Finding Average Velocity and Instantaneous Velocity from a Given Equation:

Example

A particle moves according to the equation $x = 10\ t^2$, where x is in meters and t is in seconds.

(a) Find the average velocity for the time interval from 2 s to 3 s.

(b) Find the instantaneous velocity at $t = 4\ s$.

Solution:

(a)

$$v_{avg.} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The initial and final positions (x_i and x_f) that correspond to the given initial and final times (t_i and t_f) can be found using the given equation

$$x = 10t^2$$

For t =t_i =2 s, substitute this into the equation to get x_i as: $x_i = (10)(2)^2 = 40m$

For t =t_f =3 s, substitute this into the equation to get x_f as: $x_f = (10)(3)^2 = 90m$

$$v_{avg.} = \frac{x_f - x_i}{t_f - t_i} = \frac{90m - 40m}{3s - 2s} = 50m/s$$

towards +ve x-axis

Solution:

(b)

$$v_x = \frac{dx}{dt}$$

Take the derivative of the given equation ($x = 10t^2$) with respect to t to get:

$$v_x = 20t$$

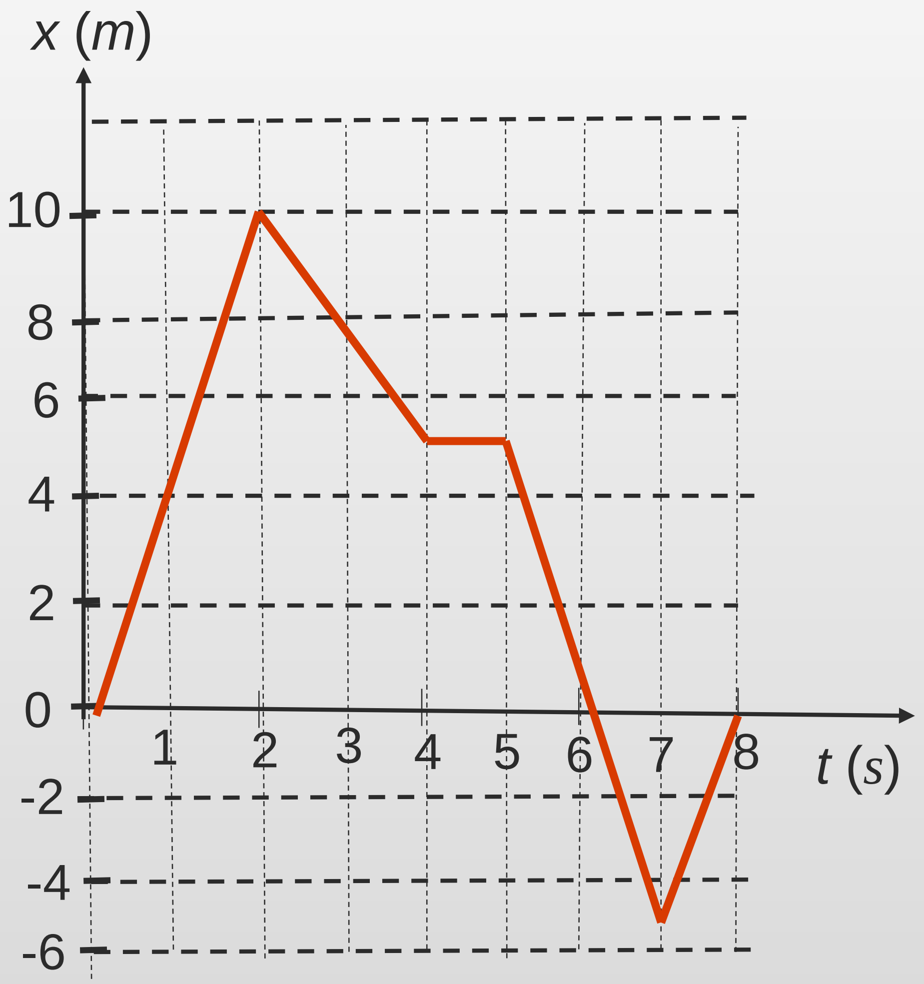
The obtained equation shows that v_x is linearly proportional to t.

At time t = 4s, the velocity is $v_x = (20)(4) = 80 m/s$ towards +ve x-axis

Finding Instantaneous Velocity from a Given Graph:

Problem

The position versus time for a certain particle moving along the x axis is shown. Find the instantaneous velocity at the following times (a) $t = 1\ s$, (b) $t = 3\ s$, (c) $t = 4.5\ s$, and (d) $t = 7.5\ s$.



Finding Instantaneous Velocity from a Given Graph:

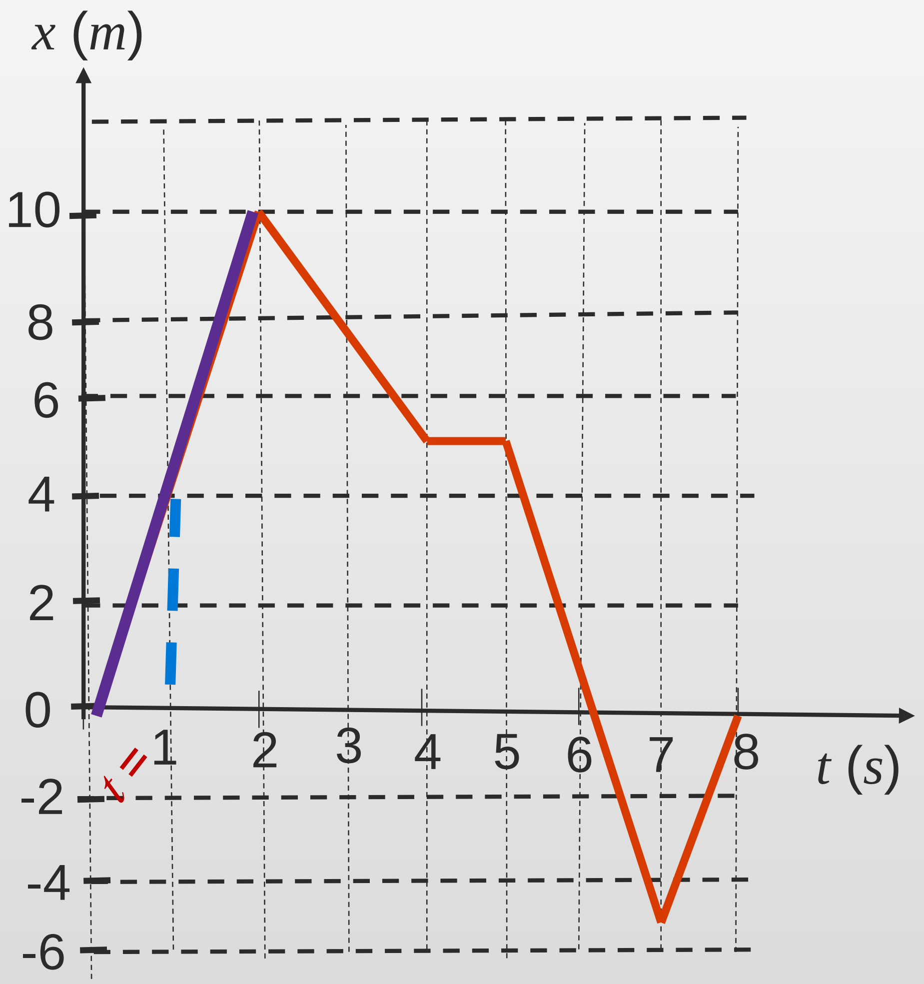
Solution:

$$v_{inst} = v_{avg.} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

For the linear relation between x and time t

(a) For $t = 1\ s$, v_{inst} is equal to v_{avg} in the time interval from $t_i = 0$ to $t_f = 2\ s$, (the slope of the purple line)

$$v_{inst} = v_{avg.} = \frac{10m - 0}{2 - 0} = +5m/s$$

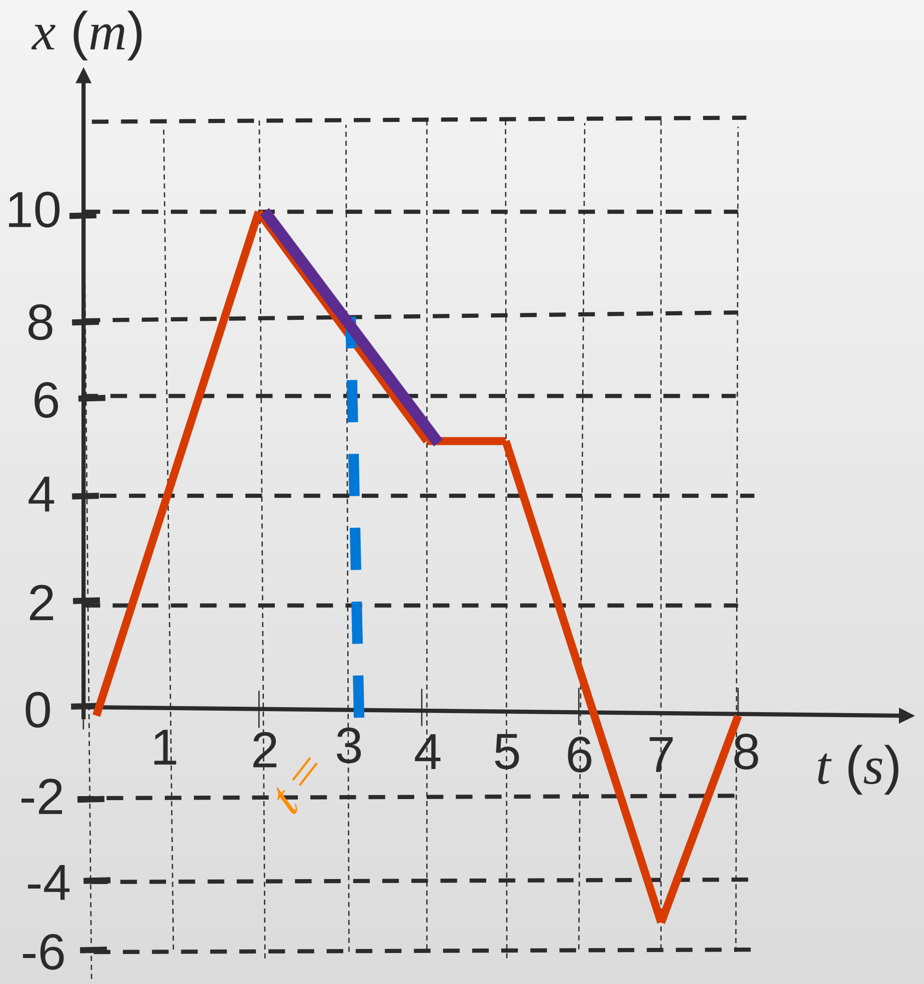


Finding Instantaneous Velocity from a Given Graph:

Solution:

$$v_{inst} = v_{avg.} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

For the linear relation between *x* and time *t*



(b) For $t = 3\ s$, v_{inst} is equal to v_{avg} in the time interval from $t_i = 2\ s$ to $t_f = 4\ s$, (the slope of the purple line)

$$v_{inst} = v_{avg.} = \frac{5m - 10m}{(4-2)s} = -2.5m/s$$

Finding Instantaneous Velocity from a Given Graph:

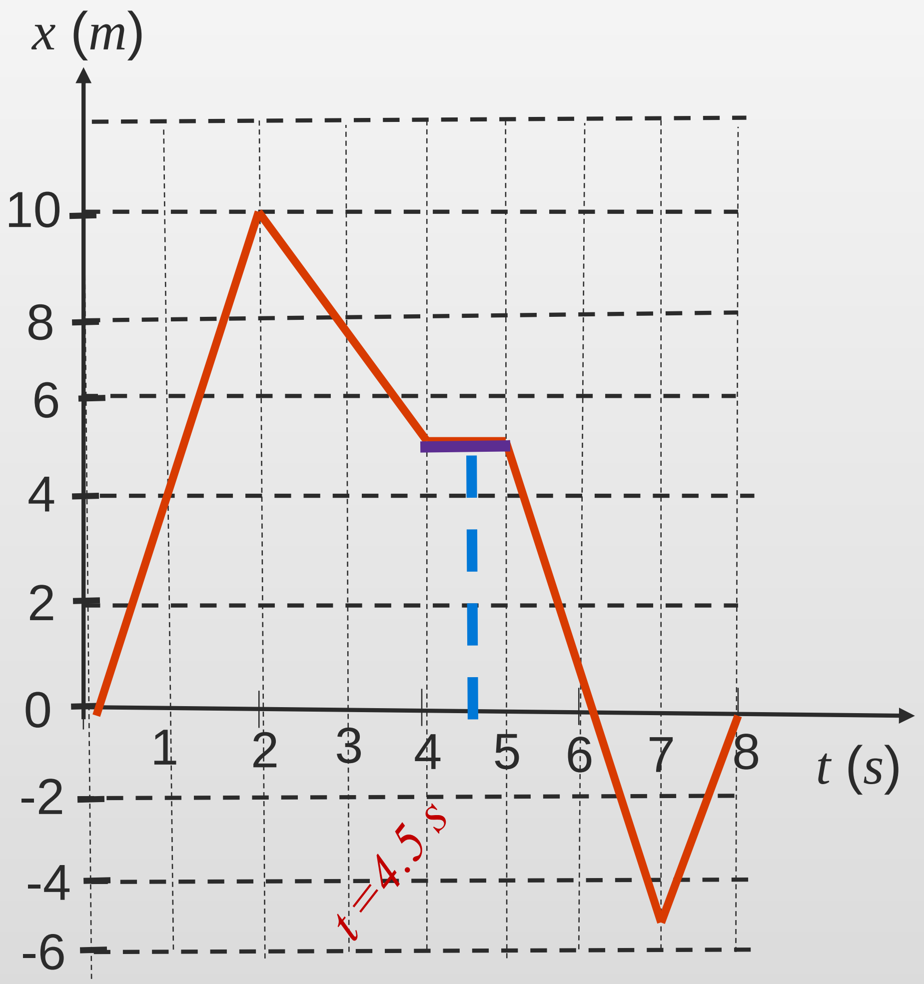
Solution:

$$v_{inst} = v_{avg.} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

For the linear relation between x and time t

(c) For $t = 4.5\ s$, v_{inst} is equal to v_{avg} in the time interval from $t_i = 4\ s$ to $t_f = 5\ s$, (the slope of the purple line)

$$v_{inst} = v_{avg.} = \frac{0-0}{(5-4)s} = 0$$



Finding Instantaneous Velocity from a Given Graph:

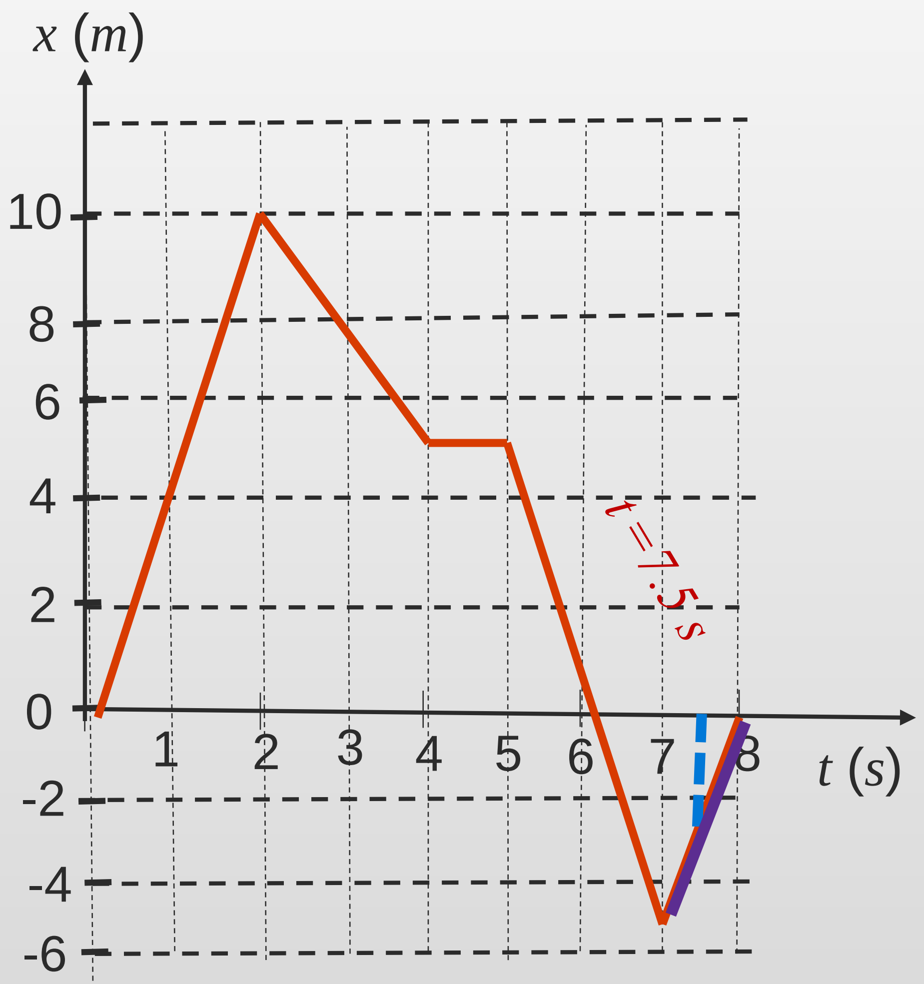
Solution:

$$v_{inst} = v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

For the linear relation between x and time t

(d) For $t = 7.5\ s$, v_{inst} is equal to v_{avg} in the time interval from $t_i = 7\ s$ to $t_f = 8\ s$, (the slope of the purple line)

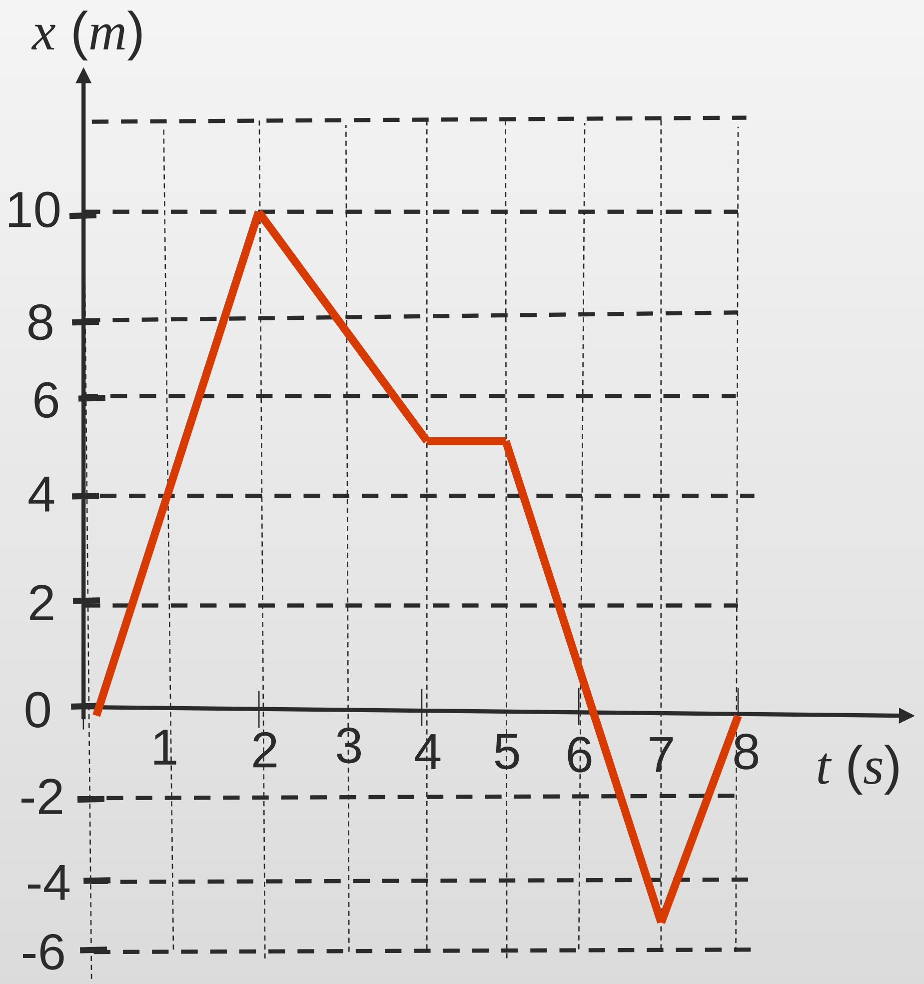
$$v_{inst} = v_{avg.} = \frac{0-(-5m/s)}{(8-7)s} = 5m/s$$



Finding Instantaneous Velocity from a Given Graph:

Exercise:

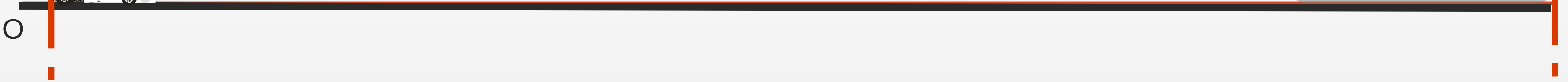
The position versus time for a certain particle moving along the x axis is shown. Find the instantaneous velocity at the following times (a) $t = 2\ s$, (b) $t = 6\ s$, and (c) $t = 7\ s$.

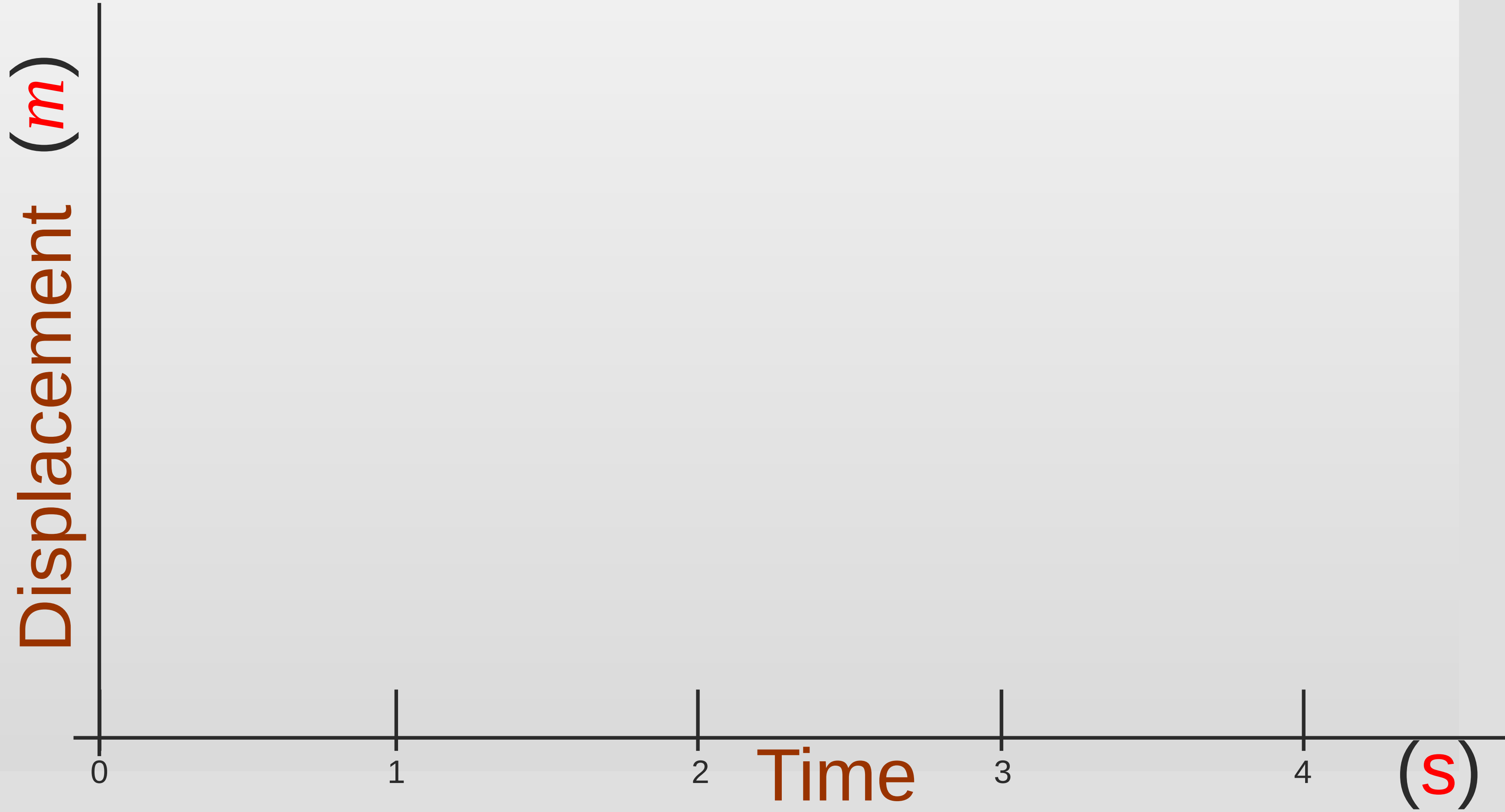


By Prof. Rashad Badran

Uniform Motion

With a constant acceleration

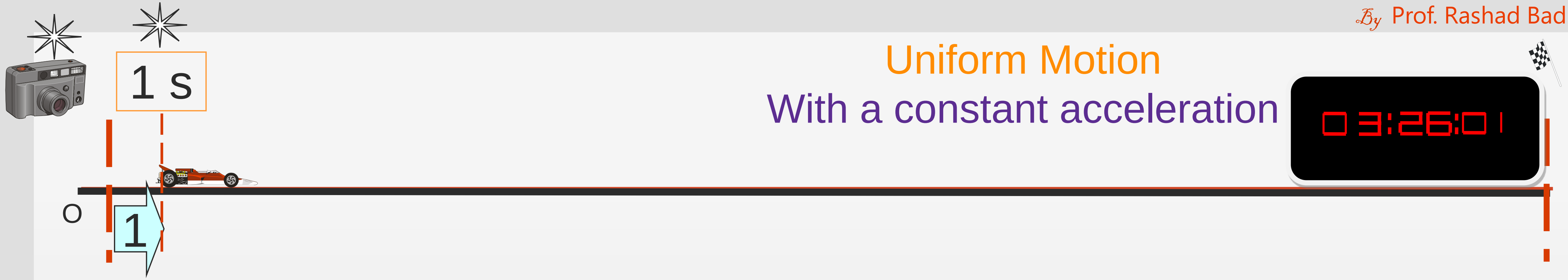


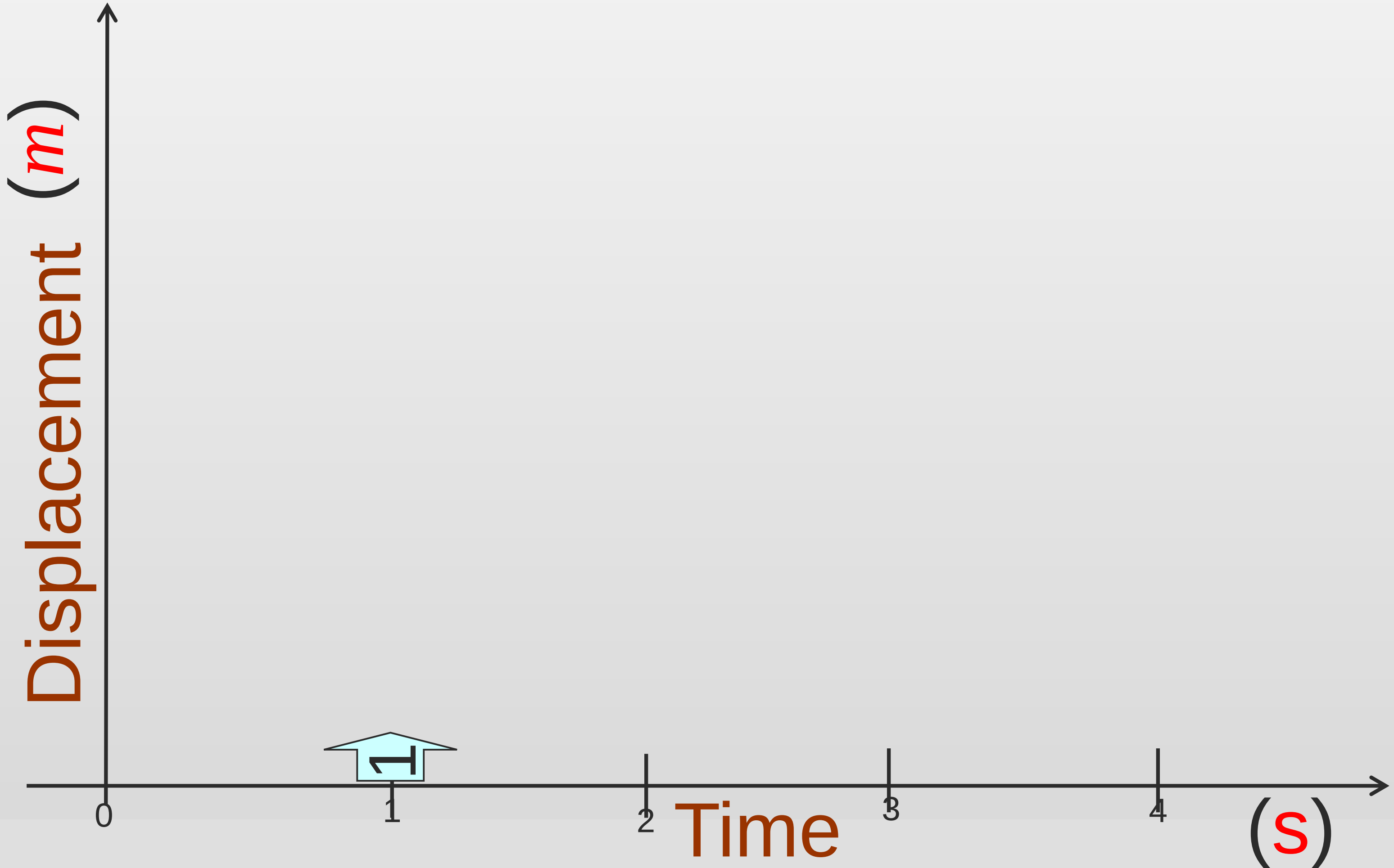


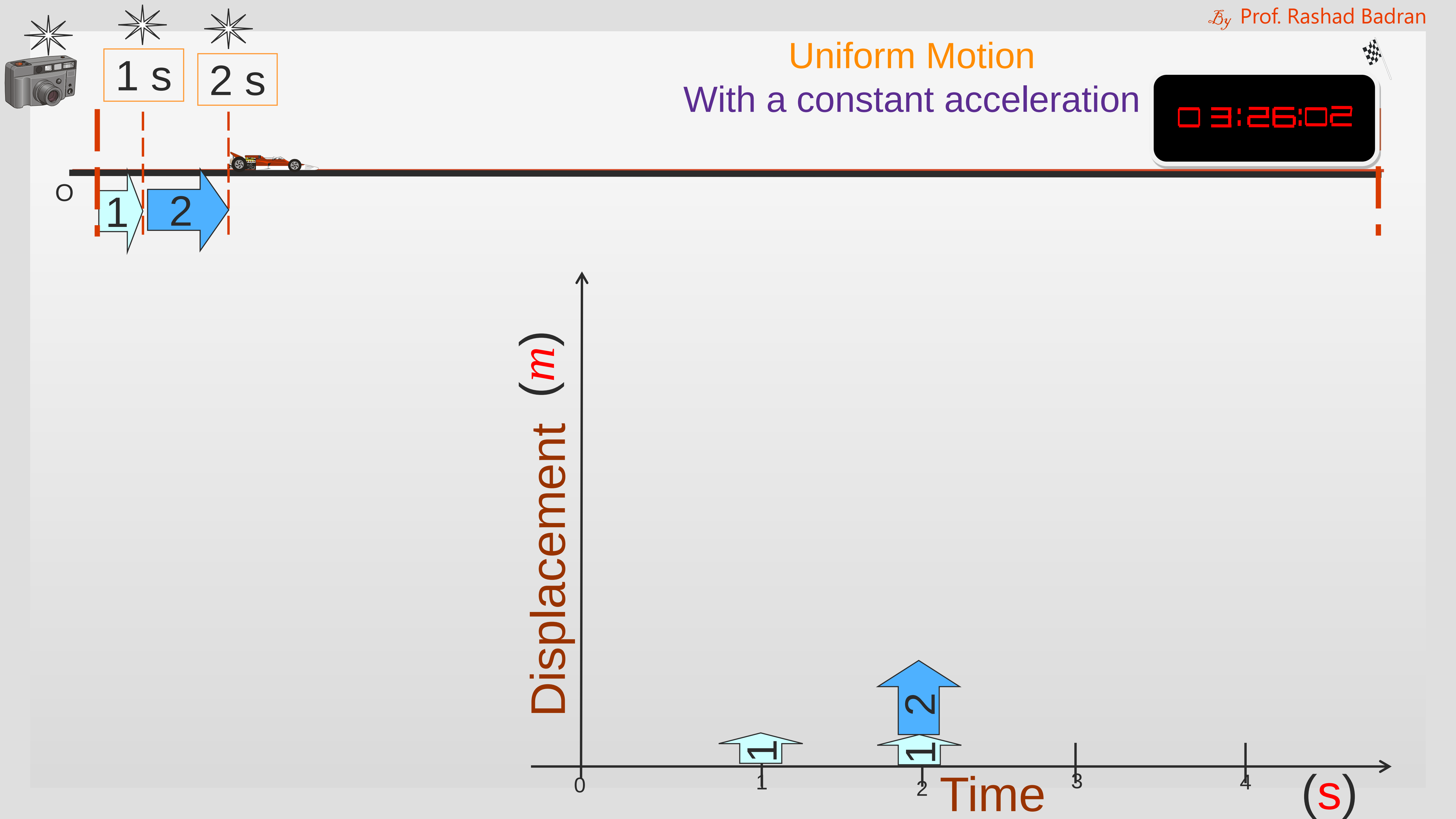
By Prof. Rashad Badran

Uniform Motion

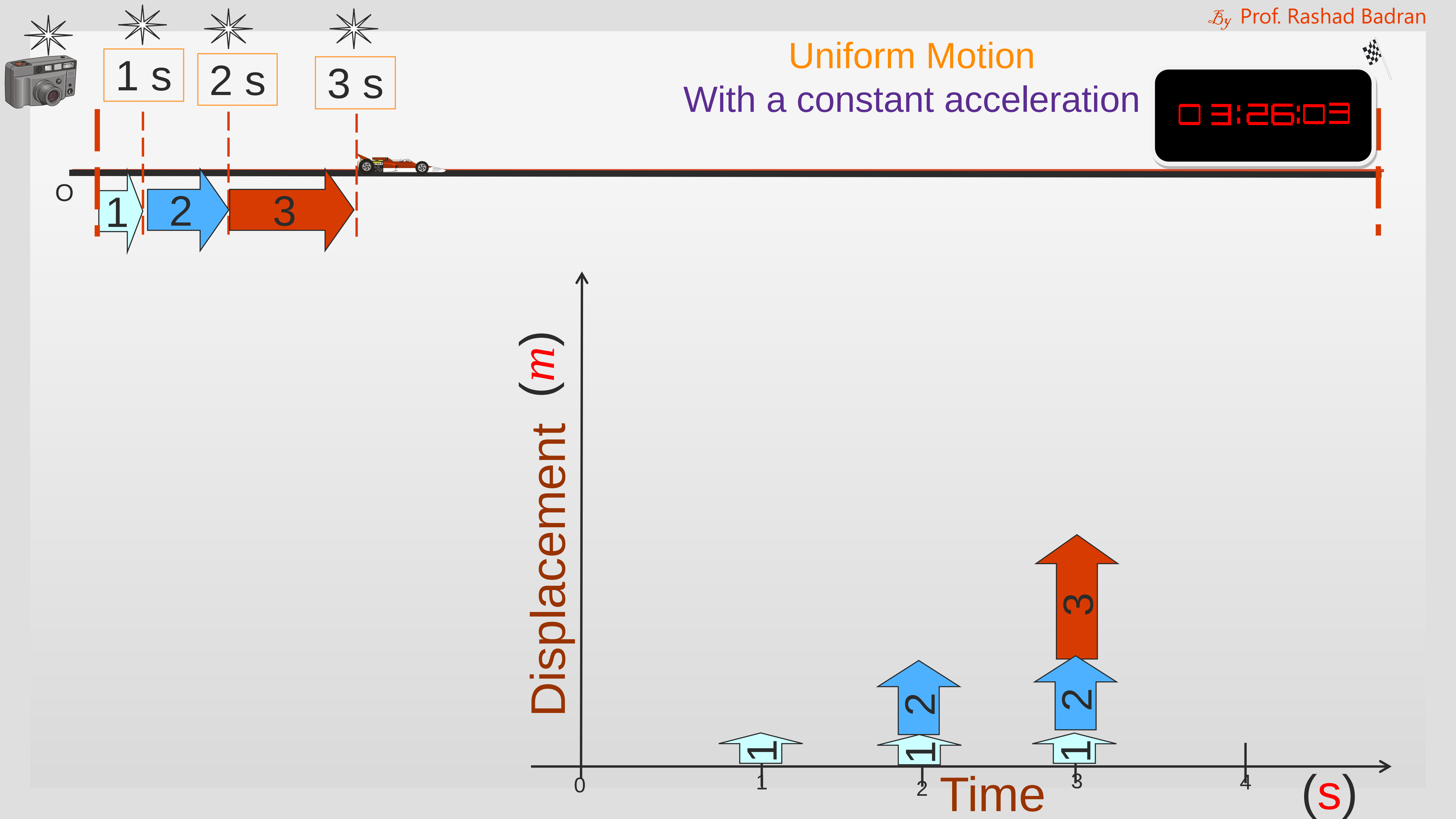
With a constant acceleration



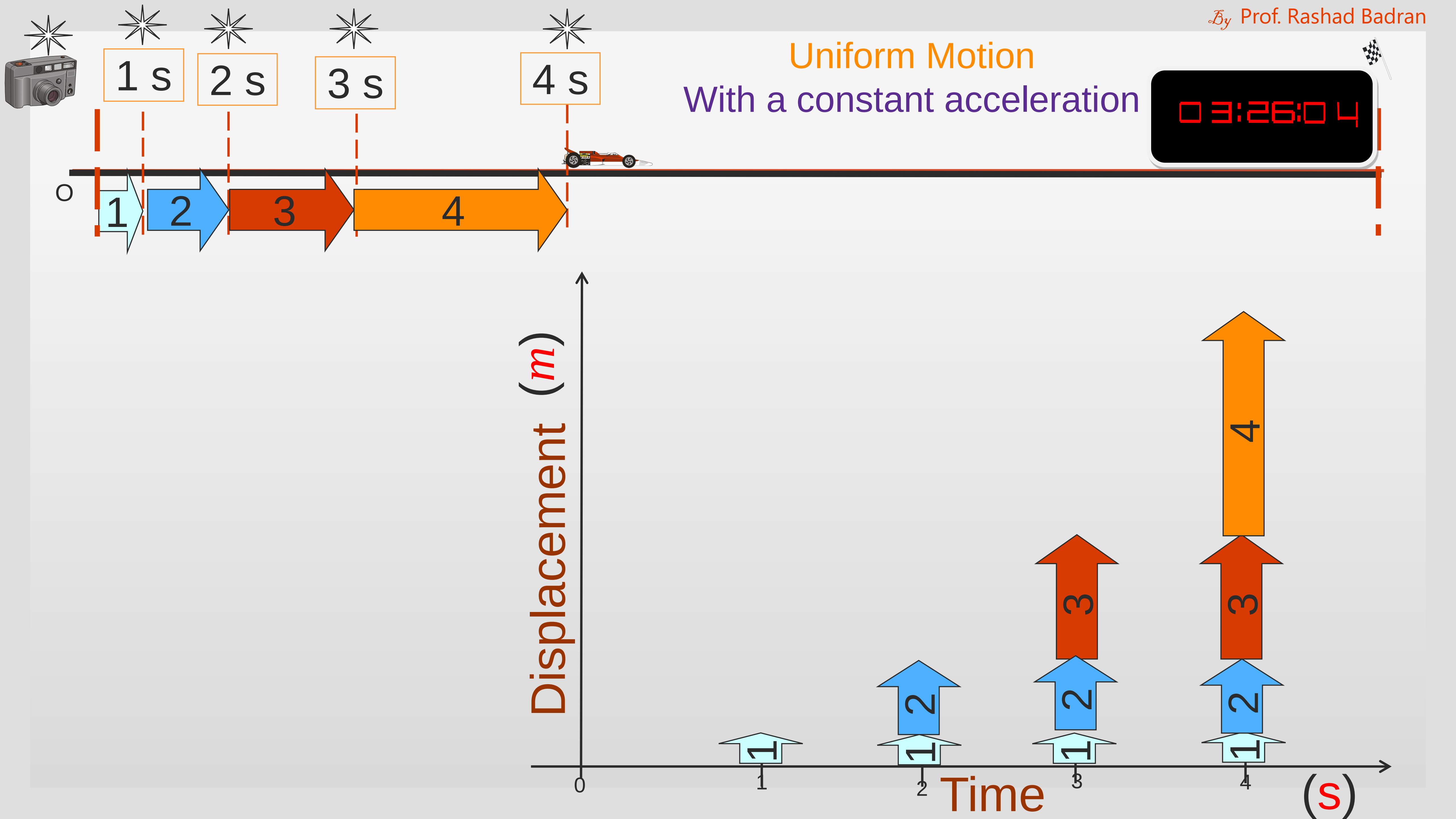




By Prof. Rashad Badran
Uniform Motion
With a constant acceleration
03:26:02
1 s
2 s
O
1
2
Displacement (m)
Time (s)
0
1
2
3
4



By Prof. Rashad Badran
Uniform Motion
With a constant acceleration
1 s
2 s
3 s
O
1
2
3
Displacement (m)
Time (s)
0
1
2
3
4



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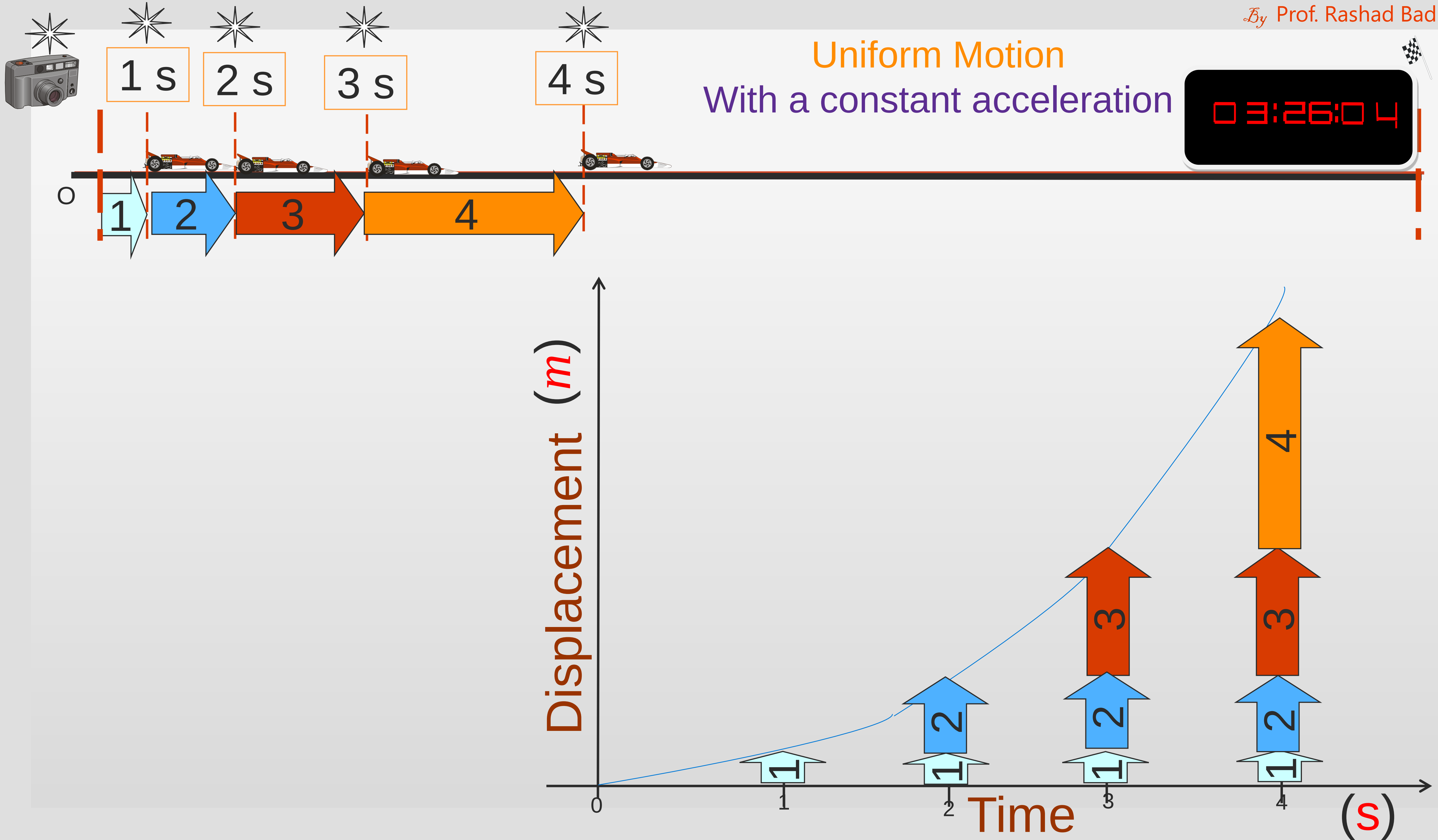
Uniform Motion
With a constant acceleration
03:26:04
1 s
2 s
3 s
4 s
O
1
2
3
4
Displacement (m)
Time (s)
0
1
2
3
4

By Prof. Rashad Badran

Uniform Motion
With a constant acceleration



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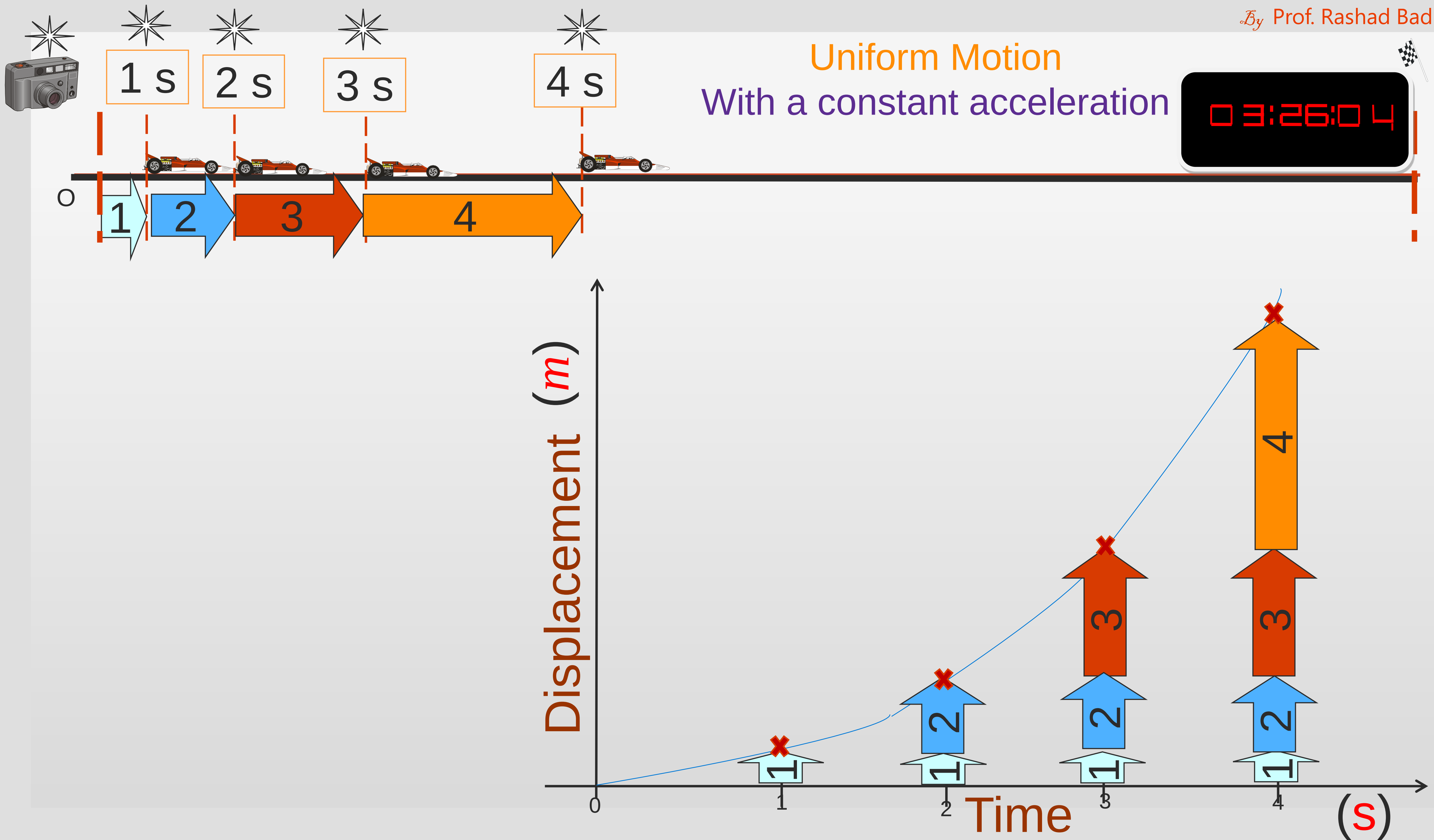
Uniform Motion
With a constant acceleration



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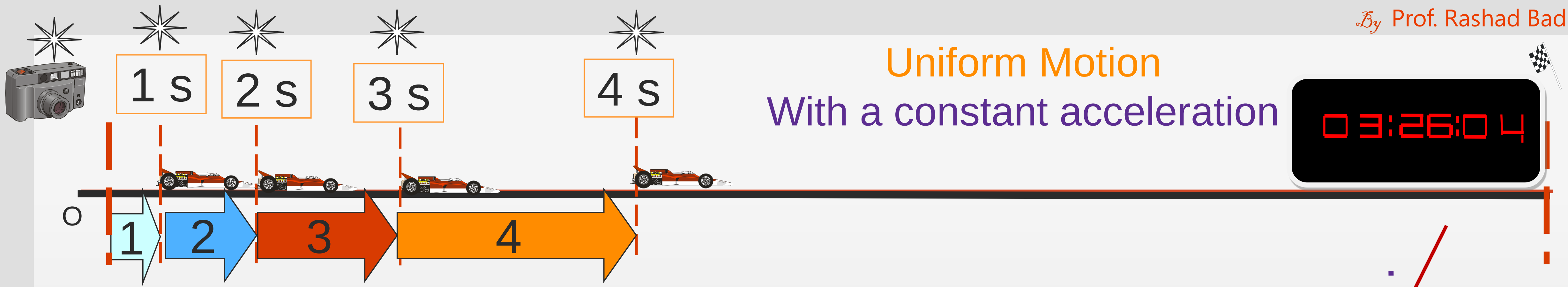
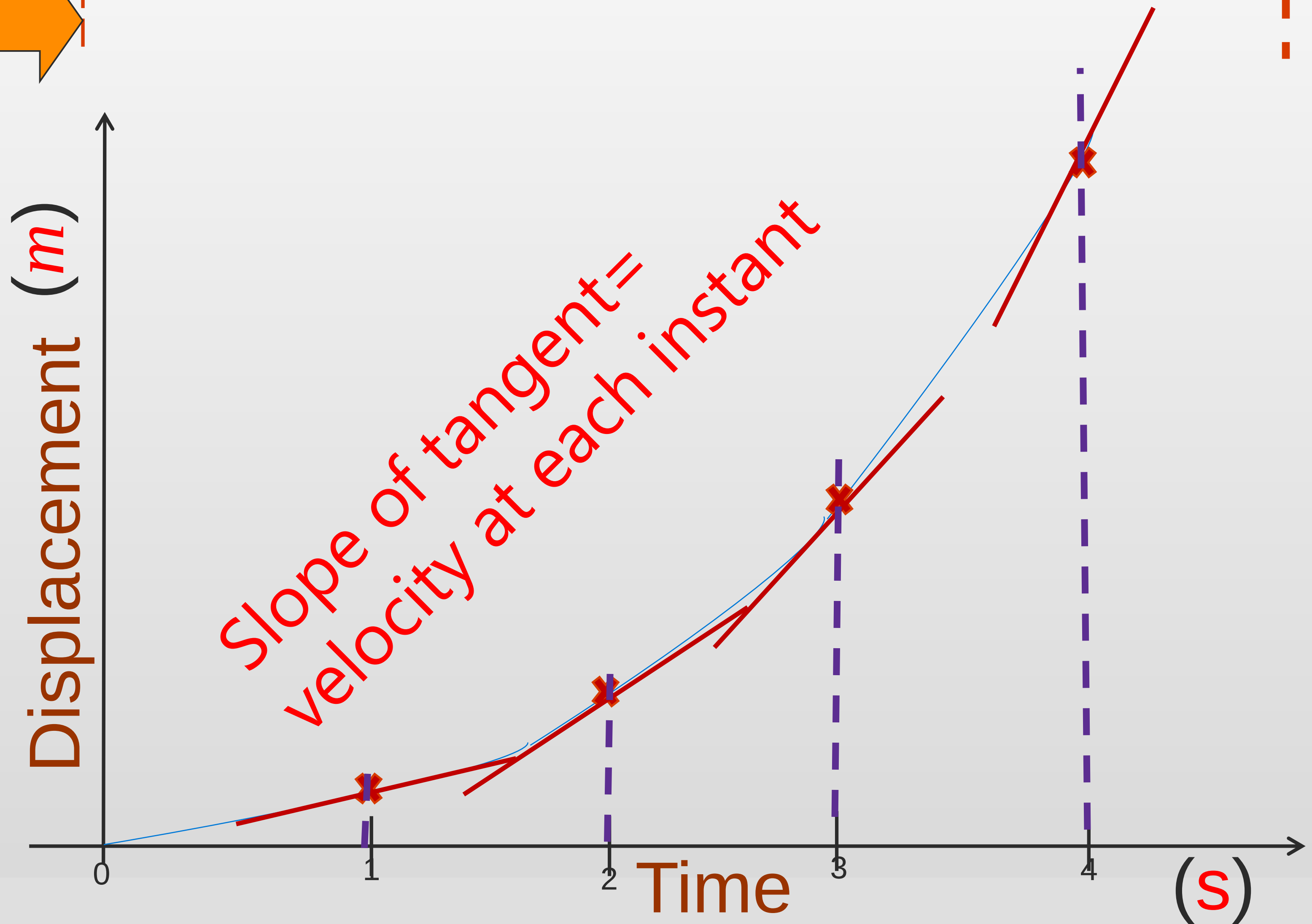
Uniform Motion
With a constant acceleration



Velocity at any instant of time can be calculated from the projection of a line drawn from Time-axis to intersect the curve (blue). Tangent at point of intersect has a slope that differs at different instants of time



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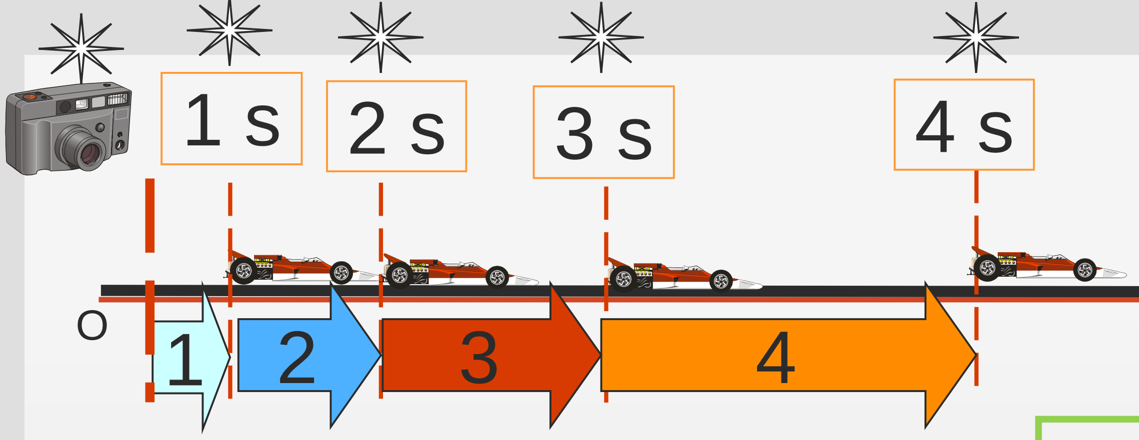
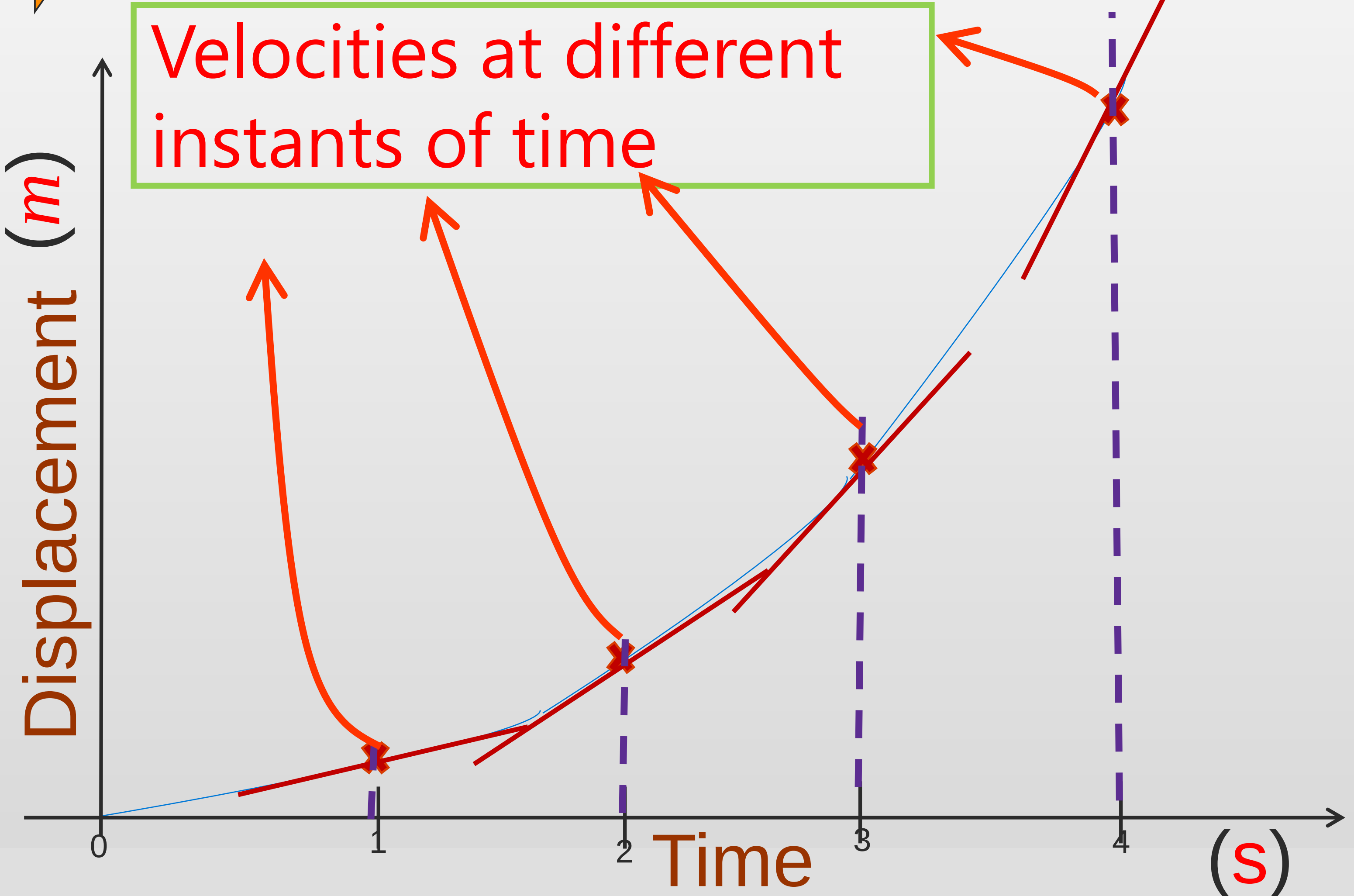
Uniform Motion

With a constant acceleration



Velocity is uniformly increasing with time



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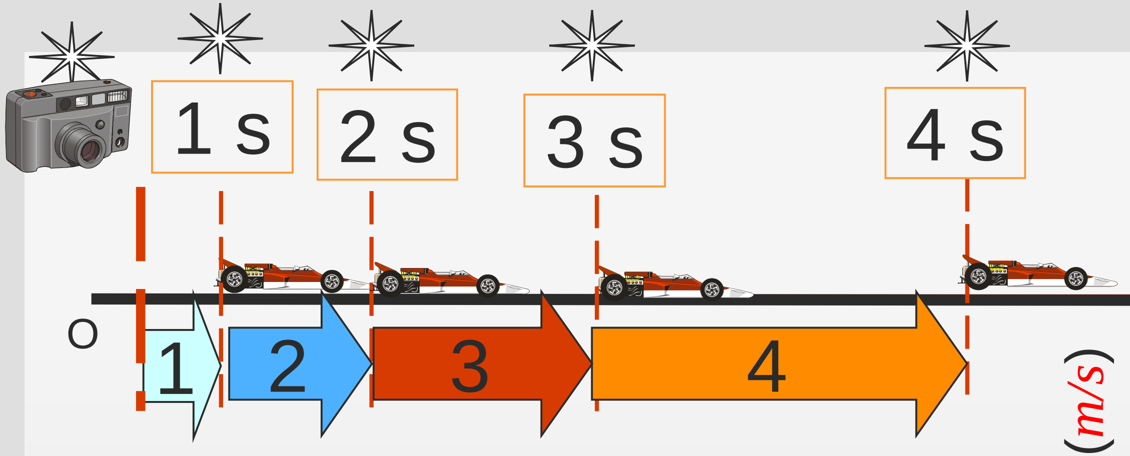
Uniform Motion

With a constant acceleration



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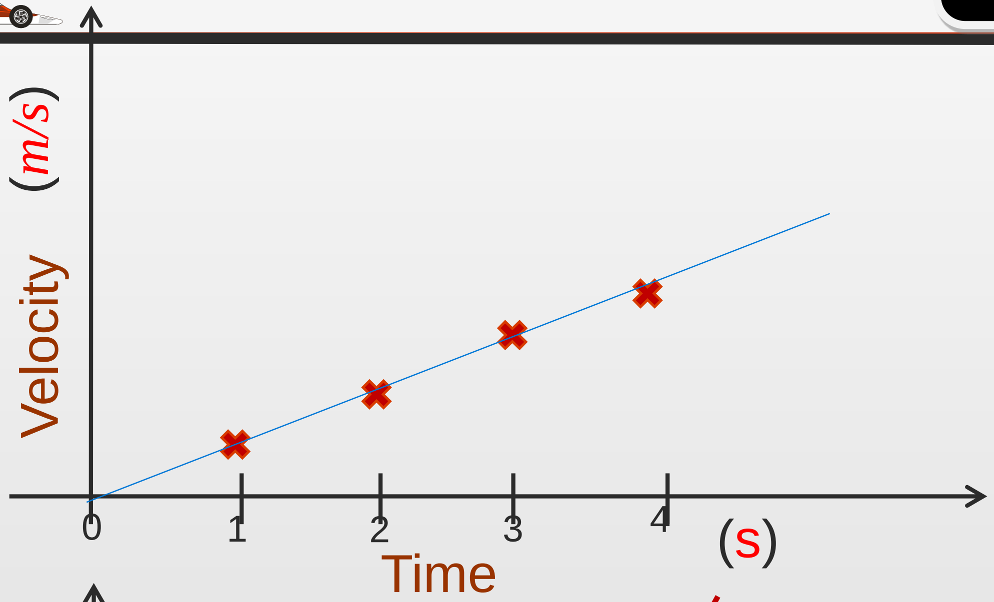

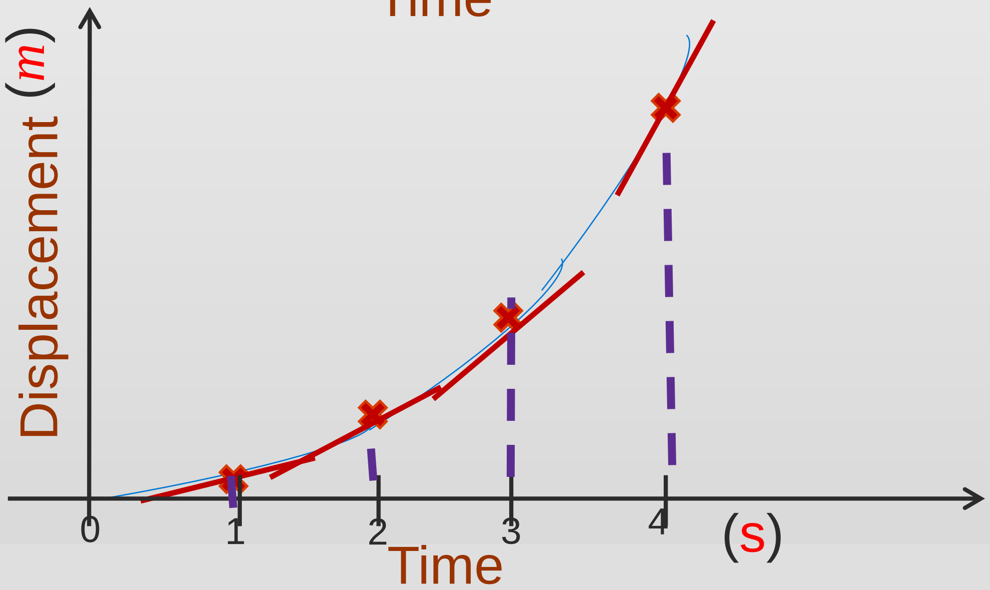

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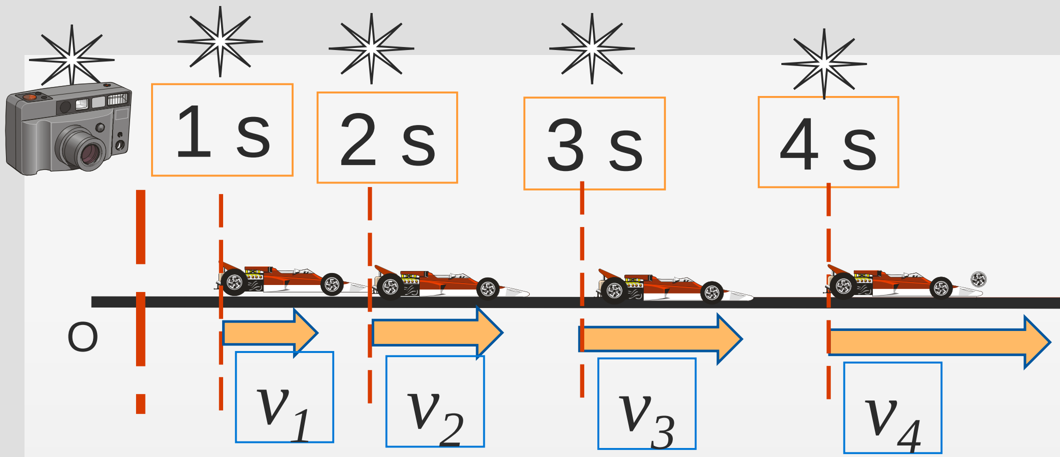
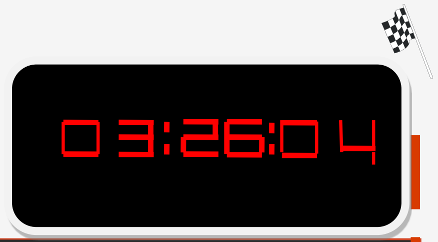
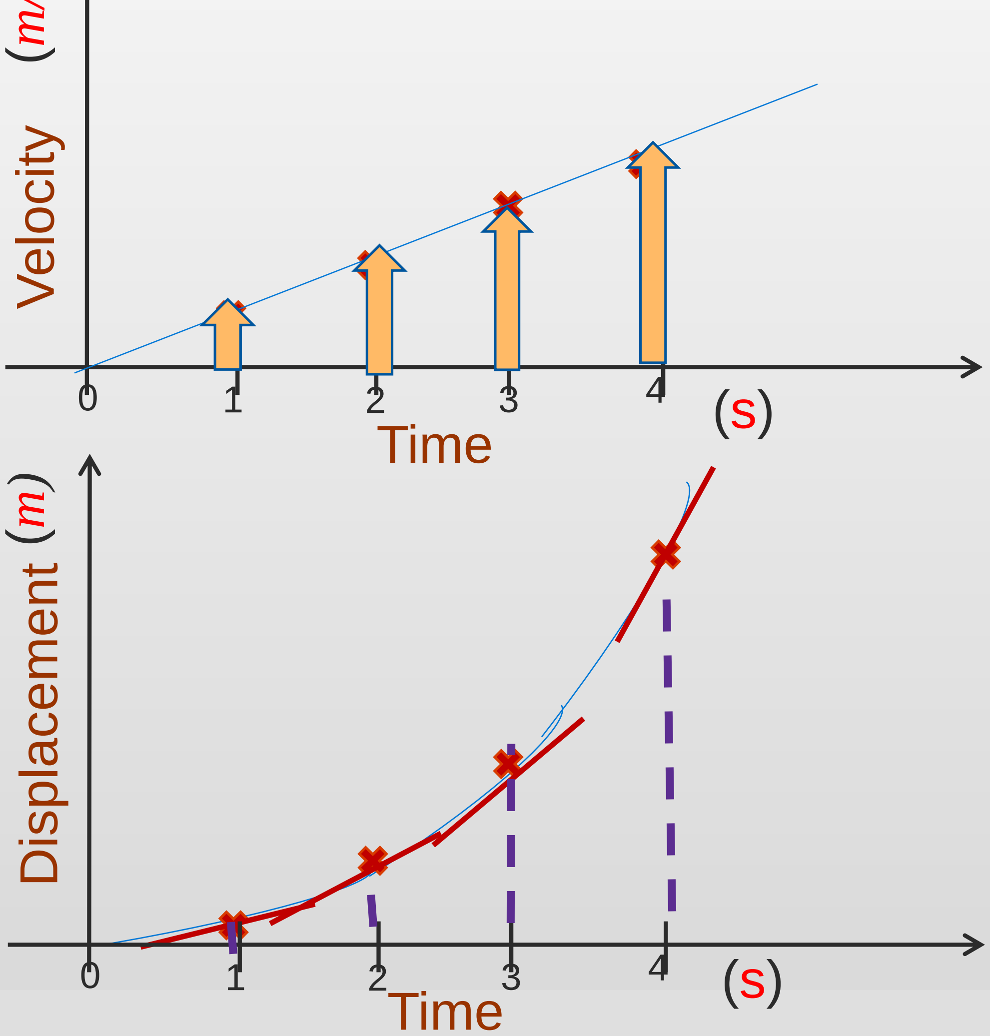
Uniform Motion

With a constant acceleration





Increasing velocities when time increases



Velocity is uniformly increasing with time

$$v = (slope) \times Time$$

$$= (Constant) \times t$$

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One Dimensional Motion:

Average Acceleration

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

In 1-D a simple form is written as:
(It has a magnitude and direction)

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Instantaneous Acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

In 1-D a simple form is written as:
(It has a magnitude and direction)

$$a = \frac{dv}{dt}$$

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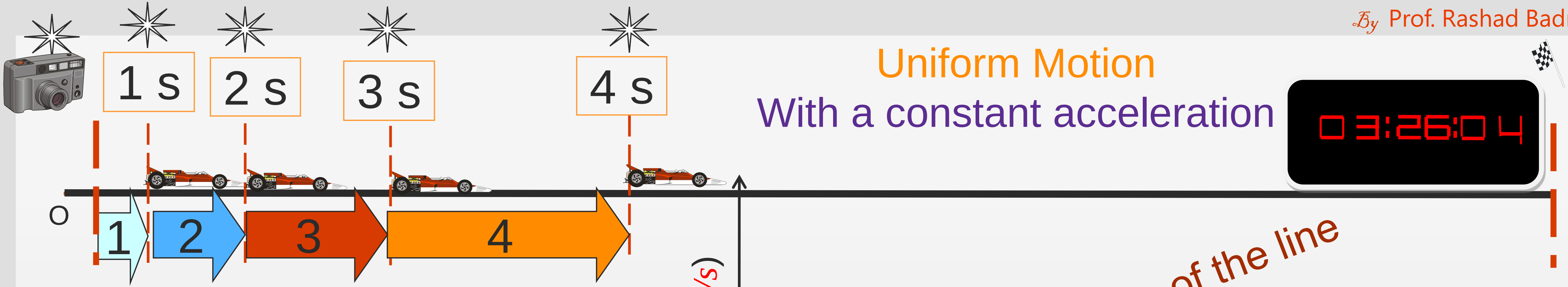
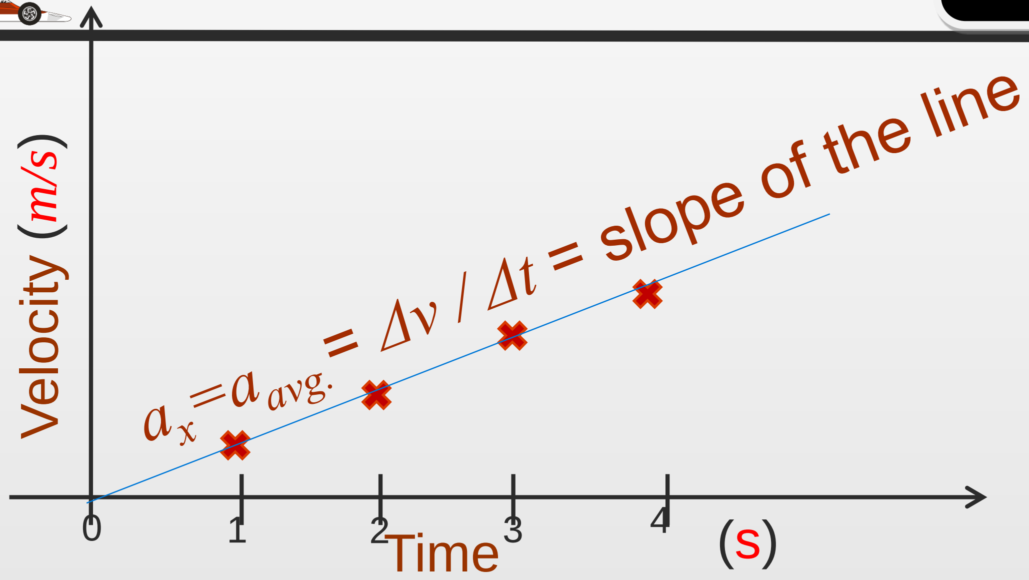
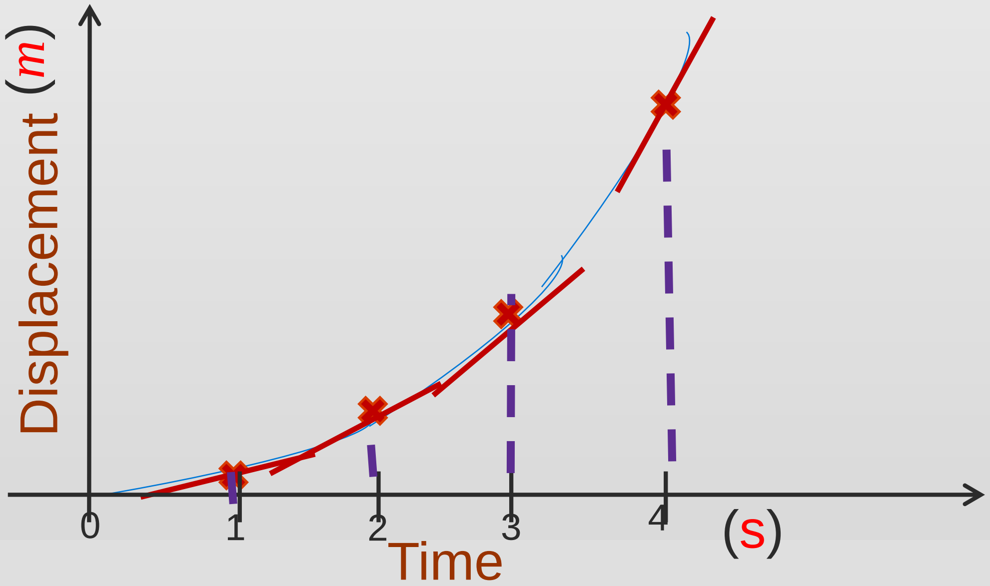
Uniform Motion

With a constant acceleration



Velocity is uniformly increasing with time





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Finding Average Acceleration and Instantaneous Acceleration from a Given Equation

Example

A particle moves according to the equation $x(t) = 10\,t^2$, where x is in meters and t is in seconds.

(a) Find the average acceleration for the time interval from 2 s to 3 s.

(b) Find the instantaneous acceleration at $t = 4$ s.

(c) At what time is the object at rest?

Solution: (a) Average acceleration for the time interval from 2 s to 3 s.

$$a_{avg.} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

To find the initial and final velocities (v_i and v_f) that correspond to the given initial and final times (t_i and t_f). One can take the first derivative with respect to time of the given equation $x = 10t^2$ to get $v_x(t) = 20t$

For $t = t_i = 2$ s substitute into the equation to get v_i as: $v_i = (20)(2) = 40 m/s$

For $t = t_f = 3$ s substitute into the equation to get v_f as: $v_f = (20)(3) = 60 m/s$

$$a_{avg.} = \frac{v_f - v_i}{t_f - t_i} = \frac{60 m/s - 40 m/s}{3s - 2s} = 20 m/s^2$$

It is in the direction of initial motion

Solution: (b) To find the instantaneous acceleration at $t = 4\ s$.

$$a_x = \frac{dv}{dt}$$

Take the second derivative of the given equation ($x(t) = 10t^2$) with respect to t to get:

$$a_x = 20 m/s^2$$

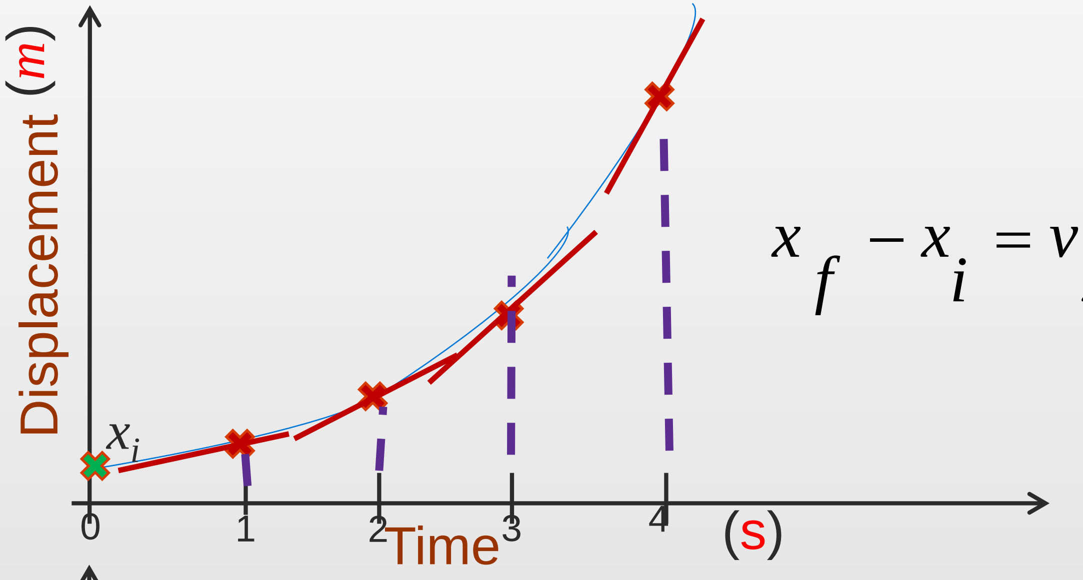
It is in the direction of initial motion

The result shows that the acceleration is constant when t varies (i.e. $a_x = a_{avg.}$)

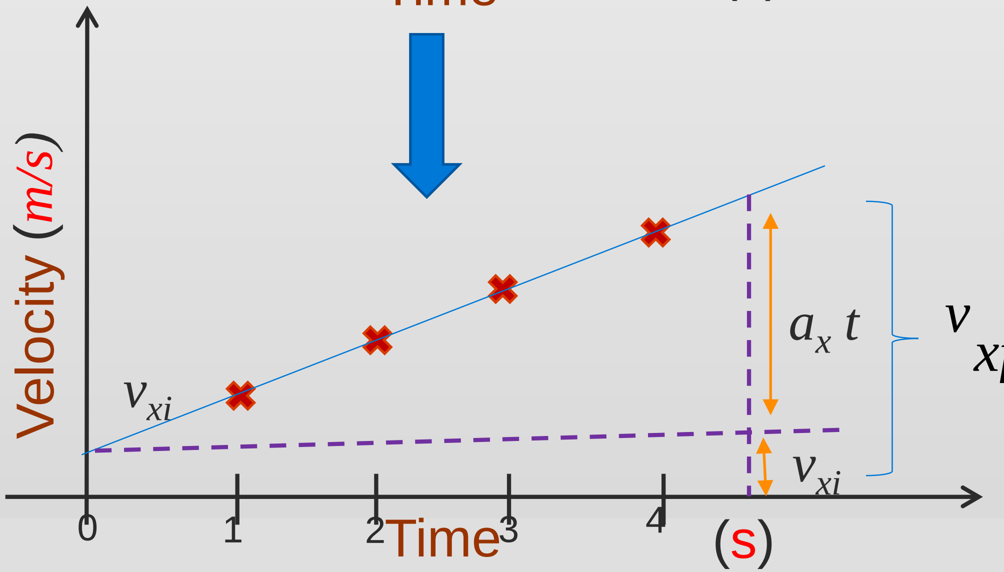
(c)To find time at which the object is at rest?

Use the obtained equation $v_x = 20t$ and put $v_x = 0$ to get the time $t = 0$

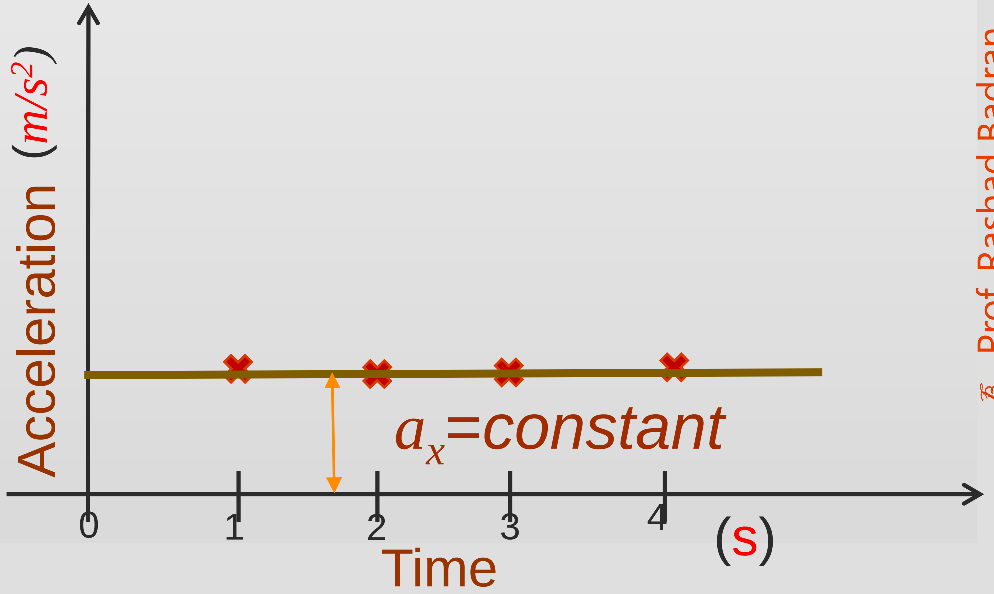
Uniform Motion with a Constant Acceleration



$$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$$



$$v_{xf} = v_{xi} + a_x t$$



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Uniform Motion with a Constant Acceleration

$$v_{xf} = v_{xi} + a_x t$$

(1) Velocity as a function of time for a constant acceleration

$$x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t$$

(2) Position as a function of velocity and time for a constant acceleration

$$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$$

(3) Position as a function of time for a constant acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x (x_f - x_i)$$

(4) Velocity as a function of position for a constant acceleration

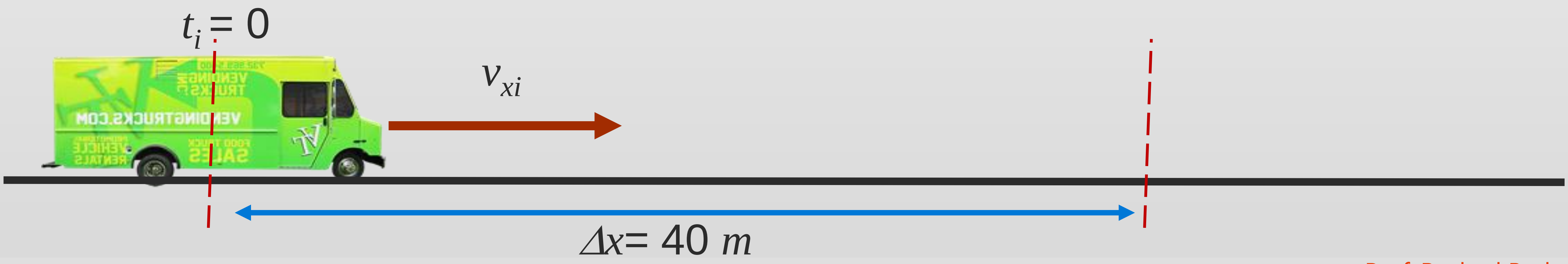
Finding Acceleration of slowing down object:

Example

A truck covers 40 m in 8.5 s while smoothly slowing down to a final speed of 2.8 m/s

(a) Find its original speed

(b) Find its acceleration.



Finding Acceleration of slowing down object:

Example

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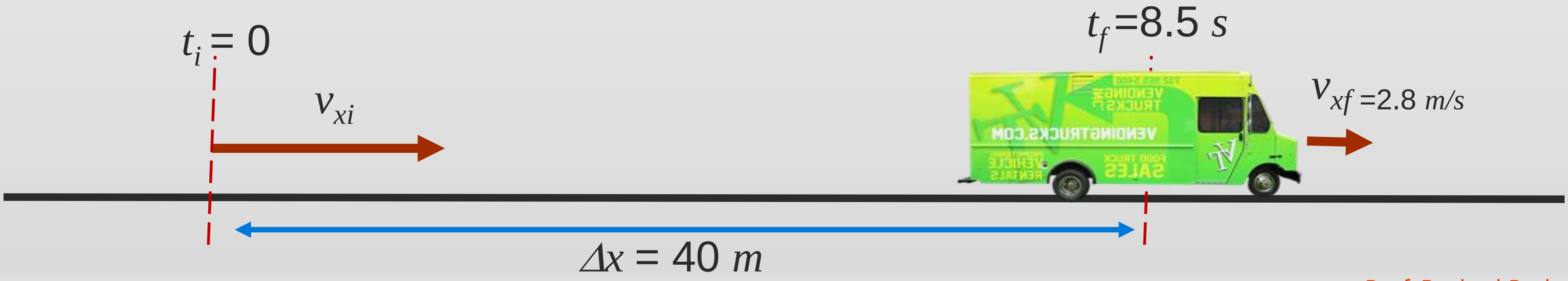
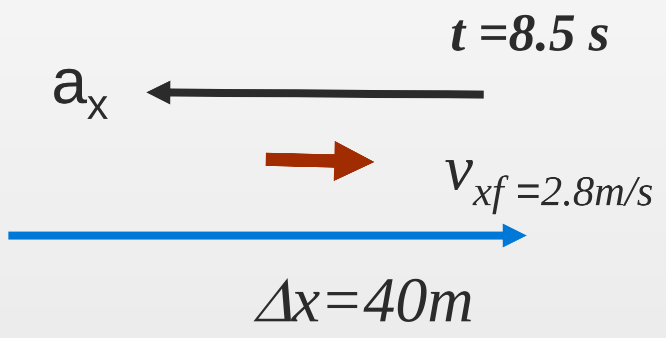


Solution:

(a)



$$\Delta x = \frac{1}{2}(v_{xf} + v_{xi})t$$

$$40m = \frac{1}{2}(2.8m/s + v_{xi})(8.5s)$$

$$\Rightarrow v_{xi} = 6.62m/s$$ due east and same as the direction of final velocity

(b)

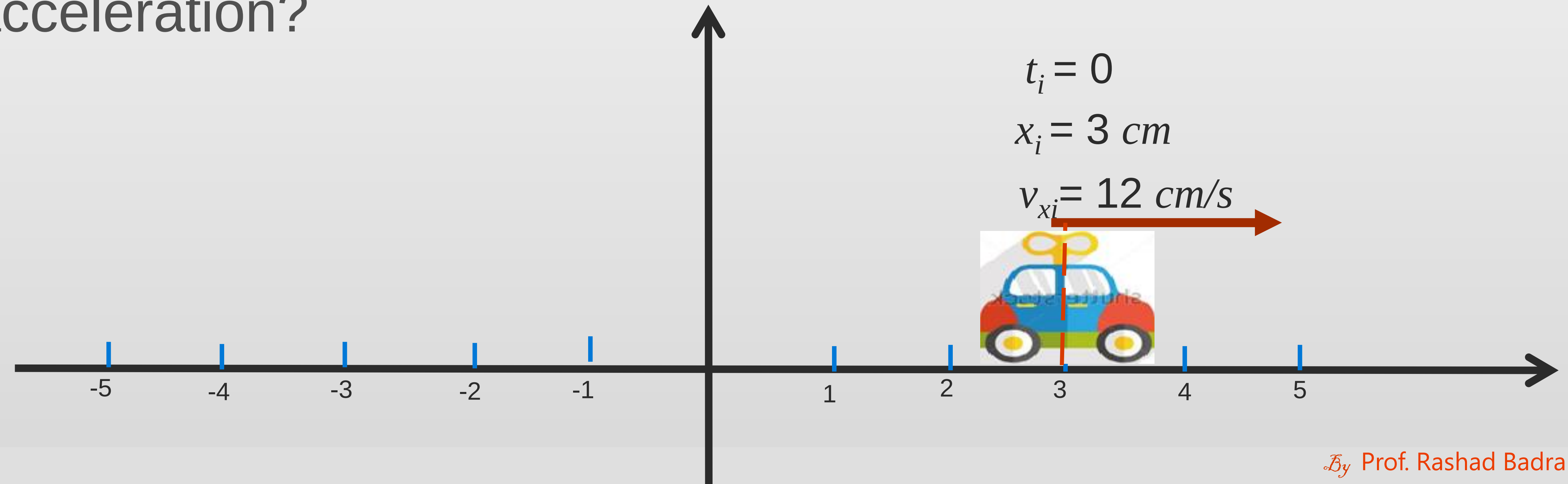
$$a_x = \frac{(v_{xf} - v_{xi})}{t} = \frac{2.8 - 6.62}{8.5s} = -0.45m/s^2$$

Conclusion: The minus sign indicates a deceleration of truck where ***a*** is directed opposite to both Δx and final motion of the truck v_{xf} .

Finding an Acceleration:

Example

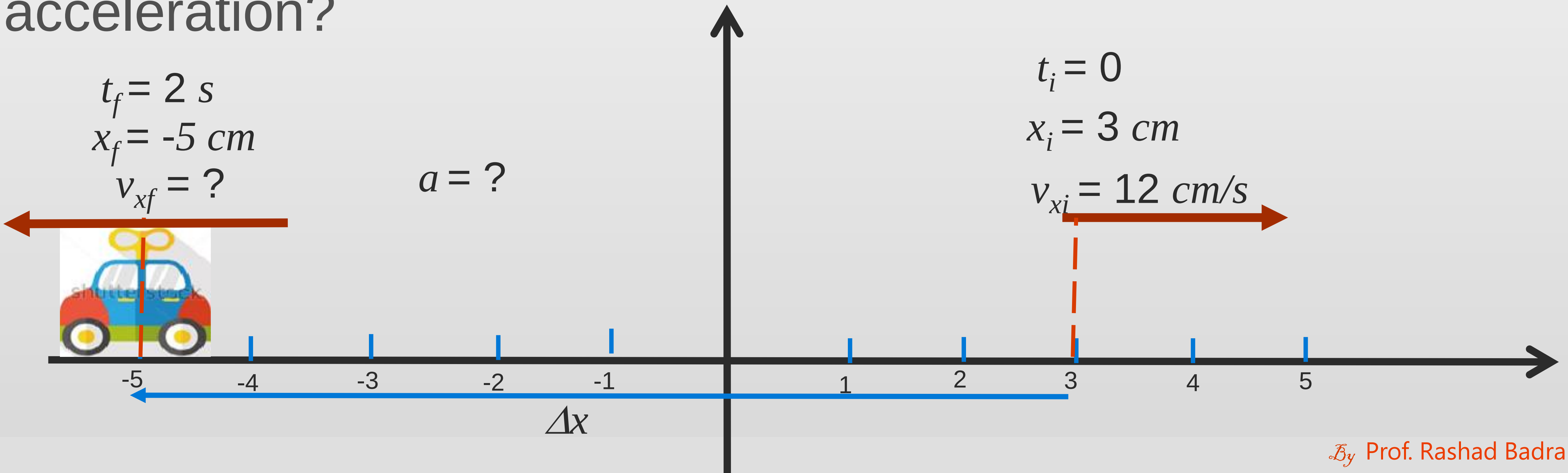
A baby toy car moving with uniform acceleration has a velocity of 12 *cm/s* in the positive direction when its *x*- coordinate is 3 *cm*. If its *x*-coordinate 2 *s* later is – 5 *cm*, what is its acceleration?



Finding an Acceleration of an object when it changes direction:

Example

A baby toy car moving with uniform acceleration has a velocity of 12 cm/s in the positive direction when its x- coordinate is 3 cm. If its x-coordinate 2 s later is – 5 cm, what is its acceleration?



Solution:

$t_i = 0$ $x_i = 3\ cm$ $v_{xi} = 12\ cm/s$ $t_f = 2\ s$ $x_f = -5\ cm$ $a_x = ?$

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

a_x

$$(-5-3)cm = (12cm/s)(2s) + \frac{1}{2}a_x(2)^2$$

$\Delta x = -8m$

$$\Rightarrow a_x = -16cm/s^2$$

due west and same as the direction of displacement

Conclusion: The minus sign indicates an acceleration towards –ve x-axis and is directed in the same direction of Δx and final velocity v_{xf} . One can check whether the final velocity is directed towards west or not as follows:

$$v_{xf} = v_{xi} + a_x t \Rightarrow \quad v_{xf} = 12cm/s + (-16cm/s^2)(2)$$

$$\Rightarrow v_{xf} = -20cm/s$$

v_{xf}

Finding position and time of an object at a turning point

Exercise

From the previous example, at what time is the car at rest? What is its position at this moment?

One Dimensional Motion:

Conceptual Question

If a car is travelling eastward, its acceleration

(a) must be varying

(b) must be eastward only

(c) can be eastward or westward

(d) The answers in (a) and (b) are correct

(e) None of above answers

One Dimensional Motion:

Conceptual Question

If the velocity of the particle is zero, its acceleration

(a) can be zero

(b) is constant but not necessarily zero

(c) cannot be zero but is increasing with time

(d) cannot be zero but is decreasing with time.

(e) The answers (a) and (b) are possible

One Dimensional Motion:

Conceptual Question

Two cars (A and B) are moving in the same direction in parallel lanes along a highway. At some instant, the velocity of car A exceeds the velocity of car B. Which of the following statements are correct?

(a) The acceleration of car A is greater than that of car B

(b) The accelerations of the two cars are the same

(c) The acceleration of car B is greater than that of car A

(d) B is an old car while car A is a new one.

(e) None of those statements is correct

By Prof. Rashad Badran

One Dimensional Motion:

Objective Question

A racing car starts from rest at $t = 0$ and reaches a final speed v at time t. if the acceleration of the car is constant during this time, which one of the following statements are true?

(a) The car travels a distant vt.

(b) The average speed of the car is $v/2$.

(c) The magnitude of the acceleration of the car is v/t.

(d) The velocity of the car remains constant.

(e) None of answers (a) through (d) is true