# Some Properties of Vectors: Addition of Vectors

2) Algebraic (or Analytical) Method

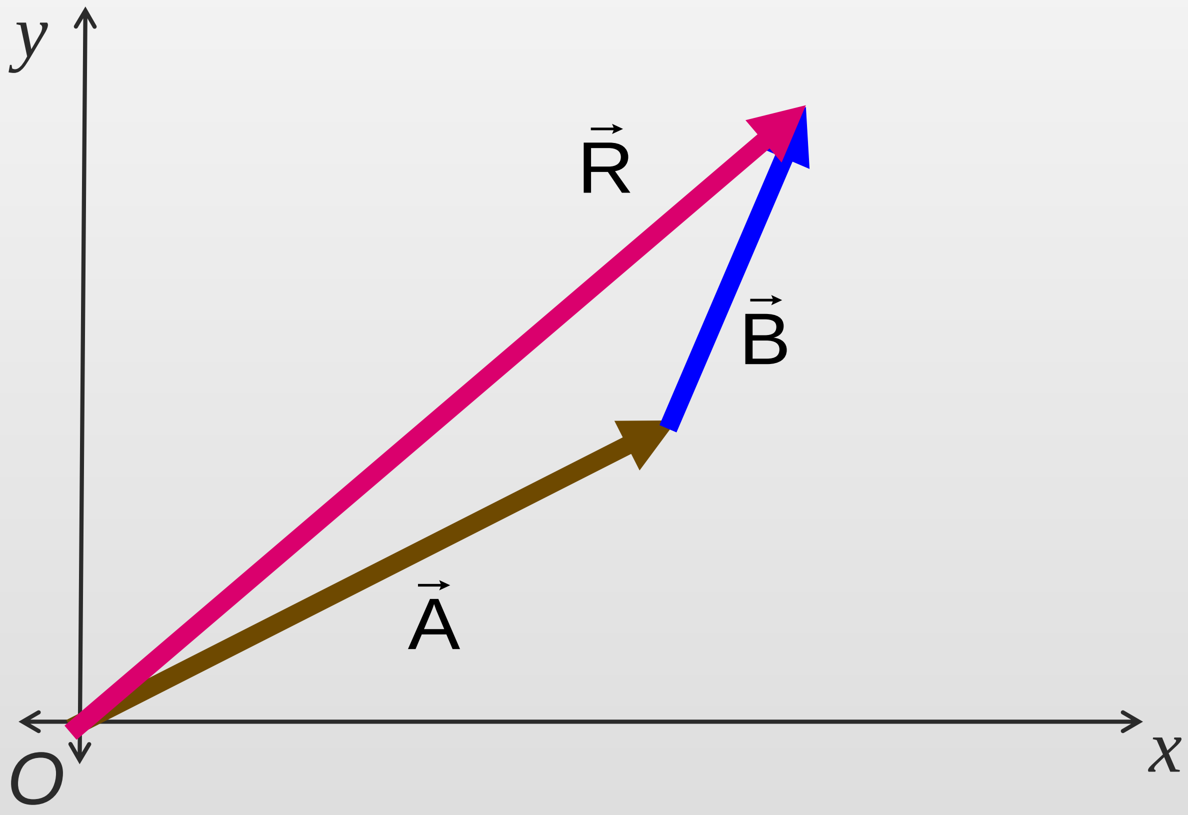
2-a) Component Method

2-b) Applying the Laws of Sines and Cosines

Notes: The component method requires the knowledge of decomposing each vector into its components.

# 2-a) Component Method of adding Vectors

Solving problem strategy can be set up to find the resultant vector (call it $\vec{R}$ ) for the shown two head-to-tail vectors $\vec{A}$ and $\vec{B}$ .



# 2-a) Component Method of adding Vectors: **Problem-Solving Strategy**

**IDENTIFY** *the relevant concepts and* **SET UP** *the problem*

Target variable (Magnitude of the sum, direction or both…)

**EXECUTE** *the solution*

1- Find $x$- and $y$-components of each vector

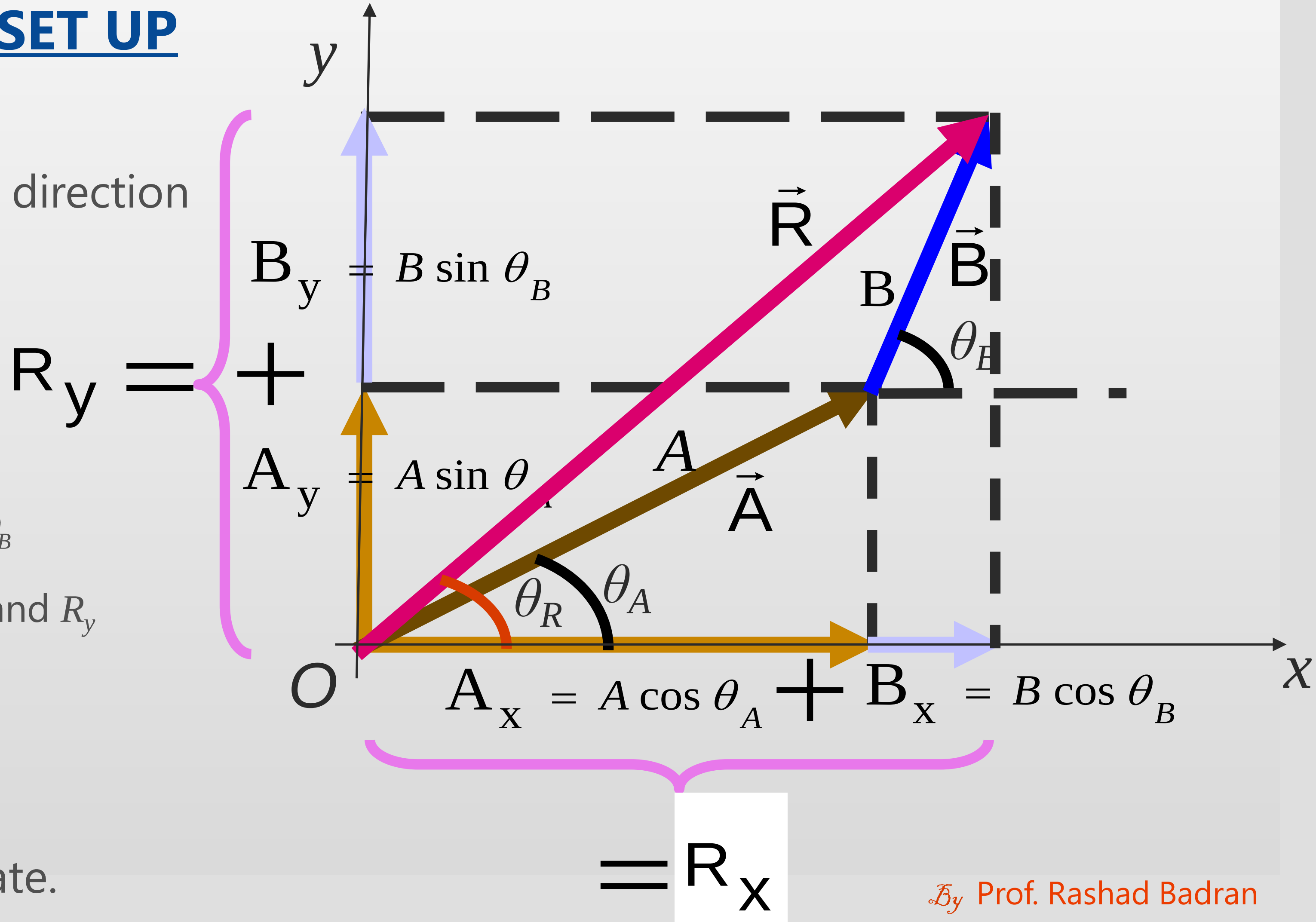
$A_x = A\cos\theta_A$, $A_y = A\sin\theta_A$, $B_x = B\cos\theta_B$, and $B_y = B\sin\theta_B$

2- Add the individual components to find $R_x$ and $R_y$

3- $R = (R_x^2 + R_y^2)^{1/2}$  $\theta_R = \arctan(R_y/R_x)$

**EVALUATE** *your answer:*

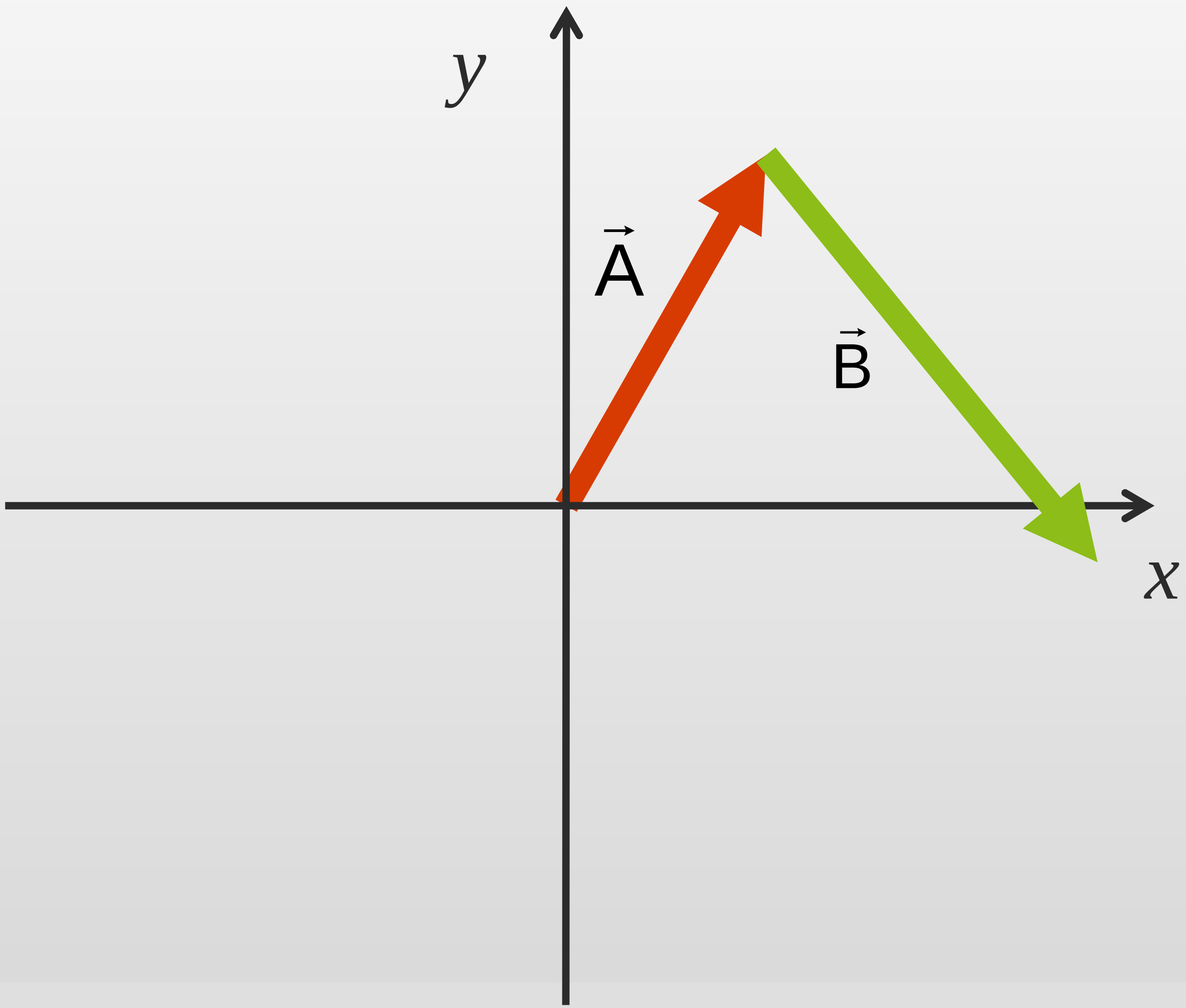
Compare your answer with your estimate.




By Prof. Rashad Badran

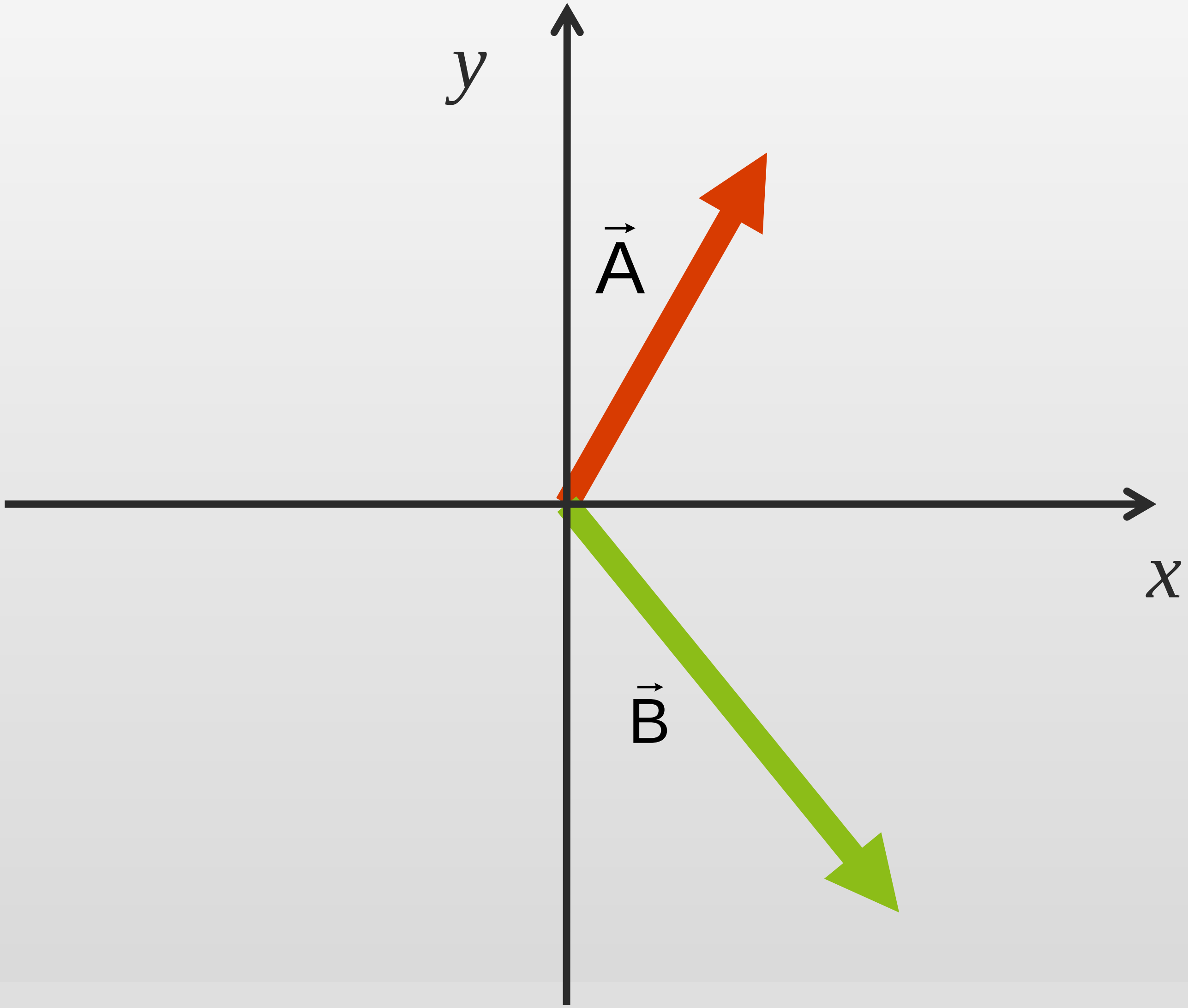
# 2-a) Component Method of adding Vectors

**Head-to-Tail Vectors**



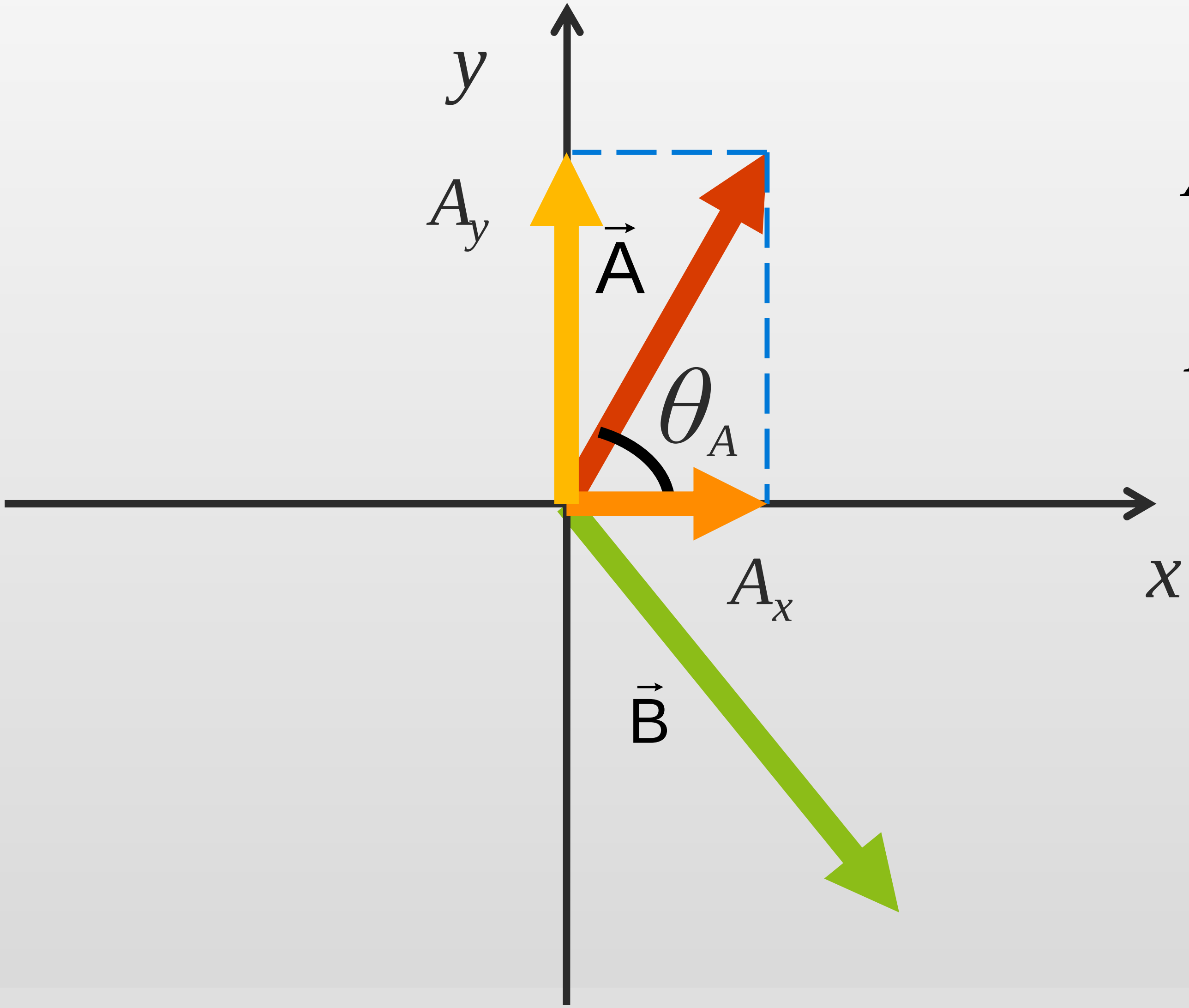
# 2-a) Component Method of adding Vectors

**Tail-to-Tail Vectors**



# 2-a) Component Method of adding Vectors

**Tail-to-Tail Vectors**




$$A_x = A\cos\theta_A$$

$$A_y = A\sin\theta_A$$

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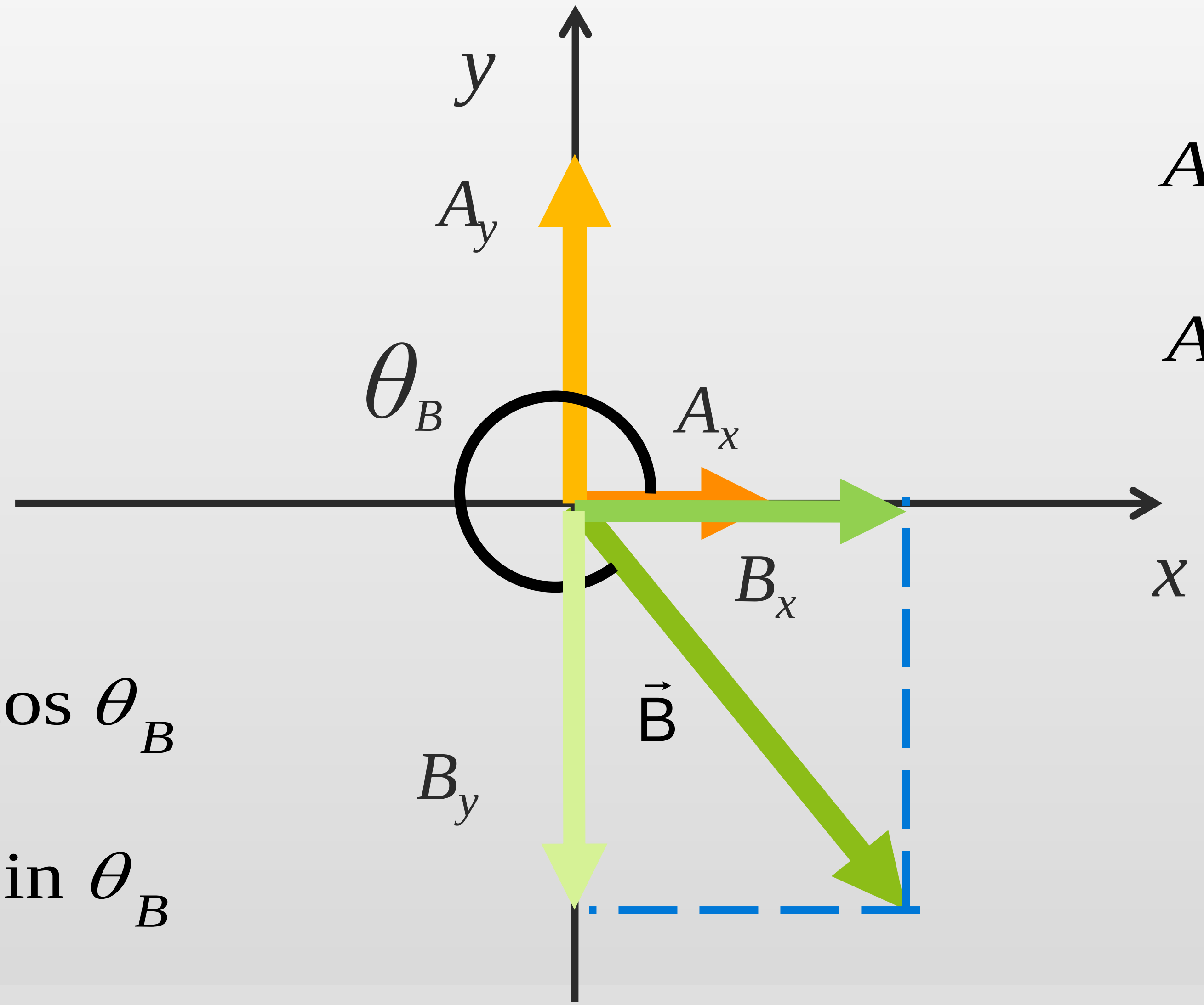
# 2-a) Component Method of adding Vectors

**Tail-to-Tail Vectors**




$$A_x = A\cos\theta_A$$

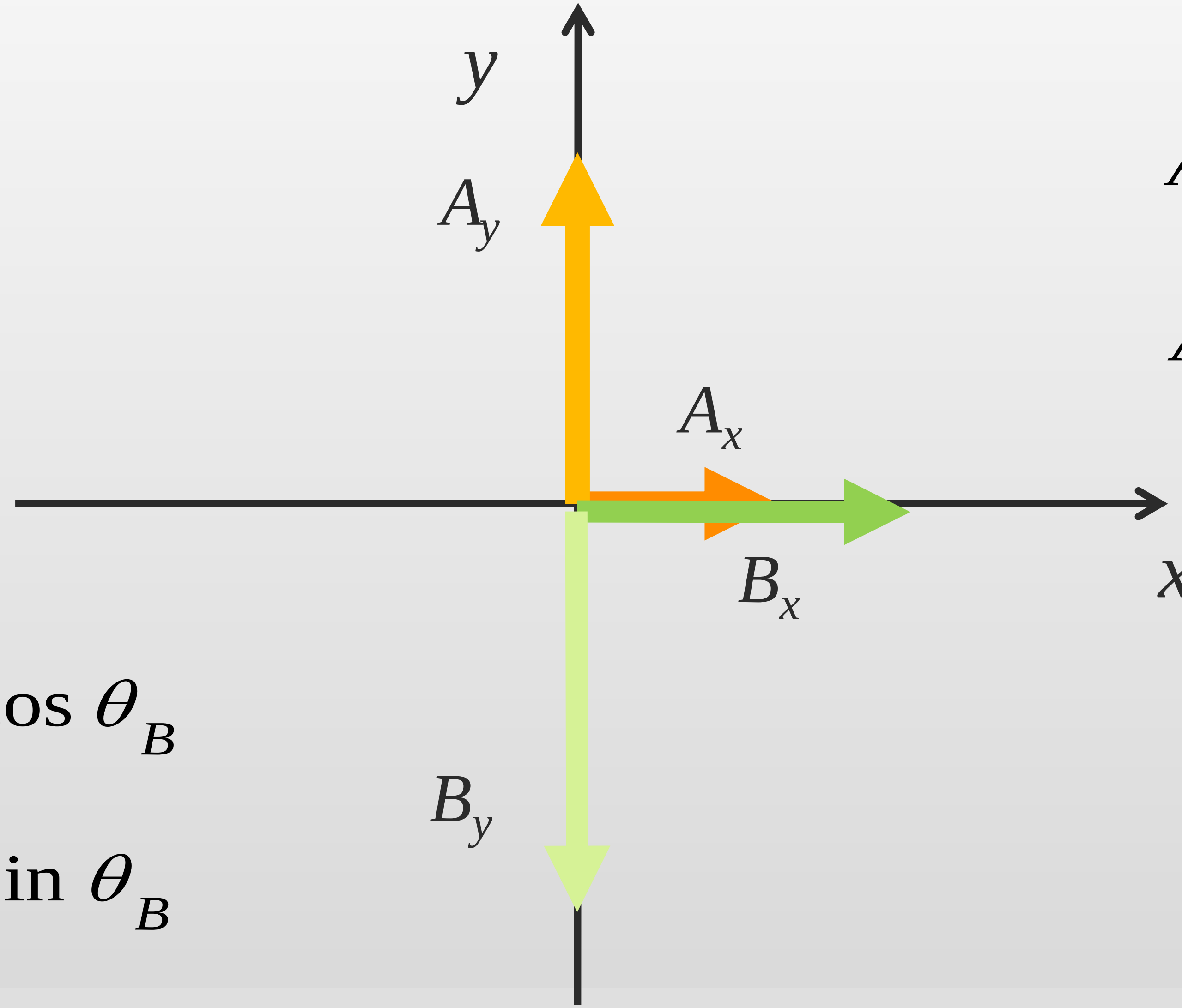
$$A_y = A\sin\theta_A$$

$$B_x = B\cos\theta_B$$

$$B_y = B\sin\theta_B$$

# 2-a) Component Method of adding Vectors

**Tail-to-Tail Vectors**




$$A_x = A\cos\theta_A$$

$$A_y = A\sin\theta_A$$

$$B_x = B\cos\theta_B$$

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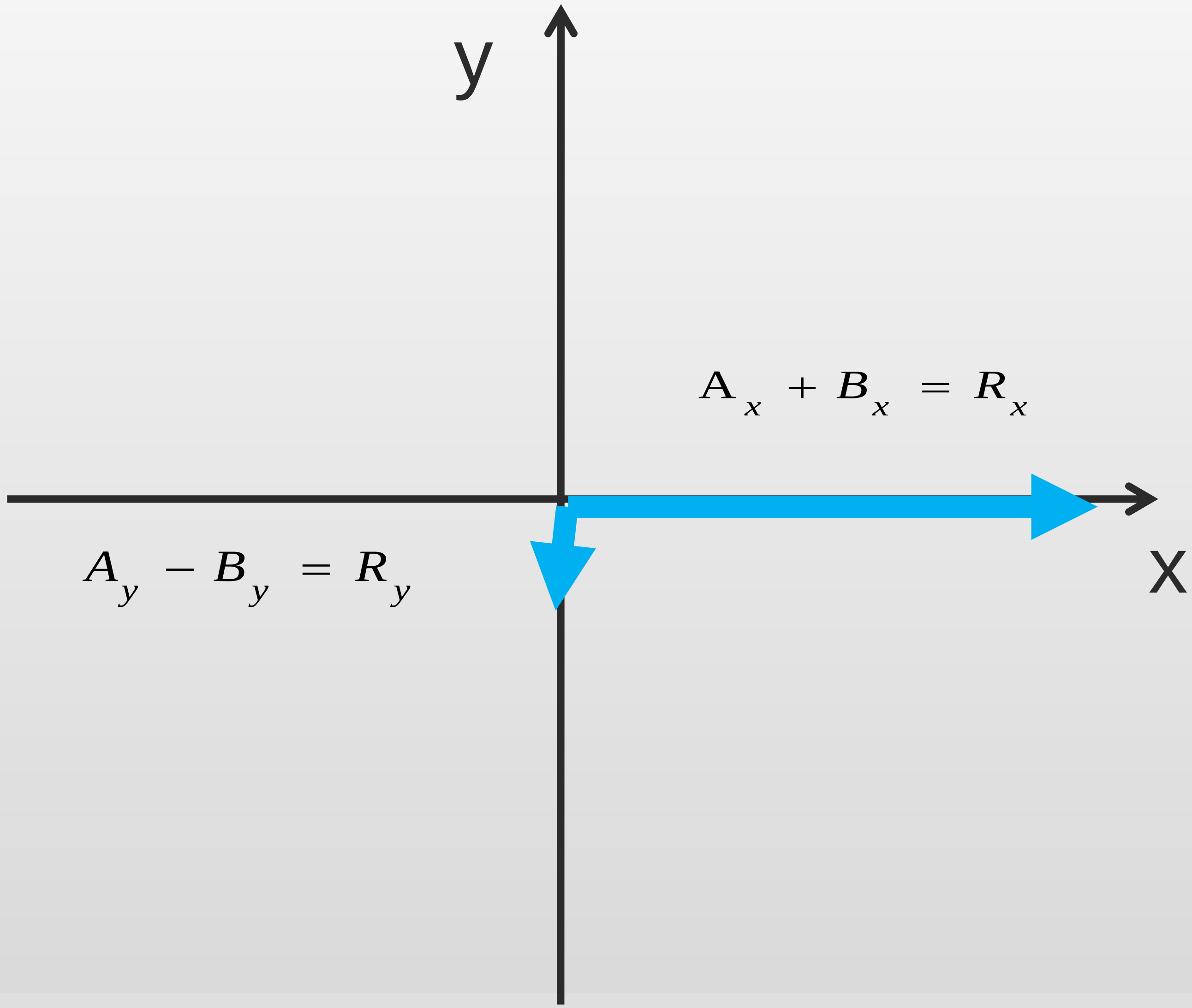
# 2-a) Component Method of adding Vectors

**Tail-to-Tail Vectors**




$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$\vec{B} = B_x\hat{i} - B_y\hat{j}$$

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y - B_y)\hat{j}$$

$$= R_x\hat{i} - R_y\hat{j}$$

# 2-a) Component Method of adding Vectors

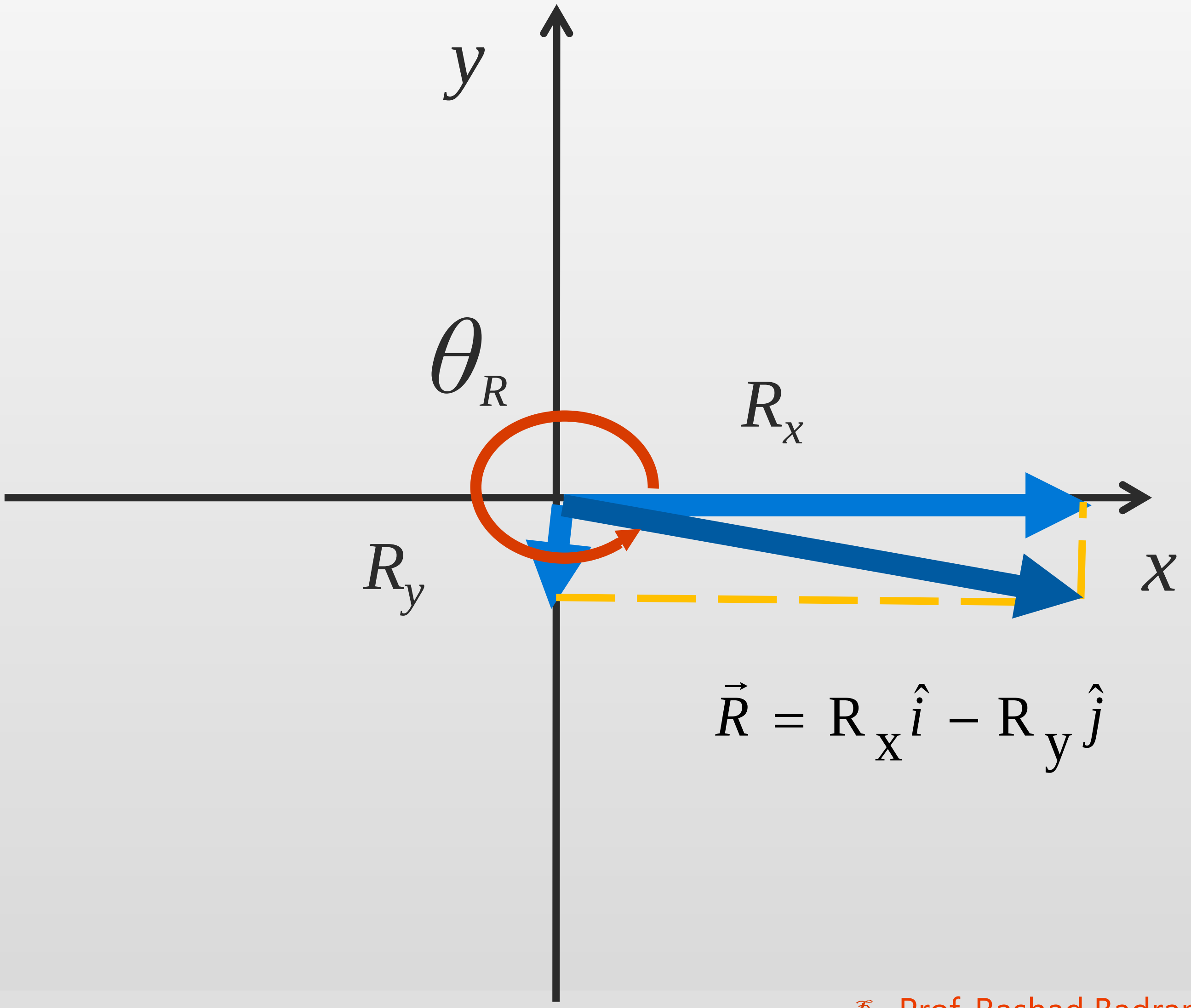
**Tail-to-Tail Vectors**

$$R_x = A_x + B_x = +ve$$

$$R_y = A_y - B_y = -ve$$

$$\left|\vec{R}\right| = \sqrt{R_x^2 + R_y^2},$$

$$\theta_R = \arctan \frac{R_y}{R_x}$$




$$\vec{R} = R_x \hat{i} - R_y \hat{j}$$

# 2-a) Component Method of adding Vectors

**Head-to-Tail Vectors**

**Example**

A cross-country skier skies *1.00 km* north and then *2.00 km* east on a horizontal snow field.

(a) How far and in what direction is she from the starting point?

(b) What are the magnitude and direction of her resultant displacement?

# 2-a) Component Method of adding Vectors

**Head-to-Tail Vectors**

**Example**

$\vec{A}$ $\vec{B}$

A cross-country skier skies $1.00\ km$ north and then $2.00\ km$ east on a horizontal snow field.

(a) How far and in what direction is she from the starting point?

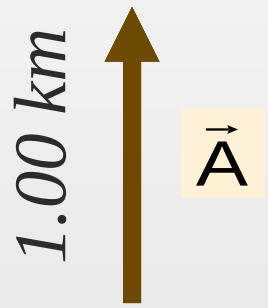
(b) What are the magnitude and direction of her resultant displacement?

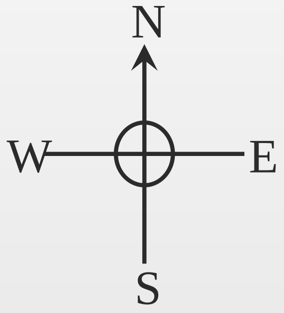
Resultant displacement $\vec{R}$ =?

Two vectors are involved in the problem!!!

**Solution** (a)

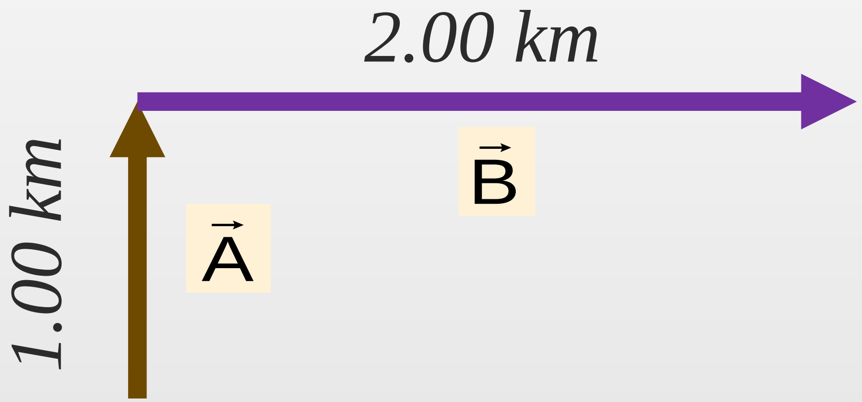
**Head-to-Tail Vectors**

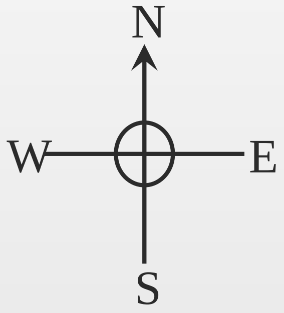


Head-to-Tail Vectors

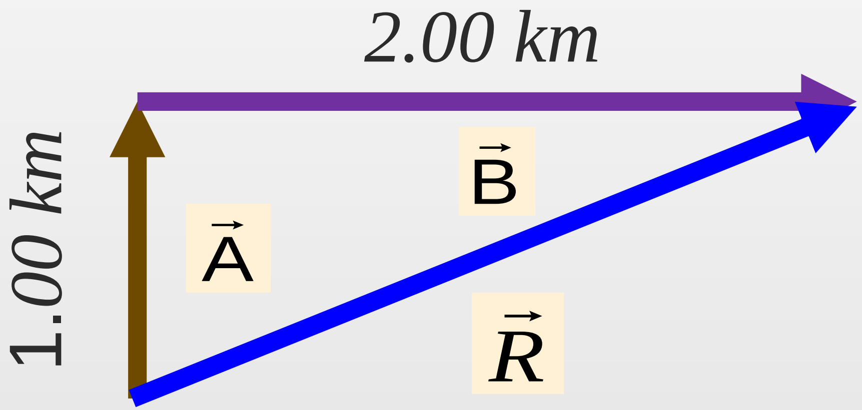
**Solution** (a)

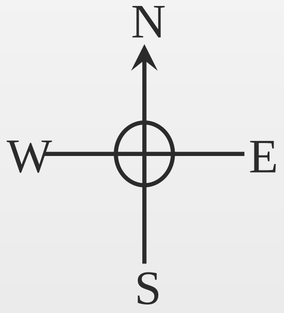


**Solution** (a)

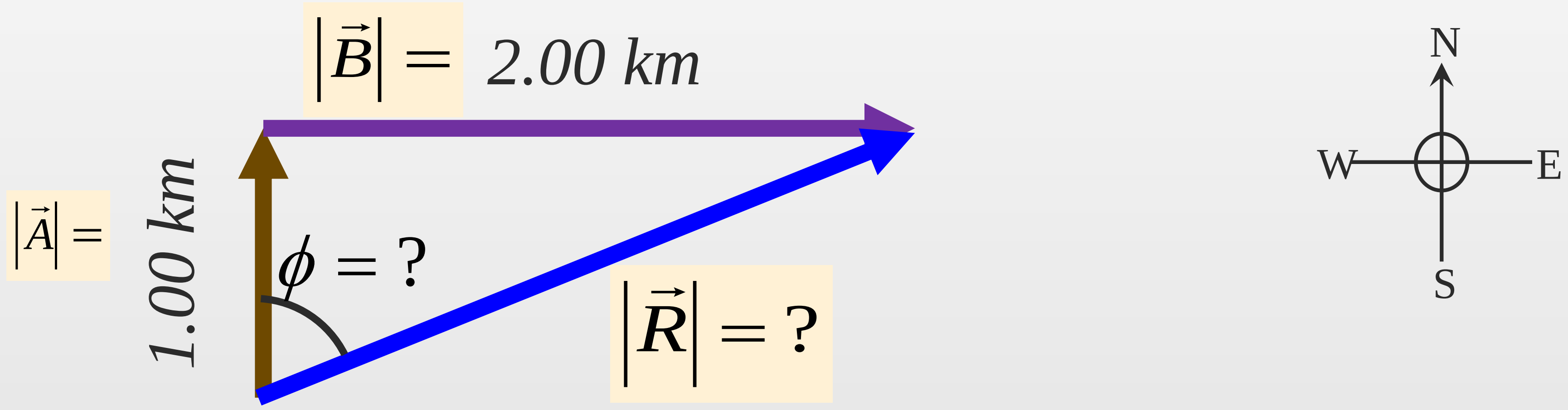
# Head-to-Tail Vectors



Solution (a)

## Head-to-Tail Vectors




The magnitude can be found using Pythagorean theory as follows

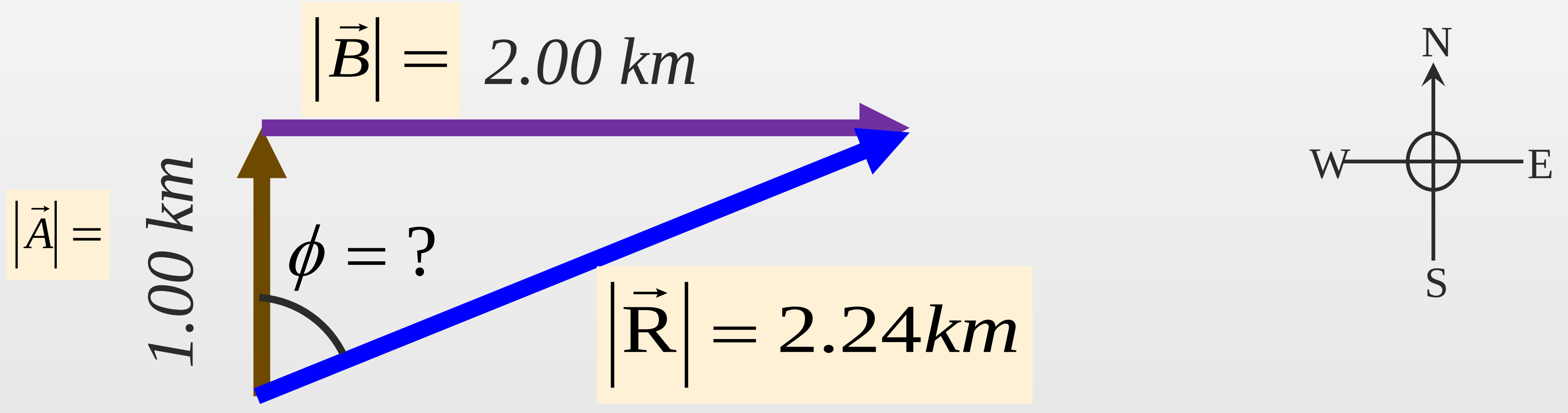
$$|\vec{R}| = \sqrt{(1.00\ \text{km})^2 + (2.00\ \text{km})^2}$$

$$= 2.24\ \text{km}$$

By Prof. Rashad Badran

# Solution (a)

## Head-to-Tail Vectors




The direction can be found using

$$\tan\phi = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{2.00\text{ km}}{1.00\text{ km}}$$

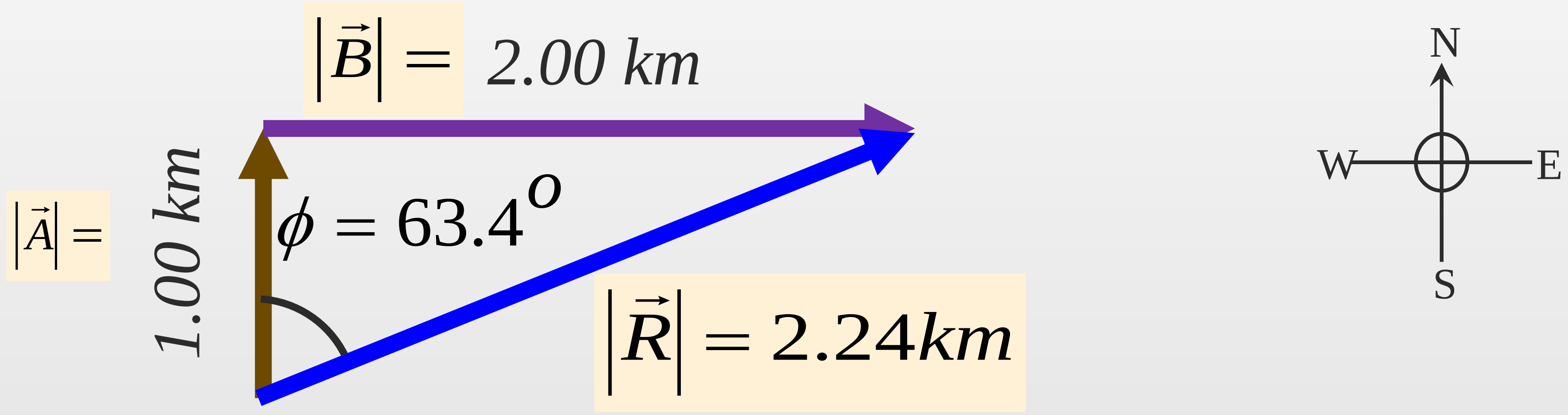
$$\phi = 63.4^{\circ}$$

She is *2.24 km* from the starting point and is directed *63.4º* east of north

## Head-to-Tail Vectors

## Solution

(b)




The magnitude of her resultant displacement is $2.24\ km$.
The direction of her resultant with respect to the positive $x$-axis is $\theta = 26.6^o$ (or $26.6^o$ north of east).

Conclusion: This is a simple example for the addition of head-to-tail vectors. The two vectors are the sides of a right angle triangle where each vector has a single component along $x$- or $y$- axis. This leads to the use of Pythagorean theorem to find the magnitude.

By Prof. Rashad Badran

# Conceptual Questions

## Conceptual Question

If the component of vector $\vec{A}$ along the direction of vector $\vec{B}$ is zero, what can you conclude about the two vectors ?

**Answer**:

(a) They are parallel vectors

(c) vector $\vec{B}$ exists but vector $\vec{A}$ doest not exist

(b) They are antiparallel vectors

(d) They are perpendicular to each other

(e) None of those answers

# 2-a) Component Method of adding Vectors

**Head-to-Tail Vectors**

**Example**

The three finalists in a contest are brought to the center of a large flat field. Each is given a meter stick, a compass, a calculator, a shovel, and (in a different order for each contestant) the following three displacements are:

$72.4\ \text{m}\ 32^\circ$ east of north called vector $\vec{A}$;

$57.3\ \text{m}\ 36^\circ$ south of west called vector $\vec{B}$;

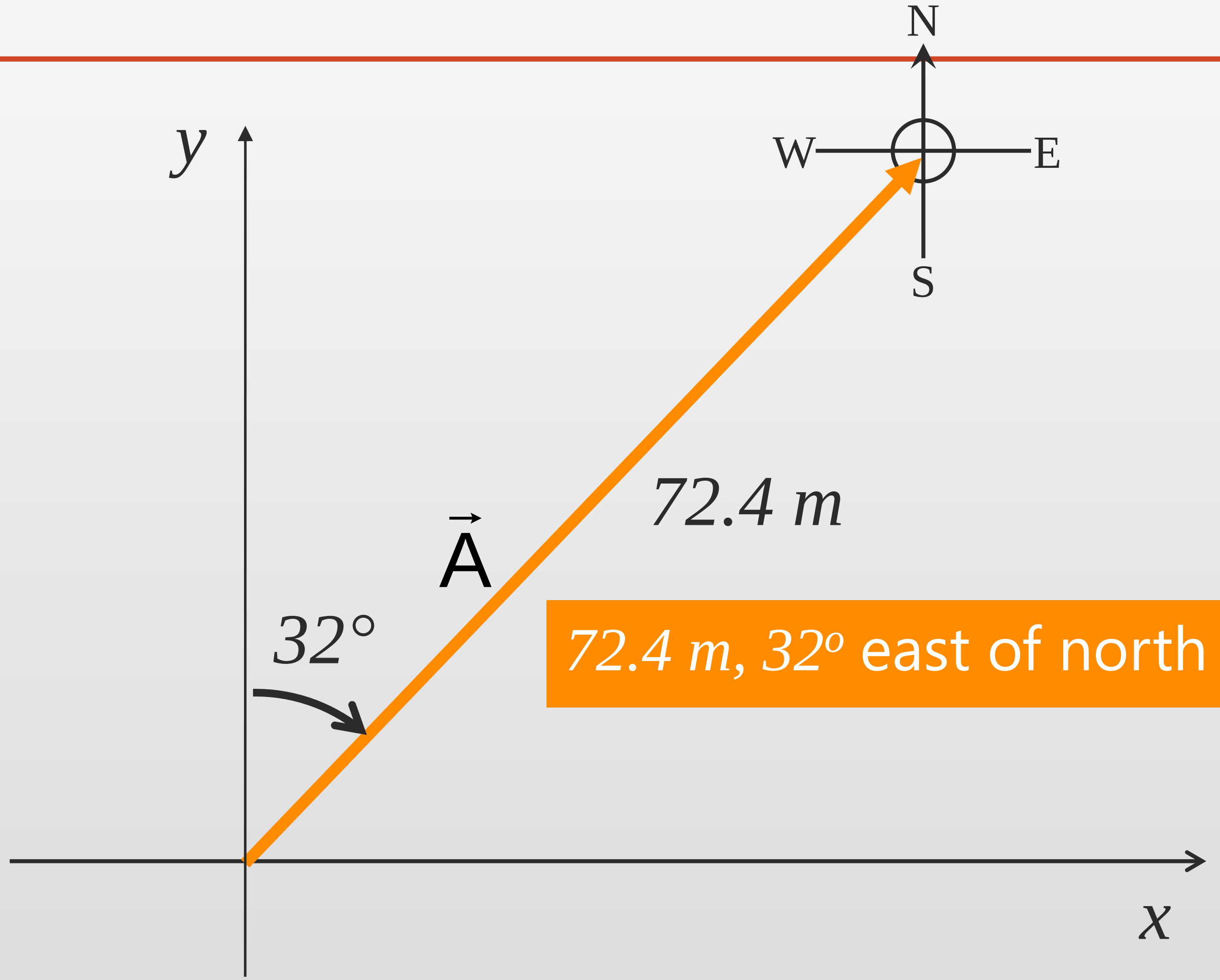
$17.8\ \text{m}$ straight south called vector $\vec{C}$;

Find the magnitude and direction of their resultant $\vec{R}$.

More than two vectors are involved in the problem!!!

# 2-a) Component Method of adding Vectors

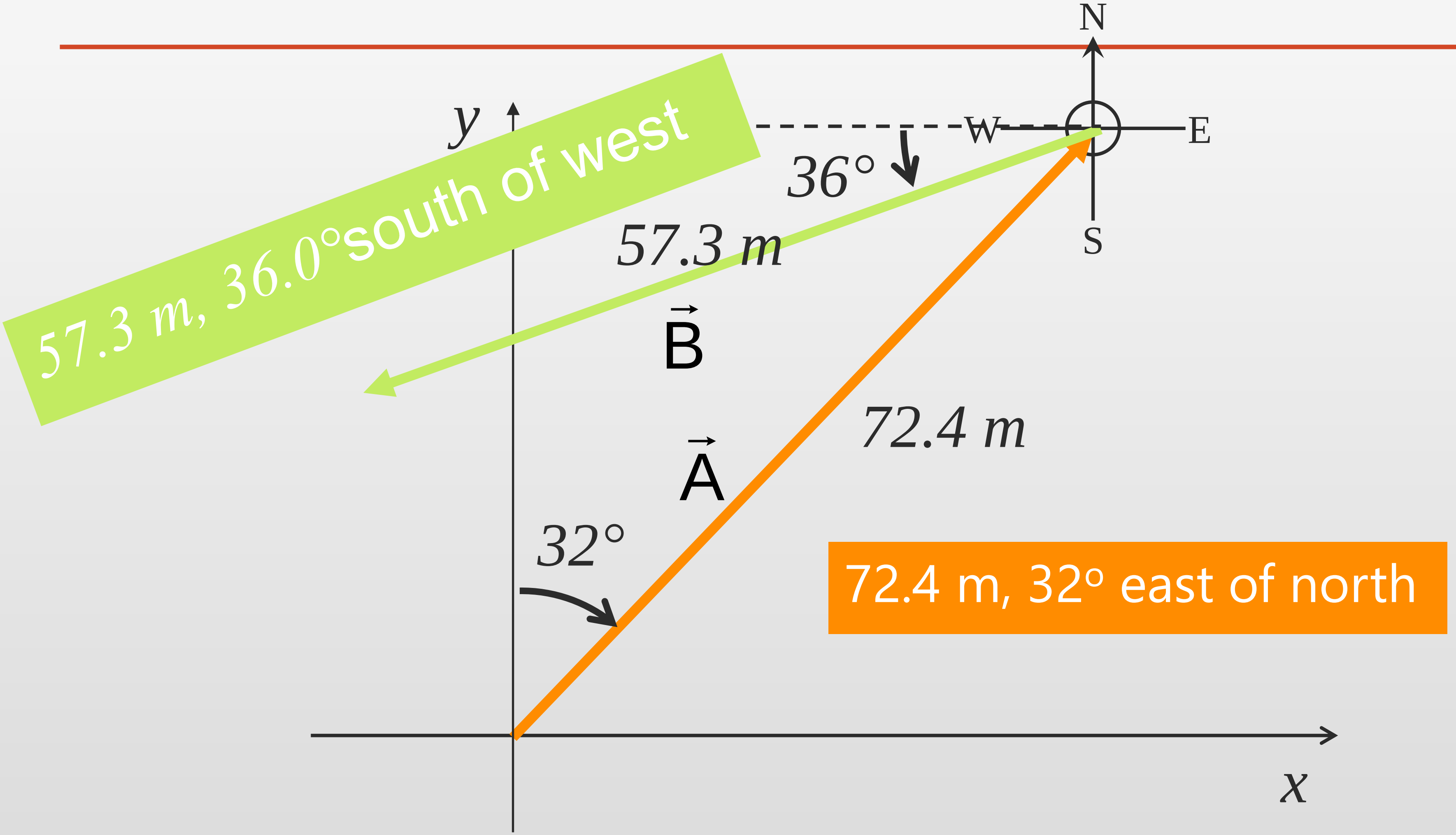
**Head-to-Tail Vectors**




72.4 m, 32° east of north

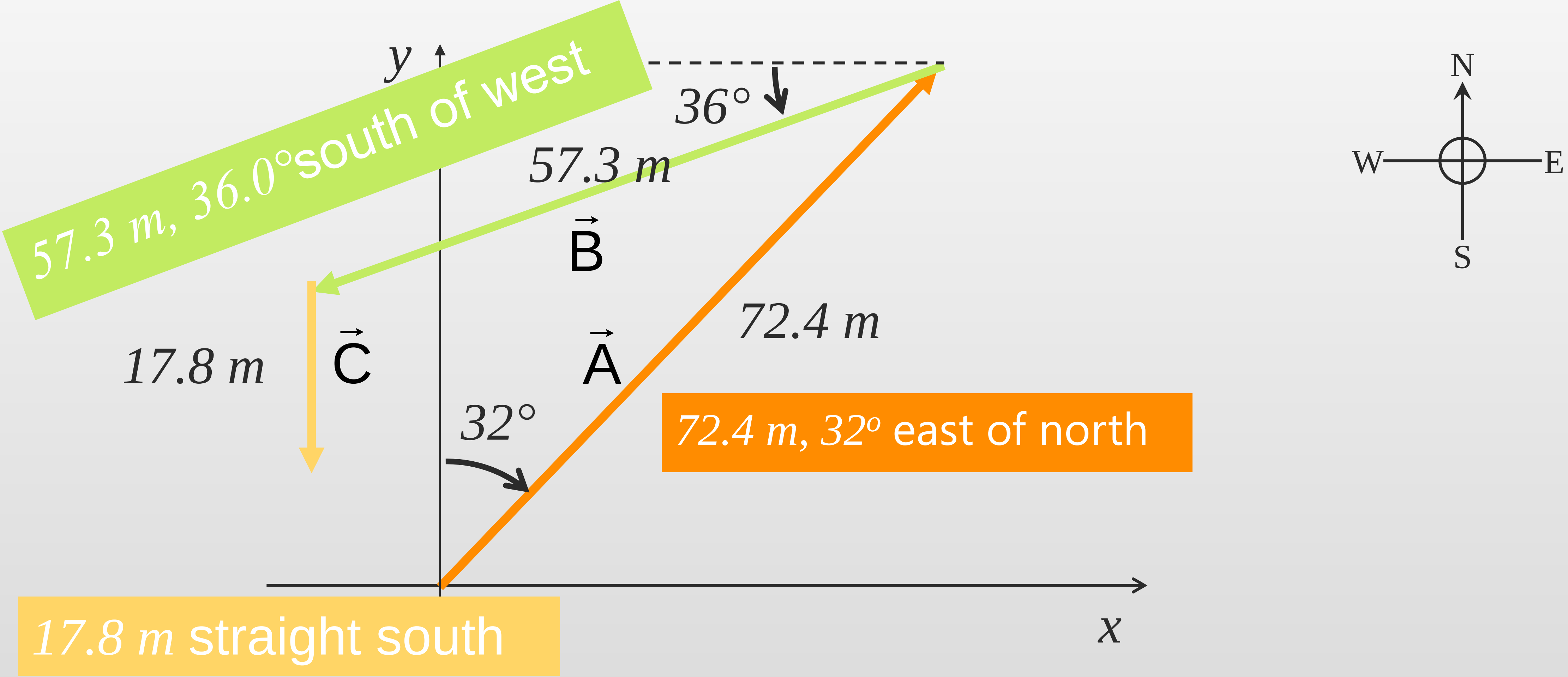
# 2-a) Component Method of adding Vectors

**Head-to-Tail Vectors**



# 2-a) Component Method of adding Vectors

**Head-to-Tail Vectors**

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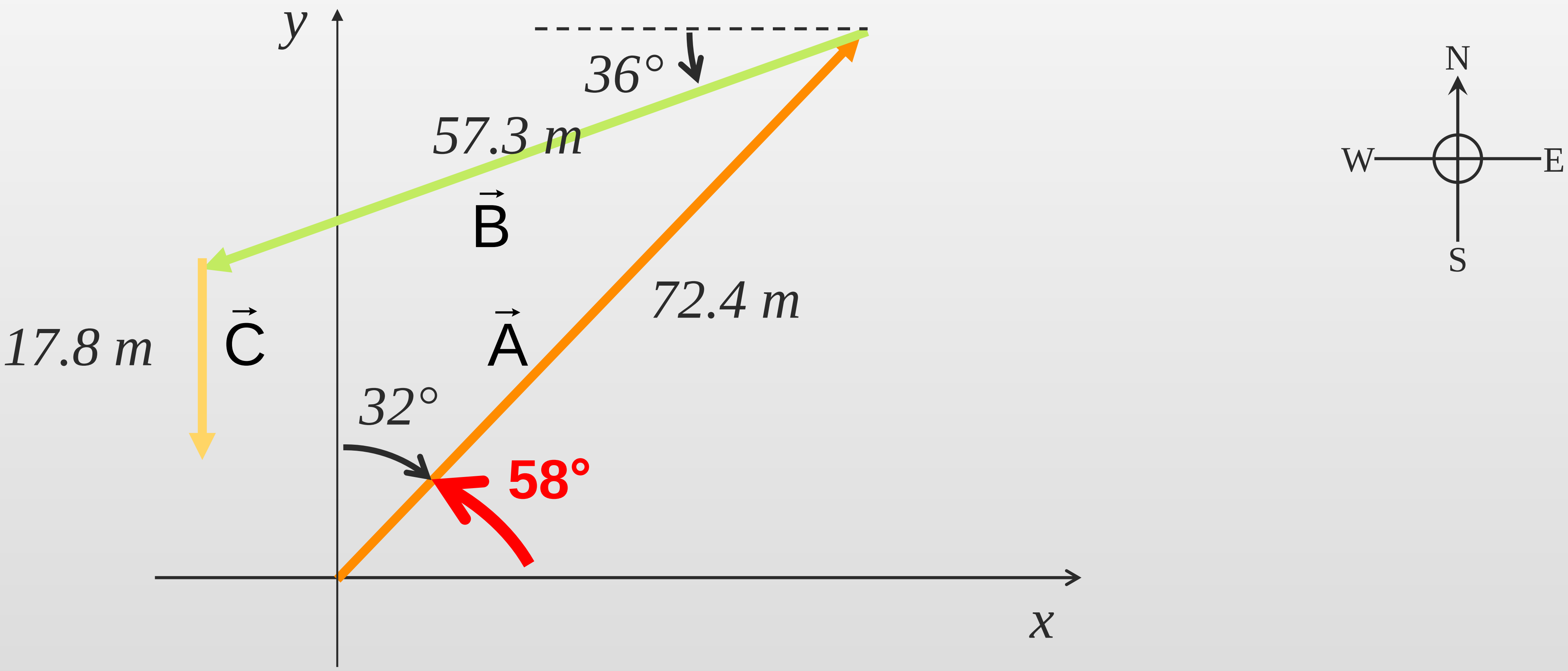
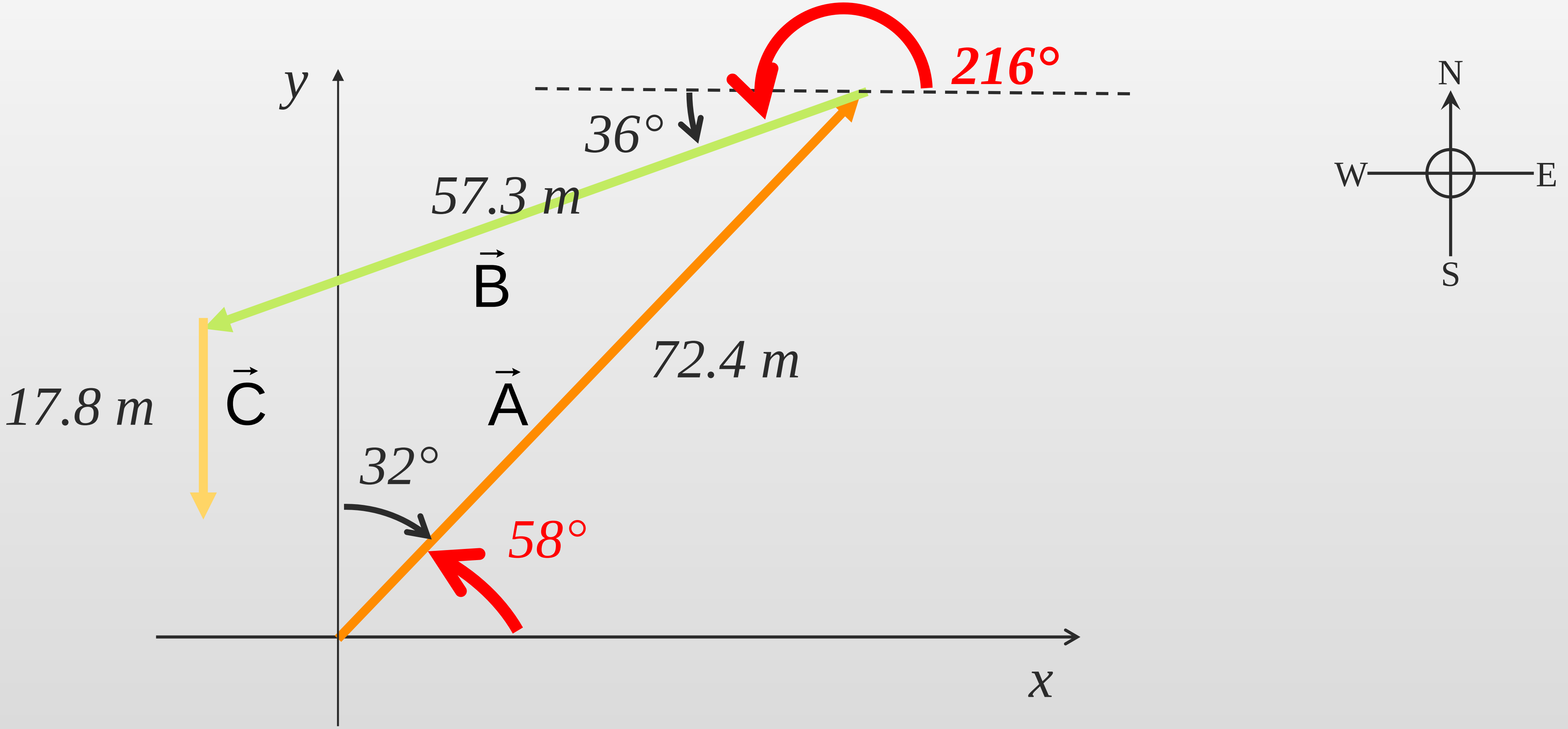
# 2-a) Component Method of adding Vectors

**Head-to-Tail Vectors**



# 2-a) Component Method of adding Vectors

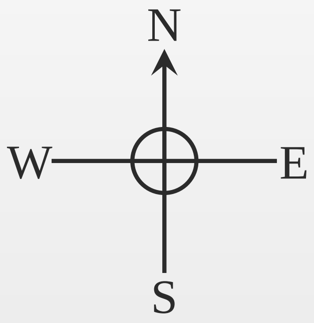
**Head-to-Tail Vectors**



# 2-a) Component Method of adding Vectors

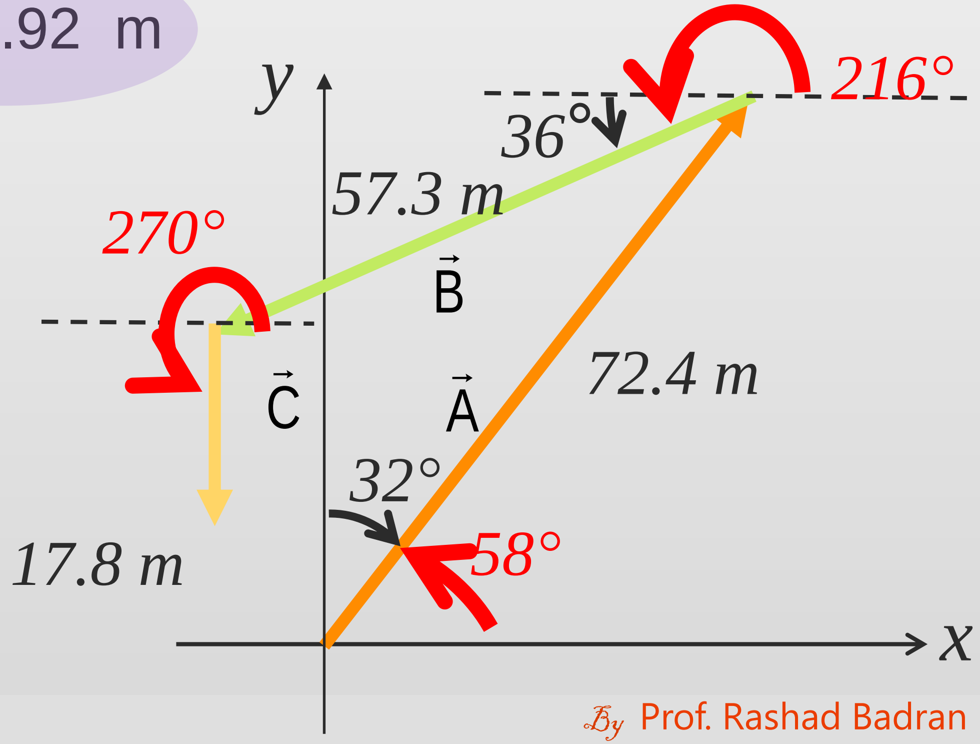
**Head-to-Tail Vectors**

| Distance | Angle | x-component | y-component |
|---|---|---|---|
| A = 72.4 m | 58.0° | 38.37 m | 61.40 m |
| B = 57.3 m | 216.0° | -46.36 m | -33.68 m |
| C = 17.8 m | 270.0° | 0.00 m | -17.80 m |
| | | $R_x$ = -7.99 m | $R_y$ = 9.92 m |




$$R = \sqrt{(-7.99\,\text{m})^2 + (9.92\,\text{m})^2} = 12.7\,\text{m}$$

$$\theta_R = \arctan\frac{9.92\,\text{m}}{-7.99\,\text{m}} = 129^\circ$$

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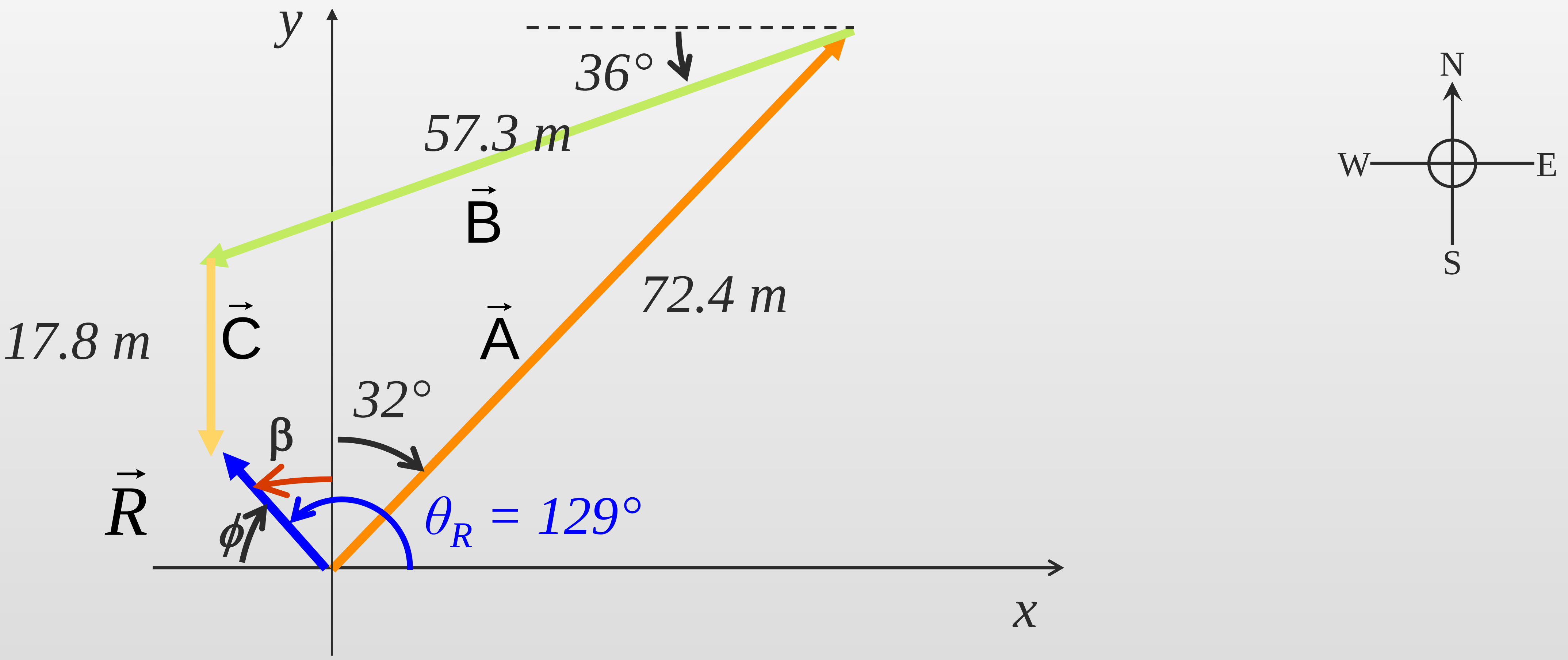
# 2-a) Component Method of adding Vectors

**Head-to-Tail Vectors**



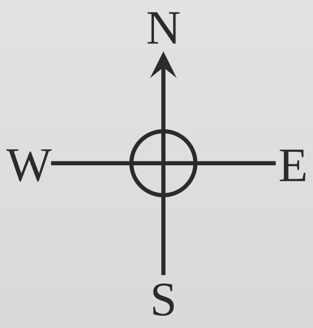

$\phi$=180-129=$51^o$ North of west

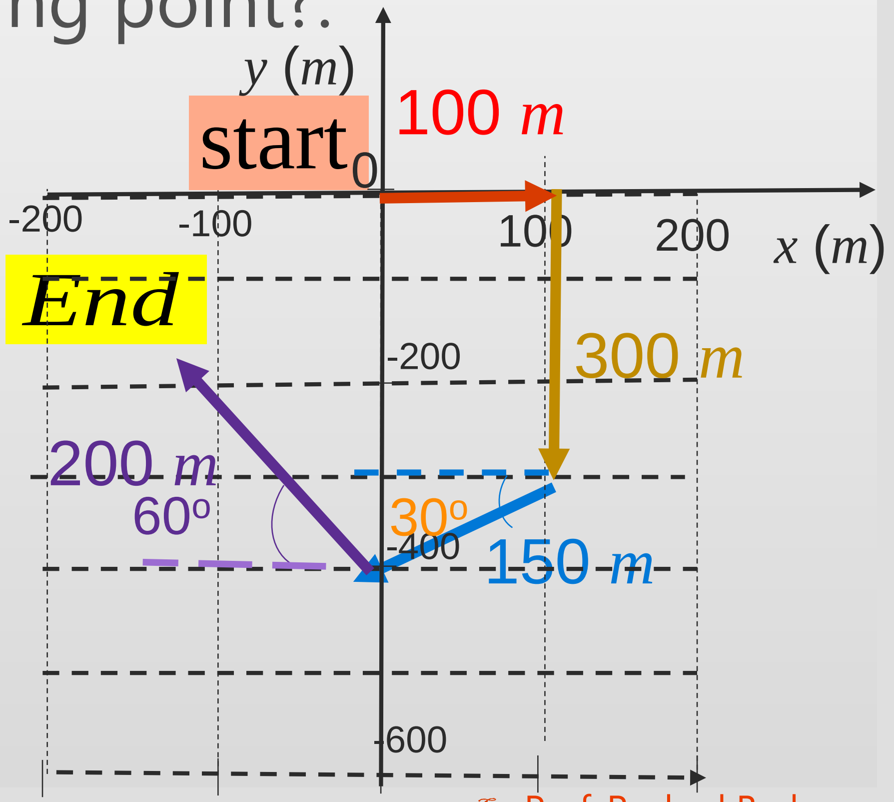
Or $\beta=39^o$ West of north

# 2-a) Component Method of adding Vectors

**Problem**

A person going for a walk follows the shown path. The total trip contains of four straight-line paths. At the end of the walk, what is the person's resultant displacement measured from the starting point?.



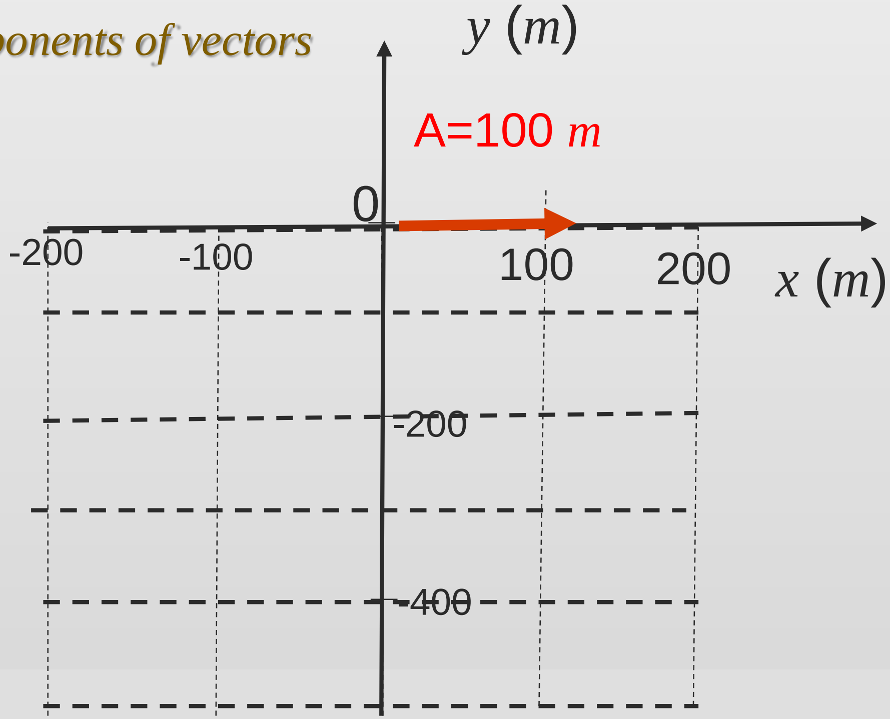



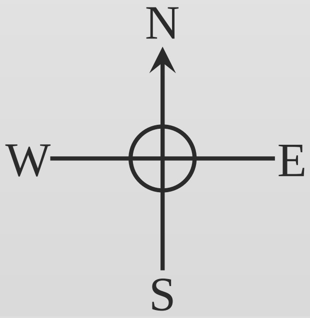
# 2-a) Component Method of adding Vectors

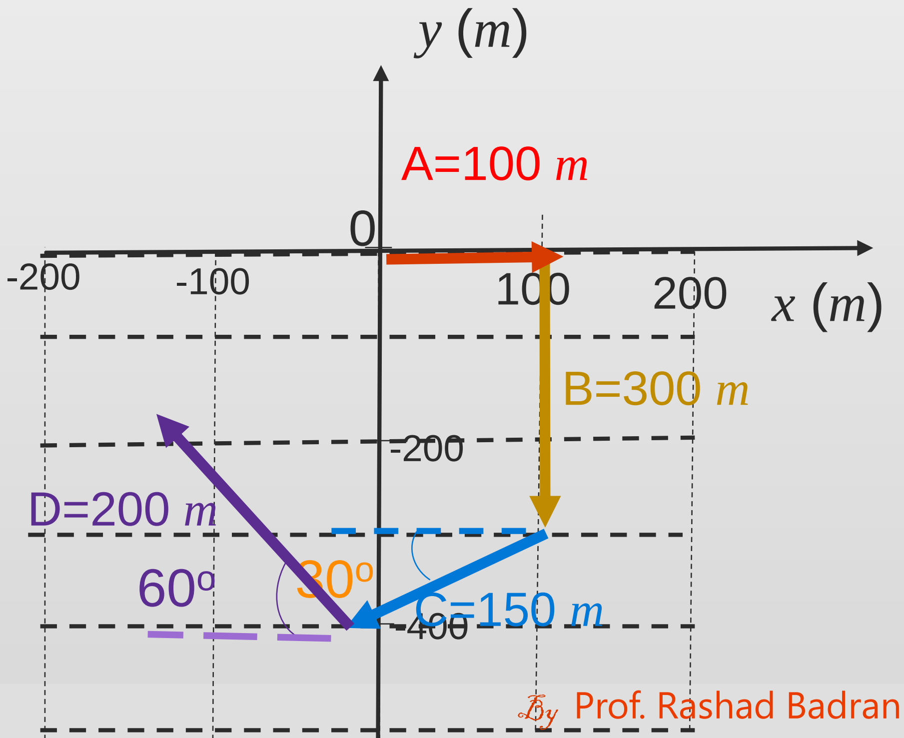
**Solution**

| | Distance | Angle | x-component | y-component |
|---|---|---|---|---|
| A = 100 m | | 0.0° | 100.0 m | 0.0 m |

*Components of vectors*



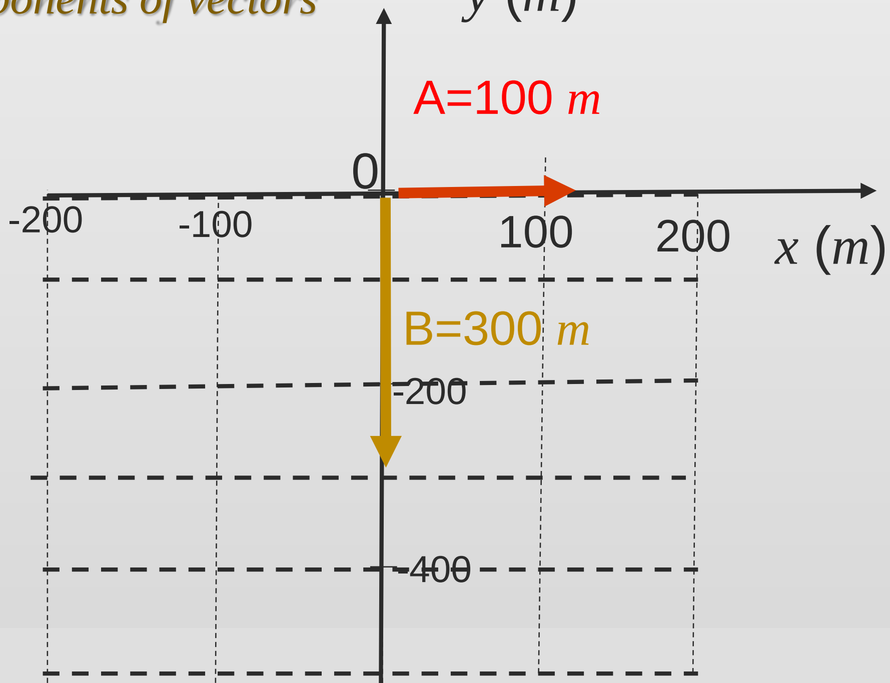



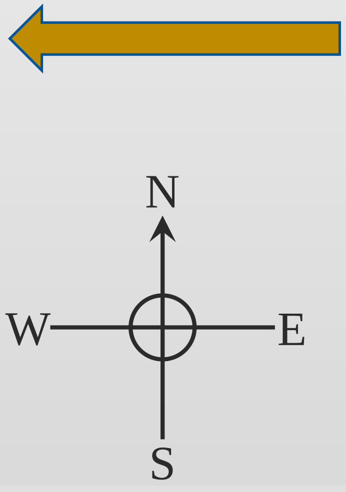
# 2-a) Component Method of adding Vectors

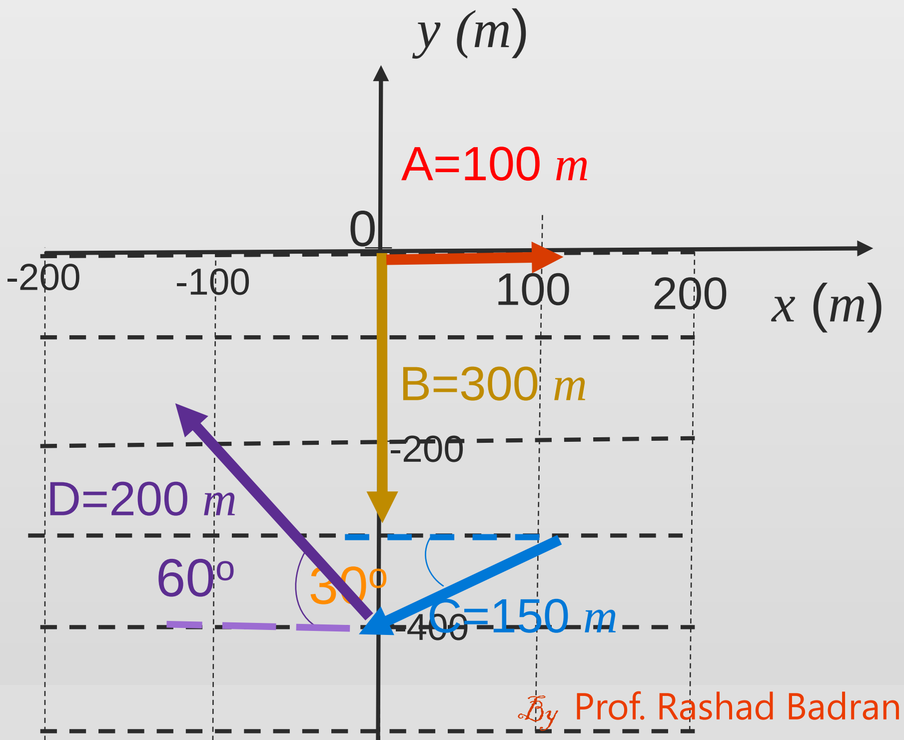
**Solution**

| | Distance | Angle | x-component | y-component |
|---|---|---|---|---|
| A = 100 m | | 0.0° | 100.0 m | 0.0 m |
| B = 300 m | | 270.0° | 0 m | -300.0 m |

*Components of vectors*

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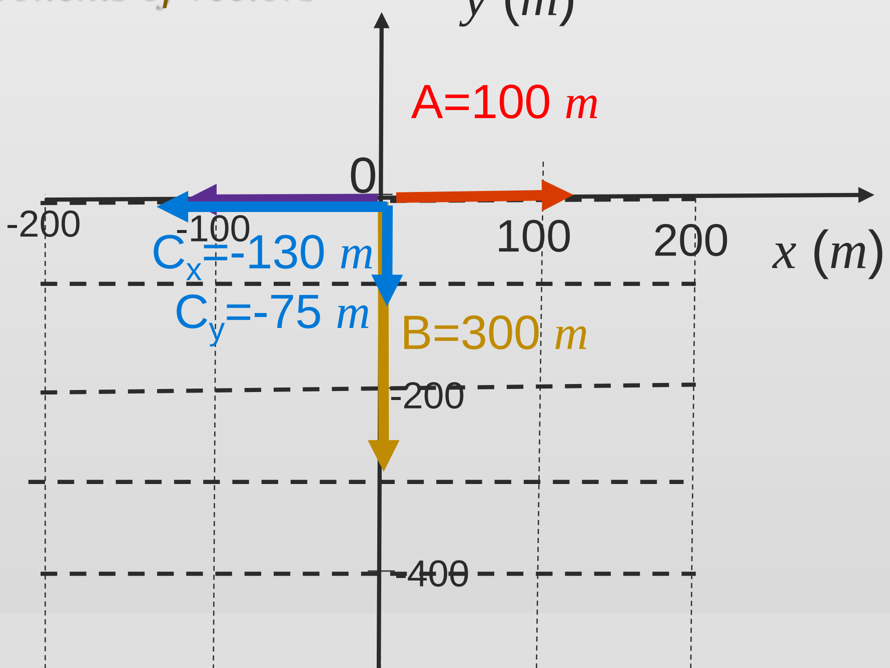
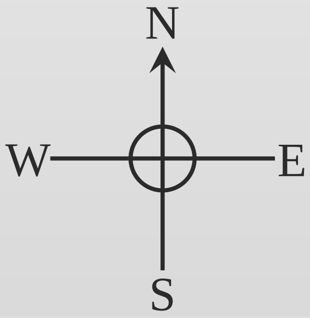
# 2-a) Component Method of adding Vectors

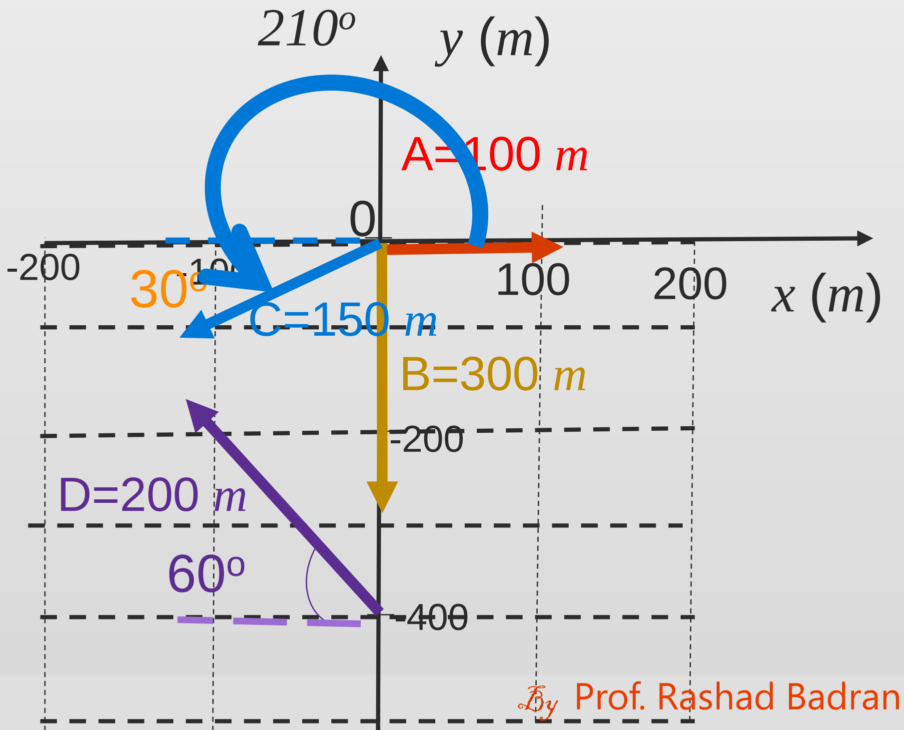
**Solution**

| | Distance | Angle | x-component | y-component |
|---|---|---|---|---|
| | A = 100 m | 0.0° | 100.0 m | 0.0 m |
| | B = 300 m | 270.0° | 0 m | -300.0 m |
| | C = 150 m | 210.0° | -130.0 m | -75.00 m |

*Components of vectors*

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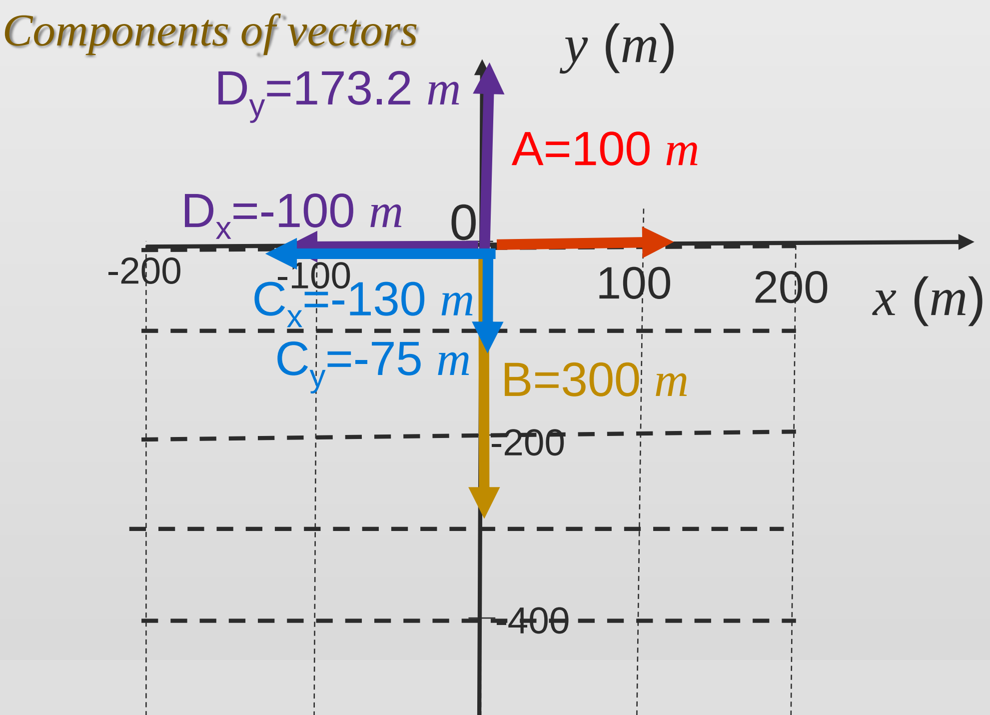
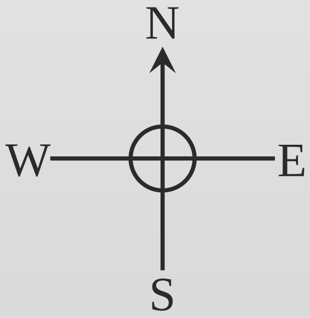
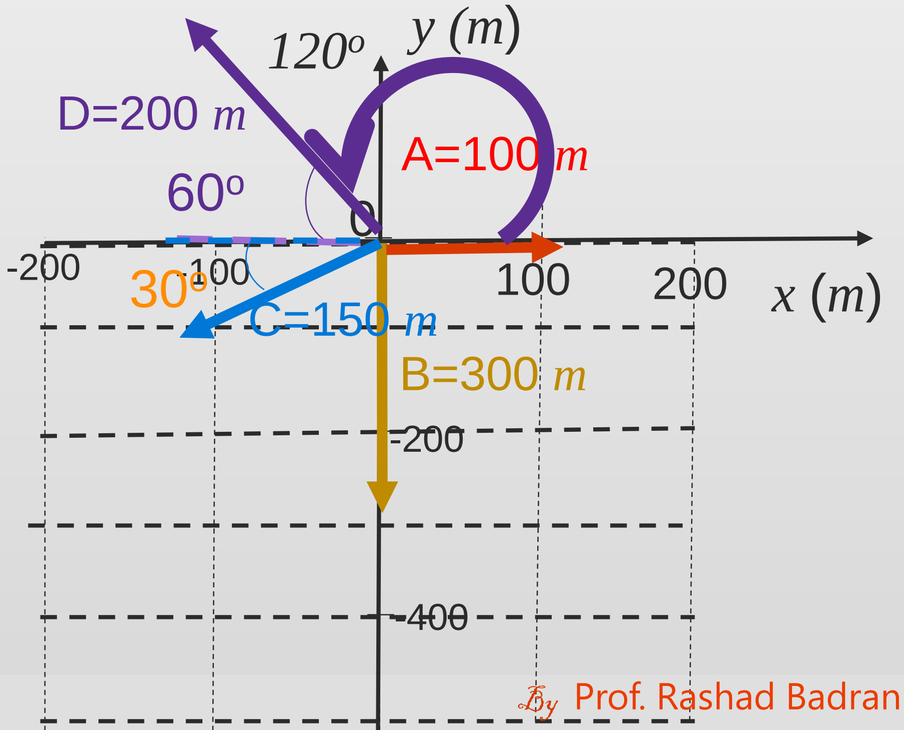
# 2-a) Component Method of adding Vectors

**Solution**

| Distance | Angle | x-component | y-component |
|---|---|---|---|
| A = 100 m | 0.0° | 100.0 m | 0.0 m |
| B = 300 m | 270.0° | 0 m | -300.0 m |
| C = 150 m | 210.0° | -130.0 m | -75.00 m |
| D = 200 m | 120.0° | -100.0 m | 173.20 m |

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# Solution

| Distance | Angle | x-component | y-component |
|---|---|---|---|
| A = 100 m | 0.0° | 100.0 m | 0.0 m |
| B = 300 m | 270.0° | 0 m | -300.0 m |
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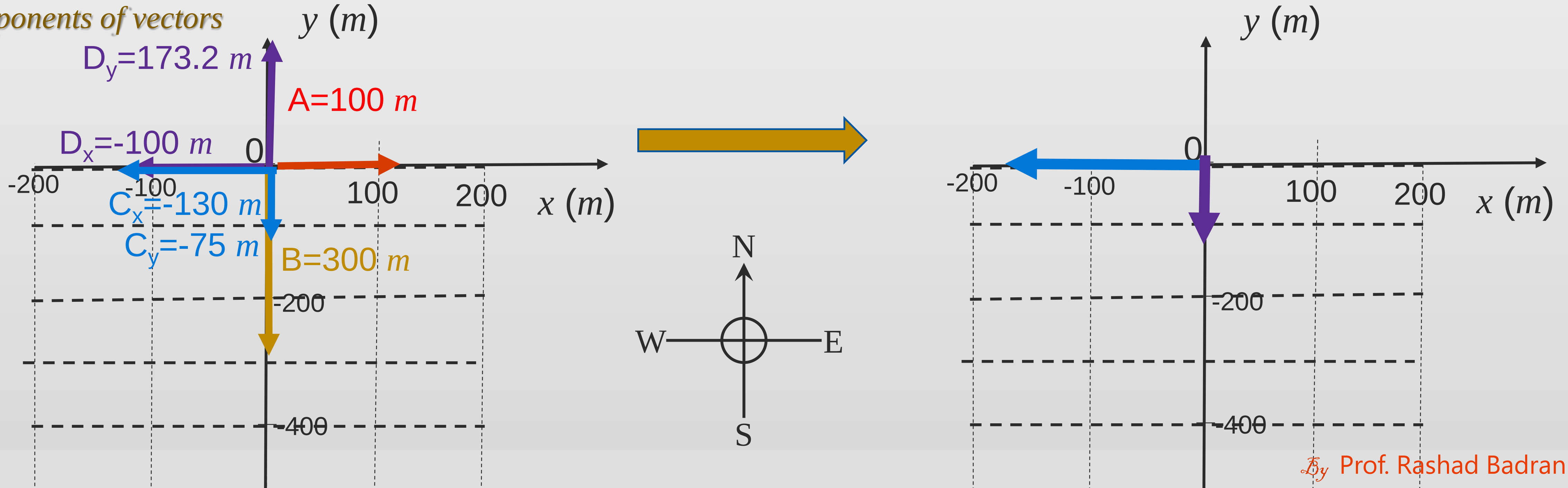
*Components of vectors*

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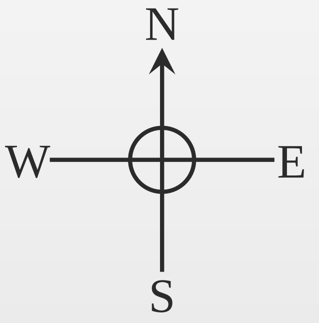
# 2-a) Component Method of adding Vectors

**Solution**

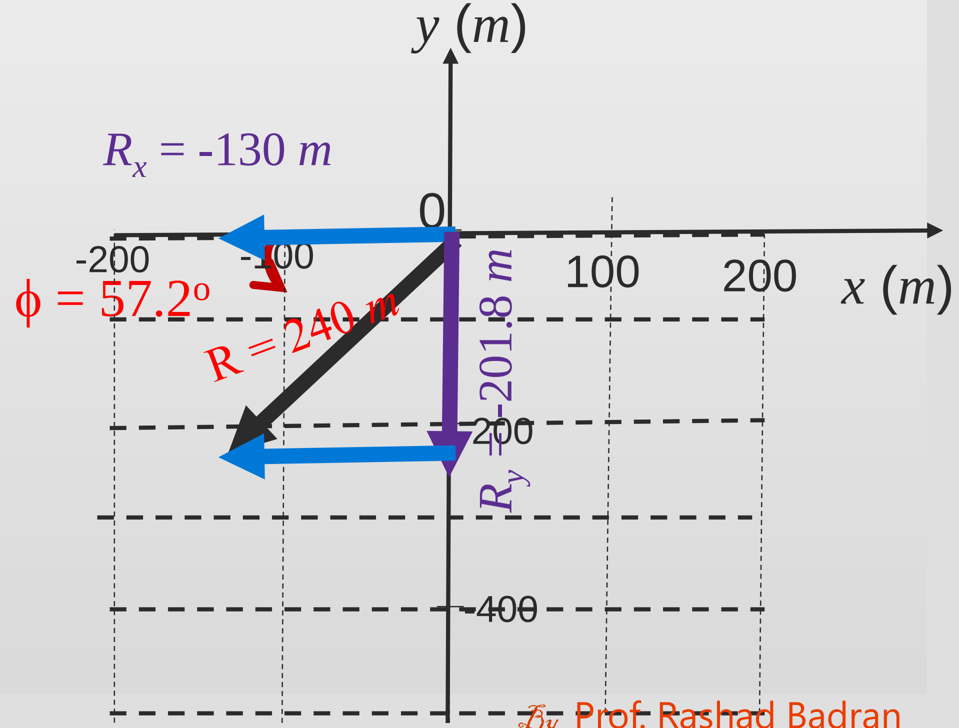
| Distance | Angle | x-component | y-component |
|---|---|---|---|
| A = 100 m | 0.0° | 100.0 m | 0.0 m |
| B = 300 m | 270.0° | 0 m | -300.0 m |
| C = 150 m | 210.0° | -130.0 m | -75.00 m |
| D = 200 m | 120.0° | -100.0 m | 173.20 m |
| | | $R_x$ = -130 m | $R_y$ = -201.8 m |




$$\vec{R} = -130\hat{i} - 201.8\,\hat{j}$$

$$\left|\vec{R}\right| = \sqrt{(-130)^2 + (201.8)^2} = \boxed{240m}$$

$$\phi = \tan^{-1}\left(\frac{-201.8}{-130}\right) = \boxed{57.2^\circ\ South\ \ of\ \ west}$$

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# 2-a) Component Method of adding Vectors

**Solution**

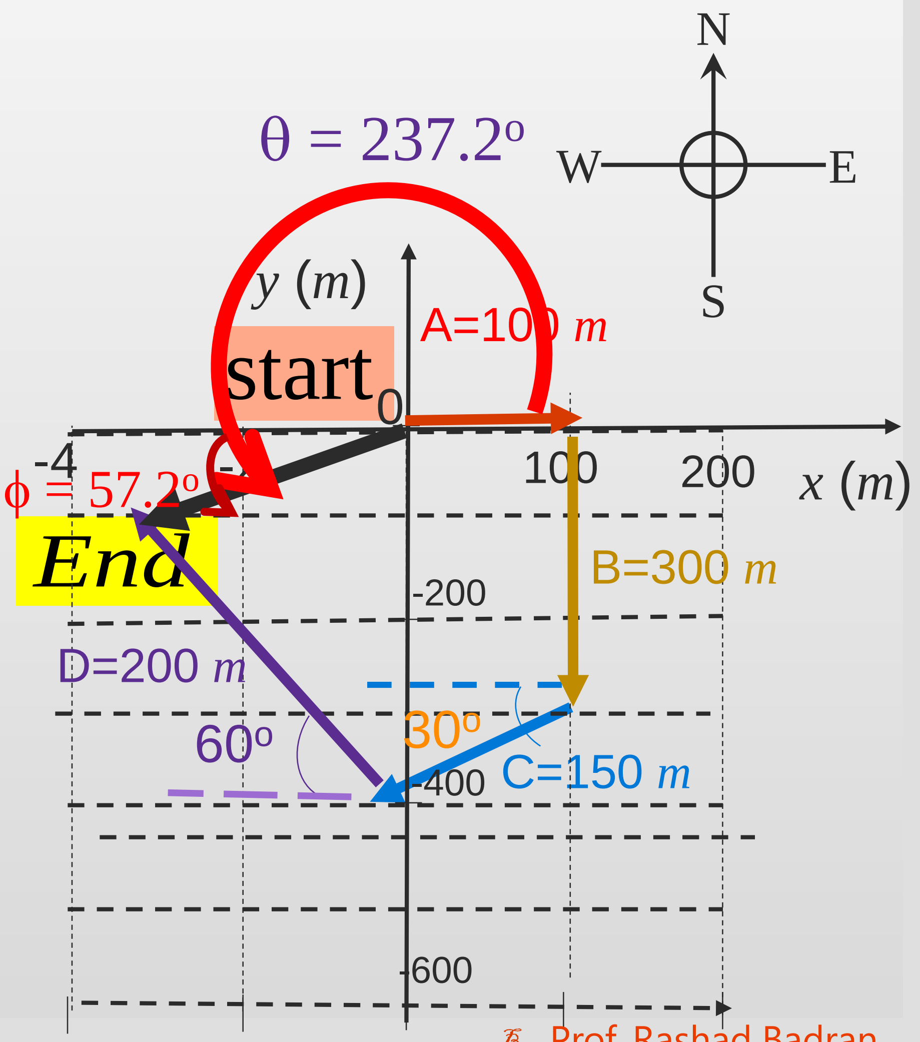
| Distance | Angle | x-component | y-component |
|---|---|---|---|
| A = 100 m | 0.0° | 100.0 m | 0.0 m |
| B = 300 m | 270.0° | 0 m | -300.0 m |
| C = 150 m | 210.0° | -130.0 m | -75.00 m |
| D = 200 m | 120.0° | -100.0 m | 173.20 m |
| | | $R_x$ = -130 m | $R_y$ = -201.8 m |

$$\vec{R} = -130\hat{i} - 201.8\,\hat{j}$$

$$\left|\vec{R}\right| = \sqrt{(-130)^2 + (201.8)^2} = 240m$$

$$\phi = \tan^{-1}\left(\frac{-201.8}{-130}\right) = 57.2^\circ\ South\ \ of\ \ west$$

$$\theta = 180^\circ + 57.2 = 237.2^\circ\ Counterclockwise$$



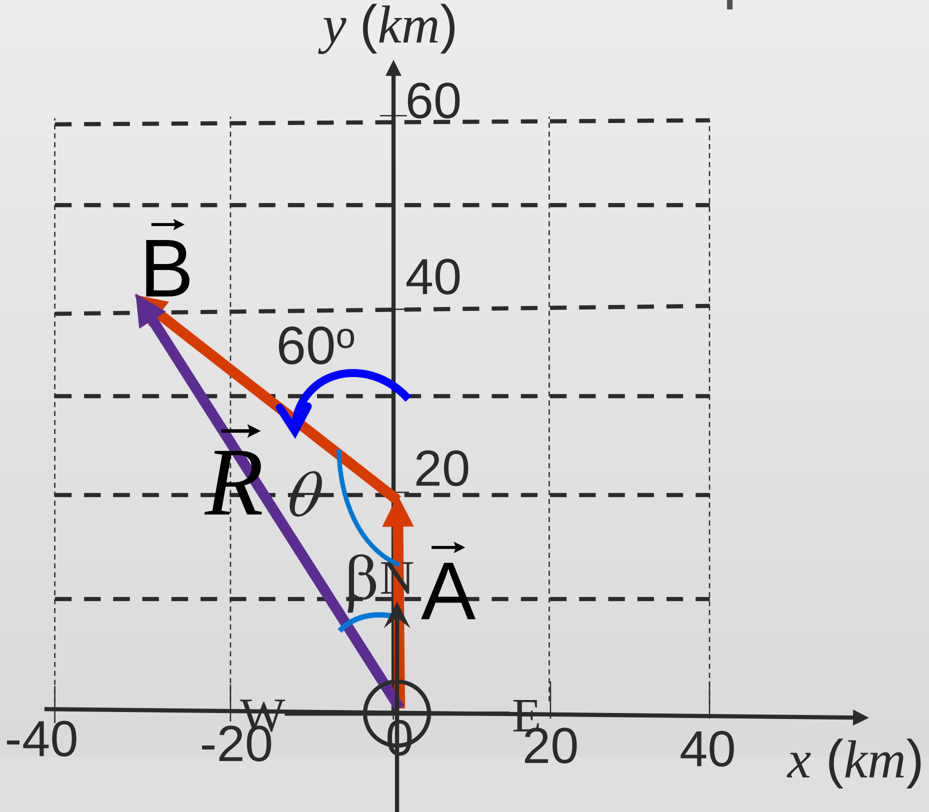
# 2-b) Applying the laws of sines and cosines

**Head-to-Tail Vectors**

**Example**

A car travels *20 km* due north and then *35 km* in a direction *60º* west of north. Find the magnitude and direction of the car's resultant displacement.

$$\left|\vec{A}\right| = 20km$$ due north




By Prof. Rashad Badran

# 2-b) Applying the laws of sines and cosines

**Head-to-Tail Vectors**

**Example**

A car travels *20 km* due north and then *35 km* in a direction *60º* west of north. Find the magnitude and direction of the car's resultant displacement.

$|\vec{A}| = 20km$ due north

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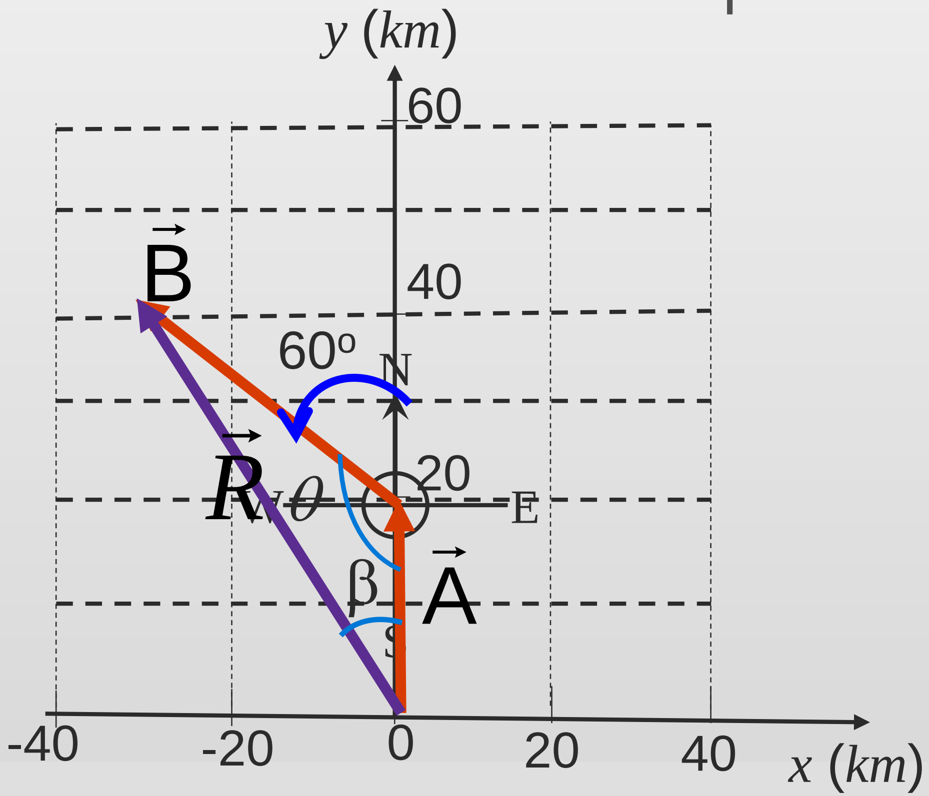



By Prof. Rashad Badran

# 2-b) Applying the laws of sines and cosines

**Head-to-Tail Vectors**

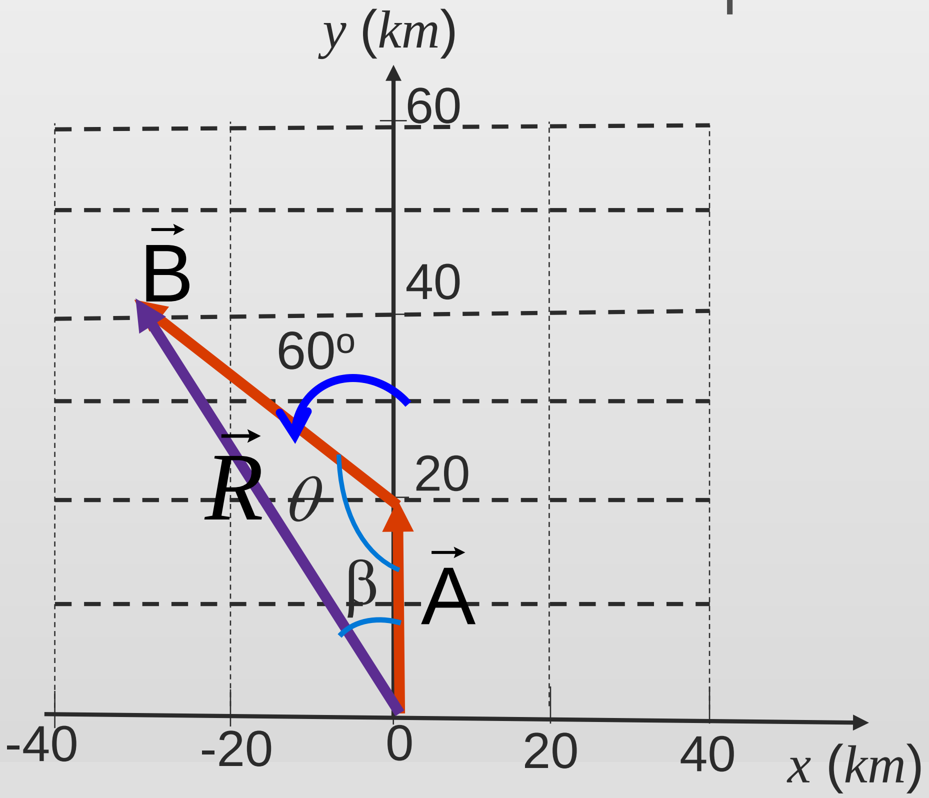
**Example**

A car travels *20 km* due north and then *35 km* in a direction *60º* west of north. Find the magnitude and direction of the car's resultant displacement.

$$\left|\vec{A}\right| = 20km \quad \text{due north}$$

$$\left|\vec{B}\right| = 35km \quad 60^o \text{ west of north}$$

$$\vec{R} = ?$$



# Head-to-Tail Vectors

## Solution

$$|\vec{A}| = 20km, \quad |\vec{B}| = 35km, \quad \phi = 60^o$$

Use the law of cosines: $R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$$\theta = 180^o - \phi = 180^o - 60^o = 120^o$$

Substitute the values of *A*, *B* and *θ* into the above relation:

$$R = \sqrt{(20km)^2 + (35km)^2 - 2(20km)(35km)\cos 120}$$

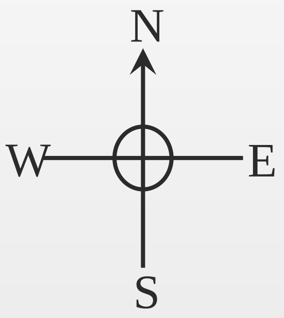
$$\Rightarrow R = 48.2km$$

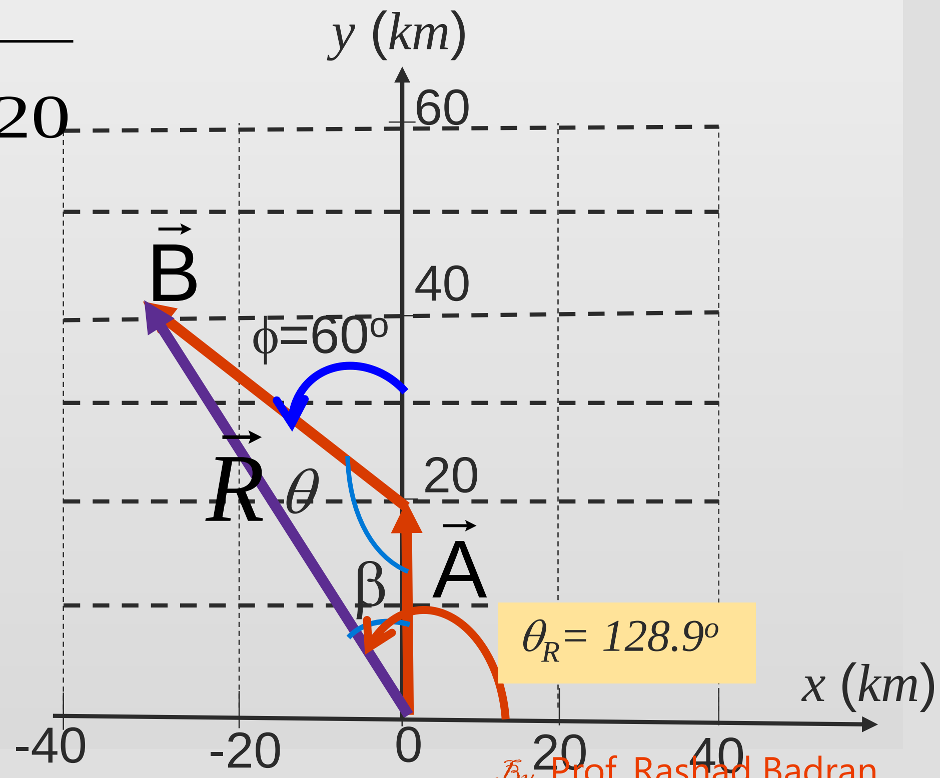
Use the law of sines: $\dfrac{\sin\beta}{B} = \dfrac{\sin\theta}{R}$

$$\sin\beta = \frac{B}{R}\sin\theta = \frac{35}{48.2}\sin 120^o = 0.629$$

$$\Rightarrow \beta = 38.9^o$$ west of north

*Or the angle of the resultant ($\theta_R$) with respect to the positive x-axis is $90^o + 38.9^o$* $= 128.9^o$

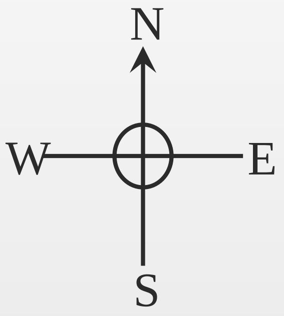



By Prof. Rashad Badran

# Solution



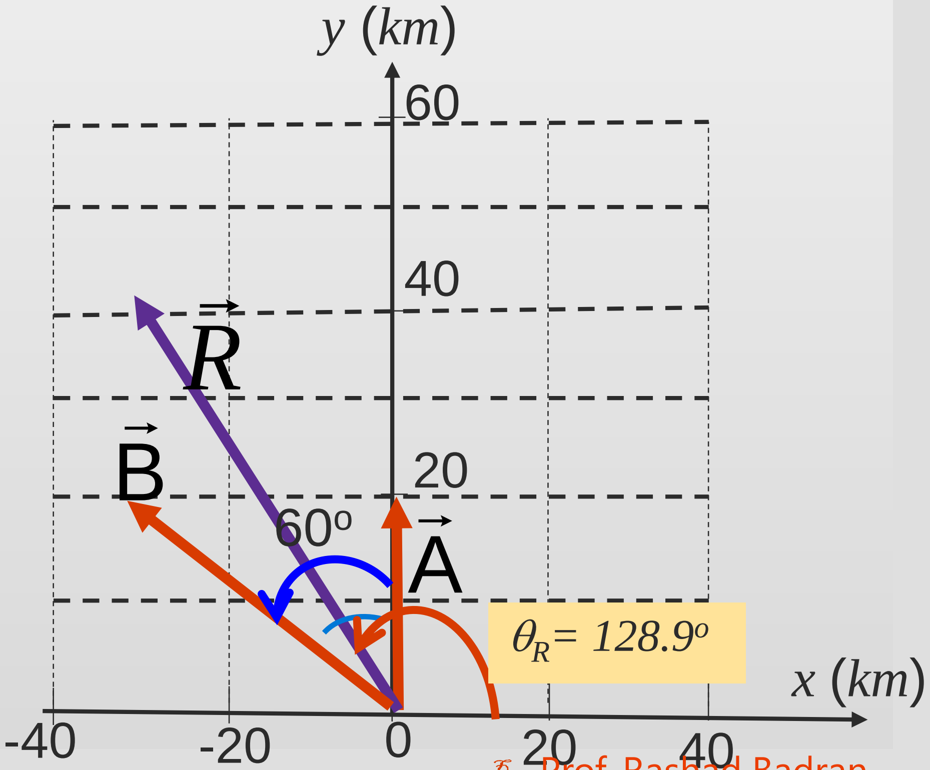



Alternatively one can use the component method

$$\left|\vec{A}\right| = 20km, \quad \left|\vec{B}\right| = 35km, \quad \theta_B = 90^o + 60^o = 150^o$$

$$\Rightarrow R = 48.2km$$

$\theta_R = 128.9^o$



# Objective Questions

## Objective Question

Let vector $\vec{A}$ point from the origin into the second quadrant of the xy plane and vector $\vec{B}$ point from the origin into the fourth quadrant. The vector $\vec{B} - \vec{A}$ must be in which quadrant :

**Answer:**

(a) the first

(b) the second

(c) the third

(d) the fourth

(e) Answers (b) and (d) are both possible

# Other Applications on Addition of Vectors

## Three dimensional vectors:

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

# Other Applications on Addition of Vectors

## Three dimensional vectors:

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$$\vec{R} = \underbrace{(A_x + B_x)}\hat{i}$$

# Other Applications on Addition of Vectors

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$R_x \quad \hat{i} \qquad R_y \qquad R_z$

# Other Applications on Addition of Vectors

## Three dimensional vectors:

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

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$$R_x \quad \hat{i} \qquad R_y \quad \hat{j} \qquad R_z$$

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$$R_x \quad \hat{i} \qquad R_y \quad \hat{j} \qquad R_z \quad \hat{k}$$

# Other Applications on Addition of Vectors

## Three dimensional vectors:

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$$\vec{R} = R_x \hat{i} + R_y \hat{j} + R_z \hat{k}$$

# Other Applications on Addition of Vectors

**Example**

Given the two displacements :

$$\vec{D} = (6\hat{i} + 3\hat{j} - \hat{k})\ \text{m} \quad \text{and} \quad \vec{E} = (4\hat{i} - 5\hat{j} + 8\hat{k})\ \text{m.}$$

Find the mangitude of the displacement $2\vec{D} - \vec{E}$

# Other Applications on Addition of Vectors

**Solution**

$\vec{D} = (6\hat{i} + 3\hat{j} - \hat{k})\ \text{m}$ and $\vec{E} = (4\hat{i} - 5\hat{j} + 8\hat{k})\ \text{m}.$

Let $\vec{F} = 2\vec{D} - \vec{E}$

# Other Applications on Addition of Vectors

**Solution**

$\vec{D} = (6\hat{i} + 3\hat{j} - \hat{k})\ m$ and $\vec{E} = (4\hat{i} - 5\hat{j} + 8\hat{k})\ m.$

$$Let \quad \vec{F} = 2\vec{D} - \vec{E}$$

$$\vec{F} = 2(6\hat{i} + 3\hat{j} - \hat{k})\,m - (4\hat{i} - 5\hat{j} + 8\hat{k})\,m$$

$$= [(12 - 4)\hat{i} + (6 + 5)\hat{j} + (-2 - 8)\hat{k}]\ m$$

$$= [8\hat{i} + 11\hat{j} - 10\hat{k}]\ m$$

# Other Applications on Addition of Vectors

**Solution**

$$\vec{D} = (6\hat{i} + 3\hat{j} - \hat{k})\,\text{m} \quad \text{and} \quad \vec{E} = (4\hat{i} - 5\hat{j} + 8\hat{k})\,\text{m}.$$

$$\text{Let} \quad \vec{F} = 2\vec{D} - \vec{E}$$

$$\vec{F} = 2(6\hat{i} + 3\hat{j} - \hat{k})\,\text{m} - (4\hat{i} \quad 5\hat{j} + 8\hat{k})\,\text{m}$$

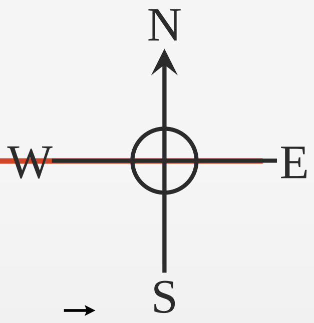
$$= [(12 - 4)\hat{i} + (6 + 5)\hat{j} + (-2 - 8)\hat{k}]\,\text{m}$$

$$= [8\hat{i} + 11\hat{j} - 10\hat{k}]\,\text{m}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= \sqrt{(8\,\text{m})^2 + (11\,\text{m})^2 + (-10\,\text{m})^2} = 17\,\text{m}$$

# Other Applications on Addition of Vectors




## Problem

Consider the two vectors $\vec{A} = 3\hat{i} - 2\hat{j}$ and $\vec{B} = -\hat{i} - 4\hat{j}$. Calculate (a) $\vec{A} + \vec{B}$, (b) $\vec{A} - \vec{B}$, (c) $|\vec{A} + \vec{B}|$, (d) $|\vec{A} - \vec{B}|$, and (e) the directions of $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$.

## Solution

$$(a)\ \vec{A} + \vec{B} = (3\hat{i} - 2\hat{j}) + (-\hat{i} - 4\hat{j}) = 2\hat{i} - 6\hat{j}$$

$$(b)\ \vec{A} - \vec{B} = (3\hat{i} - 2\hat{j}) - (-\hat{i} - 4\hat{j}) = 4\hat{i} + 2\hat{j}$$

$$(c)\ |\vec{A} + \vec{B}| = \sqrt{(2)^2 + (-6)^2} = 6.32$$

$$(e)\ Direction\ of\ \vec{A} + \vec{B} = \tan^{-1}\left(\frac{-6}{2}\right) = 288.4^o,$$

$$(d)\ |\vec{A} - \vec{B}| = \sqrt{(4)^2 + (2)^2} = 4.47$$

$$Direction\ of\ \vec{A} - \vec{B} = \tan^{-1}\left(\frac{2}{4}\right) = 26.5^o$$

# Other Applications on Addition of Vectors

## Exercise

Two vectors $\vec{A}$ and $\vec{B}$ have precisely equal magnitudes. For the magnitude $\vec{A} + \vec{B}$ to be 100 times larger than the magnitude of $\vec{A} - \vec{B}$, what must be the angle between them?

# Other Applications on Addition of Vectors

## Problem

(a) Taking $\vec{A} = (6\hat{i} - 8\hat{j})$units and $\vec{B} = (-8\hat{i} + 3\hat{j})$units, and $\vec{C} = (26\hat{i} + 19\hat{j})$, determine a and b such that a $\vec{A} + b\vec{B} + \vec{C} = 0$. (b) *A* student has learned that a single equation cannot be solved to determine values for more than one unkown in it. How would you explain to him that both a and b can be determined from the single equation used in part (a)?

# Other Applications on Addition of Vectors

## Solution

(a) The vectors $\vec{A} = (6\hat{i} - 8\hat{j})$, $\vec{B} = (-8\hat{i} + 3\hat{j})$, and $\vec{C} = (26\hat{i} + 19\hat{j})$ are in xy plane.

Thus one can substitue these vectors into the vector equation $a\vec{A} + b\vec{B} + \vec{C} = 0$.

One can have :

$$a(6\hat{i} - 8\hat{j}) + b(-8\hat{i} + 3\hat{j}) + (26\hat{i} + 19\hat{j}) = 0 \text{ - - - - - - - - } (1)$$

This equation can be written as two algebraic equations : One for the x - axis and the other for the y - axis.

# Other Applications on Addition of Vectors

**Solution**

Equate the coefficients of $\hat{i}$ to get : $6a - 8b + 26 = 0$ - - - - - - - - (2)

Equate the coefficients of $\hat{j}$ to get : $-8a + 3b + 19 = 0$ - - - - - - -(3)

To find a and b, multiply equation (2) by 4 and equation (3) by 3 to have:

$$24a - 32b + 104 = 0 \text{ - - - - - - - - - } (4)$$

$$-24a + 9b + 57 = 0 \text{ - - - - - - - - - -} (5)$$

Adding equations (4) and (5) one may have: $23b = 161 \Rightarrow$ $b = 7$

Substituting b = 7 into equation (2) to get: $a = 5$

# Other Applications on Addition of Vectors

## Solution

(b) The single vector equation is actually divided into two algebraic equations which allow us to determine the two unknowns, a and b,

**Note:** If three dimensional vectors (expressed in terms of three unit vectors) are considered in the problem instead of two dimensional vectors, then the student may be able to determine three constants which may be available in the vector equation like a, b and c.

# Other Applications on Addition of Vectors

## Exercise

Vector $\vec{A}$ has x and y components of $-8.7$ cm and 15 cm, respectively; vector $\vec{B}$ has x and y components of 13.2 cm and $-6.6$ cm, respectively. If $\vec{A} - \vec{B} + 3\vec{C} = 0$ what are the components of $\vec{C}$.

**Answer**: $C_x$ = 7.3 cm, $C_y$ = -7.2 cm