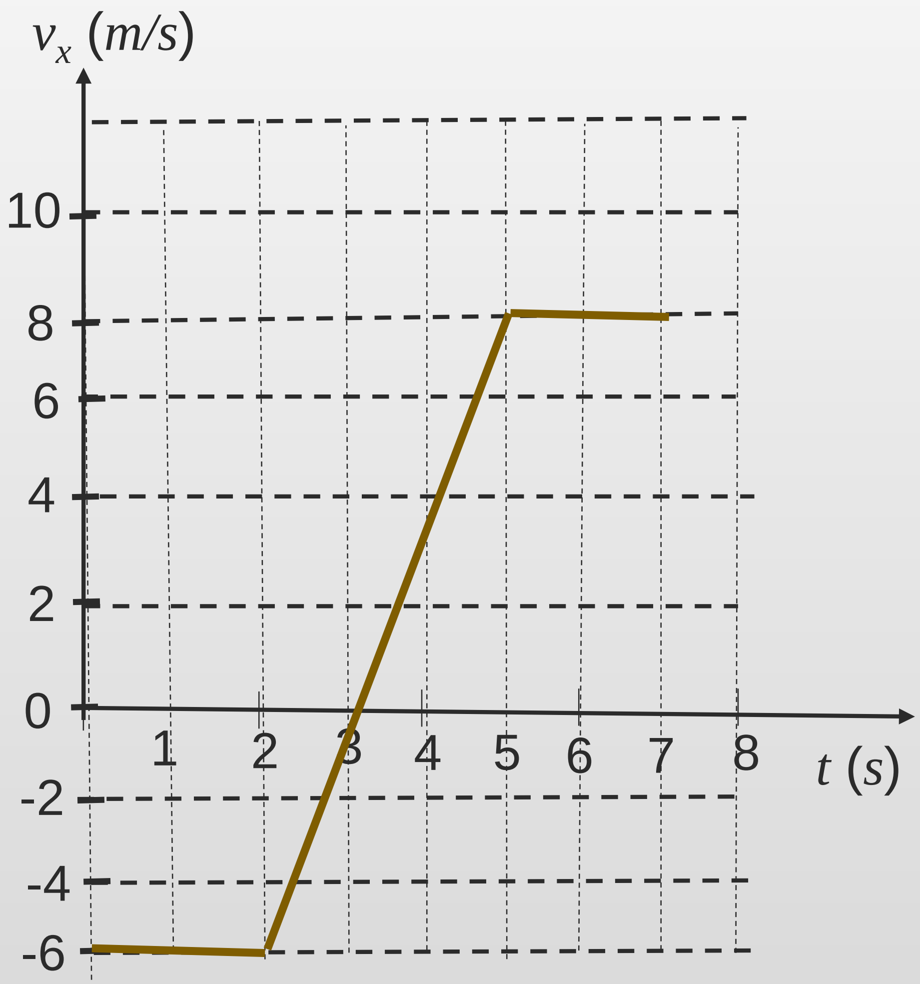
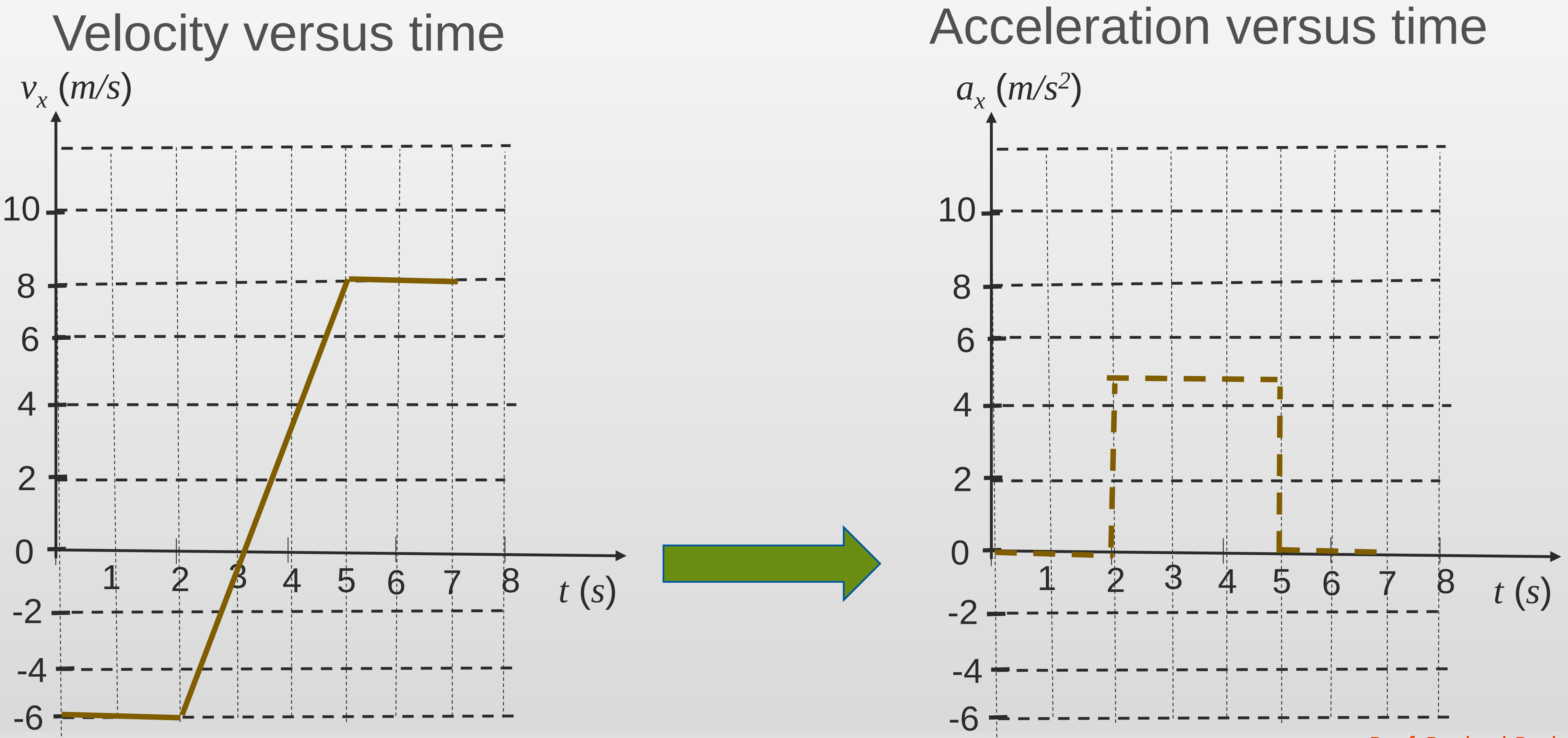
Finding Instantaneous Acceleration from a Given Graph:

Problem

The velocity versus time for a certain particle moving along the x axis is shown. Plot a graph of the acceleration versus time.



Solution:


Velocity versus time
v_x (m/s)
10
8
6
4
2
0
-2
-4
-6
1 2 3 4 5 6 7 8
t (s)
Acceleration versus time
a_x (m/s²)
10
8
6
4
2
0
-2
-4
-6
1 2 3 4 5 6 7 8
t (s)

Problem

A particle moves according to the equation $x = 3\,t^2 - 2t + 3$, where x is in meters and t is in seconds.

(a) Find the average acceleration for the time interval from $2\ s$ to $3\ s$.

(b) Find the instantaneous acceleration at $t = 2\ s$.

(c) At what time is the object at rest?

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solution

(a)

$$a_{avg.} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

To find the initial and final velocities (v_i and v_f) that correspond to the given initial and final times (t_i *and* t_f). One can take the first derivative with respect to time of the given equation $x = 3t^2 - 2t + 3$ to get $v = 6t - 2$

For $t = t_i = 2$ s, substitute this time into the equation $v = 6t - 2$ to get v_i as:

$v_i = (6)(2) - 2 = 10$ *m/s*

For $t = t_f = 3$ s, substitute this time into the equation $v = 6t - 2$ to get v_f as:

$v_f = (6)(3) - 2 = 16$ *m/s*

$$a_{avg.} = \frac{v_f - v_i}{t_f - t_i} = \frac{16m/s - 10m/s}{3s - 2s} = 6m/s^2$$

towards +ve x-axis and in the direction of motion

(b) **solution**

$$a_x = \frac{dv}{dt}$$

Take the second derivative of the given equation ($x = 3\,t^2 - 2t + 3$) with respect to t to get the acceleration as:

$a = 6\ m/s^2$ towards +ve x-axis and in the direction of motion

(c) Use the obtained equation $v = 6t - 2$ and put $v = 0$ to get the time $t = 0.33\ s$

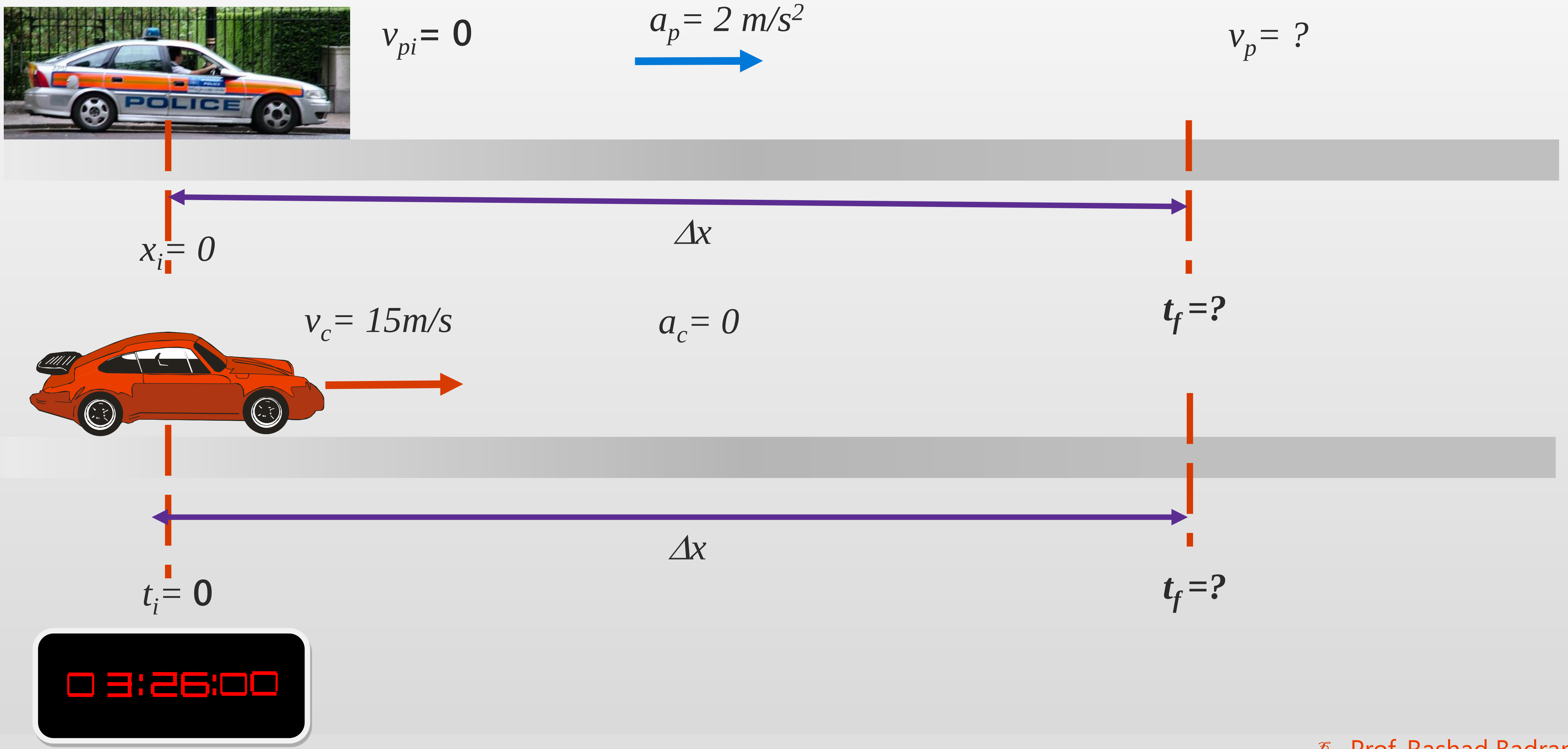
Problem

A red car travels along a straight road at a constant speed of 15 m/s. Just as the car passes a parked police car, the latter starts to accelerate at 2 m/s^2 (assuming constant for the whole chase) to overtake the red car

(a) Determine the time interval required for the police car to reach the red car. Find

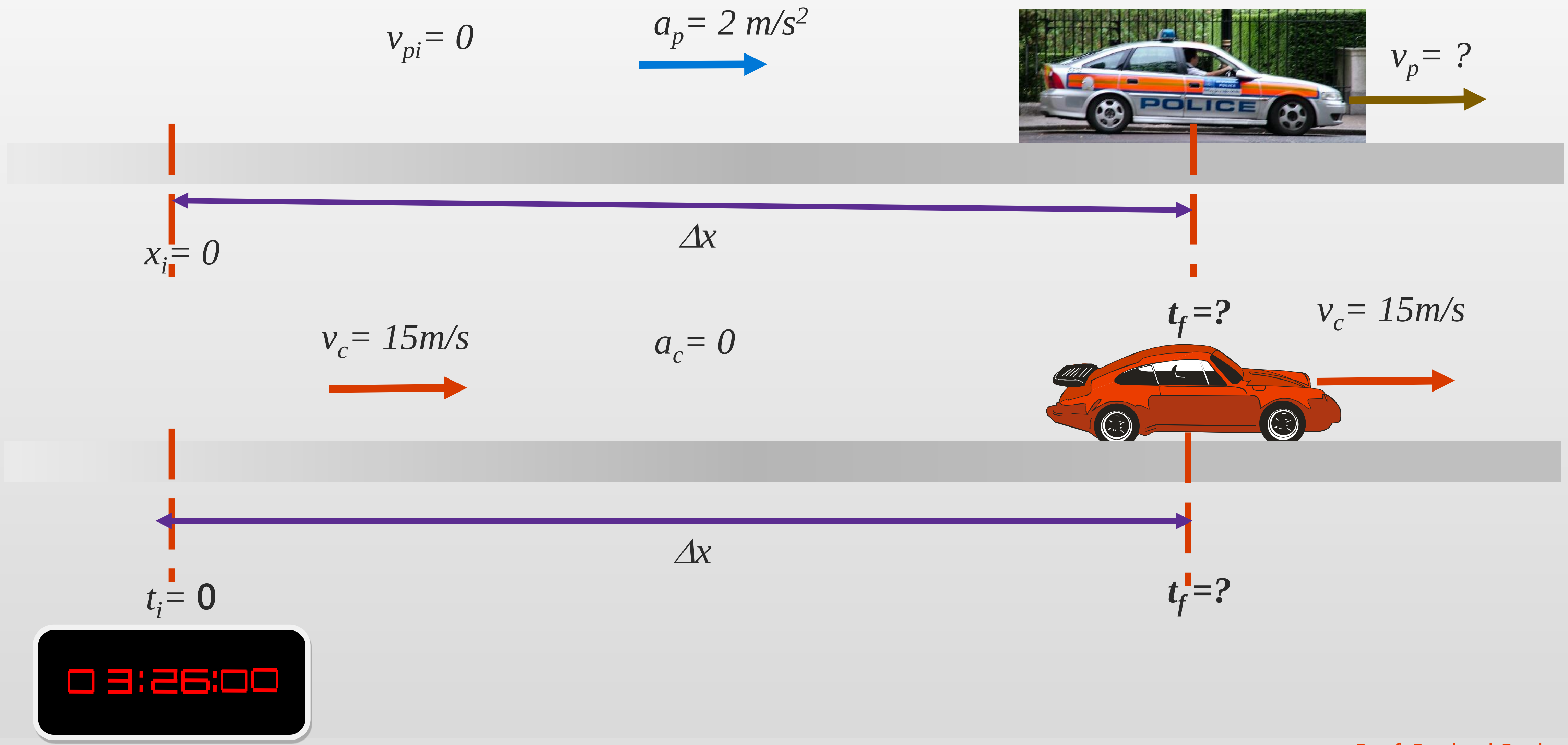
(b) the speed and (c) the total displacement of police car as it overtakes the red car.

Problem



$v_{pi} = 0$
$a_p = 2\ m/s^2$
$v_p = ?$
POLICE
$x_i = 0$
Δx
$t_f = ?$
$v_c = 15 m/s$
$a_c = 0$
Δx
$t_i = 0$
$t_f = ?$
03:26:00

Problem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POLICE
$v_p = ?$
Δx
$x_i = 0$
$t_f = ?$
$v_c = 15 m/s$
$v_c = 15 m/s$
$a_c = 0$
Δx
$t_i = 0$
$t_f = ?$
03:26:00

solution

(a)

$v_{pi} = 0$

$a_p = 2\ m/s^2$

$v_p = ?$

$$\Delta x = v_{pi}t + (0.5)at^2 = (0.5)(2m/s^2)t^2 = t^2$$

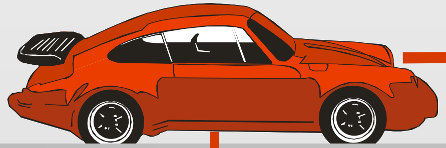
$x_i = 0$

Δx

$t_f = ?$

$v_c = 15m/s$

$v_c = 15m/s$



$$\Delta x = v_c t = (15m/s)t$$

Δx

$t_i = 0$

$t_f = ?$

03:26:00

$$15t = t^2 \Rightarrow t = 15\ s$$

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solution

(b)

$v_{pi} = 0$ $t = 15\ s$ $a_p = 2\ m/s^2$

$v_p = ?$

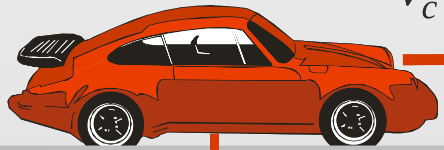
$$v_{pf} = v_{pi} + at = (2 m/s^2)(15 s) = 30 m/s$$

$x_i = 0$

$\Delta x = ?$

$t_f = 15\ s$

$v_c = 15 m/s$



$v_c = 15 m/s$

$\Delta x = ?$

$t_i = 0$

$t_f = 15\ s$

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(c)

solution

$v_{pi} = 0$ $t = 15\ s$ $a_p = 2\ m/s^2$

$v_p = ?$

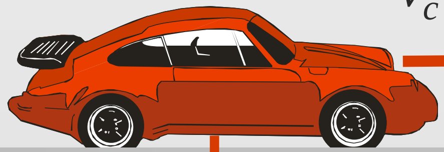
$$\Delta x = v_{pi}t + (0.5)at^2 = (0.5)(2m/s^2)(15s)^2$$

$x_i = 0$

$\Delta x = 225m$

$t_f = 15\ s$

$v_c = 15m/s$



$v_c = 15m/s$

$$\Delta x = v_c t = (15m/s)(15s)$$

$\Delta x = 225m$

$t_i = 0$

$t_f = 15\ s$

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Summary of One Dimensional Motion

1-D Motion may be classified into:

- I) Uniform Motion
- II) Non-Uniform Motion

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Summary of One Dimensional Motion

I) Uniform Motion

a) Uniform Motion with a constant velocity

Instantaneous velocity equals average velocity

b) Uniform Motion with a constant acceleration

Instantaneous acceleration equals average acceleration

Finding Displacement from Velocity Versus Time Graph:

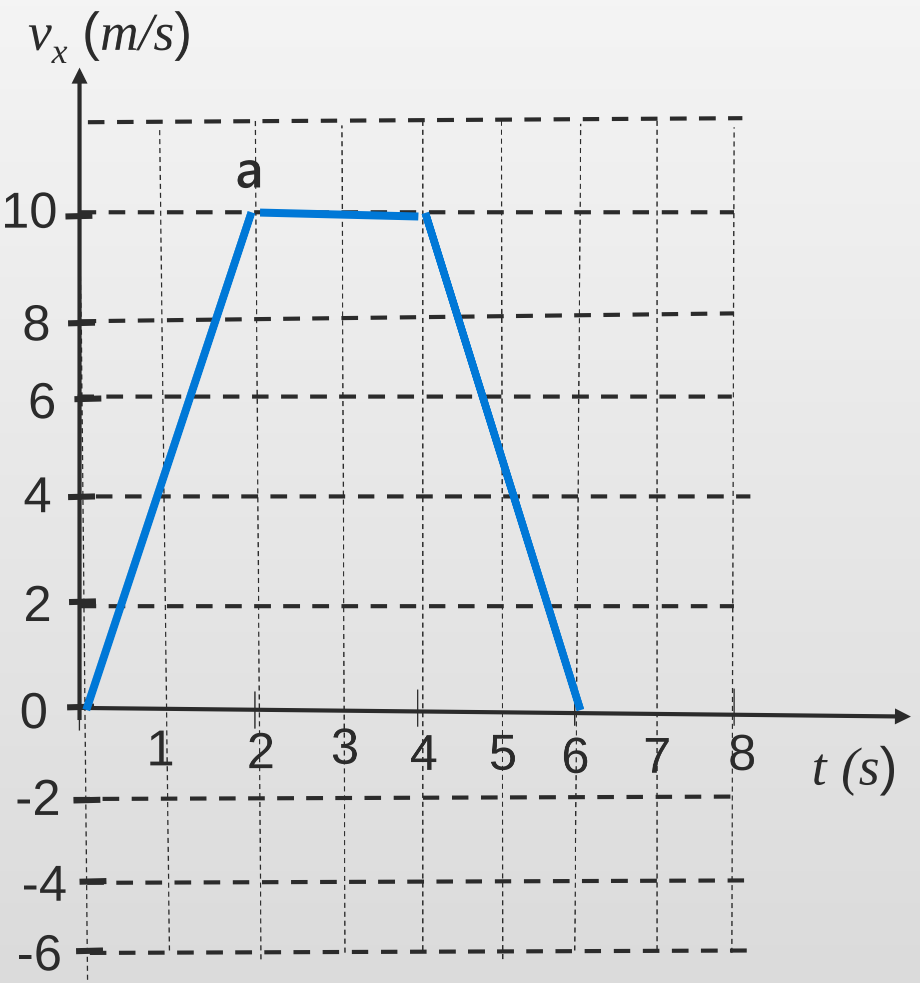
Problem

The velocity versus time for a certain particle moving along the x axis is shown.

(a) Calculate the distance travelled by the particle between the times (i) $t = 0$ and $2\ s$, (ii) $t = 2\ s$ and $4\ s$.

(b) Calculate the total distance travelled by the particle for the first six seconds time.

(c) Write an equation for x as a function of time for the first phase of motion from $t = 0$ to $t = 2\ s$.



Finding Displacement from Velocity Versus Time Graph:

Solution:

(a) To calculate the distance travelled by the particle between the times (*i*) $t = 0$ and $2\ s$, (*ii*) $t = 2\ s$ and $4\ s$, one should take the area under the curve. This is because

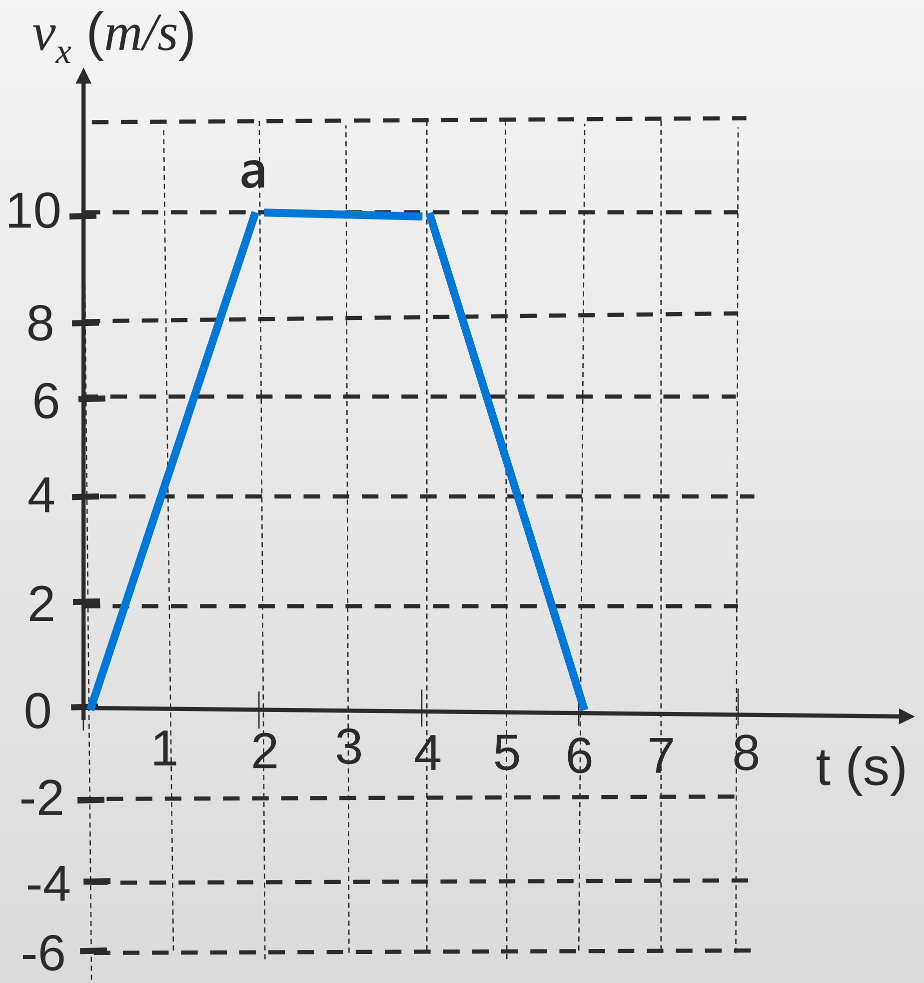
$$\because v_x = \frac{dx}{dt}$$

$$\Rightarrow dx = v_x dt$$

$$\therefore \int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v_x dt$$

$$x_f - x_i = \int_{t_i}^{t_f} v_x dt$$

Area under the curve from t_i to t_f



Finding Displacement from Velocity Versus Time Graph:

Solution:

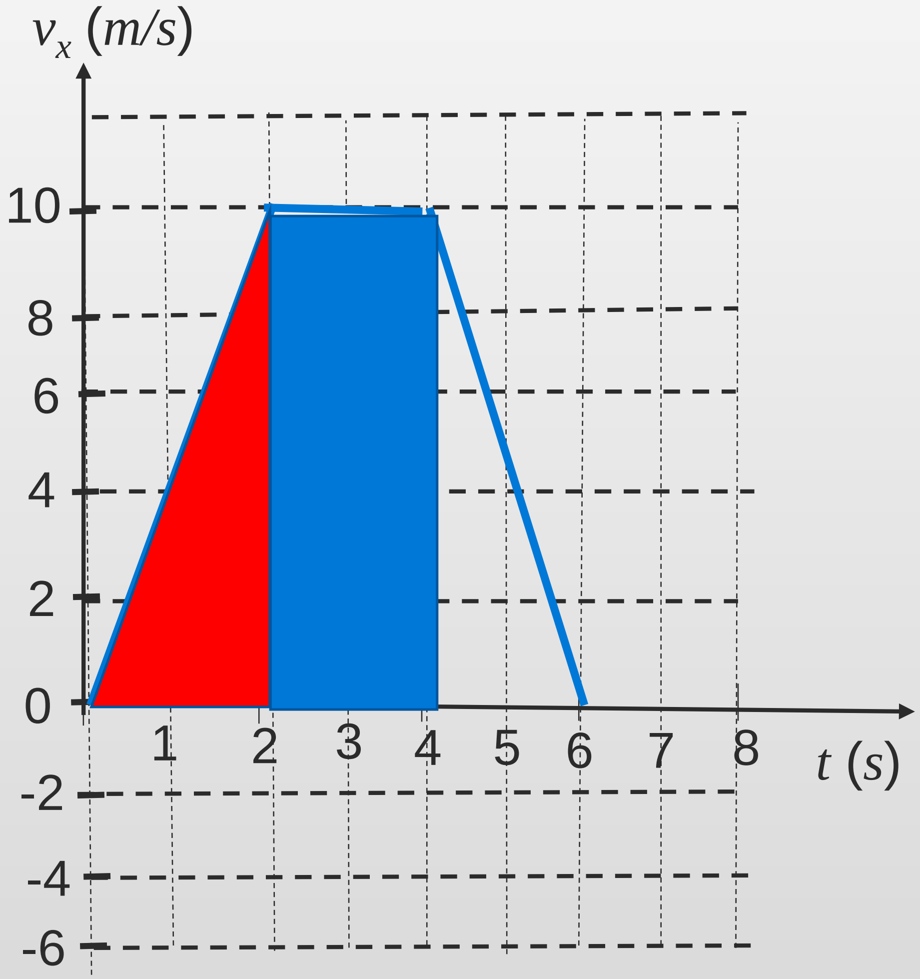
(a) (i) From $t = 0$ *and* $2\ s$:

x_f - x_i = Area of triangle (red) with base $2\ s$ and latitude $10\ m/s$, or

Distance = 0.5($2\ s$)($10 m/s$)= $10m$

(ii) From $t = 2\ s$ *and* $4\ s$:

Distance = area of rectangle with base (4 - 2)s and latitude (10m/s) = $20\ m$



Finding Displacement from Velocity Versus Time Graph:

Solution:

(b) From $t = 0$ *and* $6\ s$:

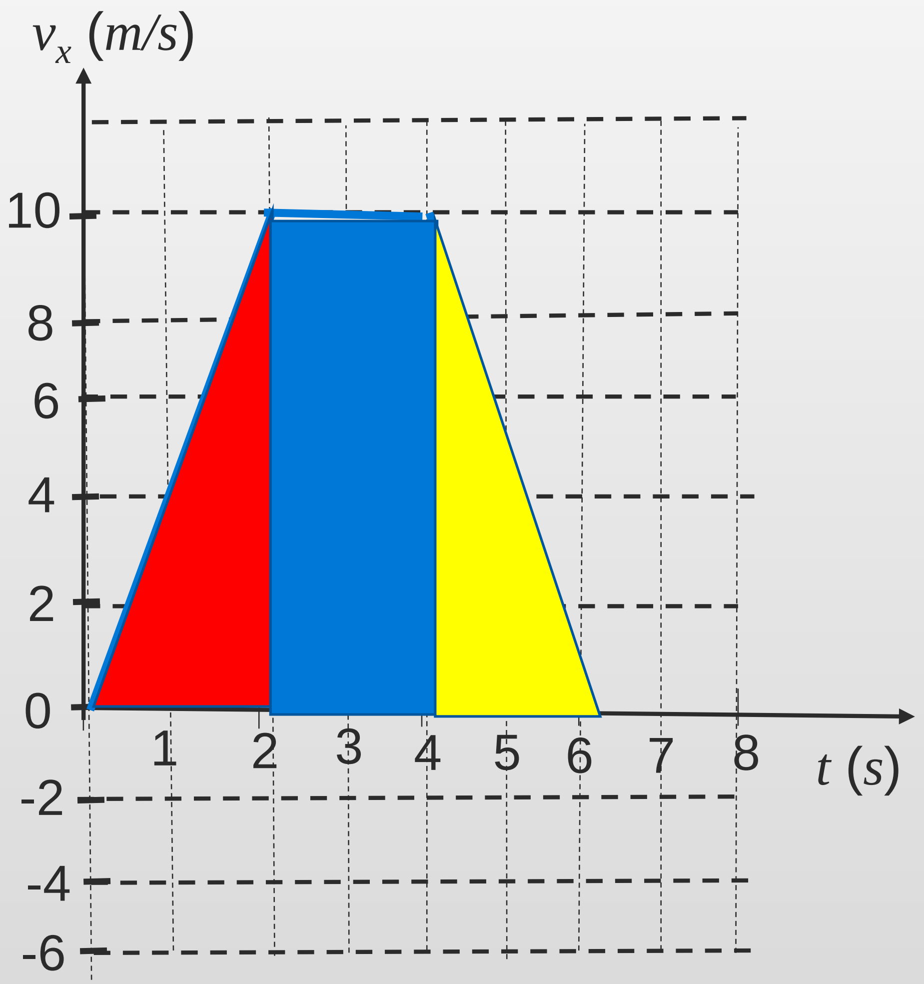
Total distance = Area of triangle (red) with base 2s and height 10m/s + area of rectangle (blue) with base (2)s and latitude (10m/s) + area of triangle (yellow) with base (6-4)s and latitude $10m/s$

$\therefore$ Total distance = $10m + 20m + 10m = 40\ m$

Or

area of trapezoid = 0.5 (base 1 + base 2)(latitude)

$= 40\ m$



Finding Displacement from Velocity Versus Time Graph:

Solution:

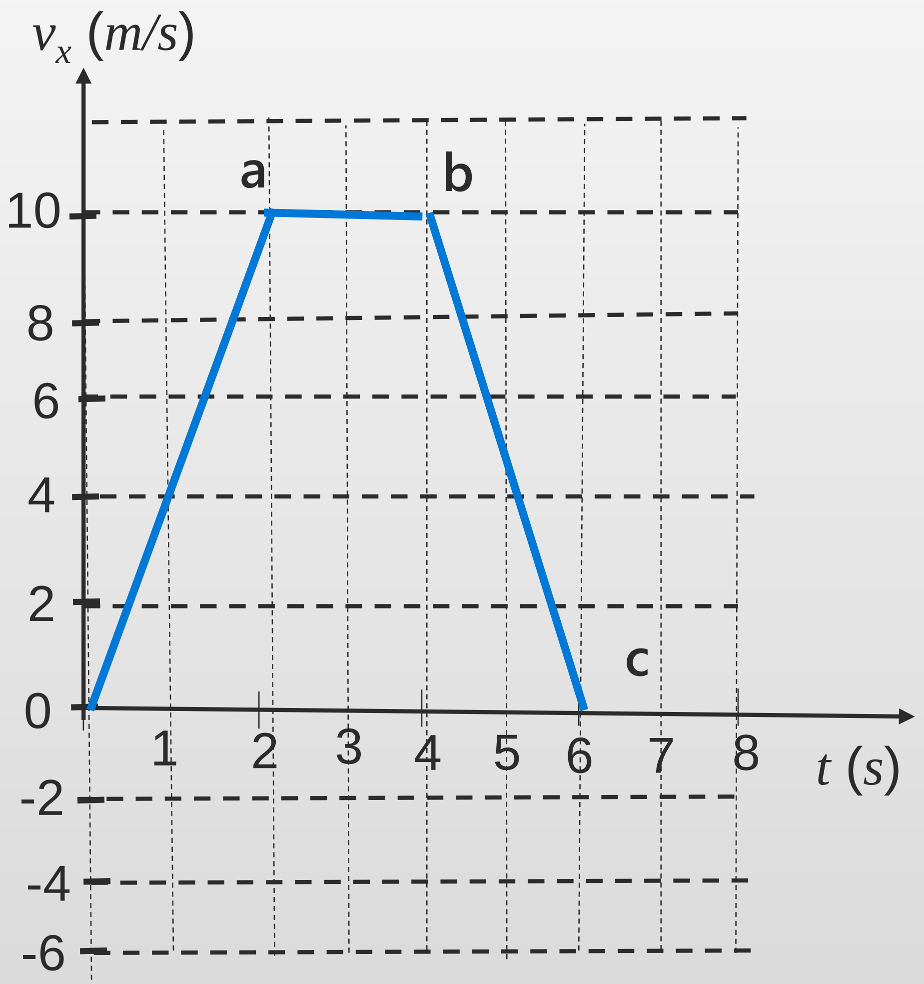
(c) To find the equation for x as a function of time for the first phase of motion from $t = 0$ to $t = 2\ s$:

one must write the equation for the line oa which is: $v_x = (slope)(t)$.

The slope of the this line is $(10\ m/s)/(2\ s) = 5\ m/s^2$.

Thus the equation of line oa becomes:

$$v_x = (5\ m/s^2)\ t$$



Finding Displacement from Velocity Versus Time Graph:

Solution:

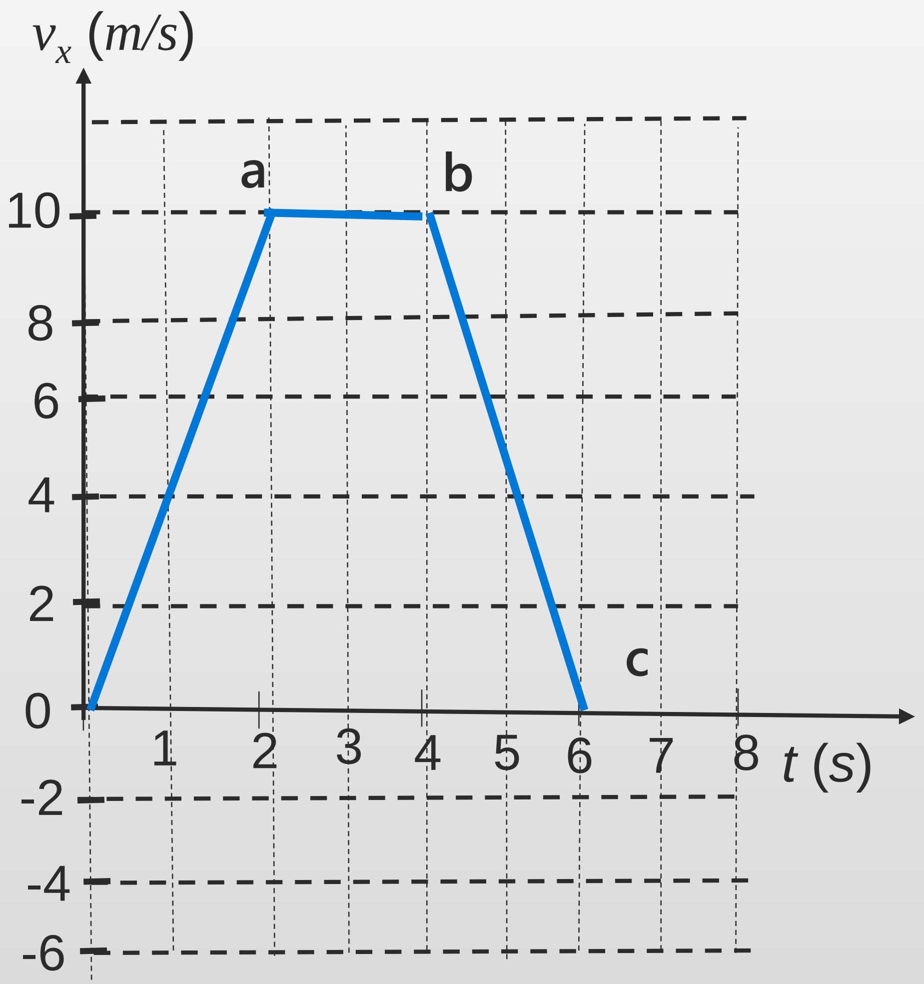
$$\because v_x = \frac{dx}{dt} = (5m/s^2)t$$

$$\Rightarrow dx = 5tdt$$

$$\therefore \int_{x_o=0}^{x_f=x} dx = 5\int_{t_o=0}^{t_f=t} tdt$$

$$x = \left.\frac{5t^2}{2}\right|_{t_o=0}^{t_f=t}$$

$$\Rightarrow x = 2.5t^2$$



Finding Displacement from Velocity Versus Time Graph:

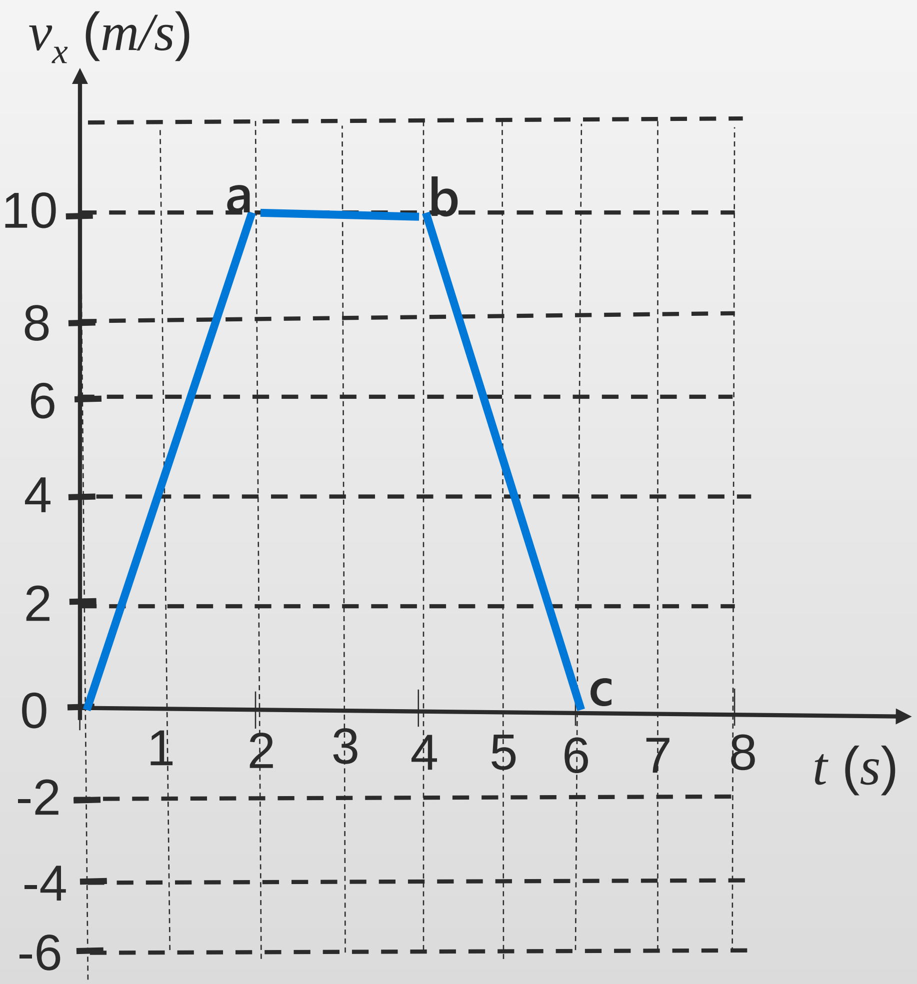
Exercise:

The velocity versus time for a certain particle moving along the x axis is shown.

(a) Write an equation for x as a function of time for the second phase of motion (a→b) from $t = 2\ s$ to $t = 4\ s$.(Answer: $x = 10t - 10$)

(b) Write an equation for x as a function of time for the third phase of motion (b→c) from $t = 4\ s$ to $t = 6\ s$.

(Answer: $x = 30t - 2.5\ t^2 - 50$)

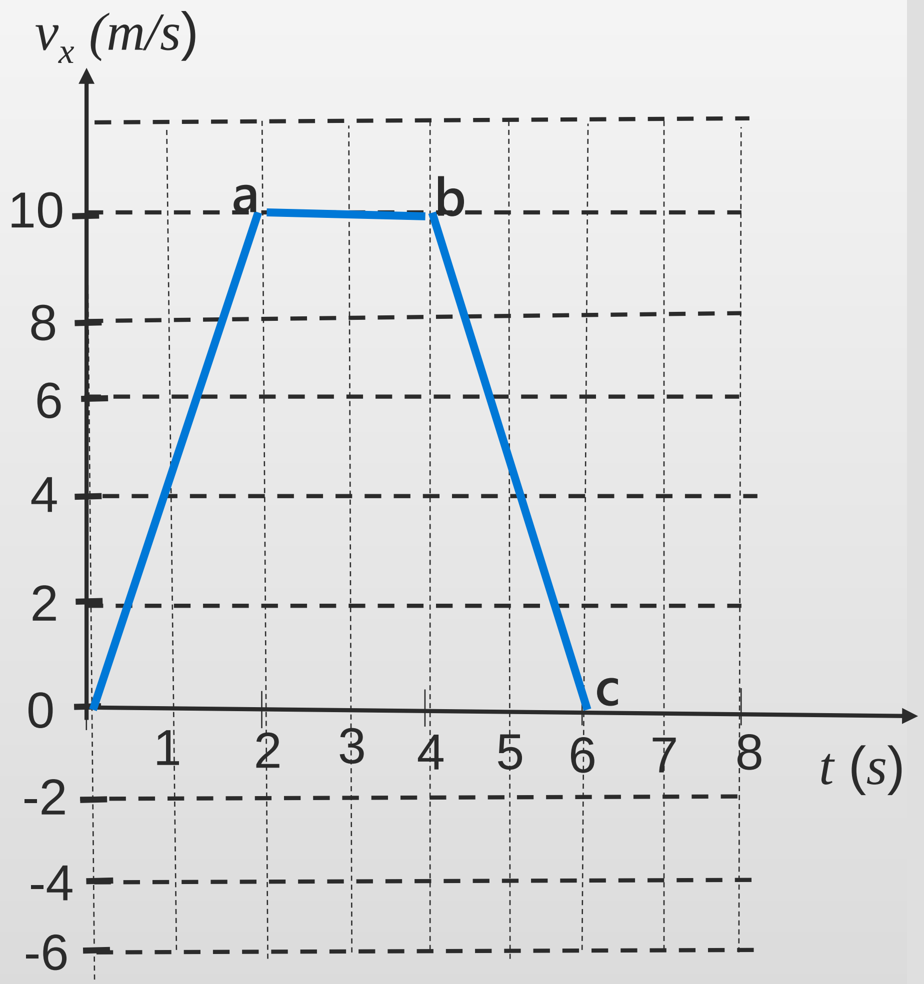


Finding Displacement from Velocity Versus Time Graph:

Solution:

(a) Use the obtained equation for the motion (0→a) from $t = 0$ to $t = 2$ s ($x = 2.5\, t^2$) to find the initial position of the second phase of motion. At $t_a = 2$ s, $x_a = 10$ m.

The speed is constant $v_x = 10$ m/s in the second phase of motion (a→b) from t_a to t_b, namely:
$v_x = 10$ m/s



Finding Displacement from Velocity Versus Time Graph:

Solution:

Use the obtained information $v_x = 10m/s$ and $x_a = 10\ m$ to find x-versus t for a→b phase

$$\because v_x = \frac{dx}{dt} = 10m/s$$

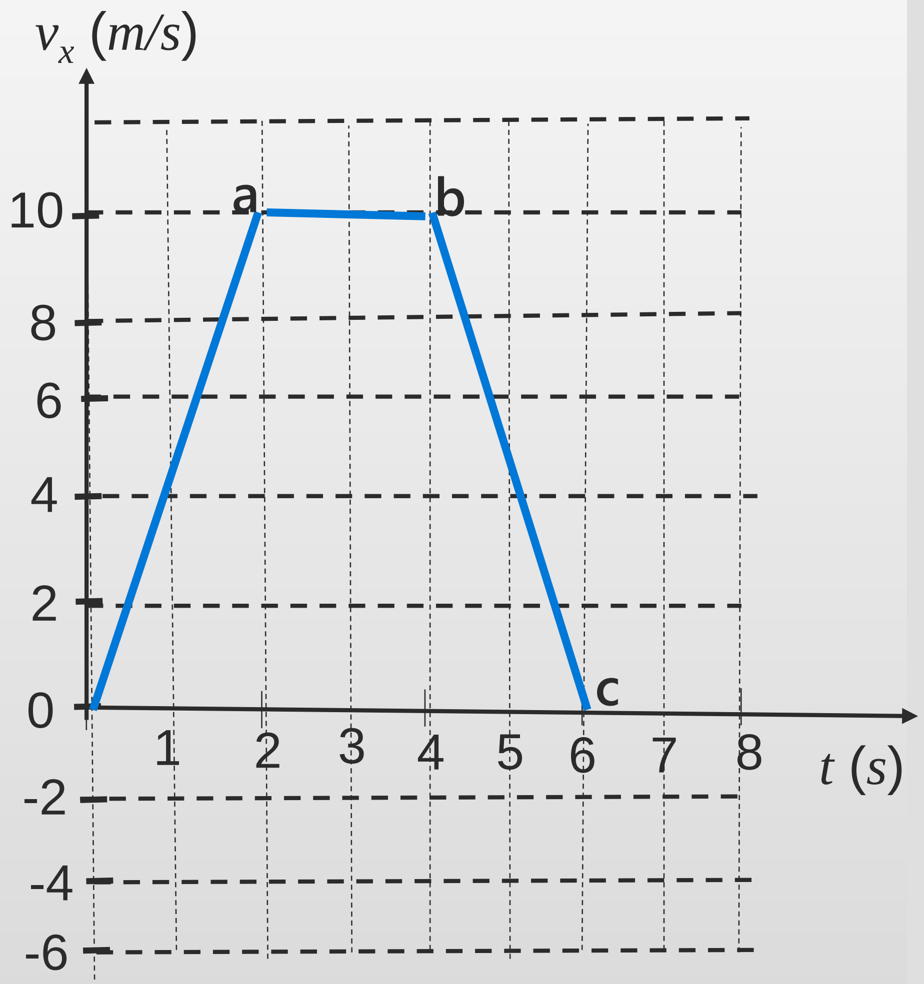
$$\Rightarrow dx = 10dt$$

$$\therefore \int_{x_a=10m}^{x_f=x} dx = 10 \int_{t_a=2s}^{t_f=t} dt$$

$$x - 10 = 10t\Big|_{t_a=2s}^{t_f=t}$$

$$\Rightarrow x - 10 = 10t - 20$$

$$x = 10t - 10$$



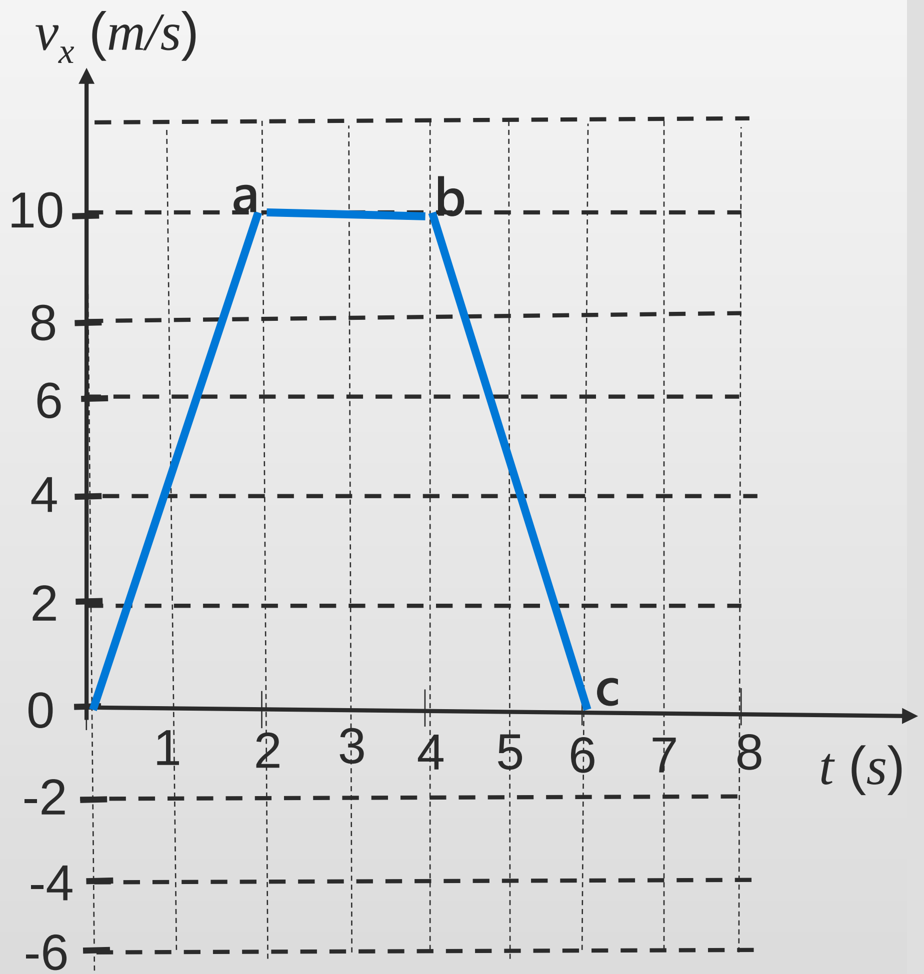
Finding Displacement from Velocity Versus Time Graph:

Solution:

(b) Use the obtained equation for the motion (a→b) from $t = 2$ s to $t = 4$ s ($x = 10t - 10$) to find the initial position of the second phase of motion. At $t_b = 4$ s, $x_b = 30$ m.

The slope of the line bc represents the acceleration $a_x = -5\ m/s^2$ in the second phase of motion (b→c) from t_b to t_c . The equation of the line bc is:

$v_x = v_{ox} - 5t$. Here v_{ox} is the intercept of this line on v_x axis which can be found when we put $v_x = 10\ m/s$ at $t_b = 4\ s \Rightarrow v_{ox} = 30\ m/s$



Thus the equation is $v_x = (30 m/s) - (5\ m/s^2)t$

Finding Displacement from Velocity Versus Time Graph:

Solution:

Use the obtained equation $v_x = (30m/s) - (5\ m/s^2)t$
and $x_b = 30\ m$ **to find** x**-versus for b→c phase**

$$\because v_x = \frac{dx}{dt} = (30m/s) - (5m/s^2)t$$

$$\Rightarrow dx = [(30m/s) - (5m/s^2)t]dt$$

$$\therefore \int_{x_b=30m}^{x_f=x} dx = \int_{t_b=4s}^{t_f=t} [(30m/s) - (5m/s^2)t]dt$$

$$x = 30t - \frac{5t^2}{2}\Bigg|_{t_b=4s}^{t_f=t}$$

$$\Rightarrow x - 30 = 30t - 2.5t^2 - 120 + 40$$

$$x = 30t - 2.5t^2 - 50$$