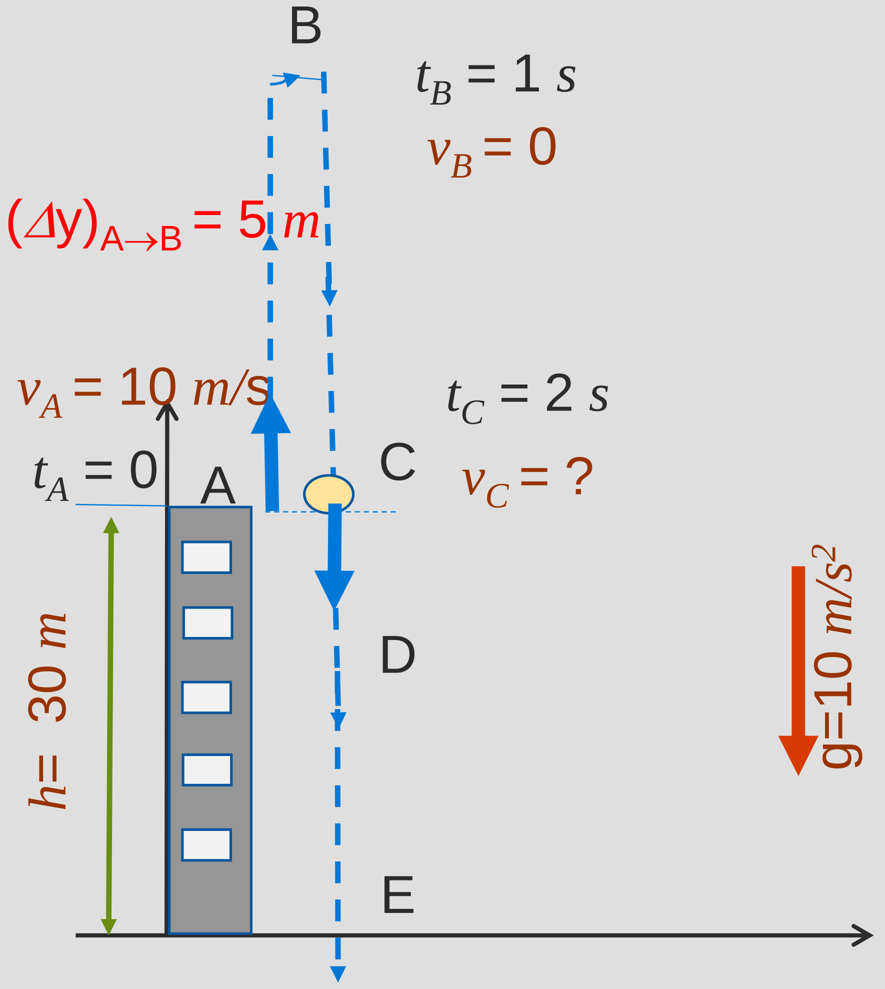
At the level of thrower's hand (point A), the ball returns to the same position, In this case $(\Delta y)_{A\rightarrow C} = 0$

$$v_{y_C}^{\;2} = v_{y_A}^{\;2} - 2g(\Delta y)_{A\rightarrow C}$$

$$v_{y_C}^{\;2} = v_{y_A}^{\;2}$$

$$v_{y_C} = -v_{y_A} = -10 m/s$$

Note: The positive sign is discarded because the ball is moving downward



By Prof. Rashad Badran

One Dimensional Motion:

Solution:

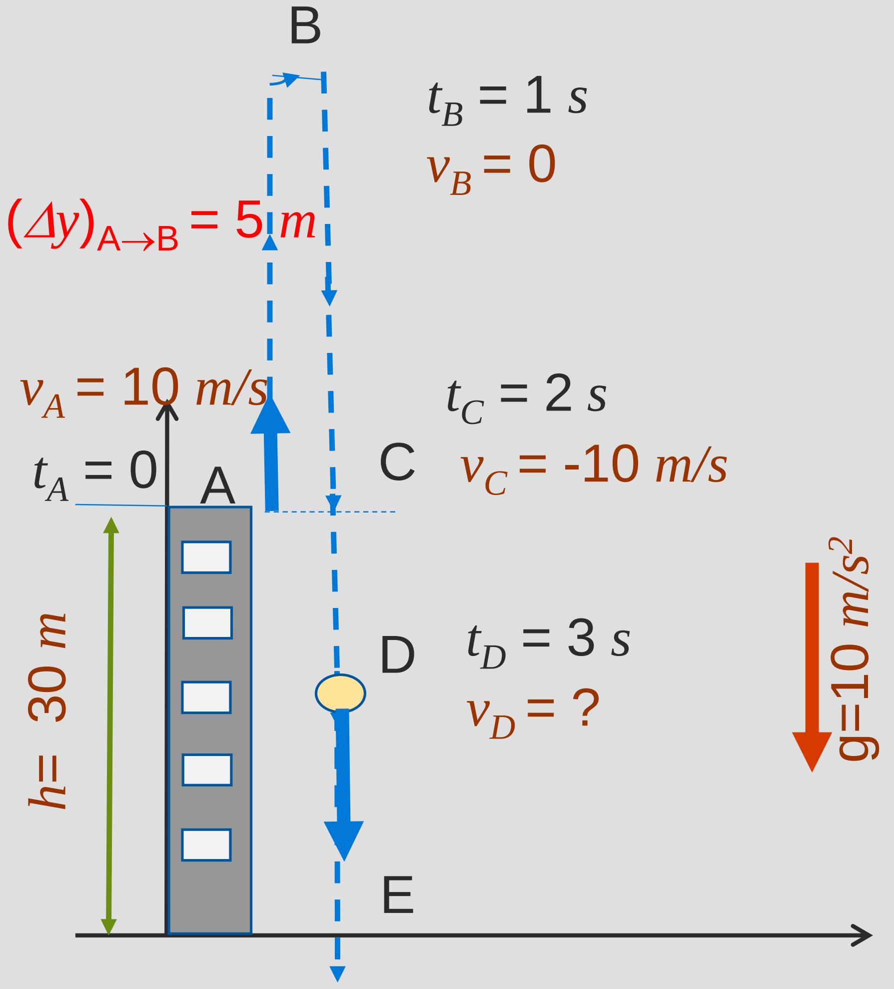
(d) The ball is launched at point A with velocity v_A= +10m/s until it reaches point D, when the time reads t_D = 3 s, and the ball is displaced downward from A to D (i.e. $(\Delta y)_{A\rightarrow D} = y_D - y_A$). One can use this information in the following equation:

$$(\Delta y)_{A\rightarrow D} = v_{y_A} t_D - \frac{1}{2} g t_D^2$$

$$(\Delta y)_{A\rightarrow D} = (10)(3) - \frac{10}{2}(3)^2$$

$$\Rightarrow (\Delta y)_{A\rightarrow D} = -15m$$

Note: The ball is 15 m below the top of building (point A).



By Prof. Rashad Badran

One Dimensional Motion:

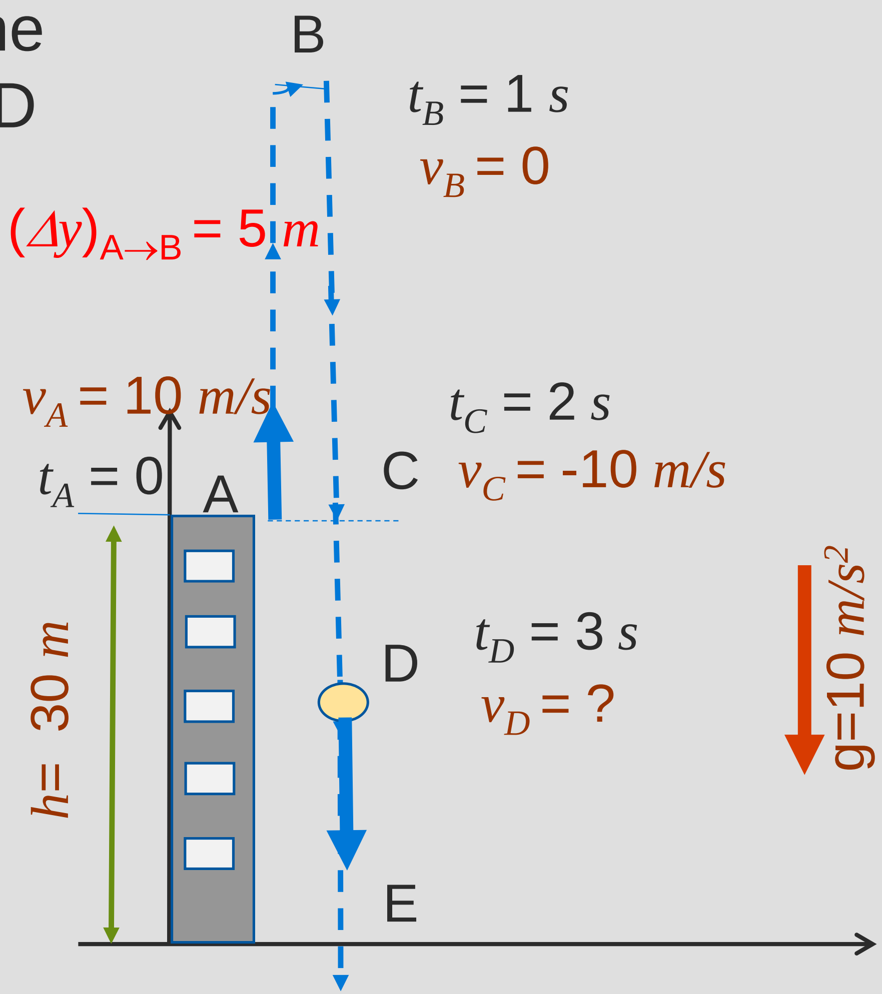
Solution:

Using the same information as before and the following equation, one can find the velocity at point D

$$v_{y_D} = v_{y_A} - gt_D$$

$$v_{y_D} = 10 - 10(3)$$

$$\Rightarrow v_{y_D} = -20 m/s$$



Note: The velocity has a magnitude of 20m/s. The minus sign indicates the direction of velocity (downward)

By Prof. Rashad Badran

One Dimensional Motion:

Solution:

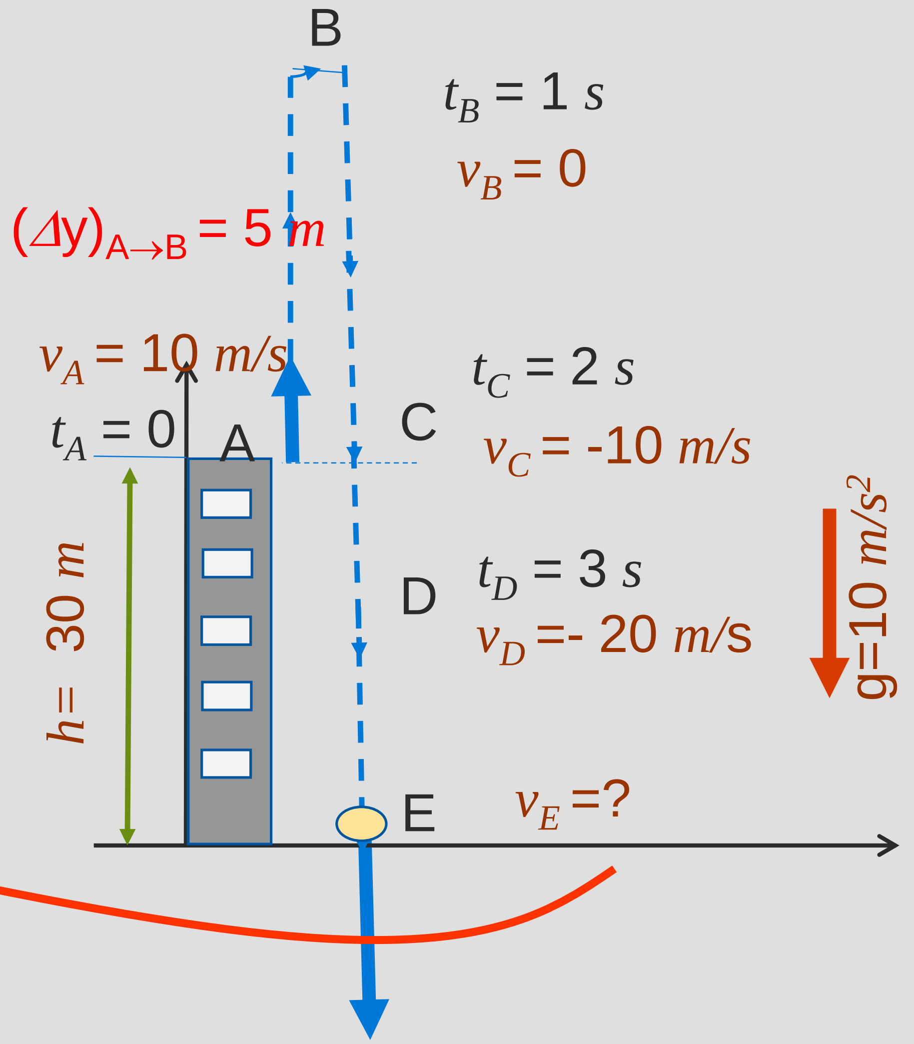
(e) To find the final velocity, use the information: [v_{yA} = +10 m/s, Δy = - 30 m, and g= 10 m/s^2] and employ the equation:

$$v_{y_E}^{\ 2} = v_{y_A}^{\ 2} - 2g(\Delta y)_{A\rightarrow E}$$

$$v_{y_E}^{\ 2} = (10)^2 - 2(10)(-30)$$

$$v_{y_E} = -26.46 m/s$$

Note: The velocity has a magnitude of 26.46m/s and its direction is downward as the minus sign indicates.



By Prof. Rashad Badran