General

Physics 101

Linear Momentum and Collisions

Prepared By

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Linear Momentum and Collisions

- Linear Momentum
- Isolated System and Conservation of Momentum
- Nonisolated System and Impulse-Momentum Theorem
- Collisions in One Dimension
- Inelastic Collision
- Applications on Inelastic Collision
- Elastic Collision
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Momentum and Impulse

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Linear Momentum

Definition of momentum:

- Momentum is a ***vector*** quantity: it has magnitude (*m***v**) and direction (the same as velocity vector)
  - Momentum of a car driving North at 20 *m/s* is different from momentum of the same car driving East at the same speed
  - Ball thrown by a major-league pitcher has greater magnitude of momentum than the same ball thrown by a child because the speed is greater

- ***Units*** of momentum (**SI**): ***mass*** × ***speed***, *kg·m/s*

Linear Momentum and Kinetic Energy

The kinetic energy and magnitude of momentum of an object are:

$$K = \frac{1}{2}mv^2 ------(1)$$

$$p = mv --------(2)$$

Multiply and divide the RHS of equation (1) by *m to get*

$$K = \frac{1}{2m}m^2v^2 ------(3)$$

Square both sides of equation (2) to get

$$p^2 = m^2v^2 --------(4)$$

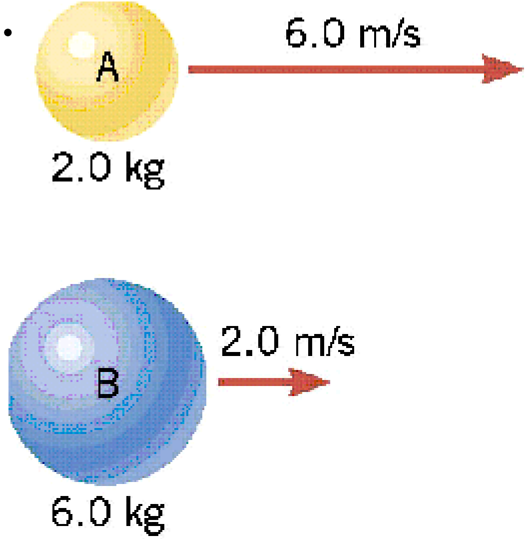
Substitute equation (4) into (3) to get

$$K = \frac{p^2}{2m} -----(5) \qquad or \Rightarrow p = \sqrt{2mK}$$

Linear Momentum

Problem

Given the mass and speed of objects A and B as shown in table and figure, find the magnitude of the momentum and the kinetic energy for each object.



	Mass	**Speed**
Object A	**2.0 kg**	**6.0 m/s**
Object B	**6.0 kg**	**2.0 m/s**

Solution

$$p_A = m_A v_A = (2.0kg)(6.0m/s) = 12kg \cdot m/s$$

$$K_A = \frac{1}{2} m_A {v_A}^2 = \frac{1}{2}(2.0kg)(6.0m/s)^2 = 36J$$

$$p_B = m_B v_B = (6.0kg)(2.0m/s) = 12kg \cdot m/s$$

$$K_B = \frac{1}{2} m_B {v_B}^2 = \frac{1}{2}(6.0kg)(2.0m/s)^2 = 12J$$

Linear Momentum

Problem An object has a kinetic energy of 275 J and a momentum of magnitude 25 kg.m/s. Find the speed and mass of the object.

Solution Use the equation that relates kinetic energy to momentum, namely:

$$K = \frac{p^2}{2m}$$

Substitute K = 275J and p = 25 kg. m/s to *get*

$$275J = \frac{(25kg.m/s)^2}{2m} \qquad \Rightarrow m = 1.136kg$$

Substitute m into momentum equation $\quad p = mv$

$$25kg.m/s = (1.136kg)v \qquad \Rightarrow v = 22m/s$$

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Momentum From Newton's 2nd Law

- We will once again re-express ***Newton's 2nd law*** in terms of momentum.

$$\sum \vec{F} = m\vec{a} \qquad \because \vec{a} = \frac{d\vec{v}}{dt} \qquad \Rightarrow \sum \vec{F} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) \qquad \therefore \sum \vec{F} = \frac{d\vec{p}}{dt}$$

- The ***linear momentum*** of a particle of mass m moving with velocity $\vec{v}$ is a vector quantity defined as the product of particle's mass and velocity: $\vec{p} = m\vec{v}$
- Newton's 2nd law is re-defined as the rate of change of momentum.
- Components of momentum:

$$\vec{p}: \quad p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z$$

Internal Forces: (An Isolated System)

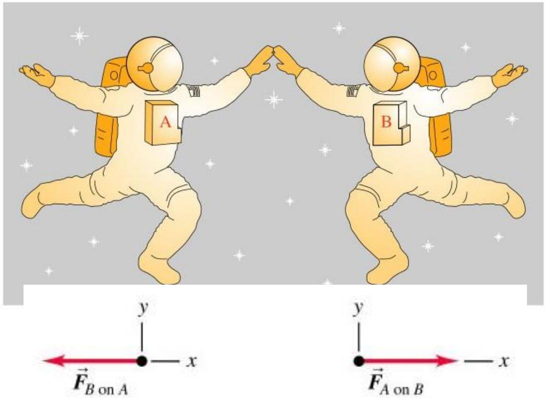
- ***Internal forces***: the forces that the particles of the system exert on each other (for any system)

- Forces exerted on any part of the system by some object outside it called ***external forces***

- For the system of two astronauts, the internal forces are $\underline{\boldsymbol{F}}_{\boldsymbol{BonA}}$, exerted by particle B on particle A, and $\underline{\boldsymbol{F}}_{\boldsymbol{AonB}}$, exerted by particle A on particle B

- There are ***NO*** external forces: we have ***isolated system***

$$\vec{F}_{BonA} = \frac{d\vec{p}_A}{dt} \qquad \vec{F}_{AonB} = \frac{d\vec{p}_B}{dt} \qquad \vec{F}_{AonB} = -\vec{F}_{BonA}$$



$$\vec{F}_{AonB} + \vec{F}_{BonA} = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \frac{d(\vec{p}_A + \vec{p}_B)}{dt} = 0$$

- ***Total momentum*** of two particles is the vector sum of the momenta of individual particles

$$p_{Tot} = p_A + p_B$$

- If system consist of any number of particles A, B, …

$$\vec{p}_{Tot} = \vec{p}_A + \vec{p}_B + \ldots = m_A\vec{v}_A + m_B\vec{v}_B + \ldots$$

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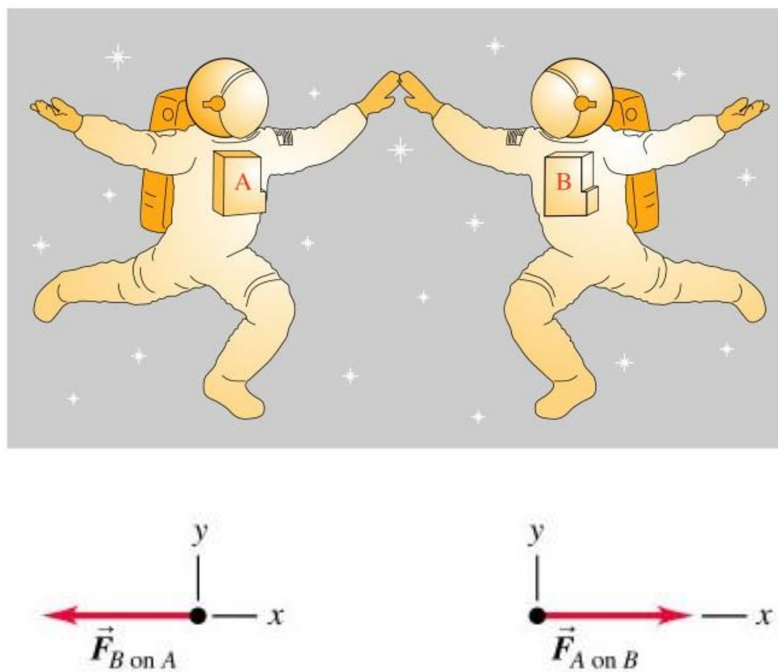
Conservation of Momentum

- Concept of ***momentum*** is particularly important in situations when you have ***two*** or ***more*** interacting ***bodies***
- Consider idealized system of two particles: two astronauts floating in the zero-gravity environment
  - Astronauts touch each other (each particle exerts force on another)
  - ***Newton's 3rd law***: these ***forces*** are equal and opposite
  - Hence, ***impulses*** that act on two particles are equal and opposite



Conservation of Total Momentum

- The time rate of change of the total momentum is zero. Hence the total momentum of the system is constant, even though the individual momenta of the particles that make up the system can change.

$$\vec{F}_{AonB} + \vec{F}_{BonA} = \frac{d\vec{p}}{dt} = 0$$

- If there are external forces acting on the system, they must be included into equation above along with internal forces.

- Then the total momentum is not constant in general. But if vector sum of external forces is zero, these forces do not contribute to the sum, and $\frac{d\vec{p}}{dt} = 0$.

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Conservation of Total Momentum

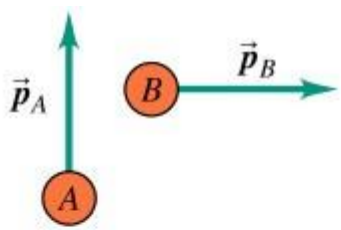
Principle of conservation of momentum:

- **If the vector sum of the external forces on a system is zero, the total momentum of the system is constant**

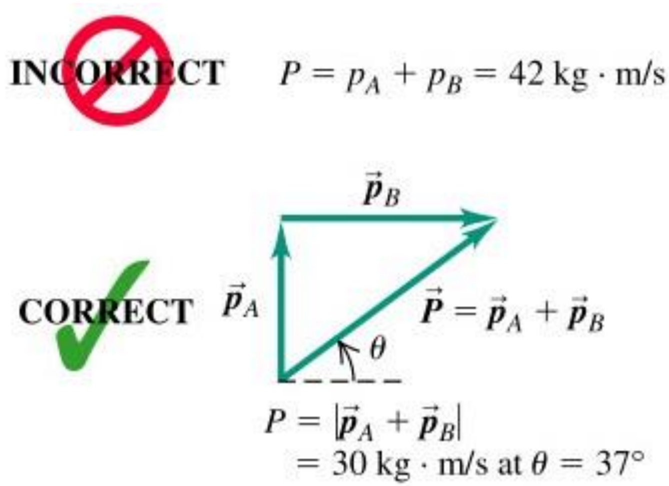
- This principle does not depend on the detailed nature of internal forces that act between members of the system: we can apply it even if we know very little about the internal forces.

- The principle acts only in internal frame of reference (because we used **Newton's 2nd law** to derive it!)

CAUTION **Vector sum !**



$p_A = 18\ \text{kg}\cdot\text{m/s}$
$p_B = 24\ \text{kg}\cdot\text{m/s}$



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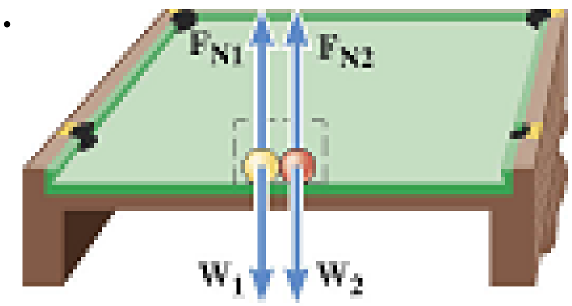
Conservation of Total Momentum

Conceptual problem: Is the total momentum conserved for the following cases:

Two balls collide on the billiard table that is free of friction.

(a) Is the total momentum of the two ball system the same before and after the collision?

(b) Answer (a) for a system that contains only one ball.

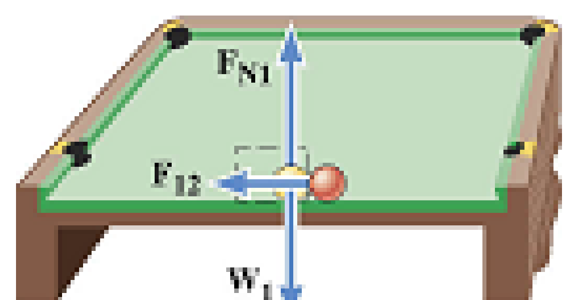


(a)

Answer:

(a) The total momentum is conserved

(b) The total momentum of one ball is NOT conserved



Conservation of Total Momentum

Problem-Solving Strategy

IDENTIFY *the relevant concepts:*

- ***Before*** applying conservation of momentum to a problem, you must first decide whether momentum ***is conserved***!
- This will be true ***only*** if the vector sum of the external forces acting on the system of particles is zero.
- If this is not the case, you ***can't use*** conservation of momentum.

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Conservation of Total Momentum

Problem-Solving Strategy

SET UP *the problem using the following steps:*

1. Define a ***coordinate system***. Make a ***sketch*** showing the coordinate axes, including the positive direction for each. Make sure you are using an inertial frame of reference. Most of the problems in this course deal with two-dimensional situations, in which the vectors have only *x*- and *y*-components; all of the following statements can be generalized to include *z*-components when necessary.
2. Treat each body as a particle. Draw "***before***" and "***after***" sketches, and include vectors on each to represent all known velocities. ***Label*** the vectors with magnitudes, angles, components, or whatever information is given, and give each unknown magnitude, angle, or component an algebraic symbol. You may use the unprimed and primed letters for velocities before and after interaction, respectively.
3. As always, identify the ***target*** variable(s) from among the unknowns.

Conservation of Total Momentum

Problem-Solving Strategy

EXECUTE *the solution as follows:*

1. Write equation in terms of symbols equating the total ***initial*** *x*-component of momentum (***before*** the interaction) to the total ***final*** *x*-component of momentum (***after***), using $\boldsymbol{p}_x = m\boldsymbol{v}_x$ for each particle.

2. Write another equation for *y*-components, using $\boldsymbol{p}_y = m\boldsymbol{v}_y$ for each particle.
   - Remember that the *x*- and *y*-components of velocity or momentum are ***never*** added together in the same equation!
   - Even when all the velocities lie along a line (such as the *x*-axis), components of velocity along this line can be positive or negative; be careful with ***signs***!

3. *Solve* these equations to determine whatever results are required. In some problems you will have to convert from the *x*- and *y*-components of a velocity to its magnitude and direction, or the reverse.

4. In some problems, energy considerations give additional relationships among the various velocities.

Problem-Solving Strategy

EVALUATE *your answer:*

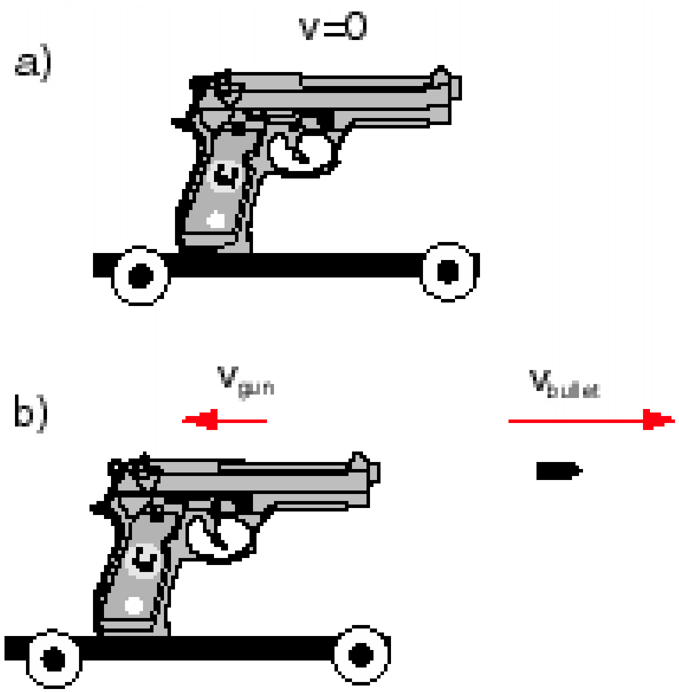
- Does your answer make physical ***sense***?
- If your target variable is a certain body's momentum, check that the ***direction*** of the momentum is ***reasonable***.

Conservation of Total Momentum

in Explosions

Problem A 2-kg gun fires a bullet of mass 0.025kg at a velocity of 300m/s as shown. What is the recoil velocity of the gun?

Solution



$$P_i = 0$$

$$P_f = m_g v_g + m_b v_b$$

gun bullet

$$P_i = P_f \qquad v_g = -v_b \frac{m_b}{m_g}$$

$$v_g = -(300)\frac{(0.025)}{(2)}$$

$$v_g = -3.75 m/s$$

Note: v_b and v_g are in opposite directions

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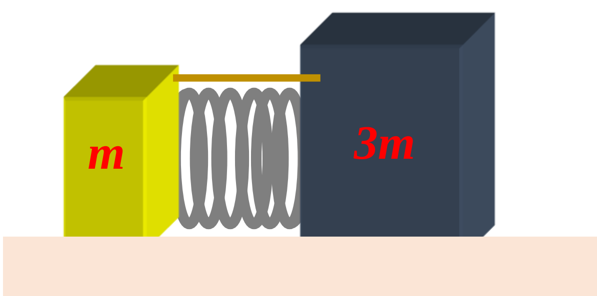
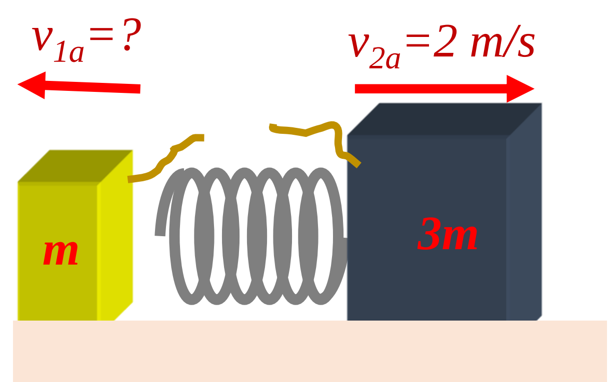
Conservation of Total Momentum

Problem

Two blocks of masses *m* and *3m* are placed on a frictionless, horizontal surface. A light spring is attached to the more massive block, and the blocks are pushed together with the spring between them. A cord initially holding the blocks together is burned: after that happens, the block of mass *3m* moves to the right with a speed of *2 m/s*. (a) What is the velocity of the block of mass *m*? (b) Find the system original elastic potential energy taking $m = 0.35$ *kg*. (c) Is the original energy in the spring or in the cord? (d) Explain your answer in part (c)



Before



After

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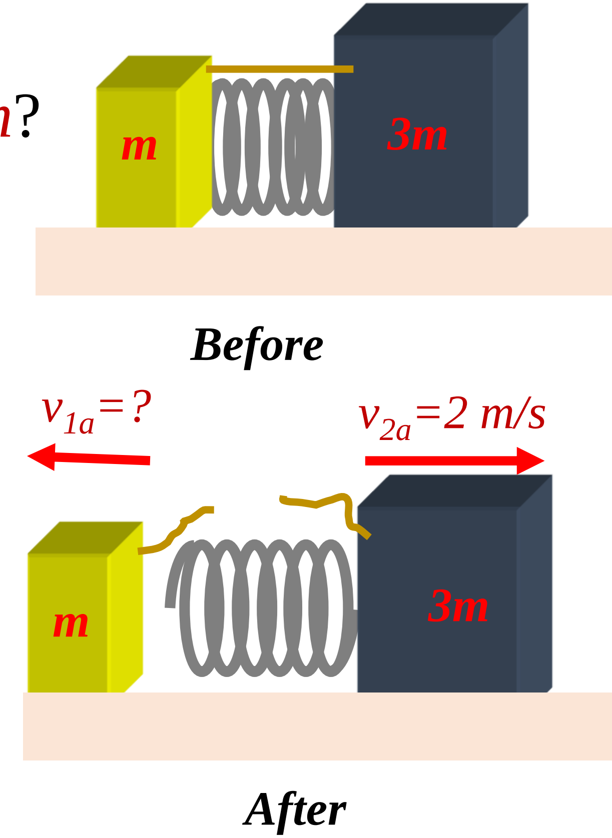
Conservation of Total Momentum

Solution

(a) What is the velocity of the block of mass *m*?



$$(\vec{p}_1 + \vec{p}_2)_{Before} = (\vec{p}_1 + \vec{p}_2)_{After}$$

$$(m_1\vec{v}_1 + m_2\vec{v}_2)_{Before} = (m_1\vec{v}_1 + m_2\vec{v}_2)_{After}$$

Conservation of linear momentum in 1-D:

$$(m_1 v_1 + m_2 v_2)_{Before} = (m_1 v_1 + m_2 v_2)_{After}$$

$m_1 = m$, $m_2 = 3m$, $v_{1b} = v_{2b} = 0$,
$v_{2a} = 2$ m/s, $v_{1a} = ?$

$$(0+0)_{Before} = (mv_{1a} + 3m(2m/s))_{After} \quad \Rightarrow v_{1a} = -6m/s$$

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Solution

(b) Find the system original elastic potential energy taking *m = 0.35 kg*.

$$K_{Tot} = \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2)$$

$m_1 = m$, $m_2 = 3m$, $v_{2a} = 2\ m/s$, $v_{1a} = -6 m/s$

$$K_{Tot} = \frac{1}{2}(m v_{1a}^2 + 3m v_{2a}^2)$$

$$= \tfrac{1}{2}(0.35kg)[(6m/s)^2 + (3)(2m/s)^2] = 8.4J$$

$$U_{Elastic} = K_{Tot} = 8.4J$$

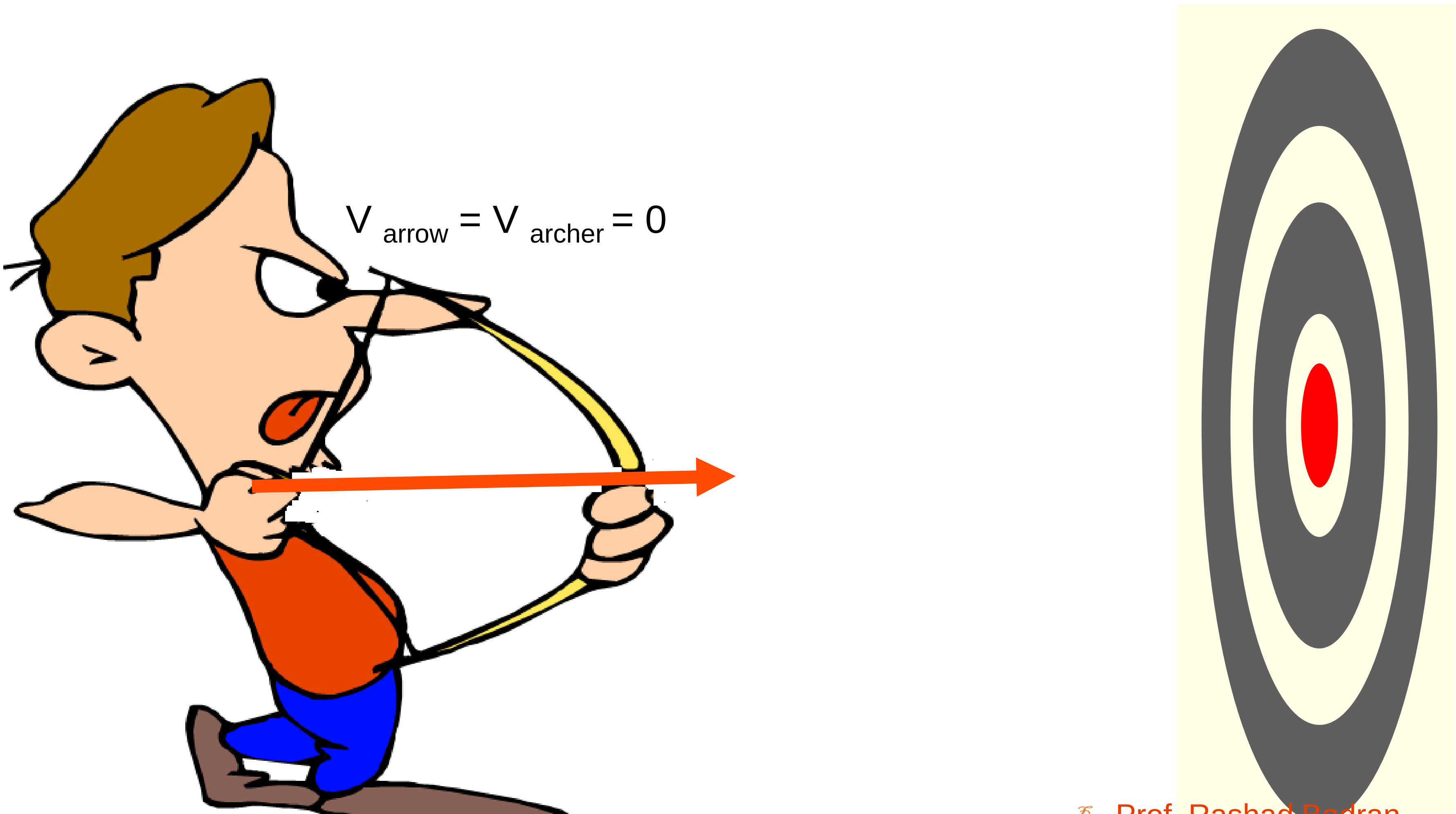
(c) Is the original energy in the spring or in the cord? It is the potential energy stored in the spring

(d) Explain your answer in part (c). This is because the spring was compressed before the release of two masses

Conservation of Momentum

Problem

A 60-kg archer stands at rest on frictionless ice and fires a 0.5 kg arrow horizontally at 50m/s with what velocity does the archer move across the ice after firing arrow?



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Conservation of Momentum

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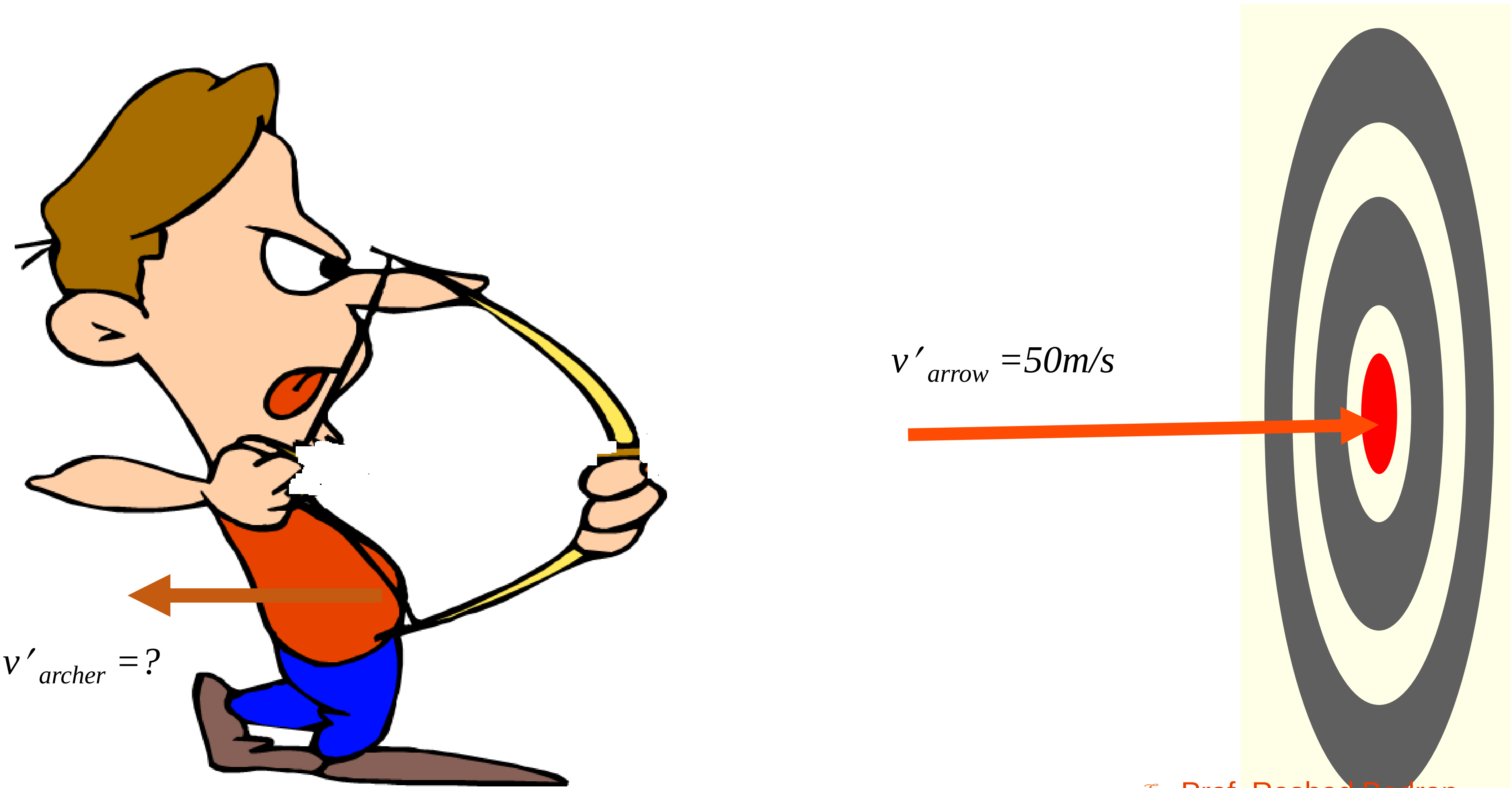

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Conservation of Momentum

A 60-kg archer stands at rest on frictionless ice and fires a 0.5 kg arrow horizontally at 50m/s with what velocity does the archer move across the ice after firing arrow?

Solution

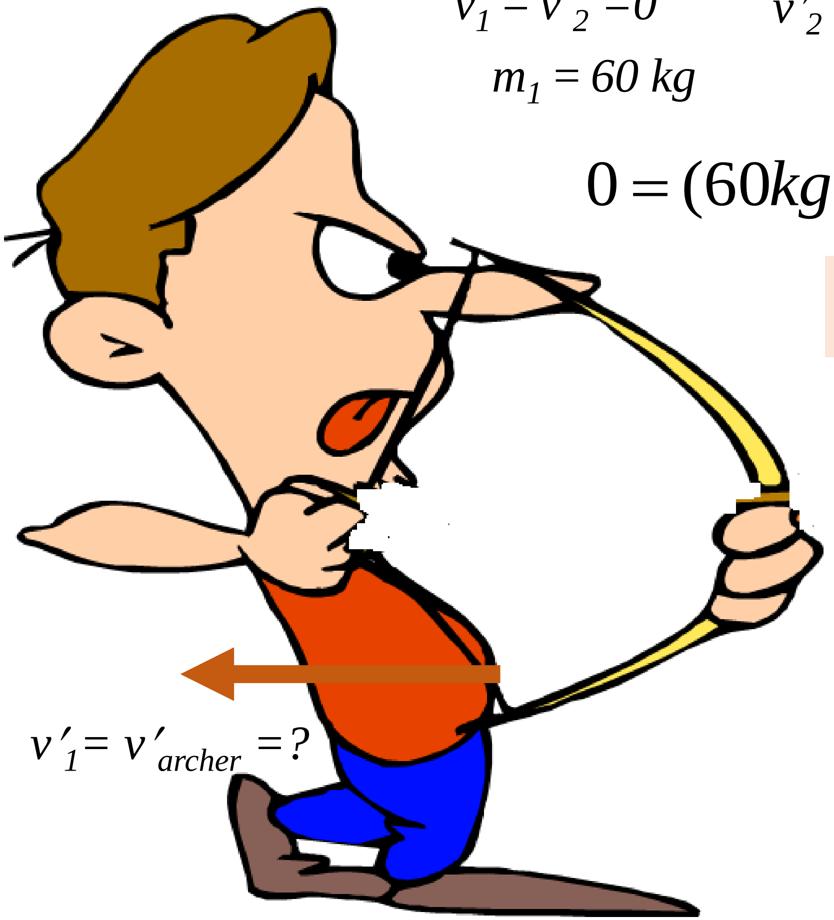
$$(m_1\vec{v}_1 + m_2\vec{v}_2) = (m_1\vec{v}_1' + m_2\vec{v}_2')$$

$v_1 = v_2 = 0$ $\quad v'_2 = v'_{arrow} = 50 m/s$ $\quad v'_1 = v'_{archer} = ?$

$m_1 = 60\ kg$ $\quad m_2 = 0.5\ kg$

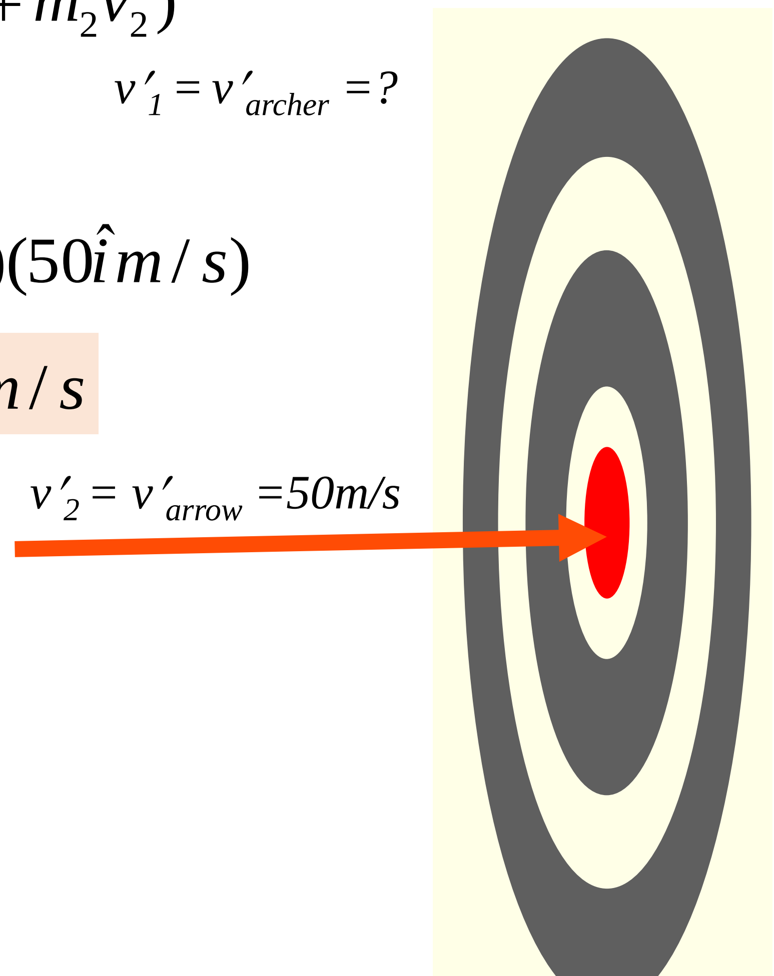
$$0 = (60kg)\vec{v}_1' + (0.5kg)(50\hat{i}\,m/s)$$

$$\vec{v}_1' = -0.42\hat{i}\,m/s$$



$v'_1 = v'_{archer} = ?$

$v'_2 = v'_{arrow} = 50 m/s$



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Review:

- In our discussion of work and energy we re-expressed ***Newton***'s ***Second Law*** in an integral form called Work-Energy Theorem (**W_{Net}** = Δ***K***) which states that the total work done on a particle equals the change in the kinetic energy of the particle.

$$dW_{Net} = F_x dx$$

From Newton's 2nd law $\quad F_x = ma_x \quad$ or $F_x = m\frac{dv_x}{dt}$

Thus $\quad dW_{Net} = m\frac{dv_x}{dt}dx$

The variable x can be changed to v_x where $\quad v_x = \frac{dx}{dt}$

$dW_{Net} = mv_x dv_x \quad$ Integrate both sides $\quad \Rightarrow \int dW_{Net} = m\int_{v_{xi}}^{v_{xf}} v_x dv_x$

$\therefore W_{Net} = \frac{1}{2}m(v_{xf}^2 - v_{xi}^2) \quad\quad \Rightarrow W_{Net} = K_f - K_i = \Delta K$

Nonisolated System:(Momentum and Impulse)

- ***Newton's 2nd law*** in terms of momentum shows that
  - Rapid change in momentum requires a large net force
  - Gradual change in momentum requires less net force
- This principle is used in the design of safety air bags in cars
  - The driver of fast-moving car has a large momentum (the product of the driver's mass and velocity).
  - If car stops suddenly in a collision, driver's momentum becomes zero due to collision with car parts (steering wheel and windshield)
  - Air bag causes the driver to lose momentum more gradually than would abrupt collision with the steering wheel, reducing the force exerted on the driver (and the possibility for injury)
- Same principle applies to the padding used to package fragile objects for shipping

Nonisolated System:(Momentum and Impulse)

- Consider a particle acted on by a ***constant*** net force $\sum \vec{F}$ during a time interval Δt from t_i to t_f. The ***impulse*** of the net force $\vec{J}$ is defined to be the product of the net force and the time interval:

$$\vec{J} = \sum \vec{F}(t_f - t_i) = \sum \vec{F}\Delta t$$

For constant net force

- ***Impulse*** is a ***vector*** quantity.
  - Its direction is the same as the net force
  - Its magnitude is the product of the magnitude of the net force and the length of time the net force acts
- ***Units*** of impulse (**SI**): ***force*** × ***time***, *N·s*
  - 1 N = 1 $kg \cdot m/s^2$ → $N \cdot s = kg \cdot m/s$ (***same*** as momentum)

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Impulse-Momentum Theorem

- If the net force is constant, then $\dfrac{d\vec{p}}{dt}$ is also constant and equals to the total change in momentum during the time interval $t_2 - t_1$, divided by this time interval:

$$\sum \vec{F} = const = \frac{d\vec{p}}{dt} = \frac{\vec{p}_f - \vec{p}_i}{t_f - t_i} \qquad \sum \vec{F}(t_f - t_i) = \vec{p}_f - \vec{p}_i$$

$$\vec{J} = \vec{p}_f - \vec{p}_i = \Delta\vec{p} \quad \text{(Impulse – Momentum Theorem)}$$

- ***Impulse – Momentum Theorem***: The change in momentum of a particle during a time interval equals the impulse of the net force that acts on the particle during that interval.

Impulse-Momentum Theorem

- We will once again re-express ***Newton's 2nd law*** in an integral form. However, in this case the ***independent variable*** will be ***time*** rather than **position** as was the case for the Work-Energy Theorem.

  - ***Impulse – Momentum Theorem*** also holds when forces are ***NOT constant***. To see this, lets integrate both sides of ***Newton's 2nd law***

$$\int_{t_1}^{t_2} \sum \vec{F} dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt} dt = \int_{t_1}^{t_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$

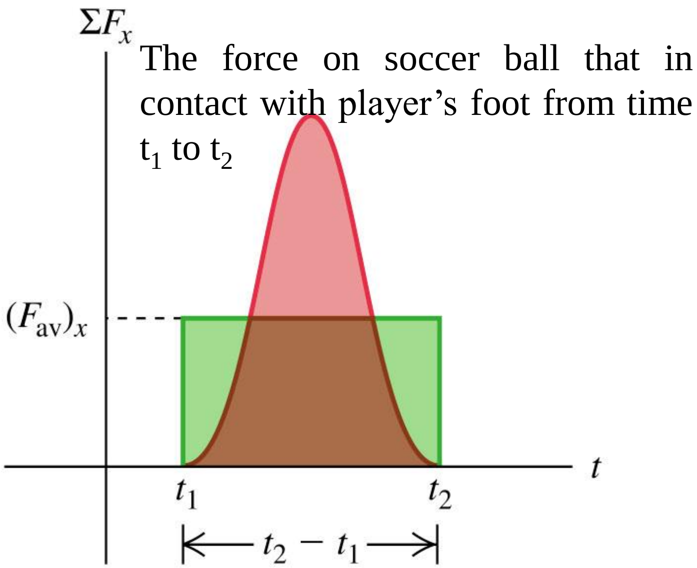
$$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F} dt \qquad \text{(General definition of impulse)}$$

Impulse-Momentum Theorem

- We can define an ***average*** net force $\sum \vec{F}$ such that even when $\sum \vec{F}$ is ***NOT constant***, the impulse $\vec{J}$ is given by

$$\vec{J} = \vec{F}_{av}(t_2 - t_1)$$

- The impulse can be interpreted as the "area" under the graph of $\boldsymbol{F}(t)$ versus t. The x-component of impulse of the net force $\Sigma \boldsymbol{F_x}$ between t_1 and t_2 equals the area under $\Sigma \boldsymbol{F_x}$–t curve, which also equals the area under rectangle with height $(\boldsymbol{F_{av}})_{\boldsymbol{x}}$



$$J_x = \int_{t_1}^{t_2} \sum F_x dt = (F_{av})_x (t_2 - t_1) = p_{2x} - p_{1x} = mv_{2x} - mv_{1x}$$

$$J_y = \int_{t_1}^{t_2} \sum F_y dt = (F_{av})_y (t_2 - t_1) = p_{2y} - p_{1y} = mv_{2y} - mv_{1y}$$

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Impulse-Momentum Theory

Problem

The magnitude of the net force exerted in the x direction on a 2.5-kg particle varies in time as shown in the figure. Find (a) the impulse of the force over the 5-s time interval, (b) the final velocity the particle attains if it is originally at rest, (c) its final velocity if its original velocity is $-2\hat{i}\,m/s$, and (d) the average force exerted on the particle for the time interval between 0 and 5 s.

Solution

(a) $J = \int_{t_1}^{t_2} \sum F dt = (F_{av})(t_2 - t_1)$

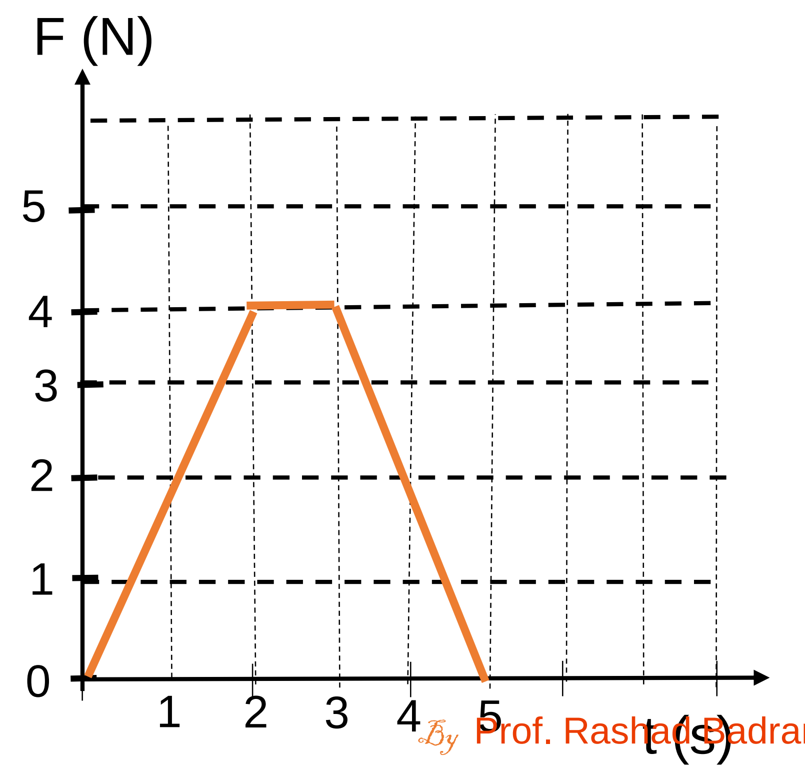
J = Area under the curve from t_1 to t_2

J = 0.5((5+1)s)(4N)= 12 N. s due east

(b) $J_x = p_{2x} - p_{1x} = mv_{2x} - mv_{1x}$

$12N.s = (2.5kg)(v_{2x} - 0)$

$v_{2x} = 4.8m/s$ due east



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Impulse-Momentum Theory

Problem

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Solution

(c) $J_x = p_{2x} - p_{1x} = mv_{2x} - mv_{1x}$

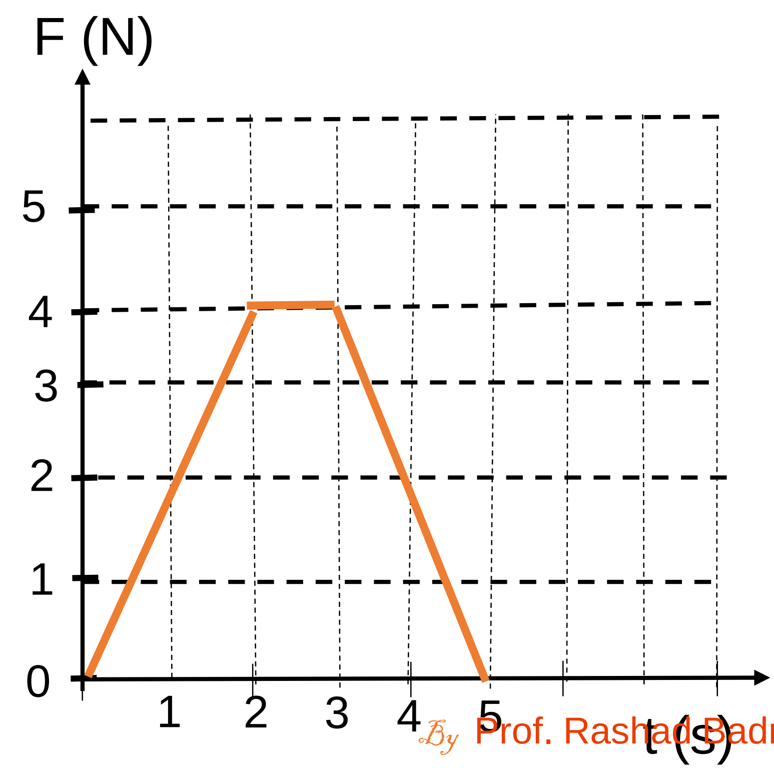
$$12N.s = (2.5kg)(v_{2x} - (-2m/s))$$

$$v_{2x} = 2.8m/s \quad \text{due east}$$

(d) $J_x = F_{x_{avg}}\Delta t = mv_{2x} - mv_{1x}$

$$12N.s = F_{x_{avg}}(5s)$$

$$F_{x_{avg}} = 2.4N \quad \text{due east}$$



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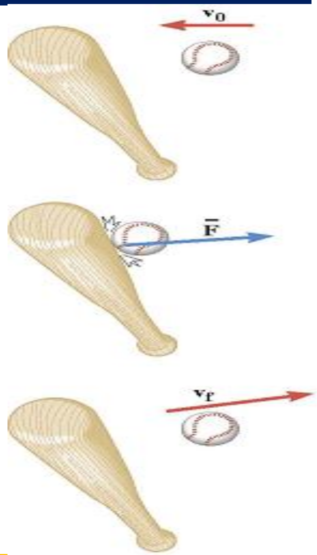
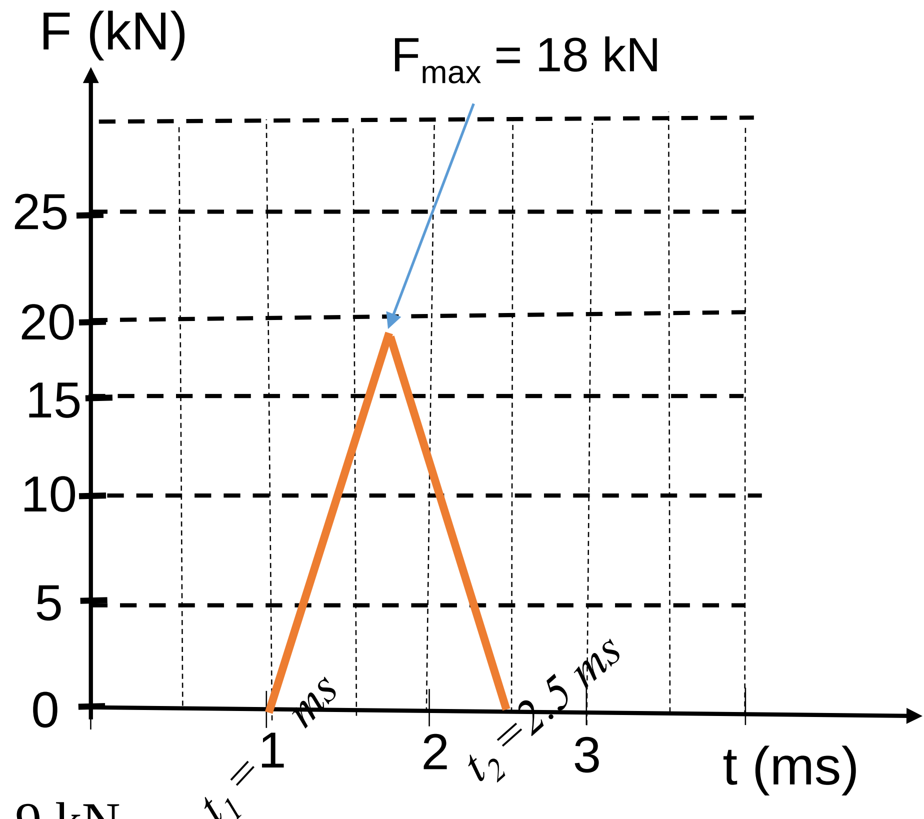
Impulse-Momentum Theory

Exercise

An estimated force-time curve for a baseball struck by a bat is shown in the figure. From this curve, determine (a) the magnitude of the impulse delivered to the ball and (b) the average force exerted to the ball.





Soution

(a) $J = \int_{t_1}^{t_2} \sum F dt = (F_{av})(t_2 - t_1)$

J = Area under the curve from t_1 to t_2

$J = (18 kN)(2.5\ ms - 1\ ms) = 13.5$ N. s

(b) F_{avg}=J/Δt= (13.5 N. s)/(1.5ms) = 9 kN

By Prof. Rashad Badran

Impulse-Momentum Theory

Exercise

A baseball (m = 0.14kg) has initial velocity of v_0 = - 38m/s as it approaches a bat. The bat applies an average force F_{avg} that is much larger than the weight of the ball, and the ball departs from the bat with a final velocity of v_f = + 58m/s.

(a) Determine the impulse applied to the ball by the bat.

(b) Assuming time of contact is Δt =1.6 × 10^{-3}s, find the average force exerted on the ball by the bat.

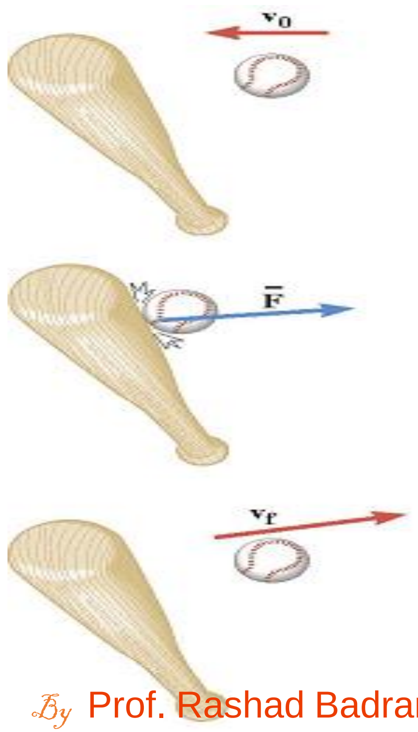
Solution

(a) $$J = mv_f - mv_0$$

$$= (0.14kg)(58m/s) - (0.14kg)(-38m/s)$$

$$\therefore J = +13.4\ kg.m/s$$

(b) $$F_{avg} = \frac{J}{\Delta t} = \frac{13.4kg.m/s}{1.6\times10^{-3}s} = +8400N$$

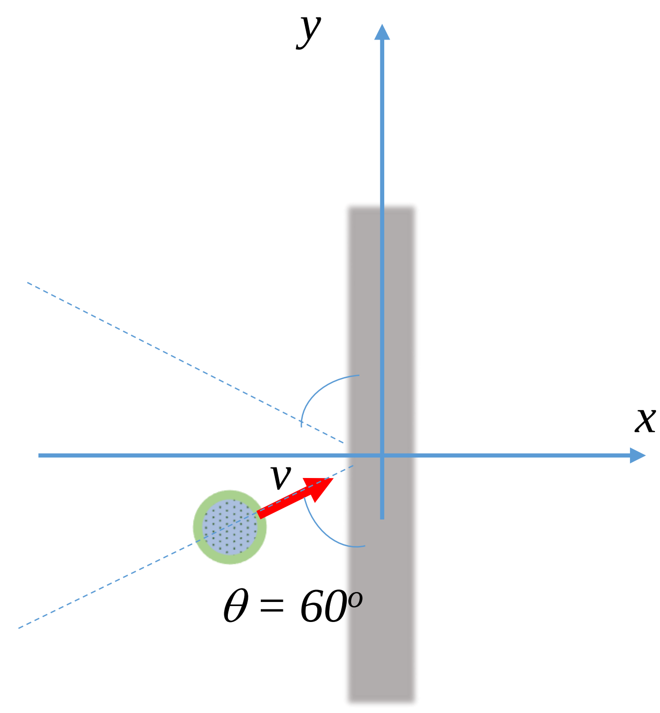


Note: Both the impulse applied to the ball and the average force are directed to the east

Impulse-Momentum Theory

Exercise

An 3-kg ball strikes a wall with a speed of 10 *m/s* at an angle of $\theta = 60^o$ with the surface. It bounces off with the same speed and angle. If the ball is in contact with the wall for 0.2 *s*, what is the average force exerted by the wall on the ball?



Impulse-Momentum Theory

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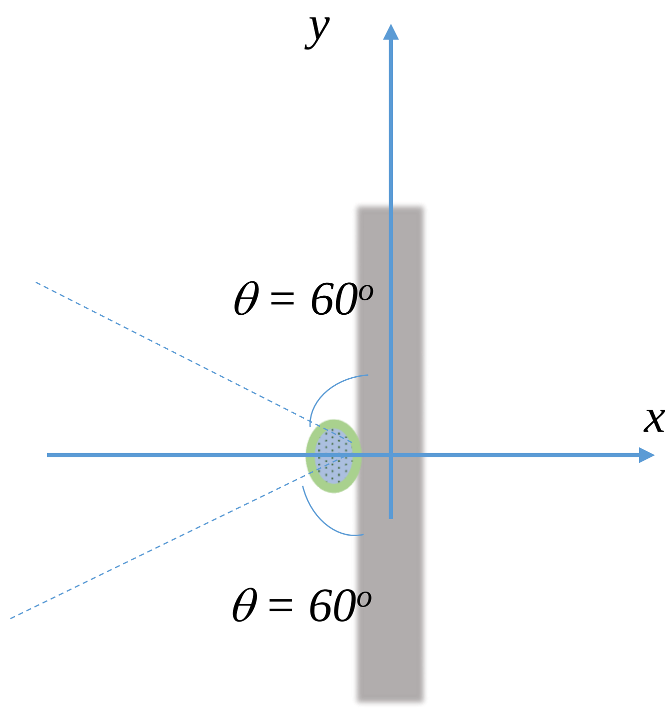

Impulse-Momentum Theory

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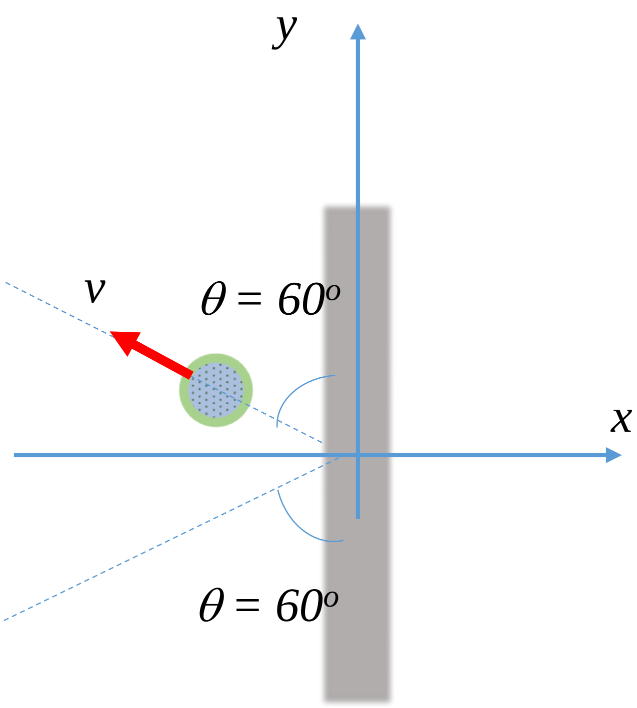

Impulse-Momentum Theory

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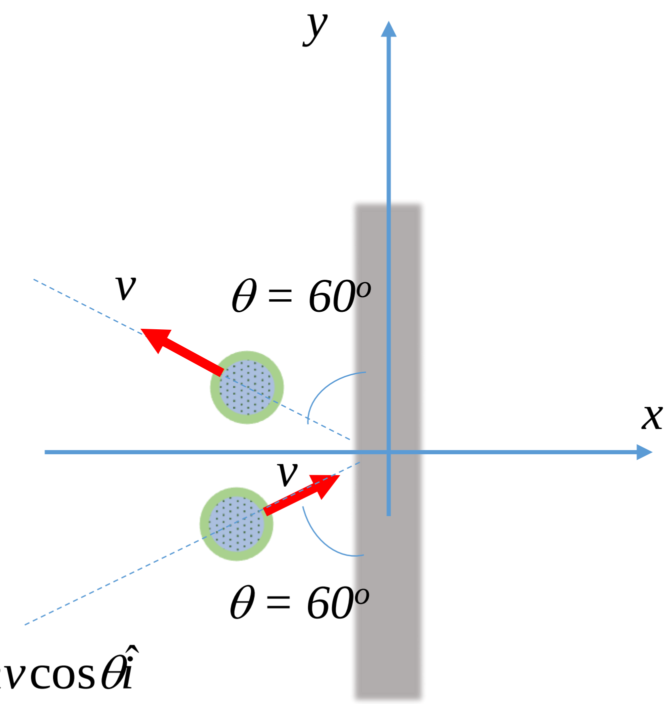


Soution

$$\vec{J} = m\vec{v}_2 - m\vec{v}_1 \qquad \vec{J} = \vec{F}_{avg}\Delta t$$

$$\vec{F}_{avg}\Delta t = m\vec{v}_2 - m\vec{v}_1$$

$$\vec{v}_1 = v_{1x}\hat{i} + v_{1y}\hat{j} = v_1\cos\theta\hat{i} + v_1\sin\theta\hat{j}$$

$$\vec{v}_2 = v_{2x}\hat{i} + v_{2y}\hat{j} = -v_2\cos\theta\hat{i} + v_2\sin\theta\hat{j}$$

$$\vec{v}_2 - \vec{v}_1 = (-v_2\cos\theta\hat{i} + v_2\sin\theta\hat{j}) - (v_1\cos\theta\hat{i} + v_1\sin\theta\hat{j})$$

$$\because v_1 = v_2 = v$$

$$\therefore m(\vec{v}_2 - \vec{v}_1) = m[(-v\cos\theta\hat{i} + v\sin\theta\hat{j}) - (v\cos\theta\hat{i} + v\sin\theta\hat{j})] = -2mv\cos\theta\hat{i}$$

$$\vec{F}_{avg}\Delta t = -2mv\cos\theta\hat{i} \qquad or \quad \vec{F}_{avg} = \frac{-2mv\cos\theta\hat{i}}{\Delta t}$$

$v = 10\ m/s,\ \theta = 60^o,\ \Delta t = 0.2\ s$

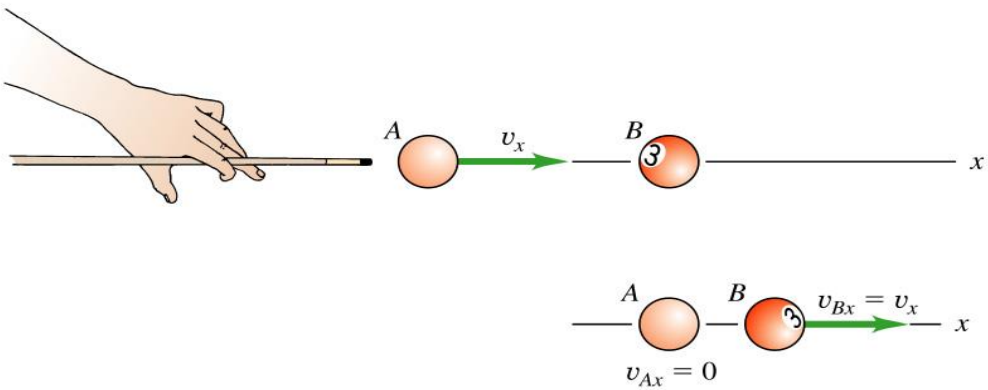
$$\Rightarrow \vec{F}_{avg} = \frac{-2(3kg)(10m/s)\cos 60}{0.2s}\hat{i} = -150\hat{i}N$$

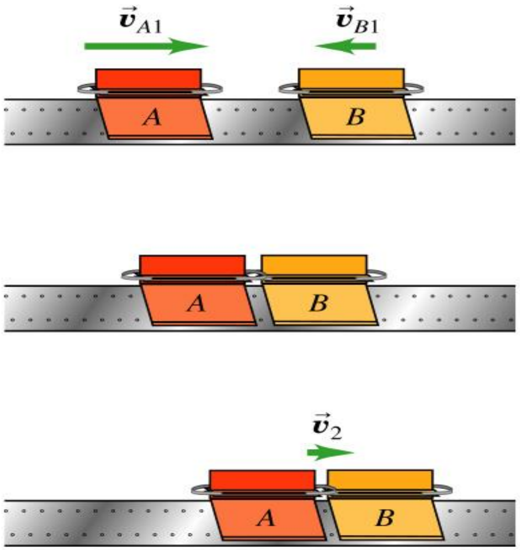
By Prof. Rashad Badran

Collisions

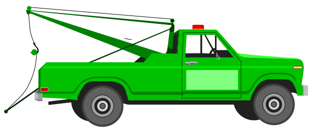
- ***Collision*** is any ***strong interaction*** between bodies that lasts a relatively ***short*** time
  - Car's collision
  - Balls colliding on a pool table
  - Neutrons hitting nuclei in reactor core
  - Bowling ball striking pins
- If the forces between the bodies are <u>much larger</u> than any <u>external forces</u>, we can neglect the external forces entirely and treat the bodies as ***isolated system***
- Since a ***collision*** constitutes an isolated system (where the net external force is zero), the ***momentum of the system is conserved*** (the same before and after the collision)
- Collision types: ***inelastic*** and ***elastic*** collisions

Collisions

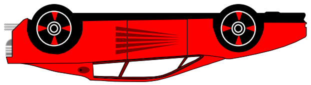


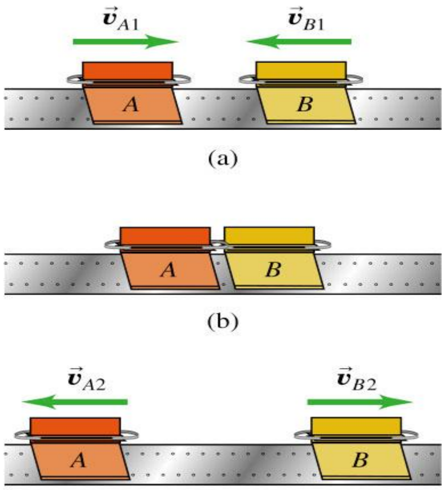


V_T



V_c





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Inelastic Collisions

- In any collision in which external forces can be neglected, ***momentum is conserved*** and the total momentum before equals the total momentum after

- Collisions are classified according to how much energy is "lost" during the collision

- ***Inelastic Collisions*** - there is a ***loss*** of ***kinetic energy*** due to the collision
  - Automobile collision is inelastic: the structure of the car absorbs as much of the energy of collision as possible. This absorbed energy cannot be recovered, since it goes into a permanent deformation of the car

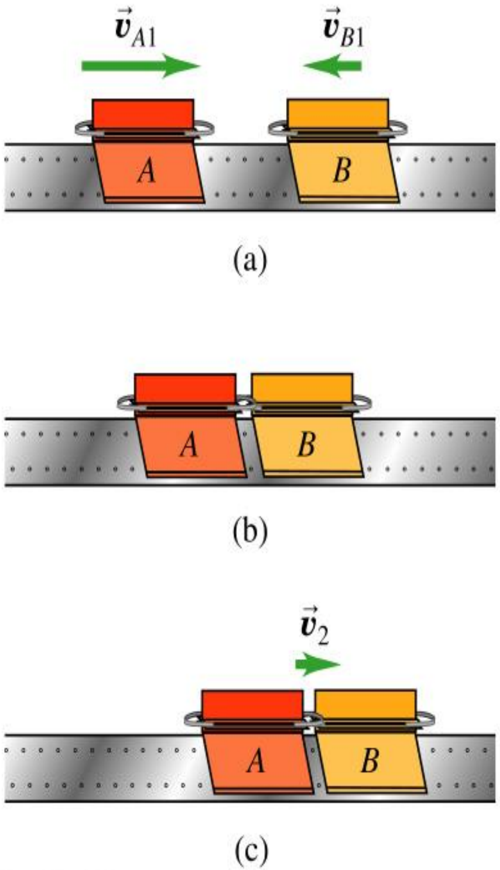
- ***Completely Inelastic Collisions*** - the loss of kinetic energy is the maximum possible. The objects ***stick together*** after the collision.

- ***Elastic collisions*** - the ***kinetic energy*** of the system is ***conserved***

Perfectly Inelastic Collision

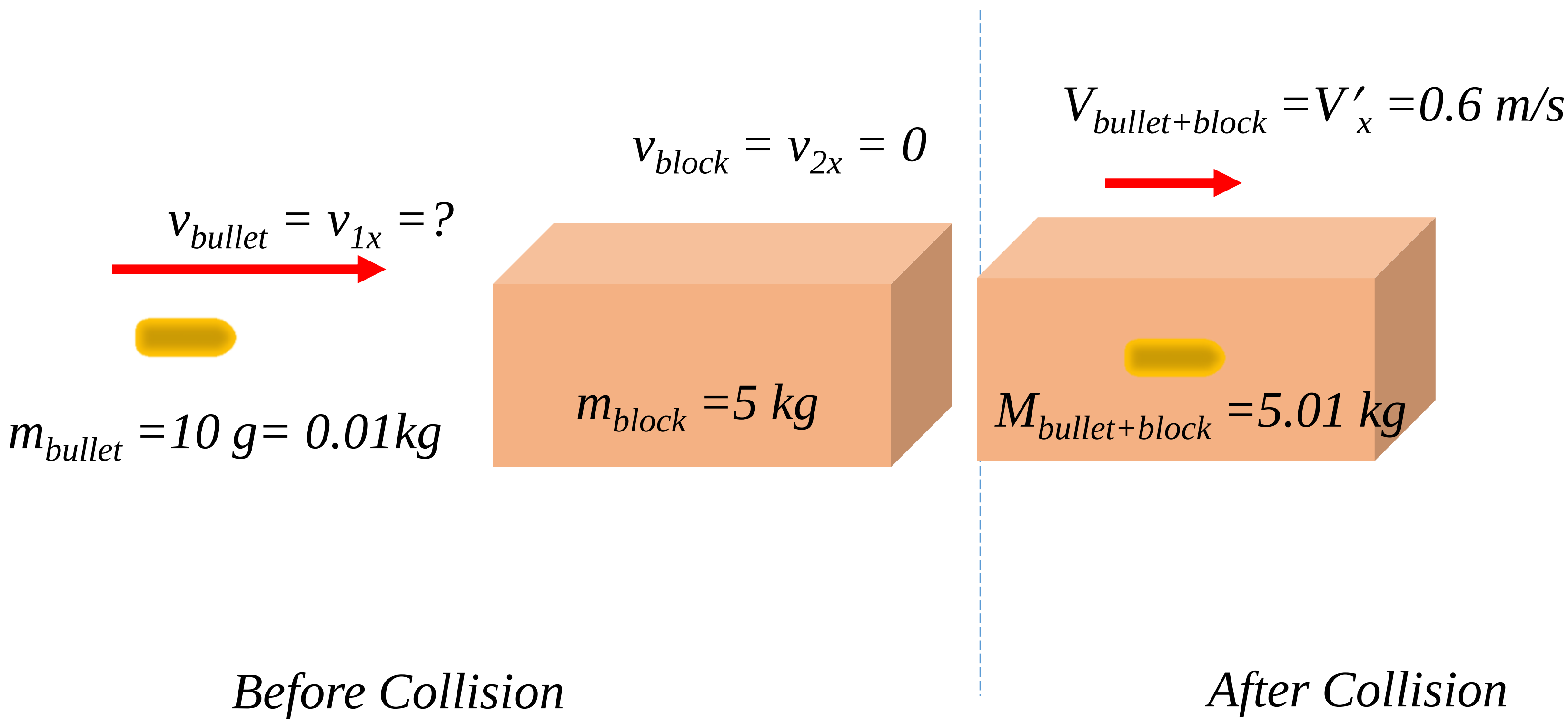
***Completely (or perfectly) inelastic collision*:**

- Glider A and glider B approach each other on a frictionless surface
- Each glider has a putty on the end, so gliders stick together after collision



Perfectly Inelastic Collision

Problem A 10-g bullet is fired into a stationary block of wood having mass *m* = 5 kg. The bullet imbeds into the block. The speed of the bullet-plus-wood combination immediately after the collision is 0.6 *m/s*. What was the original speed of the bullet?



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Perfectly Inelastic Collision

Solution

Before Collision | *After Collision*

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}$$

$$v'_{1x} = v'_{2x} = V'_x$$

$$m_1 v_{1x} + m_2 v_{2x} = (m_1 + m_2) V'_x$$

$$m_{bullet} v_{bullet} + m_{block} v_{block} = (m_1 + m_2) V'_x$$

$m_{bullet} = 0.01 kg$

$v_{bullet} = v_{1x} = ?$

$m_{block} = 5\ kg$

$v_{block} = v_{2x} = 0$

$M_{bullet+block} = 5.01\ kg$

$V_{bullet+block} = V'_x = 0.6\ m/s$

$$(0.01kg) v_{bullet} + 0 = (5.01kg)(0.6m/s)$$

$$v_{bullet} = 300.6 m/s$$

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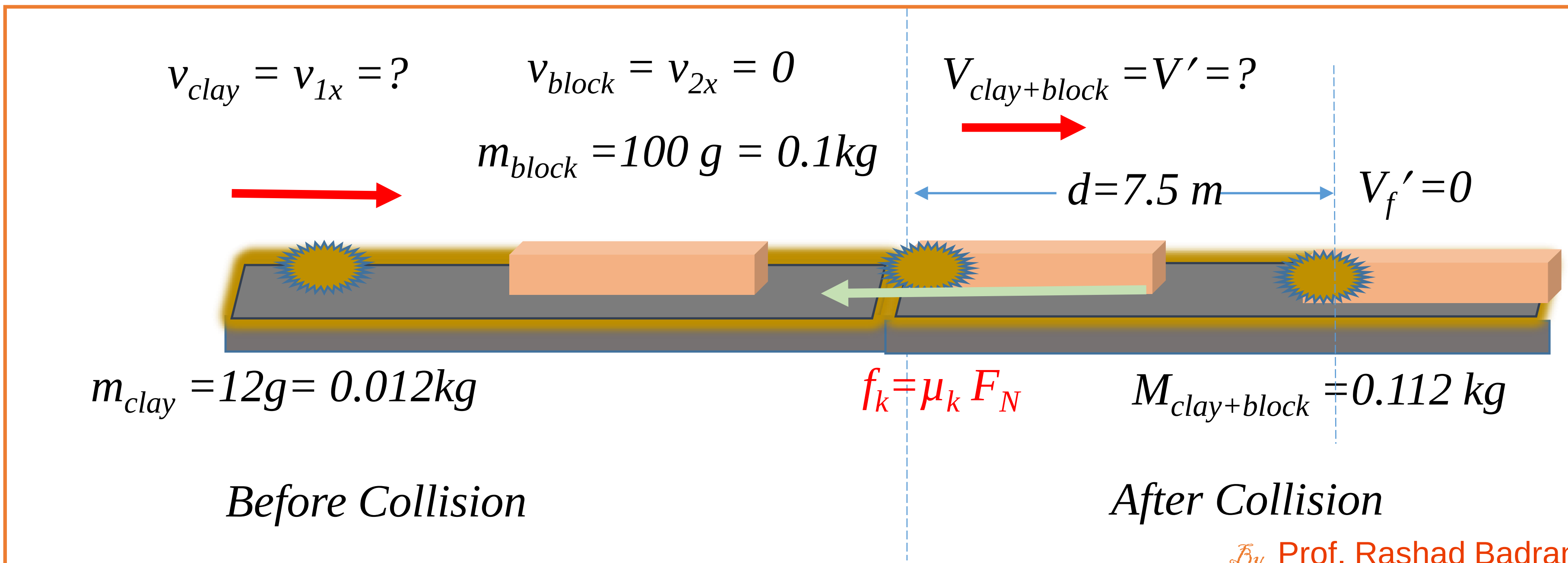
Perfectly Inelastic Collision

Problem A 12-g wad of sticky clay is hurled horizontally at 100-g wooden block initially at rest on a horizontal surface (frictionless before impact). The clay sticks to the block. After impact, the block slides 7.5 *m* before coming to rest. If the coefficient of friction between the block and the surface is 0.65, what was the speed of the clay immediately before impact?



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Perfectly Inelastic Collision

Solution

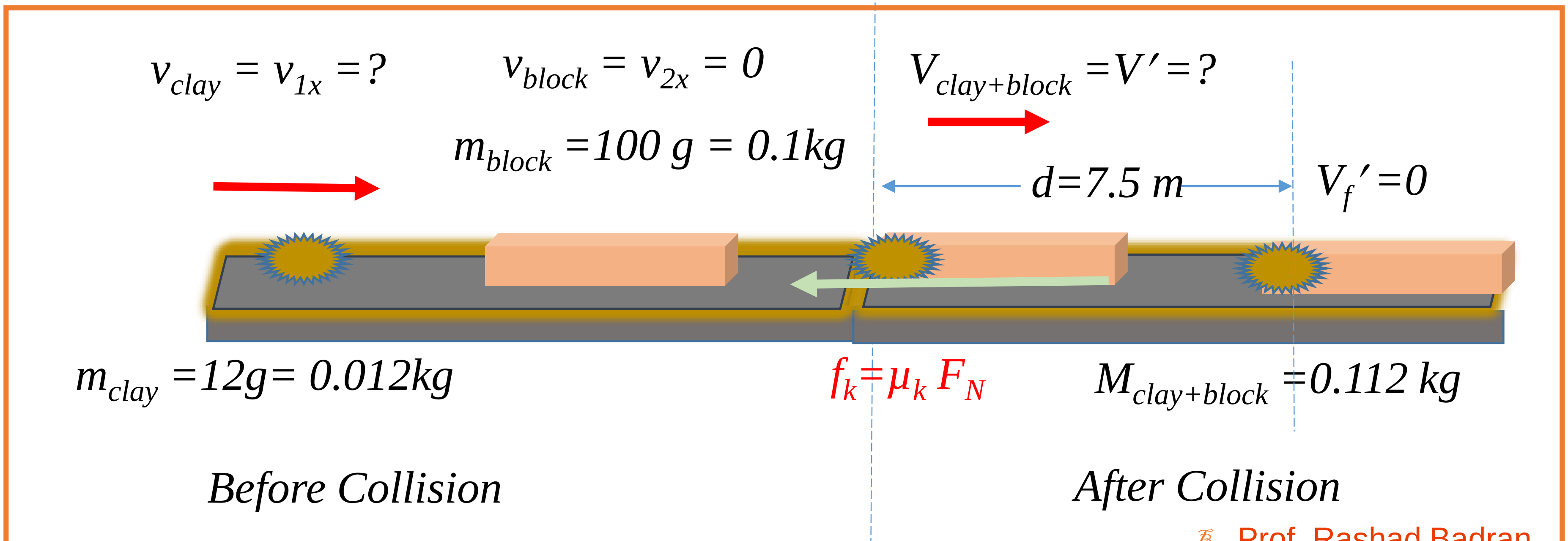
$$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}$$

$$v'_{1x} = v'_{2x} = V'_x = ?$$

$$m_{clay} v_{clay} + m_{block} v_{block} = (m_{clay} + m_{block}) V'_x$$

$$v_{block} = 0, v_{clay} = ?$$

$$(0.012kg) v_{clay} + 0 = (0.112kg) V'_x -----(1)$$



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Perfectly Inelastic Collision

Solution

$$W_f = \Delta K$$

$$-f_k d = \frac{1}{2} m_{clay+block}(V_f'^2 - V'^2)$$

$$-\mu_k F_N d = \frac{1}{2} m_{clay+block}(0 - V'^2)$$

$$-\mu_k m_{clay+block} g d = -\frac{1}{2} m_{clay+block} V'^2$$

$$\Rightarrow V' = \sqrt{2\mu_k g d}$$

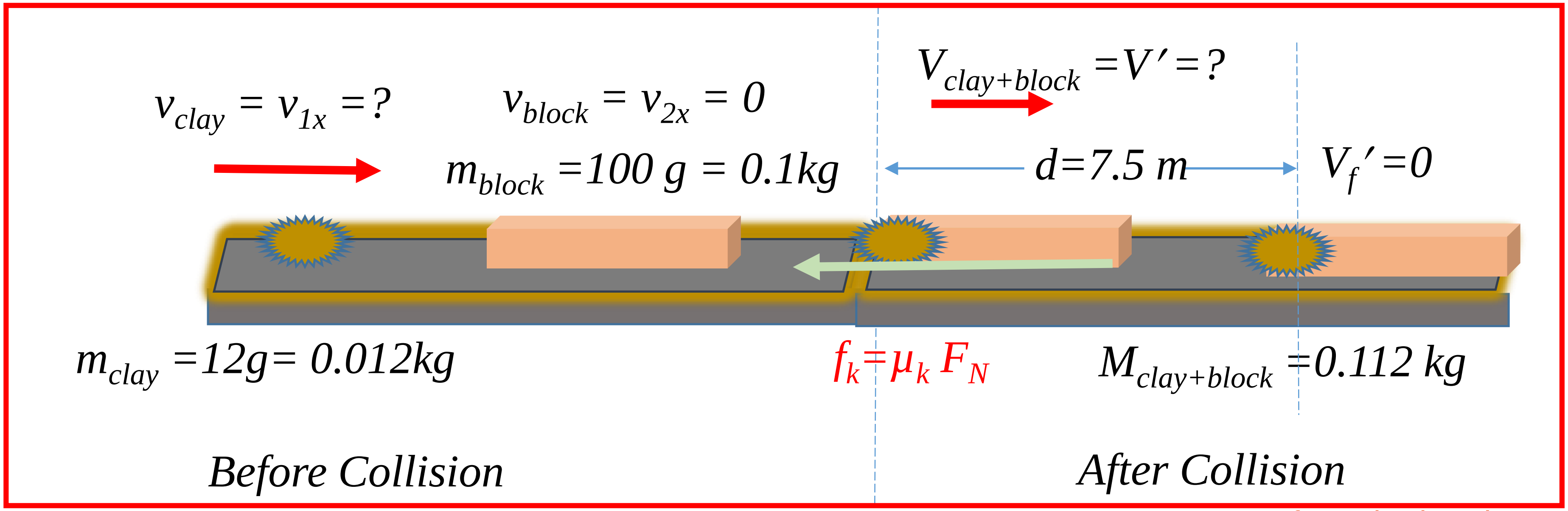
$$\Rightarrow V' = \sqrt{2(0.65)(9.8m/s^2)(7.5m)}$$

$$\therefore V' = 9.77m/s ----(2)$$

From equation (2) into equation (1):

$$(0.012kg)v_{clay} = (0.112kg)(9.77m/s)$$

$$v_{clay} = 91.2m/s$$



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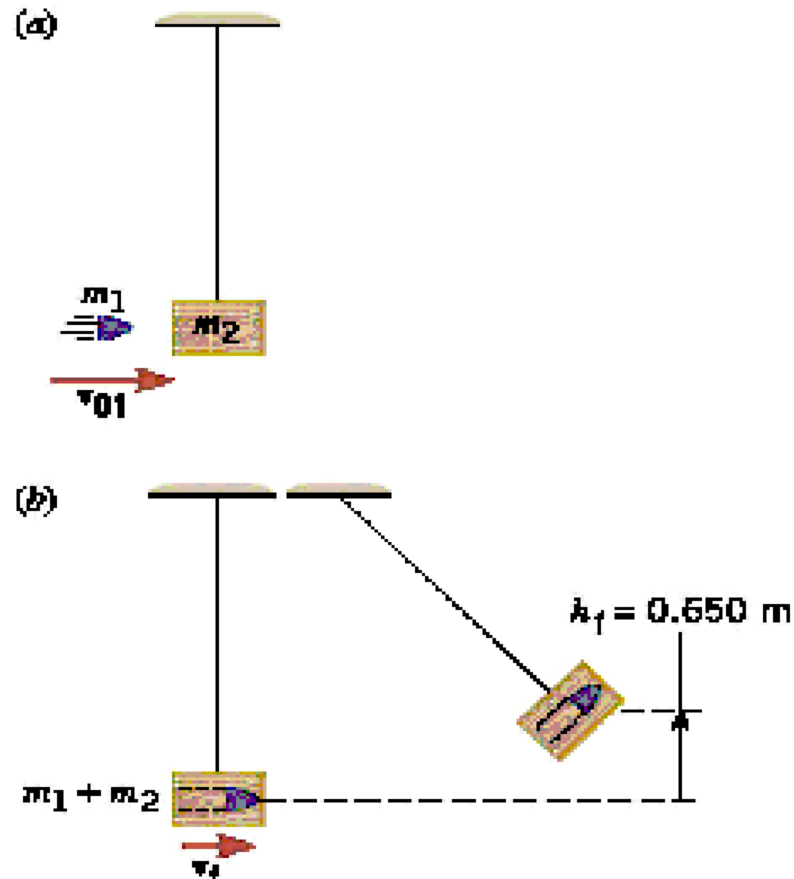
Perfectly Inelastic Collision

Problem

The ballistic pendulum consists of a block of wood (mass m_2 = 2.5kg) suspended by a wire of negligible mass. A bullet (mass m_1 = 0.01kg) is fired with a speed v_{01}. After collision, the block has a speed v_f and then swings to a maximum height of 0.65m above the initial position. Find the speed v_{01} of the bullet, assuming air resistance is negligible.



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Perfectly Inelastic Collision

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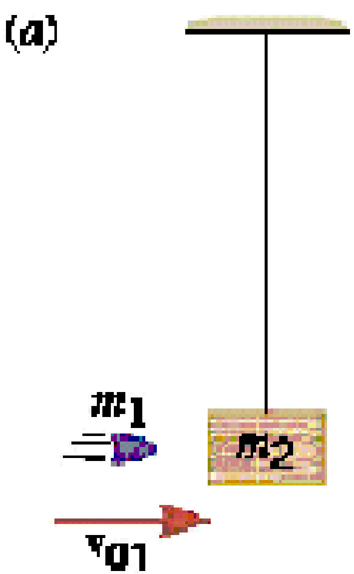
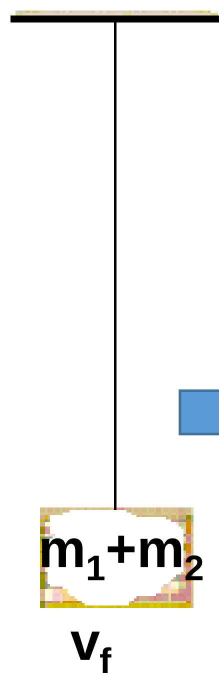
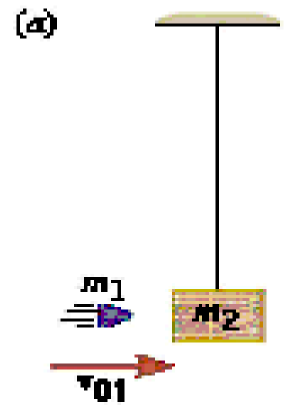
Solution

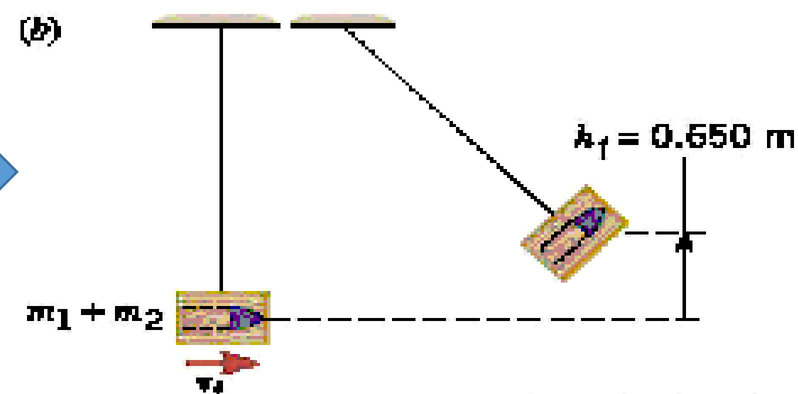
Just before collision



Just after collision







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Perfectly Inelastic Collision

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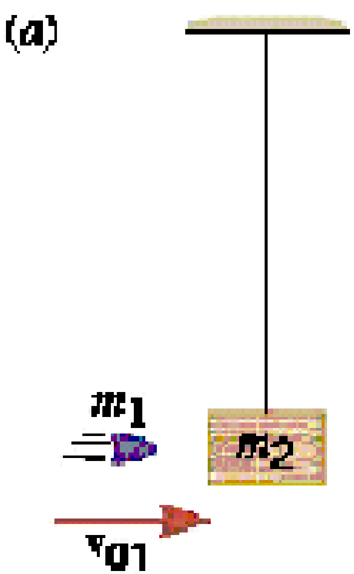
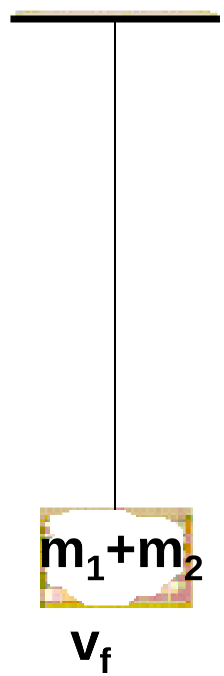
Solution

Just before collision



Just after collision



Is conservation of energy valid?

No (completely inelastic)

Is conservation of momentum valid?

Yes (no external forces $\longrightarrow$)

$$(m_1 + m_2)v_f = m_1 v_{01}$$

Total momentum after collision — Total momentum before collision

$$v_{01} = \frac{m_1 + m_2}{m_1} v_f$$

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Perfectly Inelastic Collision

The ballistic pendulum consists of a block of wood(mass m_2 = 2.5kg)suspended by a wire of negligible mass. A bullet(mass m_1 = 0.01kg) is fired with a speed v_{01}. After collision, the block has a speed v_f and then swings to a maximum height of 0.65m above the initial position. Find the speed v_{01} of the bullet, assuming air resistance is negligible.

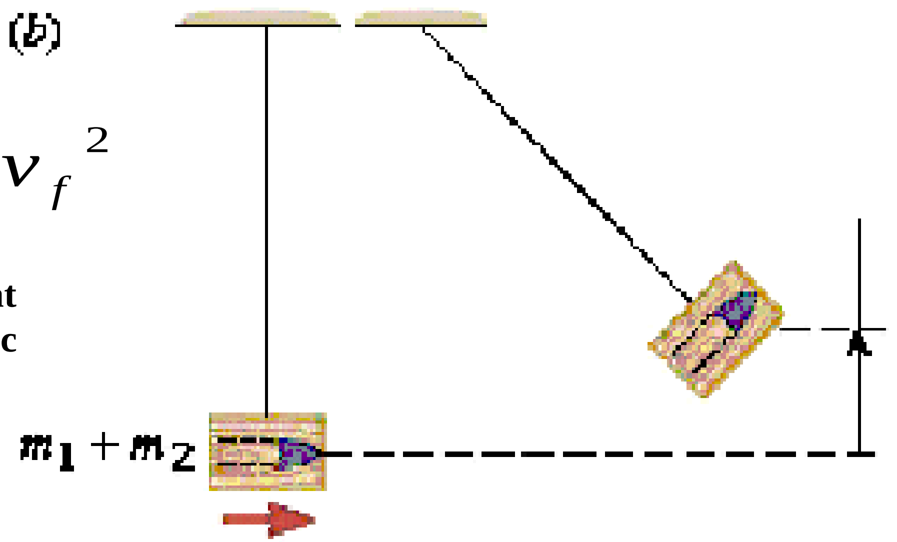
Solution

Applying conservation of energy

$$(m_1 + m_2)gh_f = \frac{1}{2}(m_1 + m_2)v_f^{\ 2}$$

Total mechanical energy at top of swing, all potential

Total mechanical energy at bottom of swing, all kinetic



$$v_{01} = (\frac{m_1 + m_2}{m_1})\sqrt{2gh_f}$$

$$= (\frac{0.0100kg + 2.50kg}{0.0100kg})\sqrt{2(9.80m/s^2)(0.650m)}$$

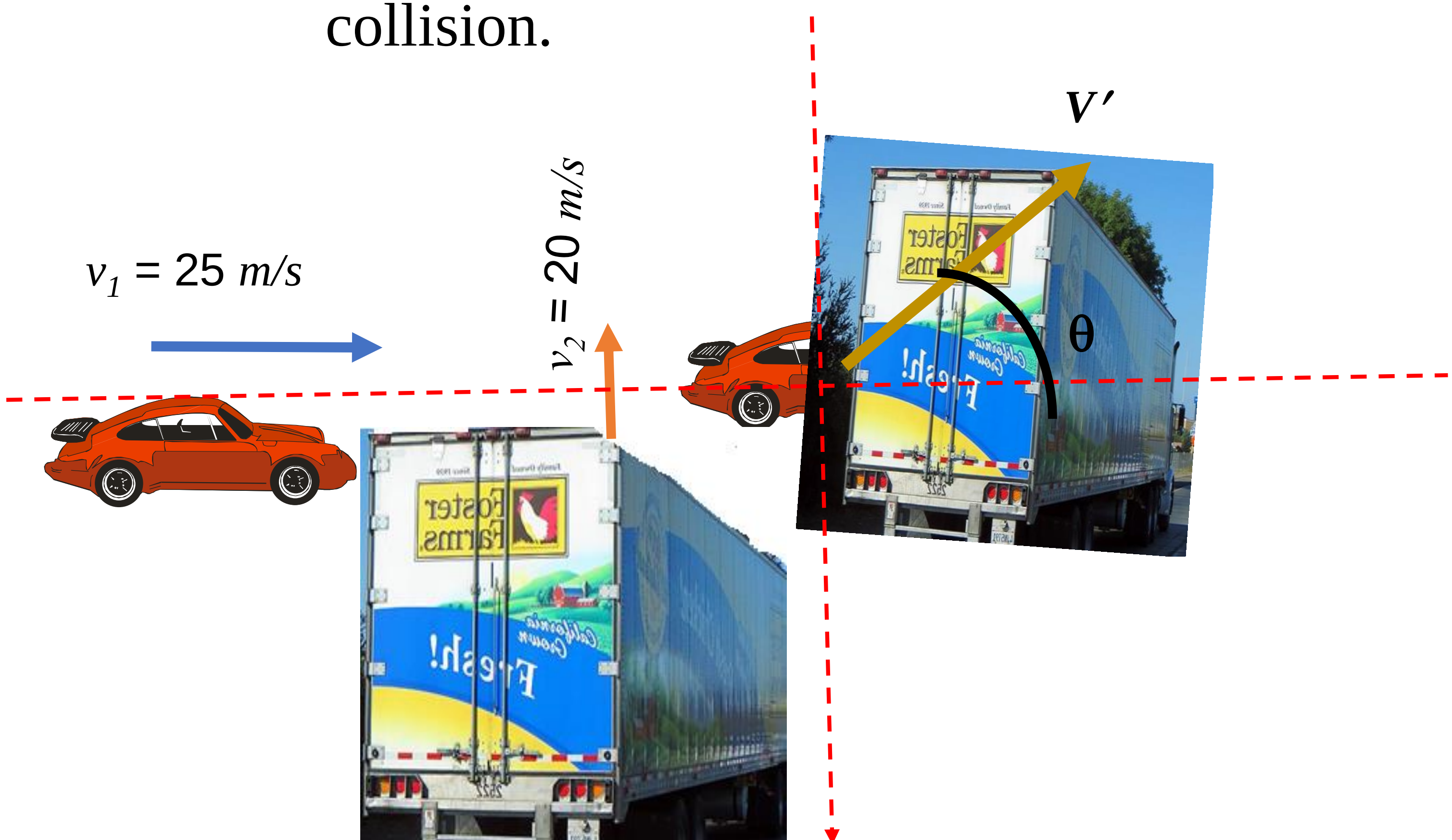
$$v_{01} = +896m/s$$

By Prof. Rashad Badran

Perfectly Inelastic Collision

Problem

A 1500-kg car travelling east with a speed of 25 *m/s* collides at an intersection with a 2500 kg truck travelling north with a speed of 20 *m/s*. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming the vehicles stick together after the collision.



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Perfectly Inelastic Collision

Solution

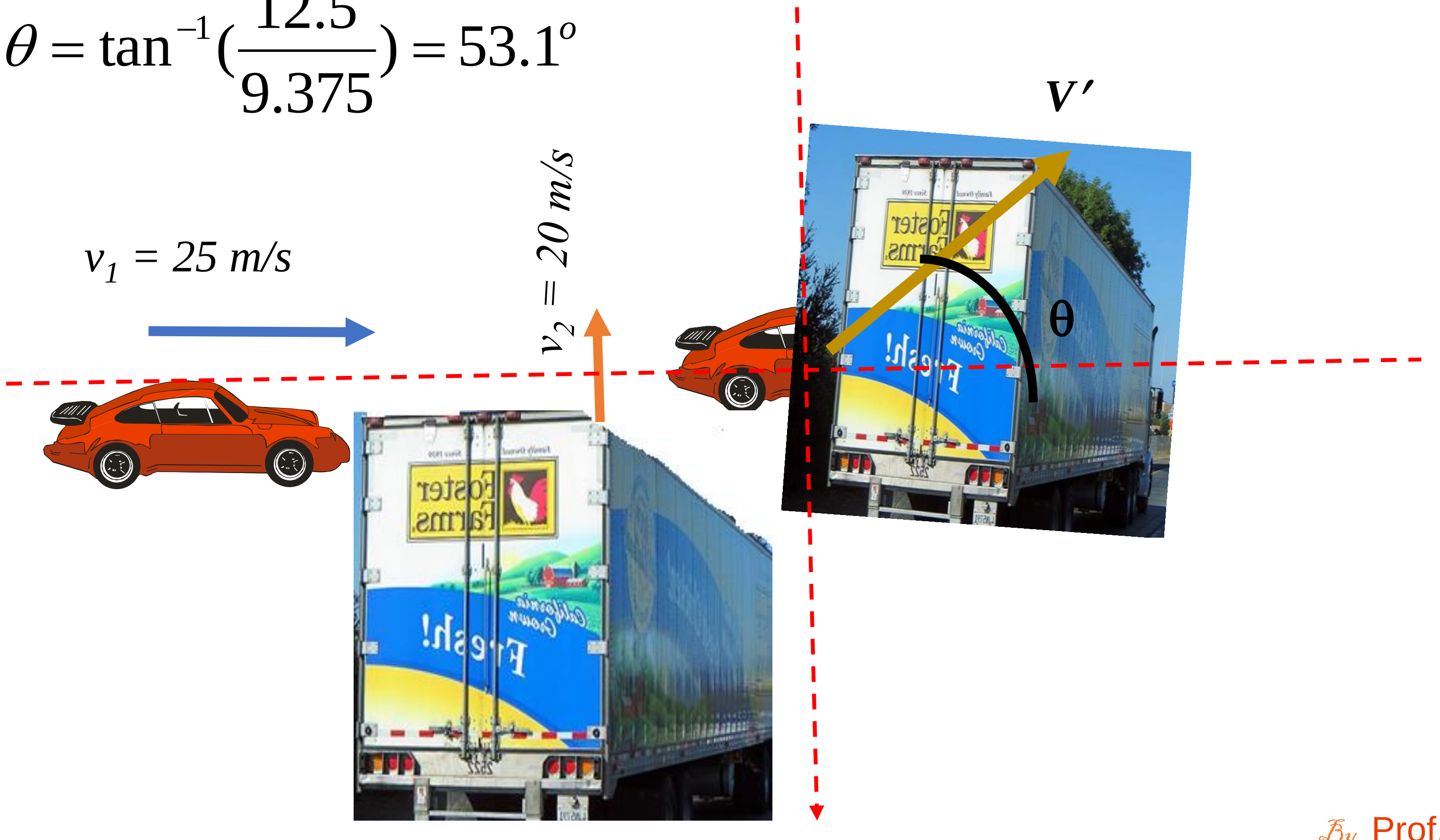
$$m_{car}\vec{v}_{car} + m_{truck}\vec{v}_{truck} = (m_{car} + m_{truck})\vec{V}'$$

$$m_{car} = 1500kg, m_{truck} = 2500kg, \quad \vec{v}_{car} = 25\hat{i}, \quad \vec{v}_{truck} = 20\hat{j}$$

$$(1500)(25\hat{i}) + (2500)(20\hat{j}) = (4000)\vec{V}'$$

$$\vec{V}' = (9.375\hat{i}) + 12.5\hat{j})m/s \qquad V' = \sqrt{(9.375)^2 + (12.5)^2} = 15.6m/s$$

$$\theta = \tan^{-1}(\frac{12.5}{9.375}) = 53.1^o$$



Perfectly Inelastic Collision

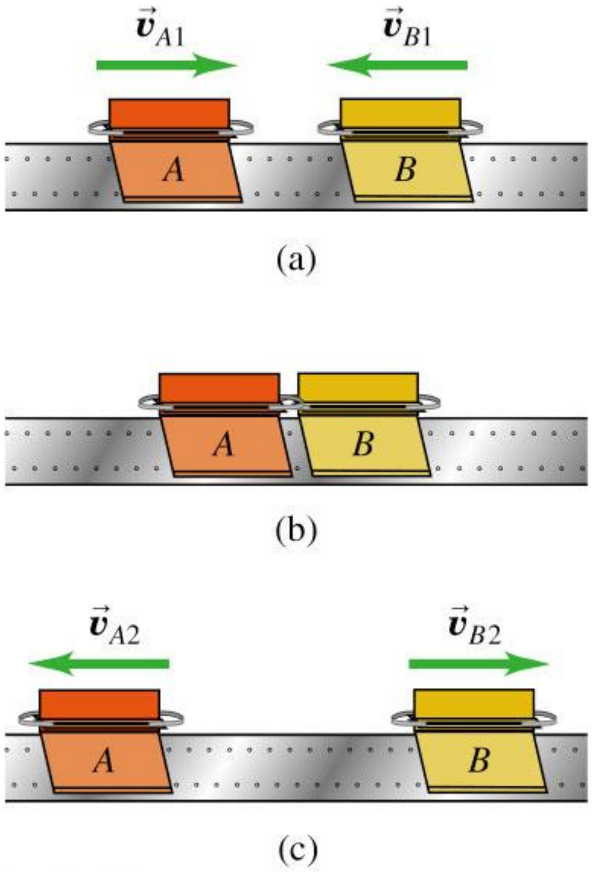
Exercise

A 90-kg fullback running east with a speed of 5 *m/s* is tackled by a 95-kg opponent running north with a speed of 3 *m/s*. (a) Explain why the successful tackle constitutes a perfectly inelastic collision. (b) Calculate the velocity of the players immediately after the tackle. (c) Determine the mechanical energy that disappears as a result of the collision. Account for the missing energy,

Elastic Collision

Elastic collision:

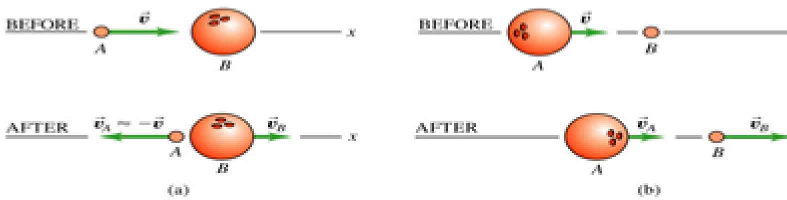
- Glider A and glider B approach each other on a frictionless surface
- Each glider has a steel spring bumper on the end to ensure an elastic collision



Elastic Collision

Elastic 1-D collision of two bodies A and B, B is at rest before collision

- ***Momentum*** of the system is ***conserved***
- ***Kinetic energy*** of the system is ***conserved***



$$m_A v_{A1x} + m_B(0) = m_A v_{A2x} + m_B v_{B2x} ---(1)$$

$$\frac{1}{2}m_A v_{A1x}^2 + \frac{1}{2}m_B(0)^2 = \frac{1}{2}m_A v_{A2x}^2 + \frac{1}{2}m_B v_{B2x}^2 --(2)$$

$$m_B v_{B2x}^2 = m_A(v_{A1x}^2 - v_{A2x}^2) = m_A(v_{A1x} - v_{A2x})(v_{A1x} + v_{A2x}) --(3)$$

From equation (1), one can have

$$m_B v_{B2x} = m_A(v_{A1x} - v_{A2x}) ----------(4)$$

Divide equations (3) by equation (4) to get

$$v_{B2x} = v_{A1x} + v_{A2x} -----(5)$$

From equation (5) into equation (4) we can have

$$m_B(v_{A1x} + v_{A2x}) = m_A(v_{A1x} - v_{A2x}) ---(6)$$

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x} -----(7)$$

From equation (7) into equation (5) one can have

$$v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x} ------(8)$$

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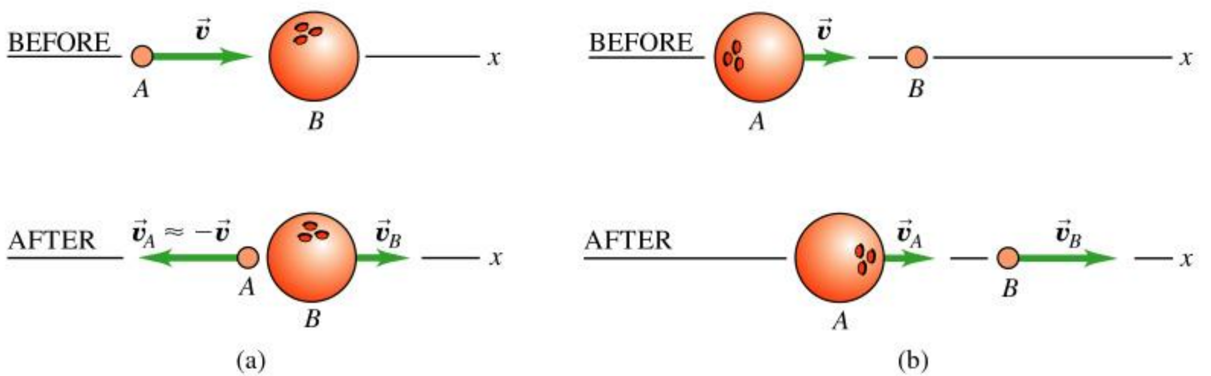
Elastic Collision

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x}$$

$$v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x}$$

Remarks on some Cases of Elastic collision

- Suppose body A is a ping-pong ball, and body B is bowling ball
- We expect A to bounce of after the collision with almost the same speed but opposite direction, and speed of B will be much smaller
- What if situation is reversed? Bowling ball hits ping-pong ball?



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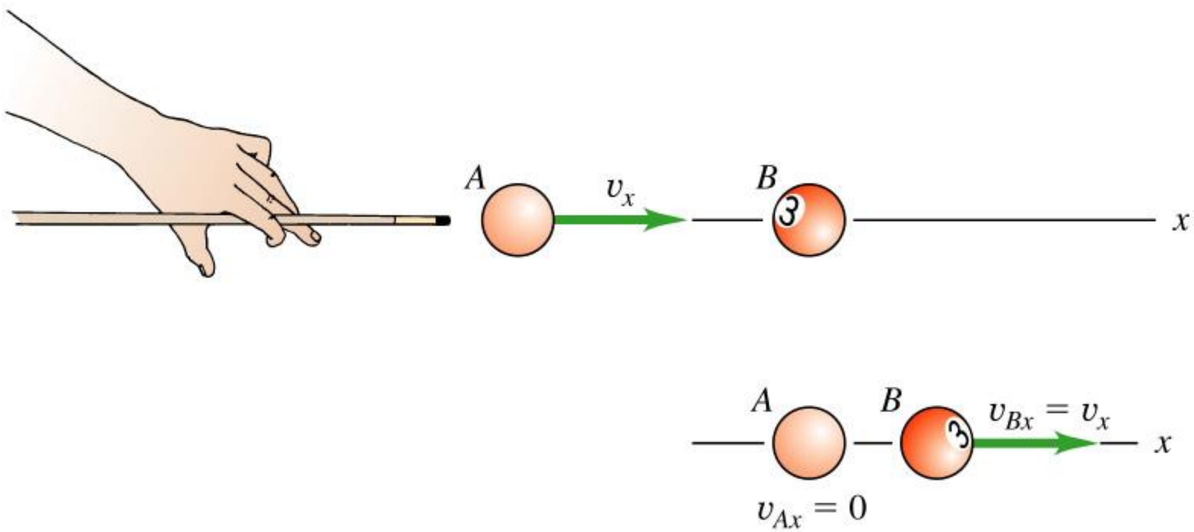
Elastic Collision

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x}$$

$$v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x}$$

The case of equal masses

- Suppose masses of bodies A and B are ***equal***
- Then $m_A = m_B$, and $V_{A2x}=0$, $V_{B2x}=V_{A1x}$



Elastic Collision

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x}$$

$$v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x}$$

Relative Velocities Before and After Collision

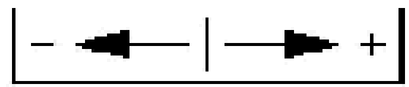
$$v_{B2x} = v_{A1x} + v_{A2x} \longrightarrow v_{B2x} - v_{A2x} = v_{A1x}$$

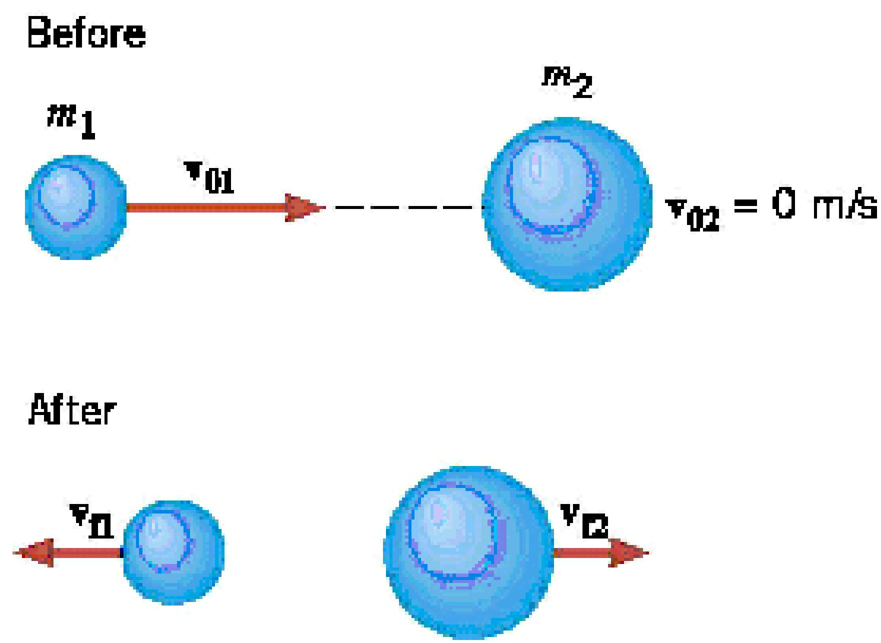
- ***v_{B2x}-v_{A2x}*** is relative velocity of B to A after the collision, and it is negative of the velocity of B relative to A before the collision
- ***In a straight-line elastic collision of two bodies, the relative velocities before and after collision have the same magnitude, but opposite sign***
- General case (when velocity of B is not zero):

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$$

Elastic Collision

Problem A ball of mass m_1 = 0.25kg and velocity v_{01}= 5*m/s* collides head-on with a ball of mass m_2 = 0.8kg that is initially at rest (v_{02} = 0). No external forces act on the balls. If the collision in elastic, what are the velocities of the balls after the collision?



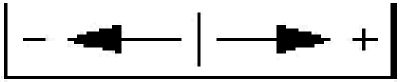


By Prof. Rashad Badran

Elastic Collision

A ball of mass m_1 = 0.25 kg and velocity v_{01} = 5 *m/s* collides head-on with a ball of mass m_2 = 0.8 kg that is initially at rest (v_{02} = 0). No external forces act on the balls. If the collision in elastic, what are the velocities of the balls after the collision?

Solution



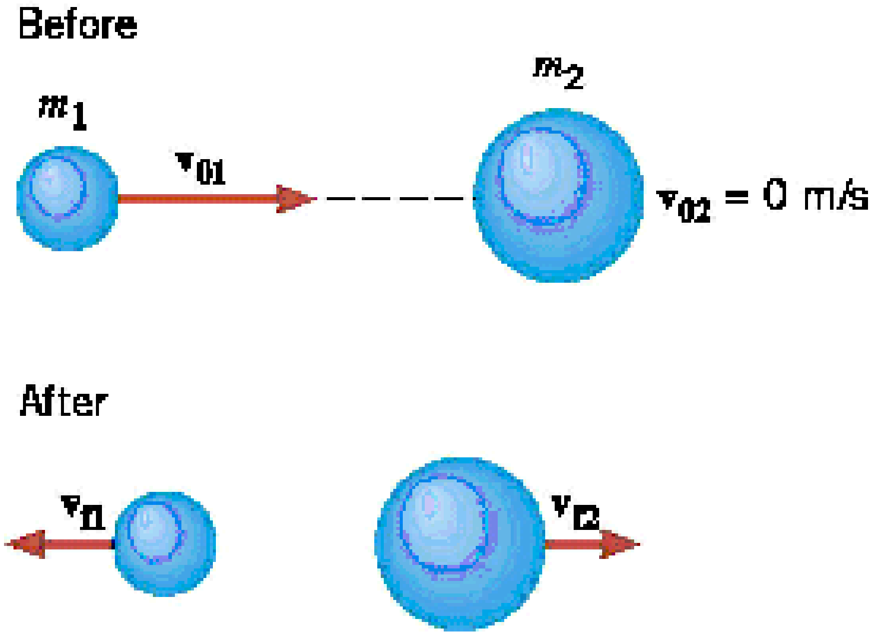
$$\underbrace{m_1 v_{f1} + m_2 v_{f2}}_{\text{after collision}} = \underbrace{m_1 v_{01} + 0}_{\text{before collision}} ---(1)$$

after collision **before collision**

$$\underbrace{\frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2}_{\text{collision}} = \underbrace{\frac{1}{2} m_1 v_{01}^2 + 0}_{\text{before collision}} ---(2)$$

collision **before collision**

$$v_{f_1} = \frac{m_1 v_{01} - m_2 v_{f_2}}{m_1} ---(3)$$



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Elastic Collision

Solution

Substitute equation (3) into (2)

$$\frac{1}{2}m_1\left(\frac{m_1v_{0_1}-m_2v_{f_2}}{m_1}\right)^2+\frac{1}{2}m_2v_{f_2}^2=\frac{1}{2}m_1v_{0_1}^2$$

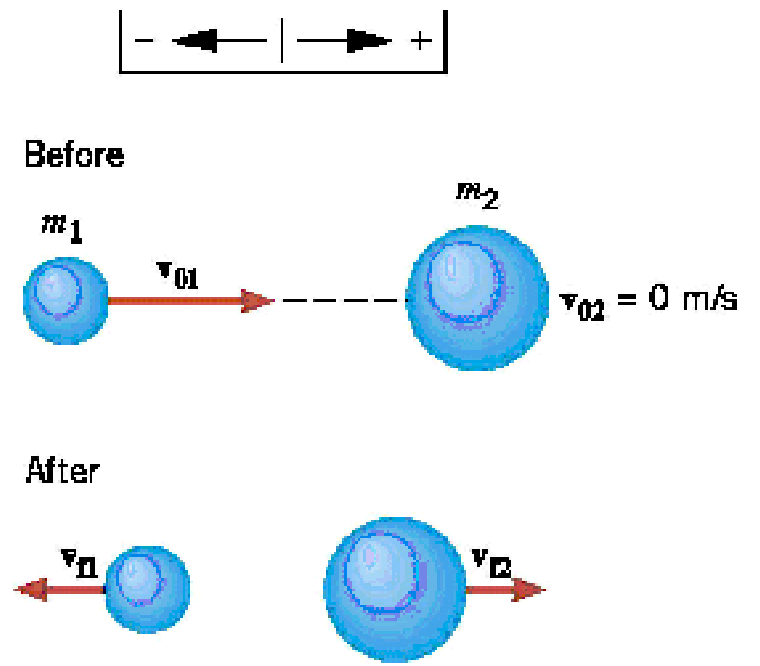
$$\frac{1}{2}\frac{m_1^2v_{0_1}^2+m_2^2v_{f_2}^2-2m_1m_2v_{0_1}v_{f_2}}{m_1}+\frac{1}{2}\frac{m_2m_1v_{f_2}^2}{m_1}=\frac{1}{2}m_1v_{0_1}^2$$

$$\frac{m_1^2v_{0_1}^2}{2m_1}-\frac{m_1v_{0_1}^2}{2}+\left(\frac{m_2^2+m_1m_2}{2m_1}\right)v_{f_2}^2-\frac{2m_1m_2v_{0_1}v_{f_2}}{2m_1}=0$$

$$\frac{m_2}{2m_1}(m_1+m_2)v_{f_2}^2=\frac{2m_1m_2}{2m_1}v_{0_1}v_{f_2}$$

$$(m_2+m_1)v_{f_2}{}^2=2m_1v_{0_1}v_{f_2}$$

$$v_{f_2}=\frac{2m_1}{m_1+m_2}v_{0_1}$$



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Elastic Collision

Solution

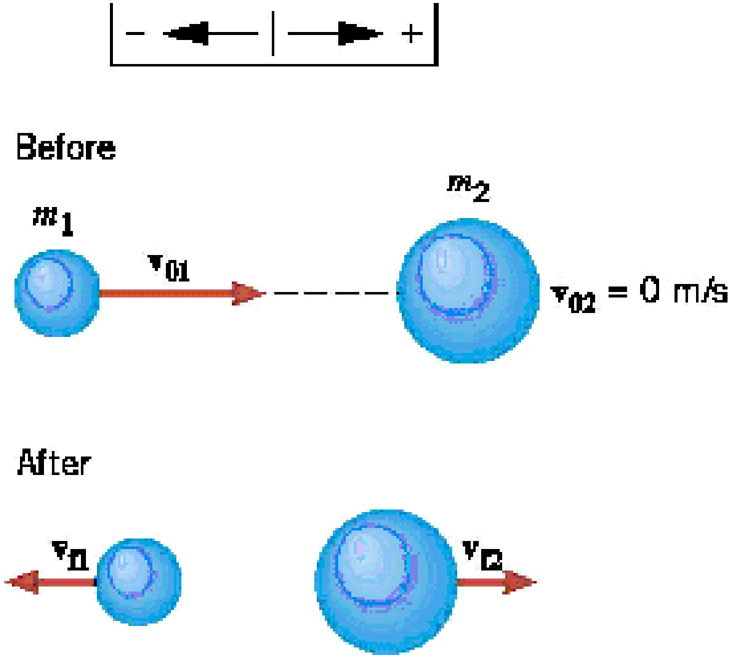
Substitute $v_{f_2} = \dfrac{2m_1}{m_1 + m_2} v_{0_1}$ into $m_1 v_{f1} = m_1 v_{01} - m_2 v_{f2} - - - (1)$

$$m_1 v_{f_1} = m_1 v_{0_1} - m_2 \frac{2m_1}{m_1 + m_2} v_{0_1}$$

$$m_1 v_{f_1} = m_1 \left(1 - \frac{2m_2}{m_1 + m_2} \right) v_{0_1}$$

$$m_1 v_{f_1} = m_1 \left[\frac{m_1 + m_2 - 2m_2}{m_1 + m_2} \right] v_{0_1}$$

$$v_{f_1} = \left[\frac{m_1 - m_2}{m_1 + m_2} \right] v_{0_1}$$

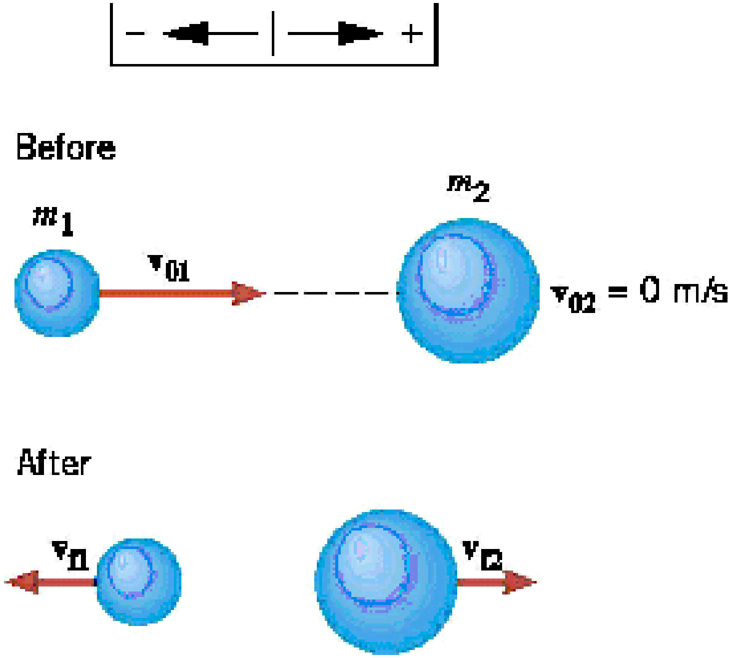


Elastic Collision

Solution

We have $$v_{f_2} = \frac{2m_1}{m_1 + m_2} v_{0_1}$$ $$v_{f_1} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{0_1}$$

$m_1 = 0.25$ kg, $m_2 = 0.8$ kg $v_{01} = 5$ m/s, $v_{02} = 0$



$$v_{f_1} = \left(\frac{0.25 - 0.8}{0.25 + 0.8}\right) 5 = -2.62 m/s$$

$$v_{f_2} = \frac{2 \times 0.25}{0.25 + 0.8} \times 5 = 2.38 m/s$$

By Prof. Rashad Badran

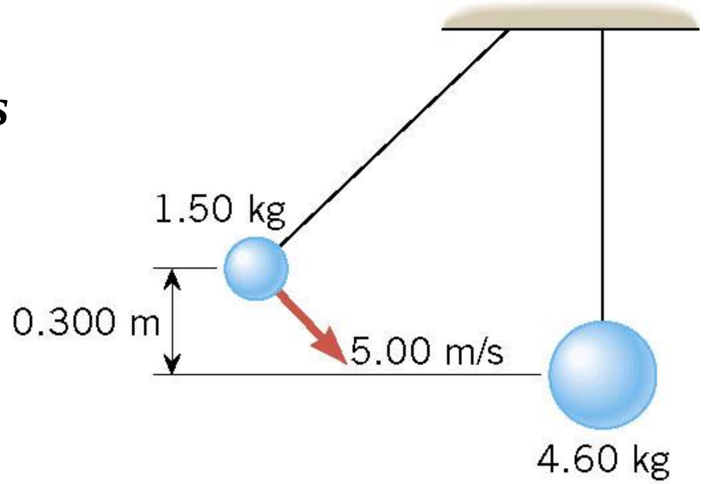
Elastic Collision

Exercise

A ball of mass m_1 = 1.5 kg attached to a string is released with a speed of 5 *m/s* from the position shown at height h = 3 *m* above the horizontal. Another ball of mass m_2 = 10 kg, initially at rest, is attached to another string. Assuming the collision is elastic, (a) find the final velocities of the two balls immediately after collision. (b) Calculate the maximum heights to which m_1 and m_2 rise after the elastic collision.

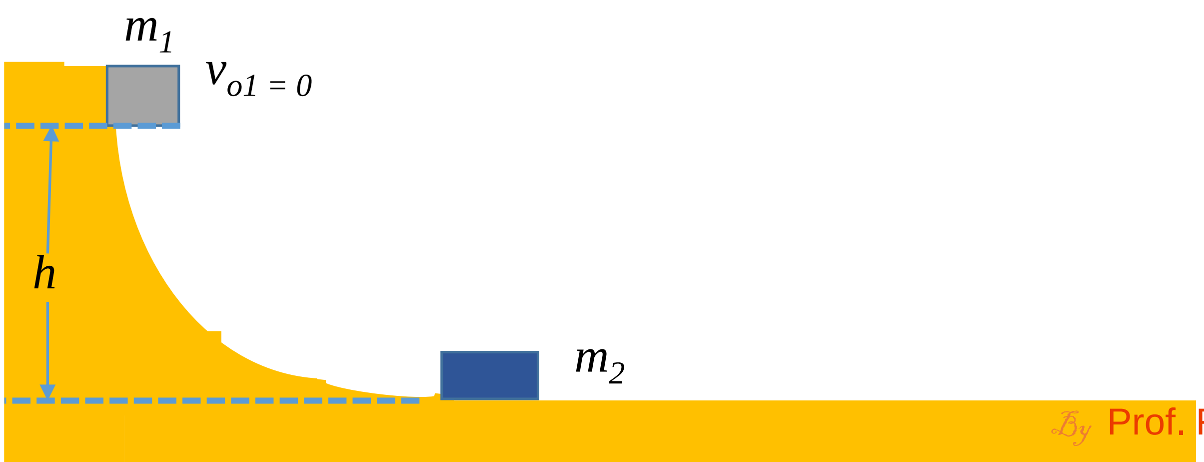
Answer: **(a)** $v_{f1} = -2.83$ ***m/s***, $v_{f2} = +2.73$ ***m/s***

(b) $h_{f1} = 0.409$ ***m***, $h_{f2} = 0.38$ ***m***



Elastic Collision

Problem Two blocks are free to slide along the frictionless, wooden track shown in the figure. The block of mass m_1 = 5 kg is released from the position shown at height h = 5 m above the flat part of the track. Protruding from its front end is the north pole of a strong magnet which repels the north pole of an identical magnet embedded in the back end of the block of mass m_2 = 10 kg, initially at rest. The two blocks never touch. Calculate the maximum height to which m_1 rises after the elastic collision.



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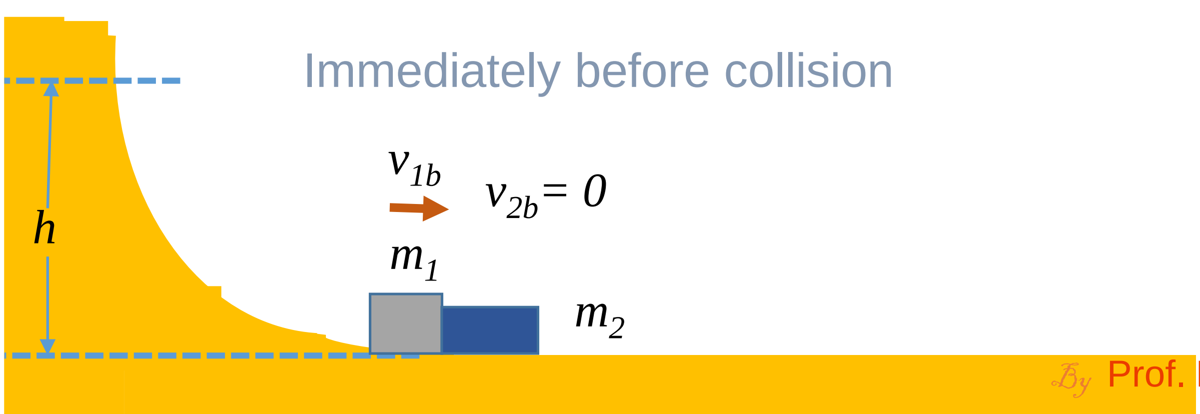

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Elastic Collision

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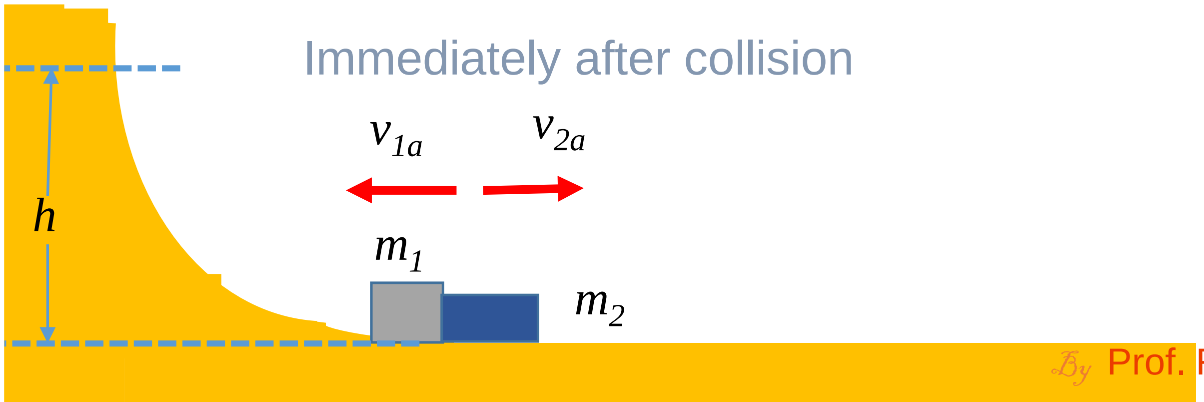

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Elastic Collision

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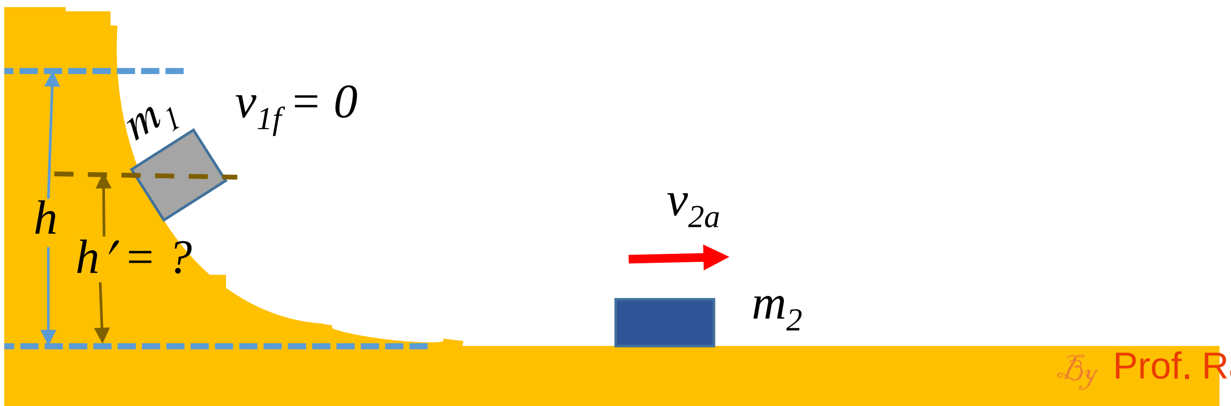


Elastic Collision

Solution

Mass m_1 is released from rest and falls down from a height $h = 5\ m$:
Apply conservation of mechanical energy

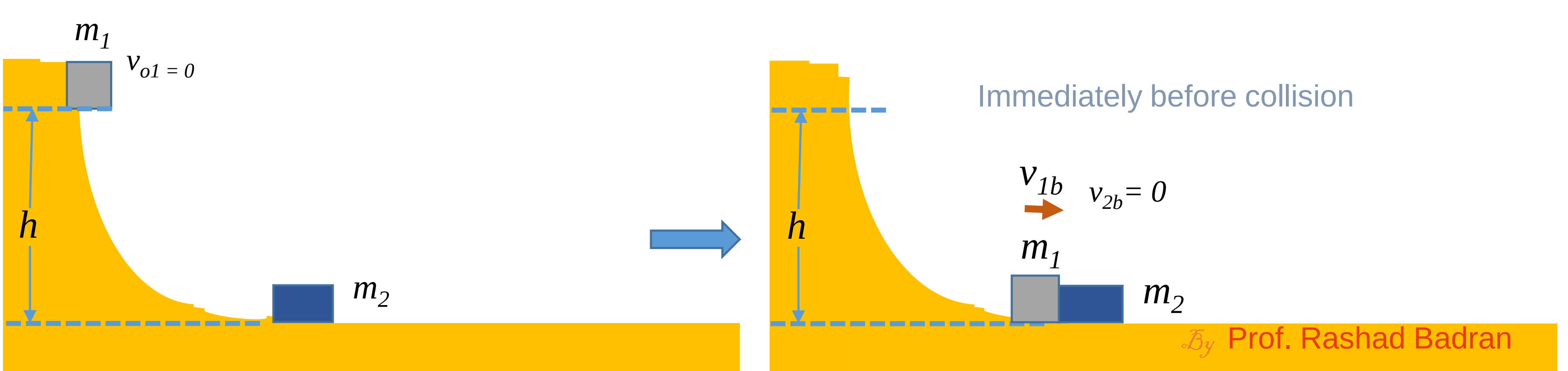
$$\Delta U + \Delta K = 0$$

$$-m_1gh + (\frac{1}{2}m_1v_{1b}^2 - \frac{1}{2}m_1v_{01}^2) = 0$$

$v_{2b} = 0,\ h = 5\ m$

$$\Rightarrow m_1gh = \frac{1}{2}m_1v_{1b}^2$$

$$\Rightarrow v_{1b} = \sqrt{2gh} = \sqrt{2(9.8m/s^2)(5m)} = 10m/s$$



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Elastic Collision

Solution

Mass m_1 with speed v_{1b} = 10 *m/s* elastically collides with mass m_2 which is at rest (v_{2b} = 0): Apply conservation of momentum

$$m_1 v_{1b} + m_2 v_{2b} = m_1 v_{1a} + m_2 v_{2a}$$

But m_1 = 5 kg, m_2 = 10 kg, v_{1b} = 10 m/s, v_{2b} = 0

$$\therefore (5kg)(10m/s) + (10kg)(0) = (5kg)v_{1a} + (10kg)v_{2a}$$

$$or \quad v_{2a} = \frac{3 - v_{1a}}{2} ---(1)$$

Apply conservation of kinetic energy

$$\frac{1}{2} m_1 v_{1b}^2 + \frac{1}{2} m_{2b} v_{2b}{}^2 = \frac{1}{2} m_1 v_{1a}^2 + \frac{1}{2} m_2 v_{2a}^2$$

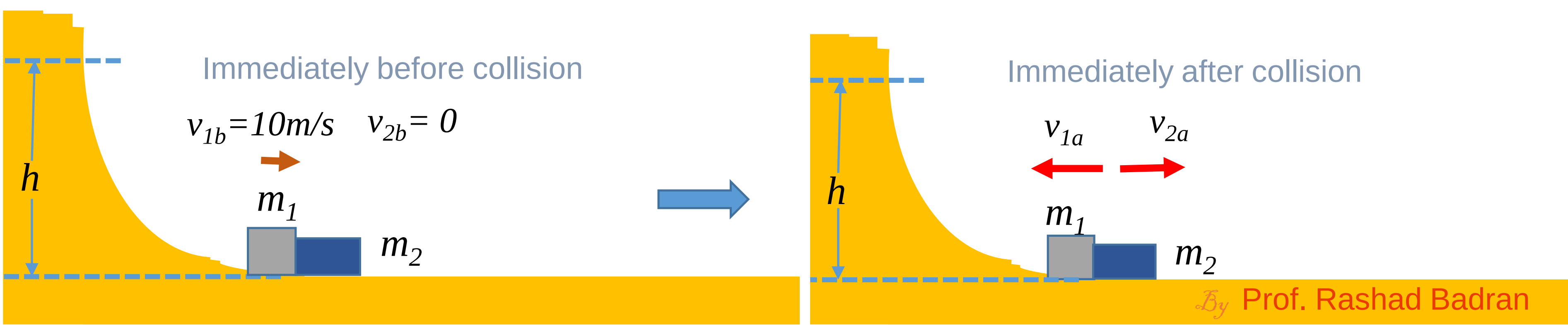
$$(5kg)(10m/s)^2 + (10kg)(0) = (5kg)v_{1a}^2 + (10kg)v_{2a}^2$$

$$or \quad 100 = v_{1a}^2 + 2v_{2a}^2 --(2)$$

From equation (1) into (2), one can get:

$$\Rightarrow 3v_{1a}^2 - 6v_{1a} - 191 = 0$$

$$\Rightarrow v_{1a} = -7m/s$$



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Elastic Collision

Solution

For mass m_1 with speed v_{1a} = - 7 m/s goes up the frictionless track until it reaches a maximum height h' (with v_{1f} = 0): Apply conservation of mechanical energy

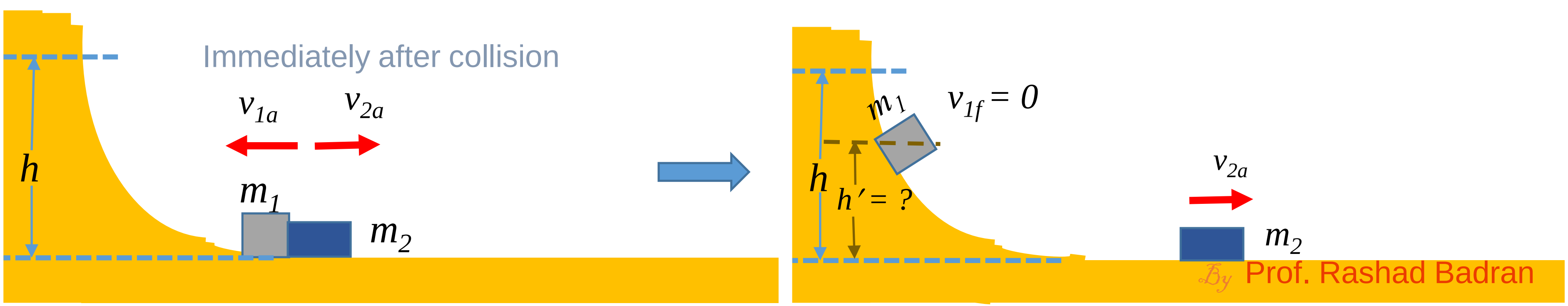
$$\Delta U + \Delta K = 0$$

$$m_1 g h' + \left(\frac{1}{2} m_1 v_{1f}^2 - \frac{1}{2} m_1 v_{1a}^2\right) = 0$$

$$v_{1a} = \text{-7 m/s},\ v_{1f} = 0$$

$$\Rightarrow g h' + \left(0 - \frac{1}{2} v_{1a}^2\right) = 0$$

$$or \quad h' = \frac{v_{1a}^2}{2g} = \frac{(7m/s)}{2(9.8m/s^2)} = 2.5m$$



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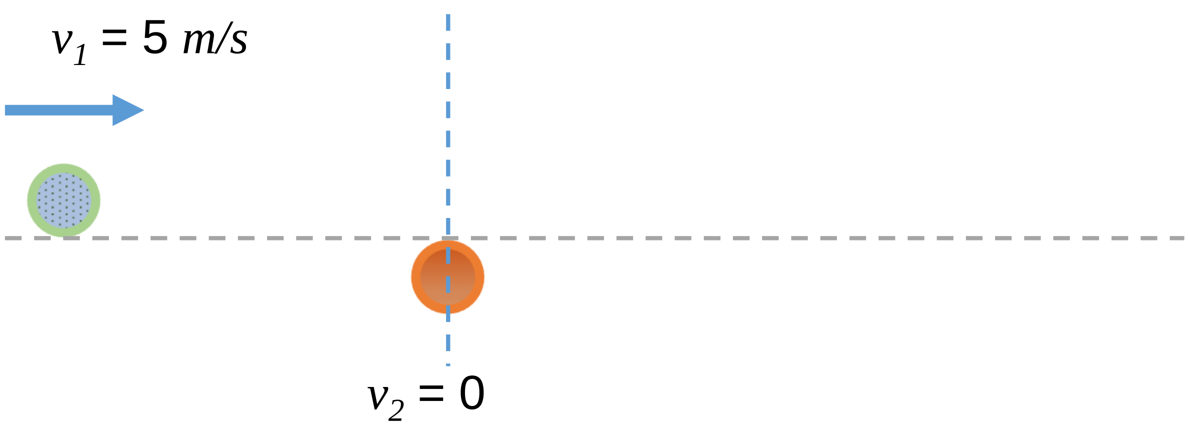
Elastic Collision

Problem A billiard ball moving at 5 *m/s* strikes a stationary ball of the same mass. After the collision, the first ball moves at 4.33 *m/s* at an angle of 30^o with respect to the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion), find the struck ball's velocity after the collision.



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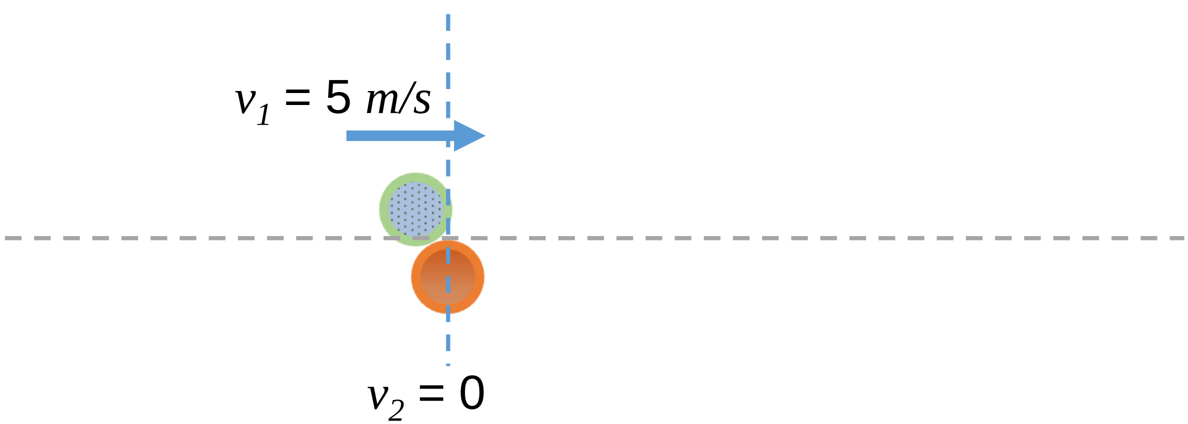

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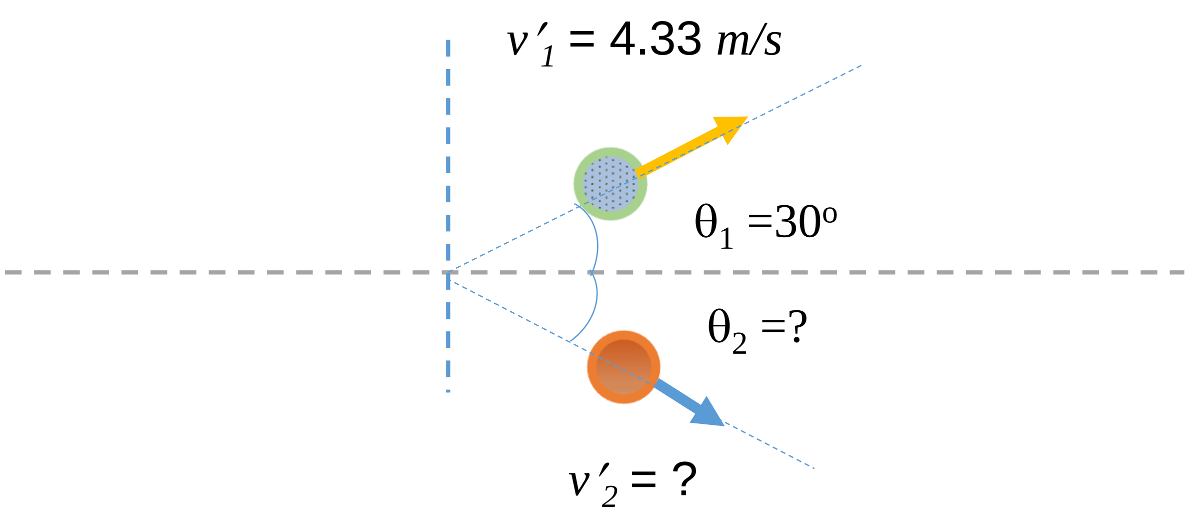

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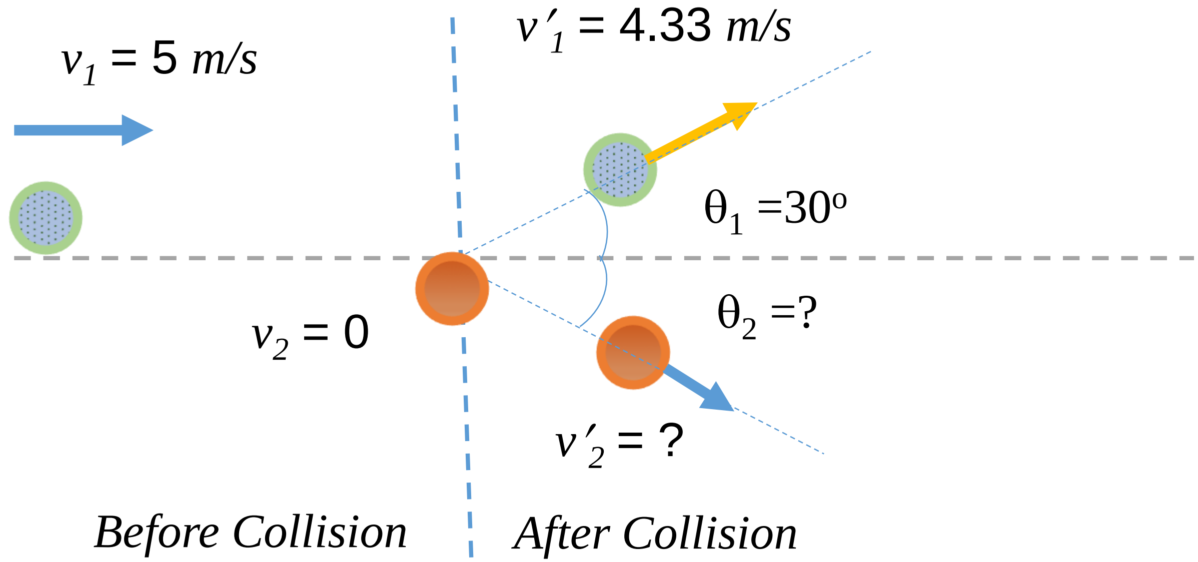

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Elastic Collision

Solution

$$(m_1\vec{v}_1 + m_2\vec{v}_2) = (m_1\vec{v}_1' + m_2\vec{v}_2')$$

$m_1 = m_2 = m$

$$(v_{1x} + v_{2x}) = (v_{1x}' + v_{2x}')$$

$$(v_{1y} + v_{2y}) = (v_{1y}' + v_{2y}')$$

$v_{1x} = v_1,\; v_{2x} = 0,\; v_{1x}' = v_1'\cos\theta_1, v_{2x}' = v_2'\cos\theta_2,\; v_{1y} = v_{2y} = 0,\; v_{1y}' = v_1'\sin\theta_1, v_{2y}' = v_2'\sin\theta_2$

$$v_1 = v_1'\cos\theta_1 + v_2'\cos\theta_2$$

$$5 = (4.33)\cos 30 + v_2'\cos\theta_2$$

$$v_2'\cos\theta_2 = 1.25 - - - (1)$$

$$0 = v_1'\sin\theta_1 - v_2'\sin\theta_2$$

$$0 = (4.33)\sin 30 - v_2'\sin\theta_2$$

$$v_2'\sin\theta_2 = 2.165 - - - (2)$$

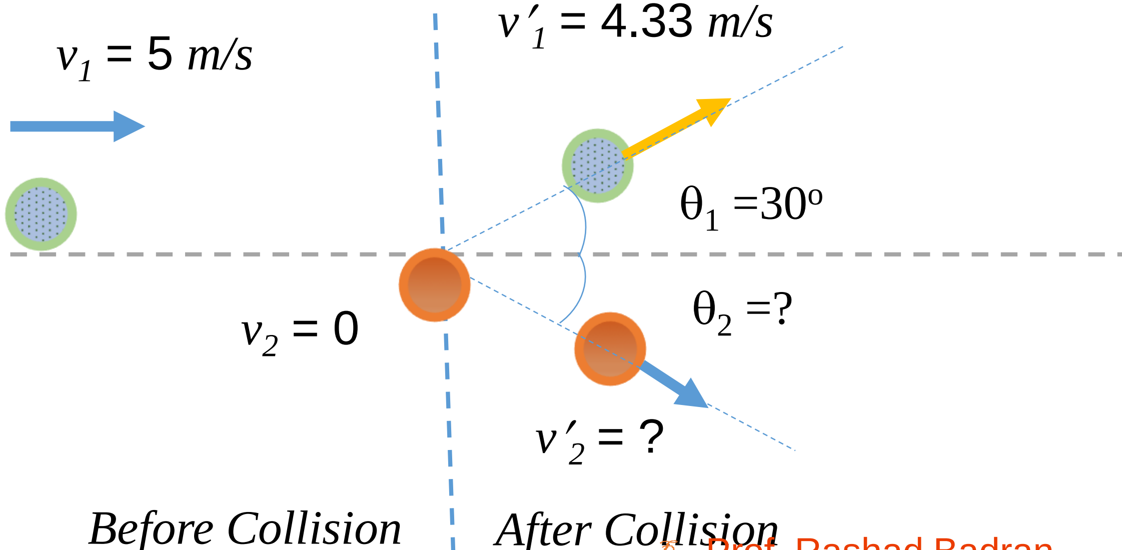
Divide equation (2) by (1)

$$\tan\theta_2 = 2.165$$

$$\theta_2 = \tan^{-1}(1.73) = 60^o$$

Substitute $\theta_2 = 60^o$ into equation (1)

$$v_2' = \frac{1.25}{\cos 60} = 2.5 m/s$$

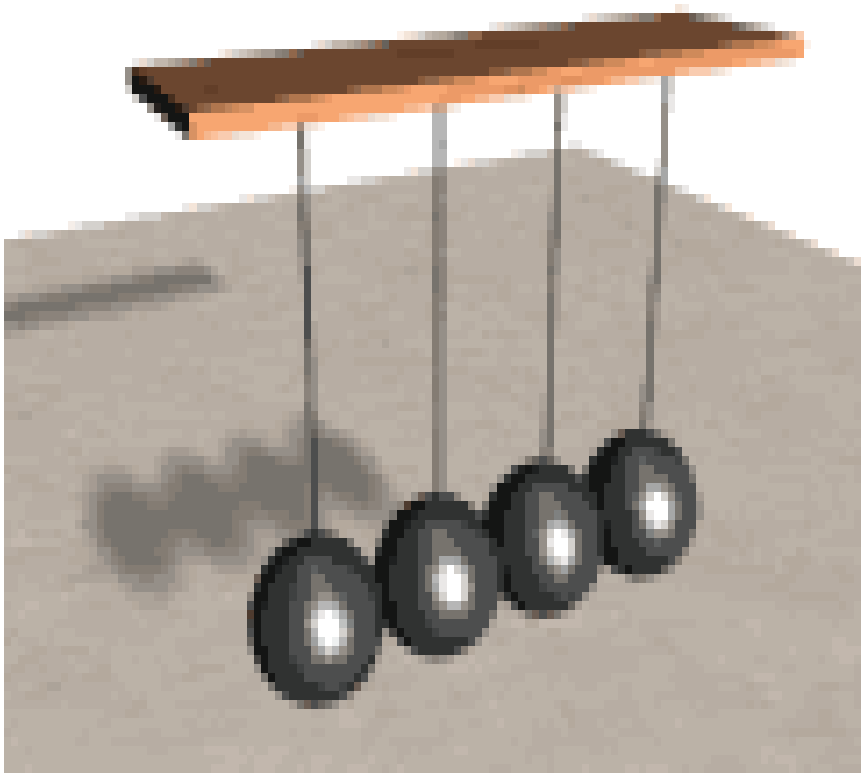


By Prof. Rashad Badran

Conservation of Total Momentum

The executive stress reliever

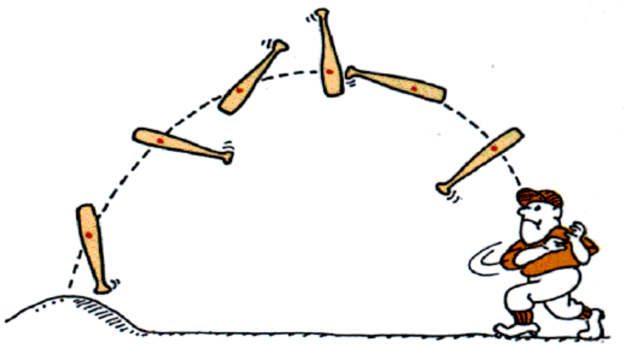
This device illustrates laws of conservation of momentum and kinetic energy. As shown it consists of four identical hard balls supported by strings of equal lengths. When ball 1 is pulled and released after almost-elastic collision between it and ball 2, ball 1 stops and ball 4 moves out as shown.

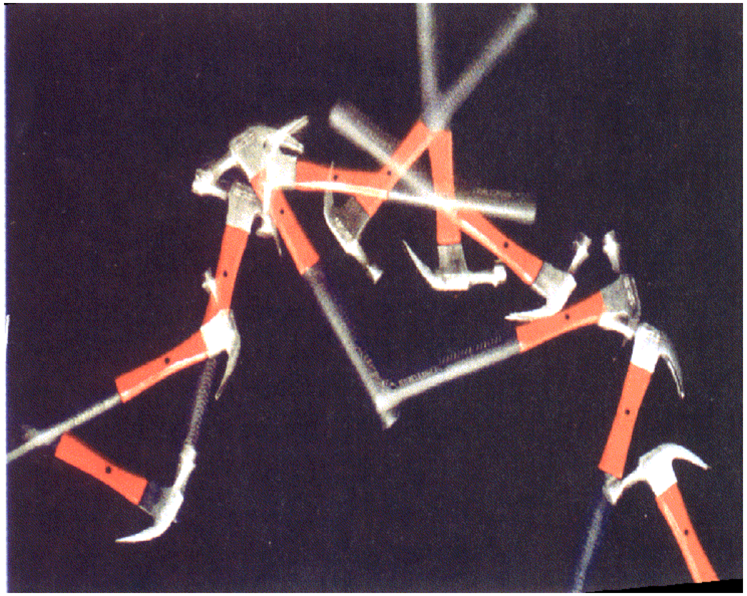


Exercise: (a)Try to do your homework by explaining why balls 2 and 3 are at rest.
(b) What is the velocity of ball 4 as it immediately leaves its position.

By Prof. Rashad Badran

Center of Mass





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Center of Mass

Consider ***several particles*** with masses m_1, m_2 and so on.

- (x_1, y_1) are coordinates of m_1, (x_2, y_2) are coordinates of m_2.
- ***Center of mass*** of the system is the point having the coordinates (x_{cm}, y_{cm}) given by

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

Center of Mass

- The center of mass of a system of *N* particles with masses m_1, m_2, m_3, etc. and positions, (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), etc. is defined in terms of the particle's position vectors using

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$$

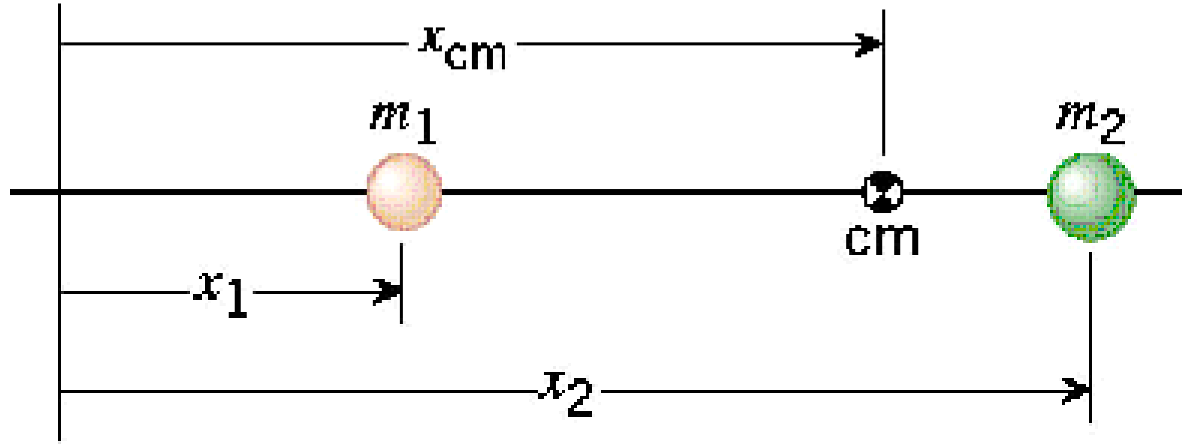
- Center of mass is a ***mass-weighted average position*** of particles
- If this expression is differentiated with respect to time then the position vectors become velocity vectors and we have

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum_i m_i\vec{v}_i}{\sum_i m_i}$$

Center of Mass

Problem

Find the *c.m.* of the system shown in the figure.



Solution

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Suppose m_1=5kg, m_2=12kg

x_1=2m, x_2=6m

$$x_{cm} = \frac{(5.0kg)(2.0m) + (12kg)(6.0m)}{5.0kg + 12kg} = 4.8m$$

Center of Mass

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum_i m_i\vec{v}_i}{\sum_i m_i}$$

Total mass $M = m_1 + m_2 + m_3 + \ldots$

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots = \vec{P}$$ Total moment of the system

- ***Total moment is equal to total mass times velocity of the center of mass***
- For a system of particles on which the net external force is zero, so that the total momentum is constant, the velocity of the center of mass is also constant

Spinning throwing knife on ice: center of mass follows straight line

By Prof. Rashad Badran

External Forces and Center-of-Mass Motion

- If we differentiate once again, then the derivatives of the velocity terms will be ***accelerations***
- Further, if we use Netwon's Second Law for each particle then the terms of the form $m_i\mathbf{a}_i$ will be the net force on particle *i*.

- When these are summed over all particles any forces that are ***internal*** to the system (i.e. a force on particle *i* caused by particle *j*) will be included in the sum twice and these ***pairs*** will ***cancel*** due to Newton's Third Law. The result will be

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots = \sum \vec{F}_{ext}$$

Body
or collection of particles

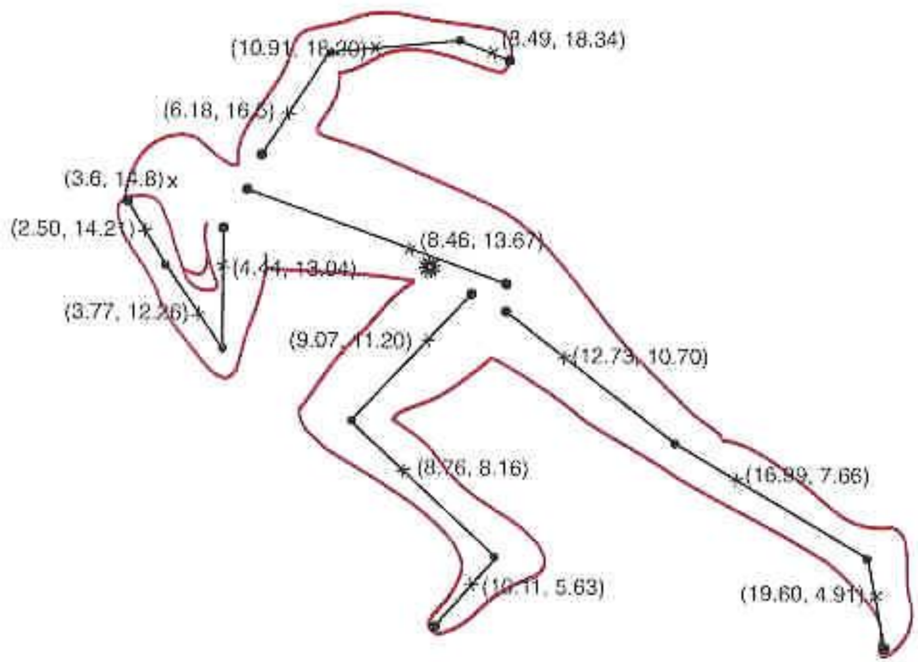
- It states that the ***net external force*** on a system of particles ***equals*** the ***total mass*** of the system ***times*** the ***acceleration of the center of mass*** (Newton's Second Law for a system of particles)

- It can also be expressed as the net external force on a system equals the time rate of change of the total linear momentum of the system.

Center of Mass

The segmental method involves computation of the segmental *cm's*

The whole body *cm* is computed based on the segmental *cm's*



By Prof. Rashad Badran

Center of Mass

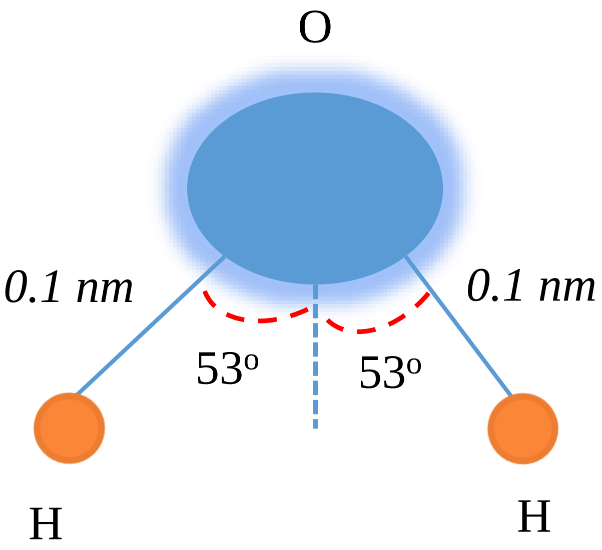
Problem A water molecule consists of an oxygen atom with two hydrogen atoms bound to it. The angle between the two bounds is 106°. If the bounds are 0.1 *nm* long, where is the center of mass of the molecule?

Solution

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$



Center of Mass

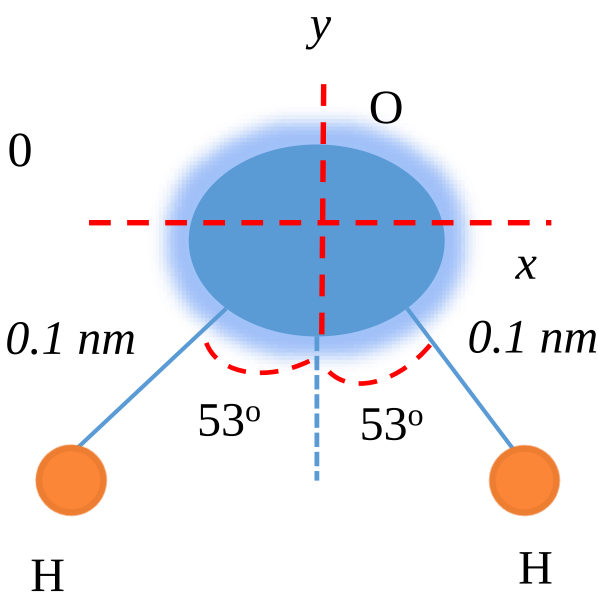
Problem

A water molecule consists of an oxygen atom with two hydrogen atoms bound to it. The angle between the two bounds is 106°. If the bounds are 0.1 *nm* long, where is the center of mass of the molecule?

Solution

$M_O = 16u,\ M_H = 1u,\ x_{HL} = (-0.1)\sin 53),\ x_{HR} = (0.1)\sin 53)$

$$x_{cm} = \frac{(16)(0) + (1)(0.1)\sin 53 - (1)(0.1)\sin 53}{1+1+16} = \frac{0.0798 - 0.0798}{18} = 0$$



$M_O = 16u,\ M_H = 1u,\ y_{HL} = (-0.1)\cos 53),\ y_{HR} = (-0.1)\cos 53)$

$$y_{cm} = \frac{(16)(0) + (1)(-0.1)\cos 53 + (1)(-0.1)\cos 53}{18} = \frac{-0.12}{18} = -0.0066nm$$

$$\therefore r_{cm} = (x_{cm} = 0, y_{cm} = -0.0066nm)$$

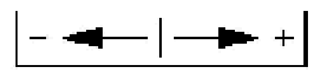
By Prof. Rashad Badran

Center of Mass

Problem A ball of mass m_1= 0.25kg and velocity v_{01}= 5m/s collides head-on with a ball of mass m_2 = 0.8kg that is initially at rest (v_{02}= 0). No external forces act on the balls. If the collision in elastic, what are the velocities of the center of mass before and after the collision?

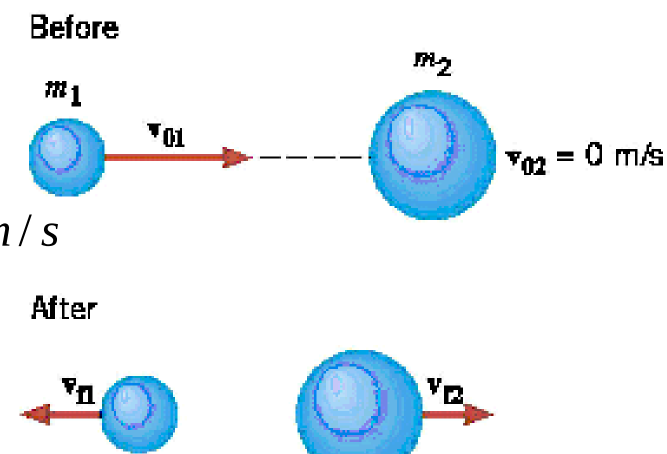
Solution **Before collision**

$$v_{cm} = \frac{(0.250kg)(+5.00m/s) + (0.800kg)(0m/s)}{0.250kg + 0.800kg} = +1.19m/s$$



After collision

$$v_{cm} = \frac{(0.250kg)(-2.62m/s) + (0.800kg)(+2.38m/s)}{0.250kg + 0.800kg} = +1.19m/s$$



Center of Mass

Problem

A 2-kg particle has a velocity $\vec{v}_{2kg} = (2\hat{i} - 3\hat{j})m/s$ and a 3-kg particle has a velocity $\vec{v}_{3kg} = (1\hat{i} - 6\hat{j})m/s$ Find (a) the velocity of the center of mass and (b) the total momentum of the system.

Solution

(a)
$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\sum_i m_i\vec{v}_i}{\sum_i m_i}$$

$$\vec{v}_{cm} = \frac{(2kg)(2\hat{i} - 3\hat{j})m/s + (3kg)(1\hat{i} - 6\hat{j})}{(2+3)kg} = \frac{7}{5}\hat{i} - \frac{24}{5}\hat{j}$$

(b)
$$\therefore M\vec{v}_{cm} = (2+3)(kg)(\frac{7}{5}\hat{i} - \frac{24}{5}\hat{j})m/s = (7\hat{i} - 24\hat{j})kg.m/s$$

Center of Mass

Problem

Solution

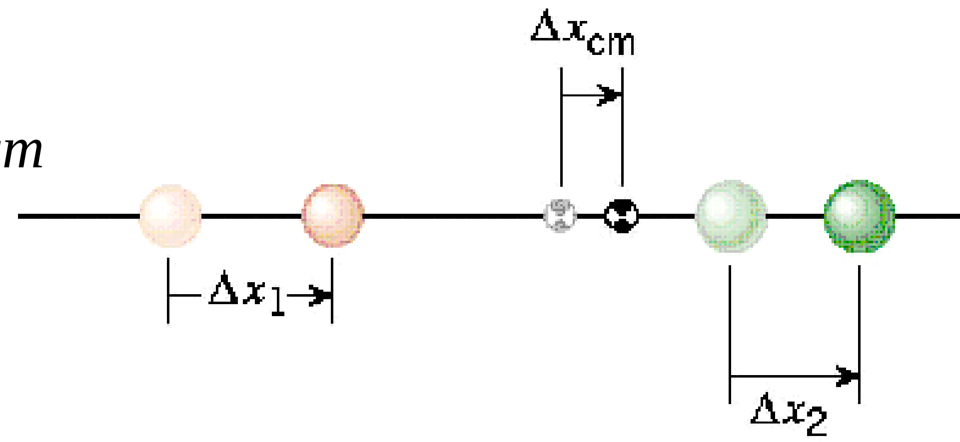
During a time interval Δt a particle of mass m_1 is displaced along the x-axis by Δx_1 with a velocity v_1, while another particle of mass m_2 is displaced by Δx_2 and with a velocity v_2. Find (a) the displacement of the center of mass, and (b) the velocity of center of mass of the system.

[Given Δt = 0.2 s, $\Delta x_1 = 4cm$, $\Delta x_2 = 2cm$, $m_1 = 2kg$, $m_1 = 3kg$]

(a)

$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

$$\Delta x_{cm} = \frac{(2kg)(4cm) + (3kg)(2cm)}{(2+3)kg} = 2.8cm$$



(b)

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$v_{cm} = \frac{(2kg)(20cm/s) + (3kg)(10cm/s)}{(2+3)kg} = 14cm/s$$