General

Physics 101

Conservation of Energy

Prepared By

Prof. Rashad Badran

Conservation of Energy

- Conservation of Mechanical Energy (Absence of Nonconservative Forces)
- Conservation of Energy (Presence of Nonconservative Forces and Other Forces)

Conservative of Energy: Involving Conservative Forces Only

- A force is conservative if the work it does on an object moving between two points is independent of the path the object takes between the points
  - ✓ The work depends only upon the initial and final positions of the object (it does not depend on the movement path)
  - ✓ The work done by this force in a closed path is zero
  - ✓ Any conservative force can have a potential energy function associated with it
  - ✓ Work done by gravity $W_g = PE_i - PE_f = mgy_i - mgy_f$ or $W_g = U_{gi} - U_{gf} = mgy_i - mgy_f$
  - ✓ Work done by spring force $W_s = PE_{si} - PE_{sf} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$ or $W_s = U_{si} - U_{sf} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

$$\therefore W_{conservativeforce} = -(U_f - U_i) = -\Delta U$$

Conservative of Energy: Involving Conservative Forces Only

$$W_{net} = K_f - K_i = \Delta K$$

$$W_{conservativeforce} = -(U_f - U_i) = -\Delta U$$

$$W_{net} = W_{conservativeforce} = W_c$$

The conservation of mechanical energy is

$$\Delta U + \Delta K = 0$$

Or it can be rewritten as:

$$U_i + K_i = U_f + K_f$$

The mechanical energy E is the sum of potential and kinetic energies,

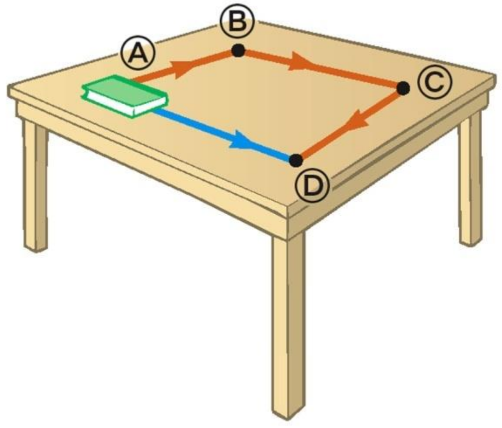
$$\therefore E_i = E_f$$

Non-Conservative Forces

- A force is nonconservative if the work it does on an object depends on the path taken by the object between its final and starting points.
  - The work depends upon the movement path
  - For a non-conservative force, potential energy can NOT be defined
  - Work done by a nonconservative force

$$W_{nc} = \sum \vec{F} \cdot \vec{d} = -f_k d + \sum W_{otherforces}$$

  - It is generally dissipative. The loss of energy takes the form of heat or sound



Conservation of Energy: All types of Forces

- The work-energy theorem can be written as:

$$W_{net} = K_f - K_i = \Delta K ----(1)$$

$$W_{net} = W_{nc} + W_c --------(2)$$

  - W_{nc} represents the work done by nonconservative forces
  - W_c represents the work done by conservative forces

- Any work done by conservative forces can be accounted for by changes in potential energy

  - Gravity work $W_g = PE_{gi} - PE_{gf} = mgy_i - mgy_f = U_i - U_f$
  - Spring force work $W_s = PE_{si} - PE_{sf} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = U_i - U_f$

$$\because W_c = -(U_f - U_i) = -\Delta U ----(3)$$

Conservation of Energy: All types of Forces

Thus Conservation of energy law can be obtained from equations (1), (2) and (3) as:

$$W_{nc} = \Delta U + \Delta K$$

But $$W_{nc} = \sum \vec{F} \cdot \vec{d} = -f_k d + \sum W_{otherforces}$$

Thus $$-f_k d + \sum W_{otherforces} = \Delta U + \Delta K$$

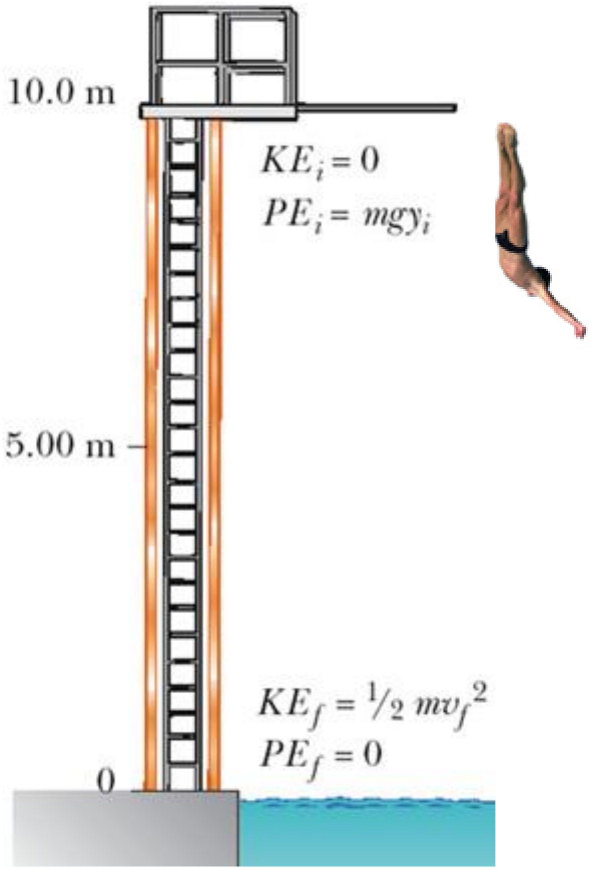
If there are two conservative forces (like force of gravity and spring force) and one moving object in addition to different nonconservative forces in the problem one gets

$$-fd + \sum W_{otherforces} = (mgy_f - mgy_i) + (\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2) + (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2)$$

Conservation of Energy

Example

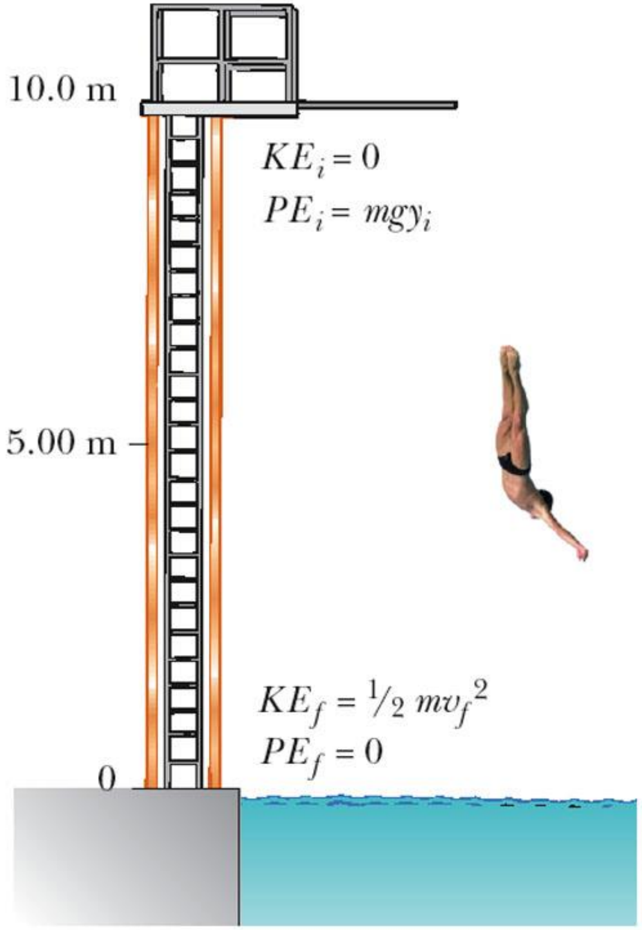
- A diver of mass m drops from a board 10.0 m above the water's surface. Neglect air resistance.
- (a) Find is speed 5.0 m above the water surface
- (b) Find his speed as he hits the water



Conservation of Energy

Example

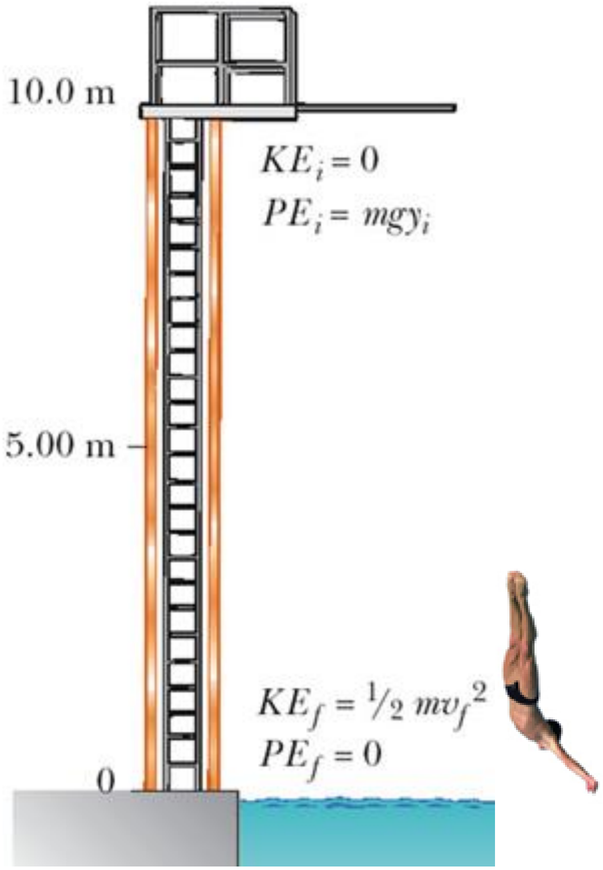
- A diver of mass m drops from a board 10.0 m above the water's surface. Neglect air resistance.
- (a) Find is speed 5.0 m above the water surface
- (b) Find his speed as he hits the water



Conservation of Energy

Example

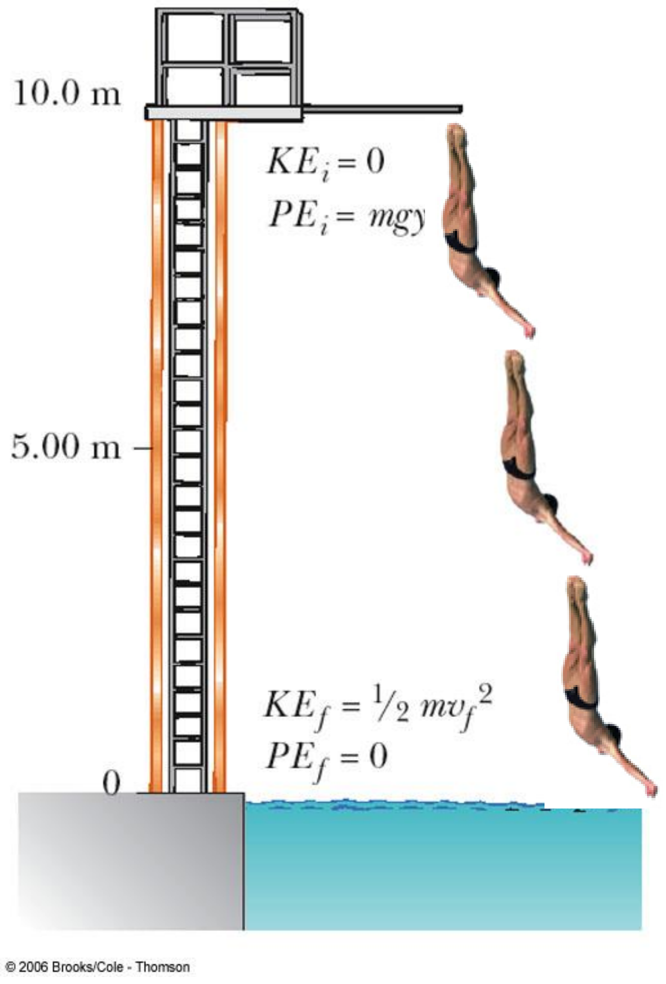
- A diver of mass m drops from a board 10.0 m above the water's surface. Neglect air resistance.
- (a) Find is speed 5.0 m above the water surface
- (b) Find his speed as he hits the water



Conservation of Energy

Example

- A diver of mass m drops from a board 10.0 m above the water's surface. Neglect air resistance.
- (a) Find is speed 5.0 m above the water surface
- (b) Find his speed as he hits the water



Solution

Conservation of Energy

- (a) Find his speed 5.0 m above the water surface (at point B)

$$\Delta U + \Delta K = 0$$

$$(mgy_f - mgy_i) + (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2) = 0$$

$$gy_{fB} - gy_{iA} + (\frac{1}{2}v_{fB}^2 - 0) = 0$$

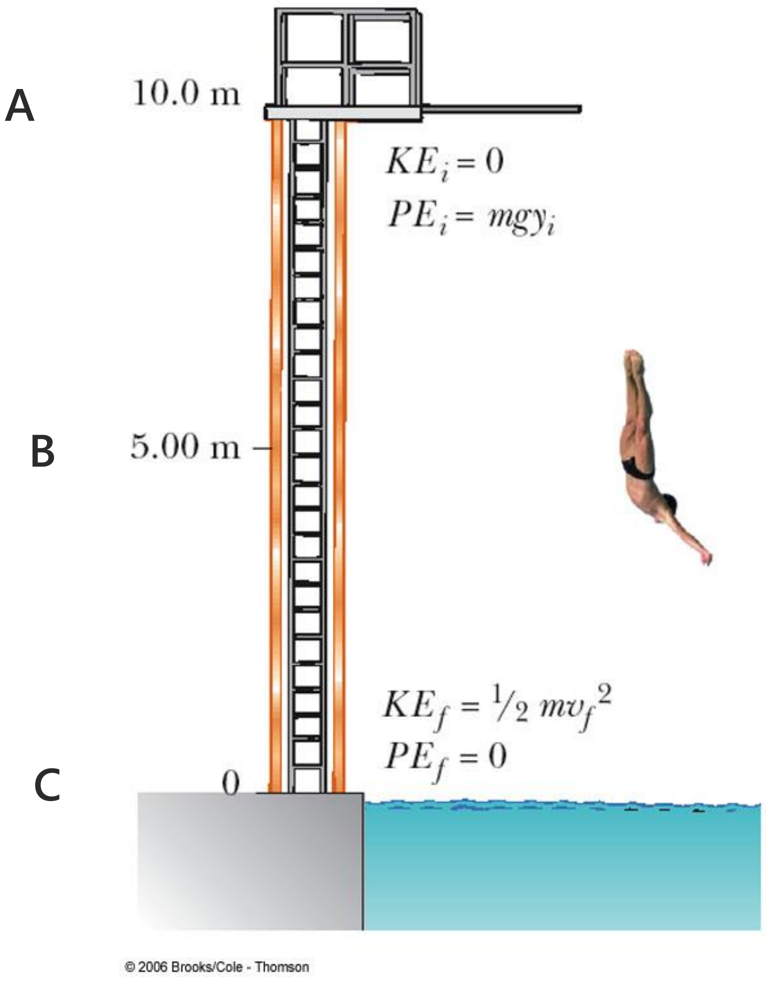
$$or \ -g(\Delta y)_{A\to B} + (\frac{1}{2}v_{fB}^2 - 0) = 0$$

$$(\Delta y)_{A\to B} = h_{AB}$$

$$\Rightarrow \quad gh_{AB} = \frac{1}{2}v_{fB}^2$$

$$v_{fB} = \sqrt{2gh_{AB}}$$

$$= \sqrt{2(9.8m/s^2)(5m)} = 9.9m/s$$



Solution

Conservation of Energy

- (b) Find his speed 10.0 m above the water surface (at point C)

$$\Delta U + \Delta K = 0$$

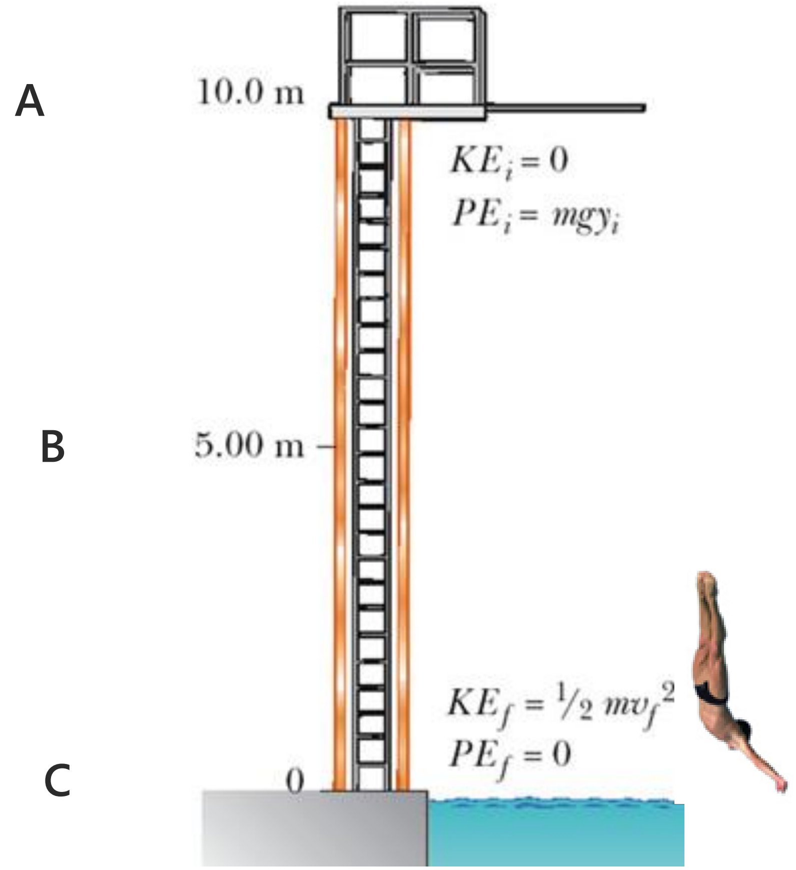
$$(mgy_f - mgy_i) + (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2) = 0$$

$$-mg(\Delta y)_{A\to C} + (\frac{1}{2}mv_{fC}^2 - 0) = 0$$

$$(\Delta y)_{A\to C} = h_{AC}$$

$$mgh_{AC} = \frac{1}{2}mv_{fC}^2$$

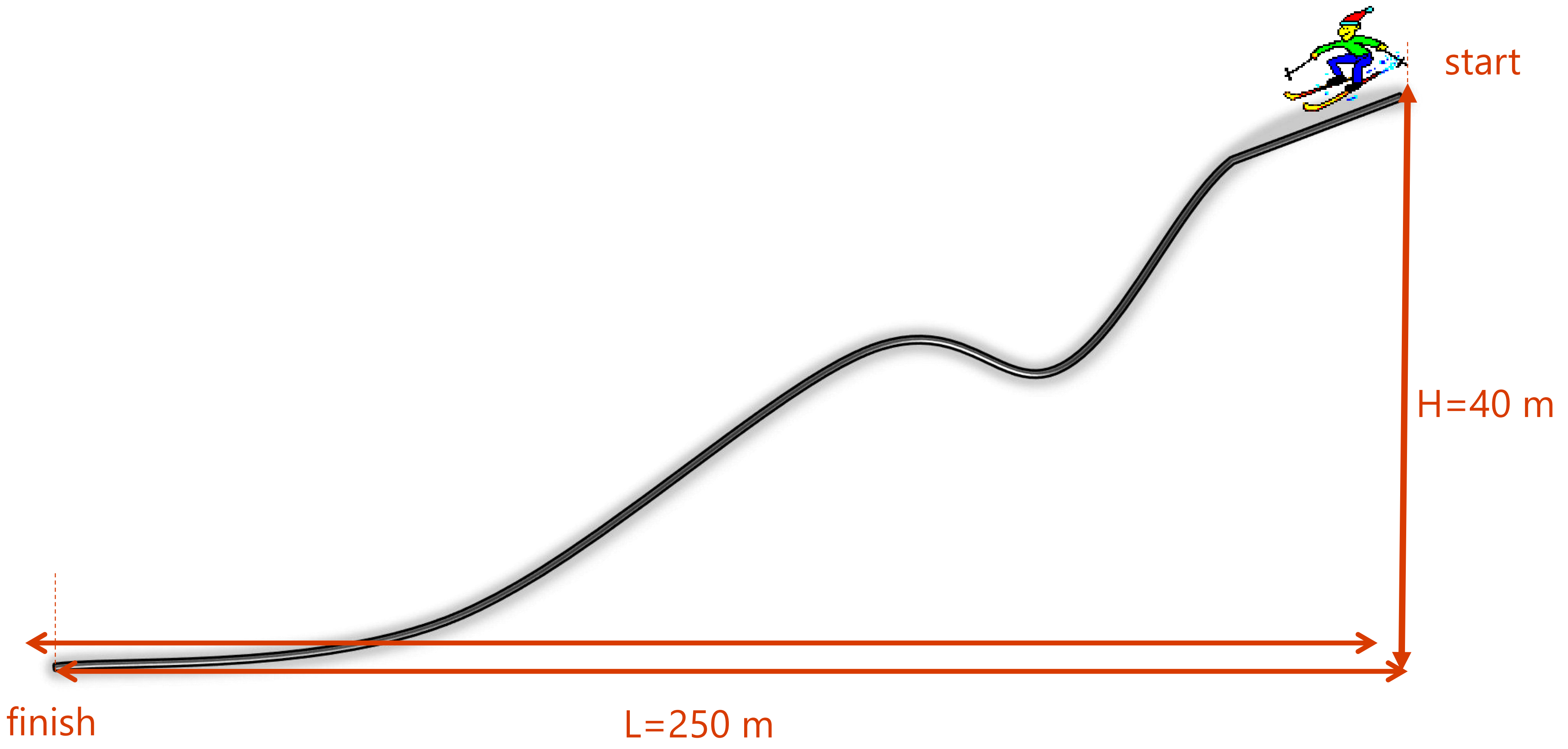
$$v_{fC} = \sqrt{2gh_{AC}} = 14m/s$$



Conservation of Energy

Example

A skier slides down the frictionless slope as shown. What is the skier's speed at the bottom?

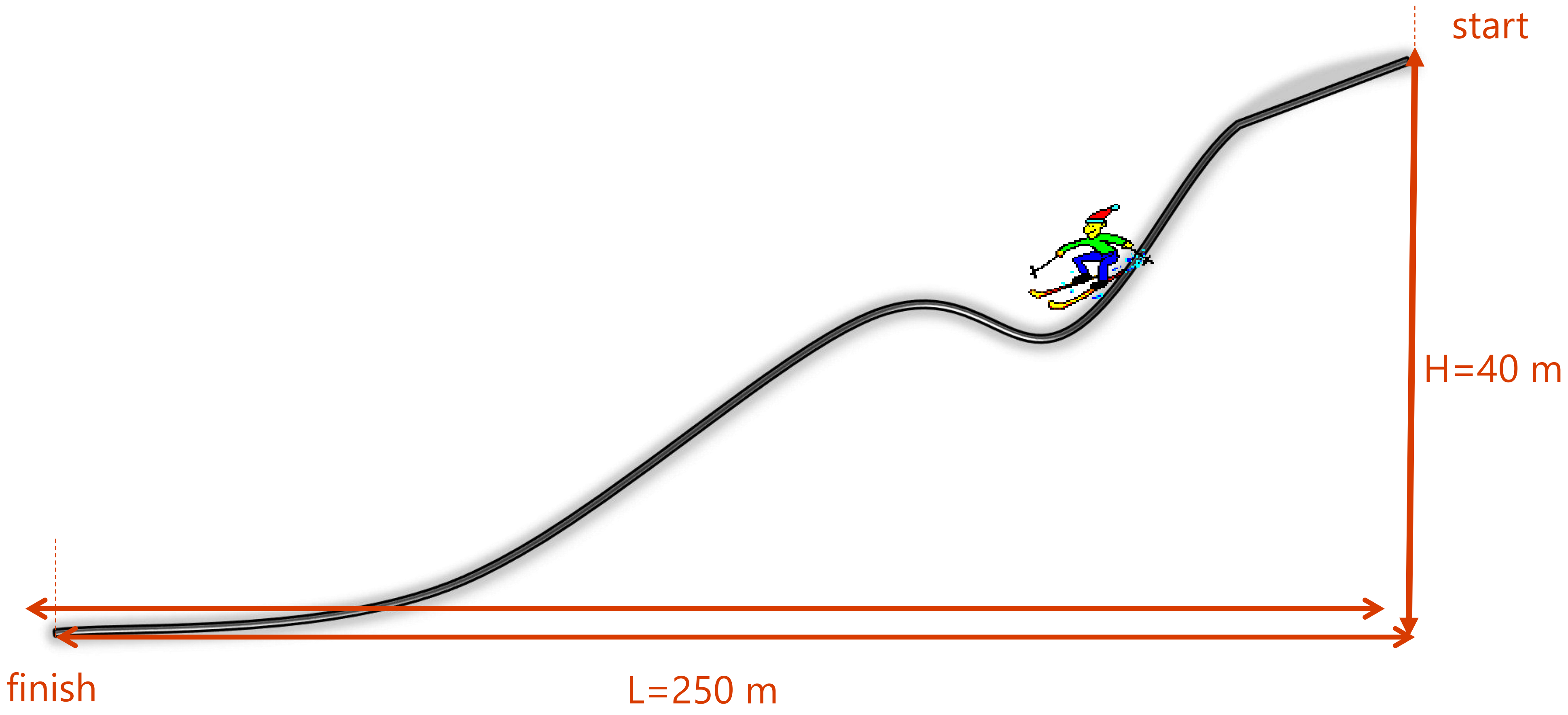


Answer: 28.0 m/s

Conservation of Energy

Example

A skier slides down the frictionless slope as shown. What is the skier's speed at the bottom?

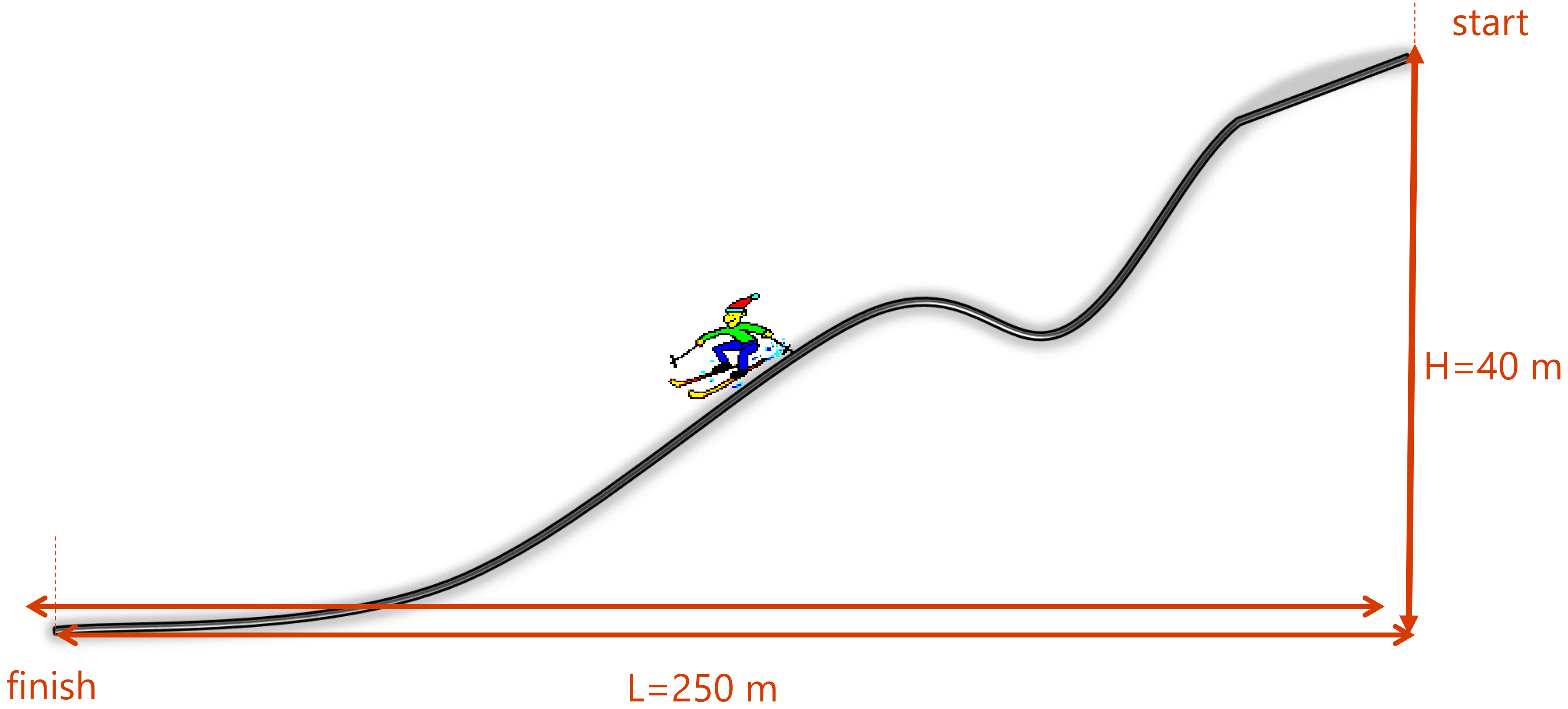


Answer: 28.0 m/s

Example

Conservation of Energy

A skier slides down the frictionless slope as shown. What is the skier's speed at the bottom?

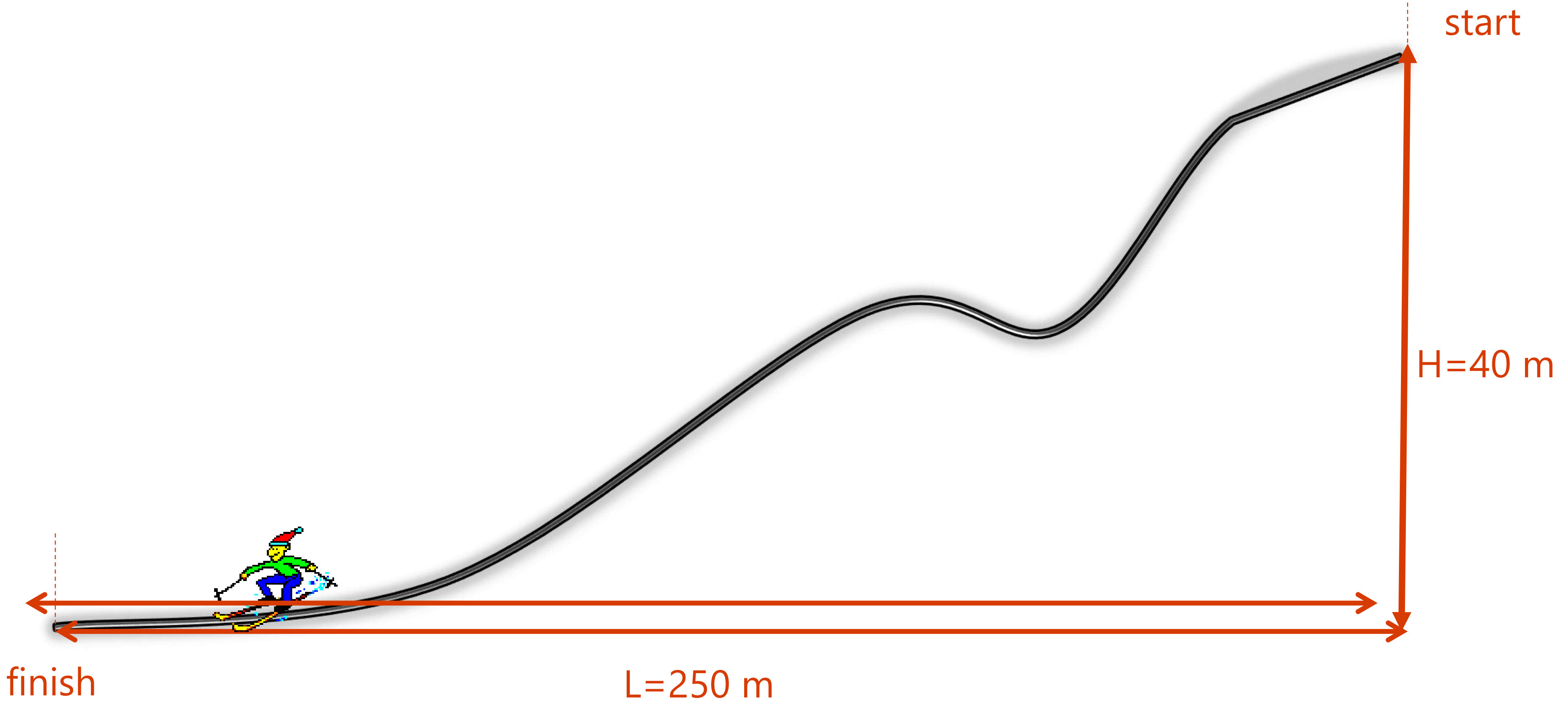


Answer: 28.0 m/s

Example

Conservation of Energy

A skier slides down the frictionless slope as shown. What is the skier's speed at the bottom?

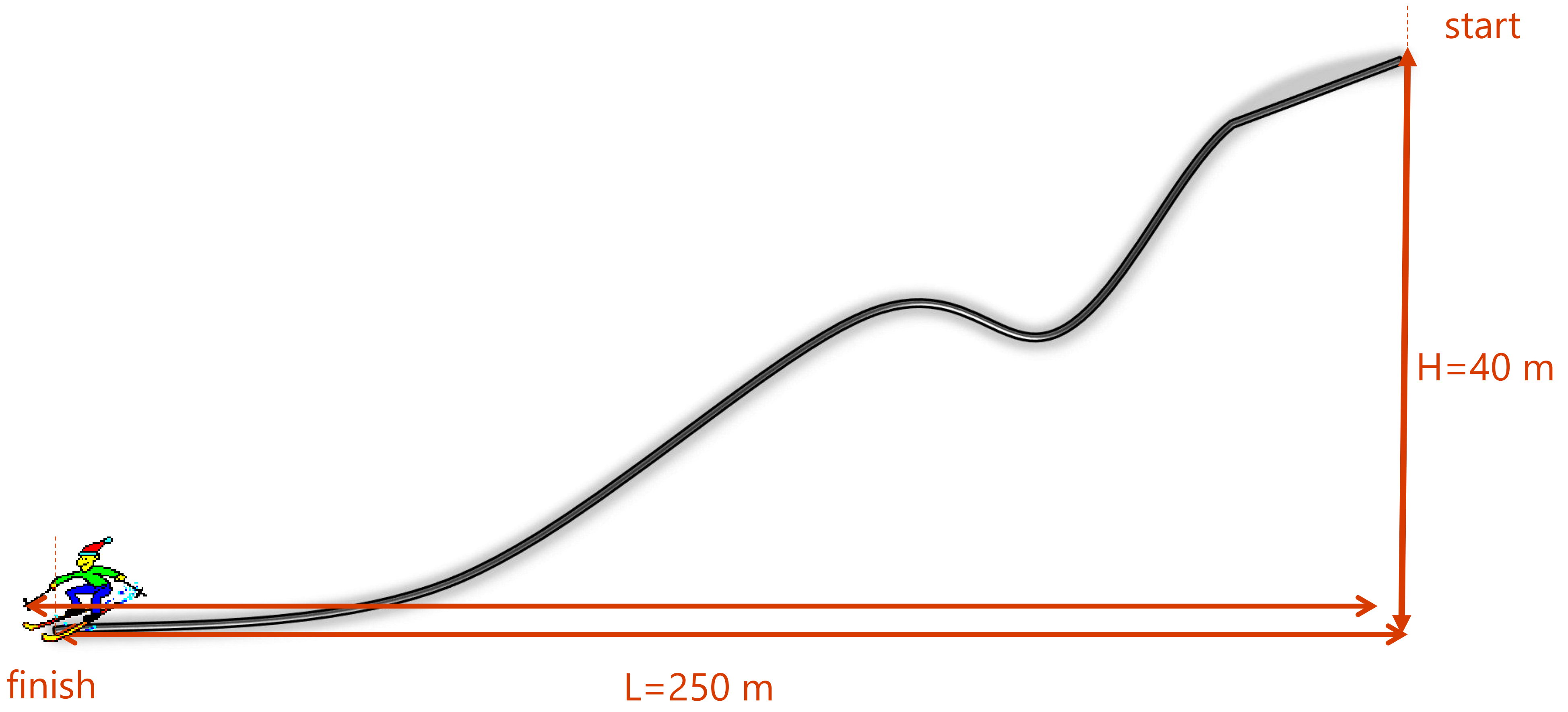


Answer: 28.0 m/s

Example

Conservation of Energy

A skier slides down the frictionless slope as shown. What is the skier's speed at the bottom?



Answer: 28.0 m/s

Conservation of Energy

Solution

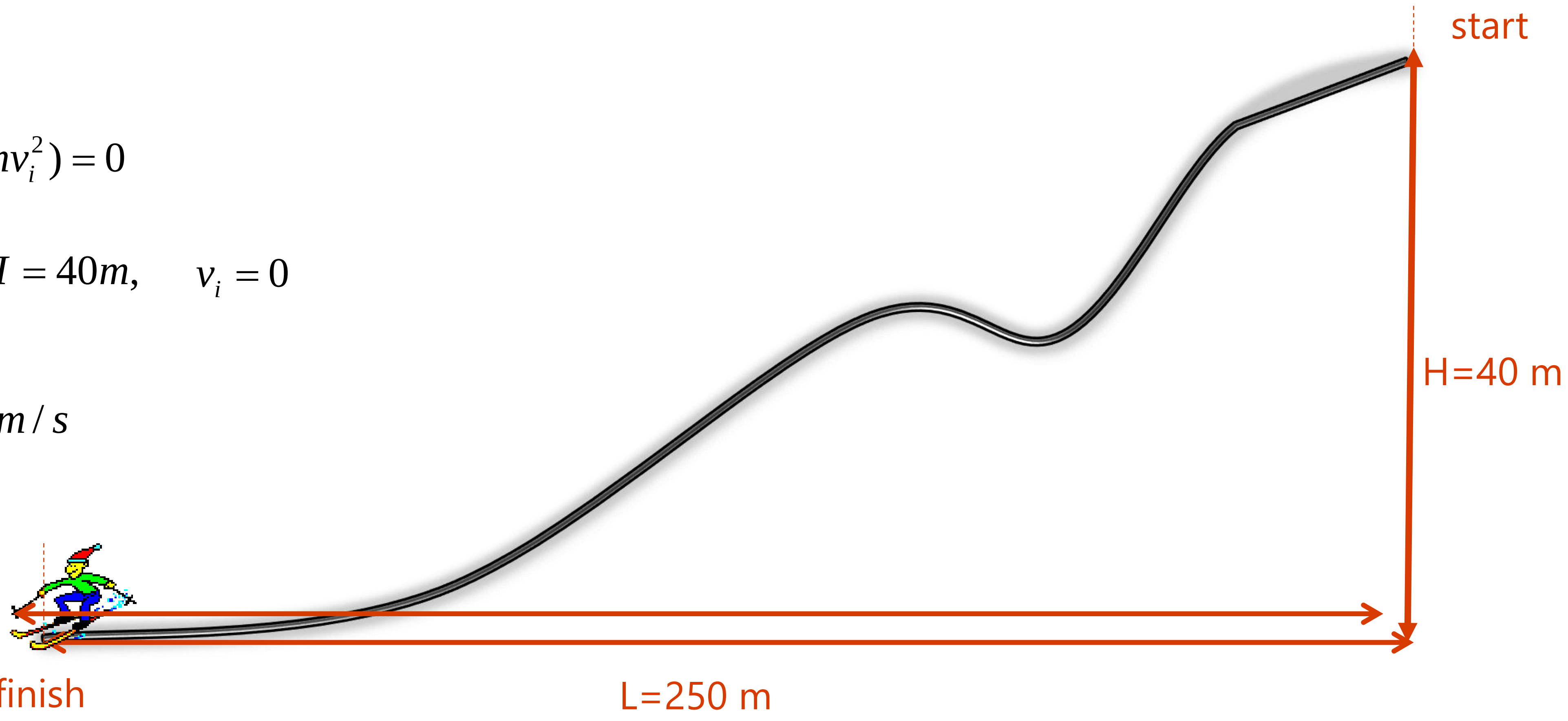
A skier slides down the frictionless slope as shown. What is the skier's speed at the bottom?

$$\Delta U + \Delta K = 0$$

$$-mg\Delta y + (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2) = 0$$

Given: $\Delta y = H = 40m, \quad v_i = 0$

$$\Rightarrow v_f = \sqrt{2gH} = 28m/s$$

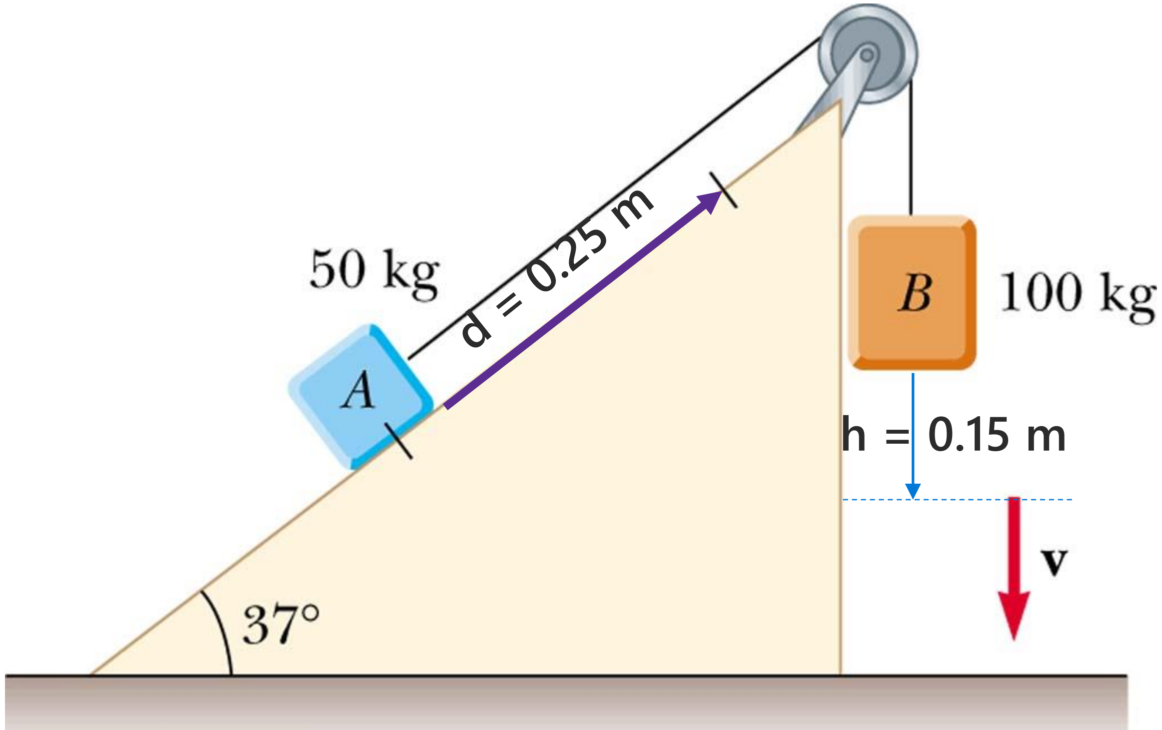


Answer: 28.0 m/s

Example

Conservation of Energy

Two blocks, *A* and *B* (m_A=*50 kg* and m_B=100 kg), are connected by a string as shown. If the blocks begin at rest, what will their speeds be after *A* has slid a distance *d* = 0.25 m? [Hint: Assume the pulley and incline are frictionless.]



Answer: 1.0 m/s

Conservation of Energy

Solution

Given: $m_A = 50\ kg$, $m_B = 100\ kg$, $v_{iA} = v_{iB} = 0$, $\theta = 37^o$ and $d = 0.25$ m.

[Hint: Assume the pulley and incline are frictionless.]

$$(\Delta U)_A + (\Delta U)_B + (\Delta K)_A + (\Delta K)_B = 0$$

$$m_A g(\Delta y)_{up} - m_B g(\Delta y)_{down} + (\frac{1}{2}m_A v_{fA}^2 - \frac{1}{2}m_A v_{iA}^2) + (\frac{1}{2}m_B v_{fB}^2 - \frac{1}{2}m_B v_{iB}^2) = 0$$

$$(\Delta y)_{up} = (\Delta y)_{down} = h = d\sin\theta = (0.25)\sin 37^o = 0.15m$$

$$v_{fA} = v_{fB} = v \qquad v_{iA} = v_{iB} = 0$$

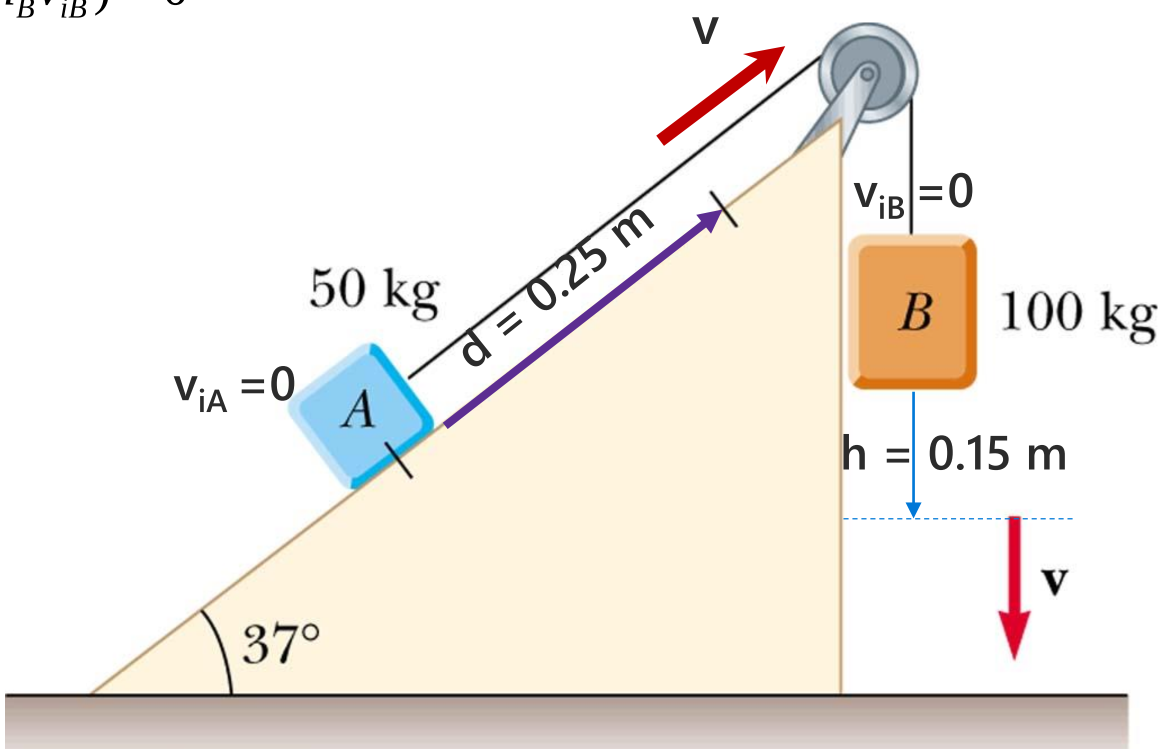
$$(m_A - m_B)gh + \frac{1}{2}(m_A + m_B)v^2 = 0$$

$$(50kg - 100kg)(9.8m/s^2)(0.15m) + \frac{1}{2}(50kg + 100kg)v^2 = 0$$

$$(50kg)(9.8m/s^2)(0.15m) = \frac{1}{2}(150kg)v^2$$

$$v_{fA} = v_{fB} = v = 1m/s$$

Answer: 1.0 m/s



Conservation of Energy

Example

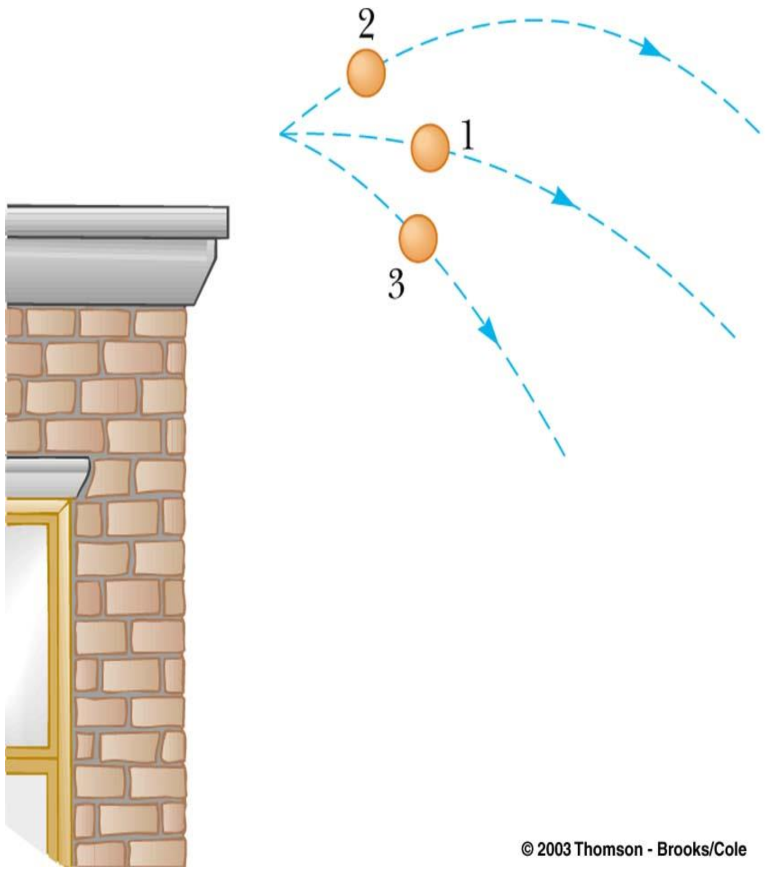
Three identical balls are thrown from the top of a building with the same initial speed. Initially,

Ball 1 moves horizontally.

Ball 2 moves upward.

Ball 3 moves downward.

Neglecting air resistance, which ball has the fastest speed when it hits the ground?



A) Ball 1

B) Ball 2

C) Ball 3

D) All have the same speed.

Example

Conservation of Energy

Tarzan swings from a vine whose length is 12 m. If Tarzan starts at an angle of 30 degrees with respect to the vertical and has no initial speed, what is his speed at the bottom of the arc?

Solution

$$v_i = 0, \qquad v_f = ?$$

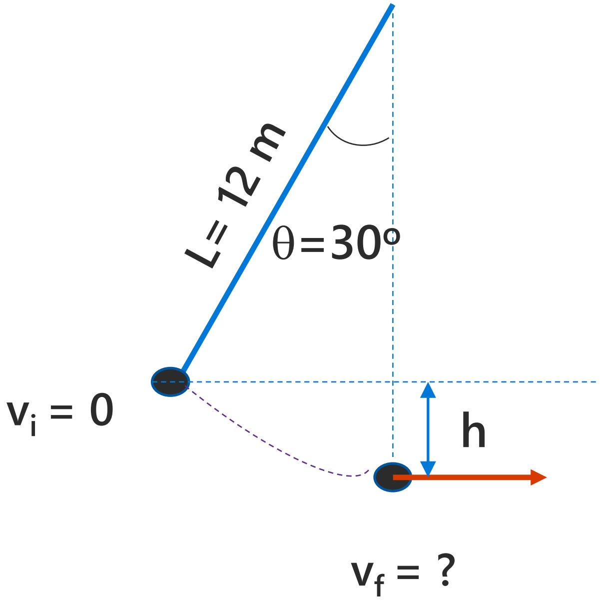
$$\Delta U + \Delta K = 0$$

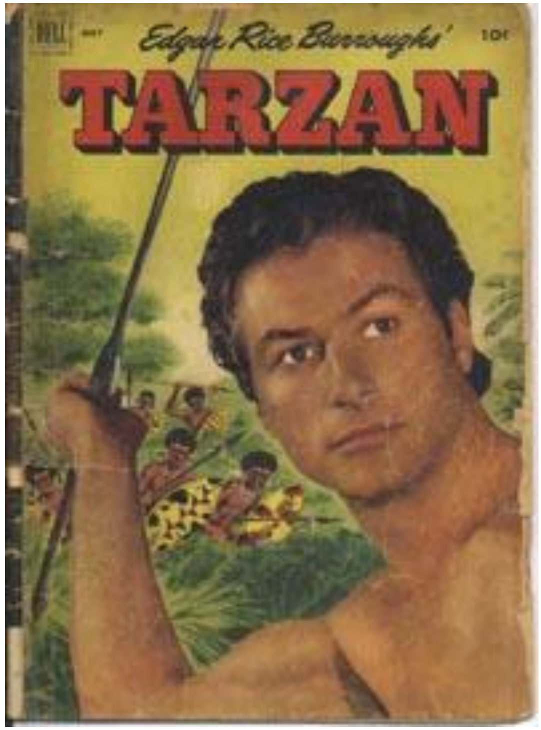
$$-mg\Delta y + \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) = 0$$

$$\Delta y = h$$

$h = L - L\cos\theta =$ (12)(1-cos30°)=1.6 m

$$\Rightarrow v_f = \sqrt{2gh} = 32.15 m/s$$





Conservation of Energy

Example

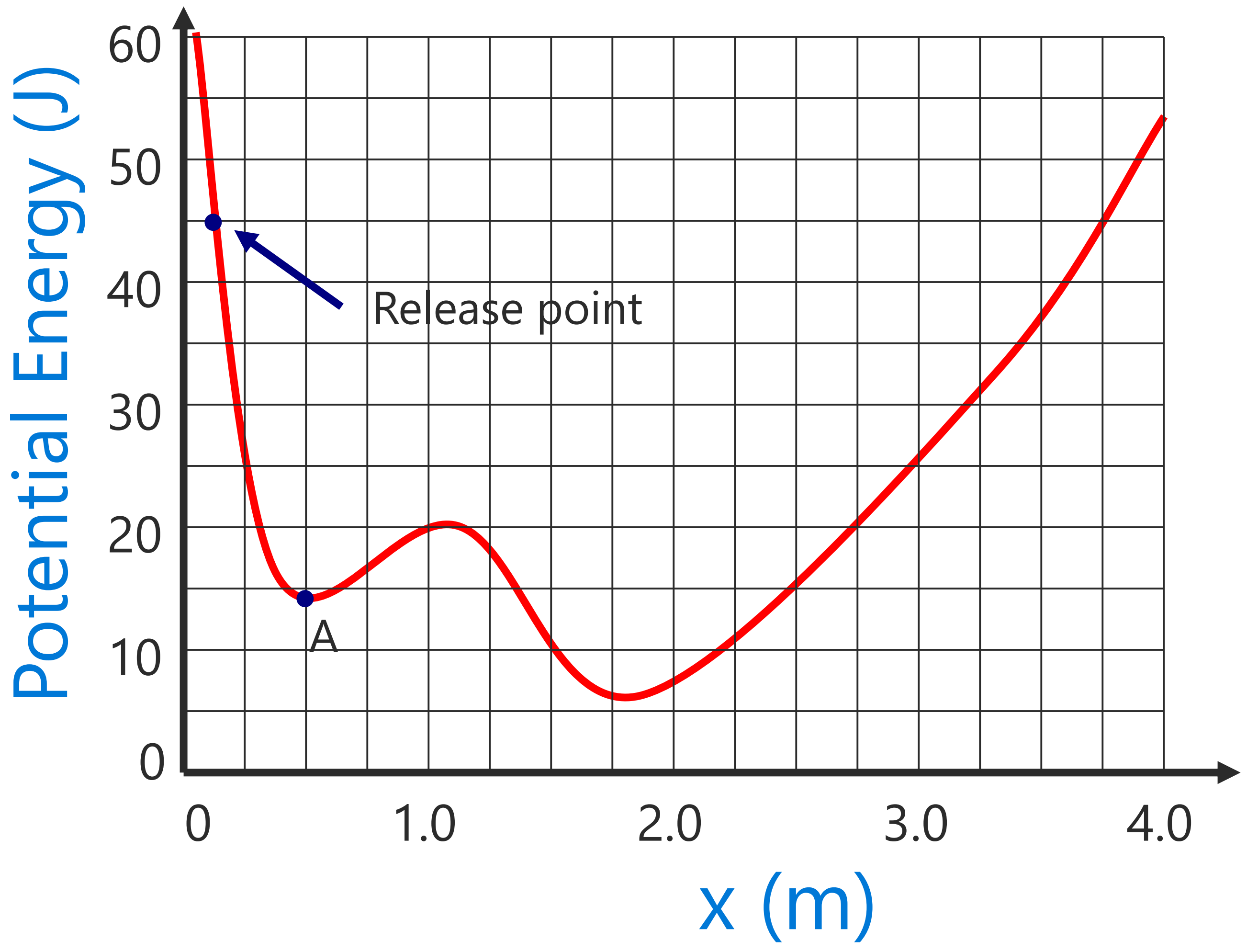
At point 'A', which are zero?

a) force

b) acceleration

c) force and acceleration

d) velocity



Conservation of Energy

Example

At point 'B', which are/is zero?

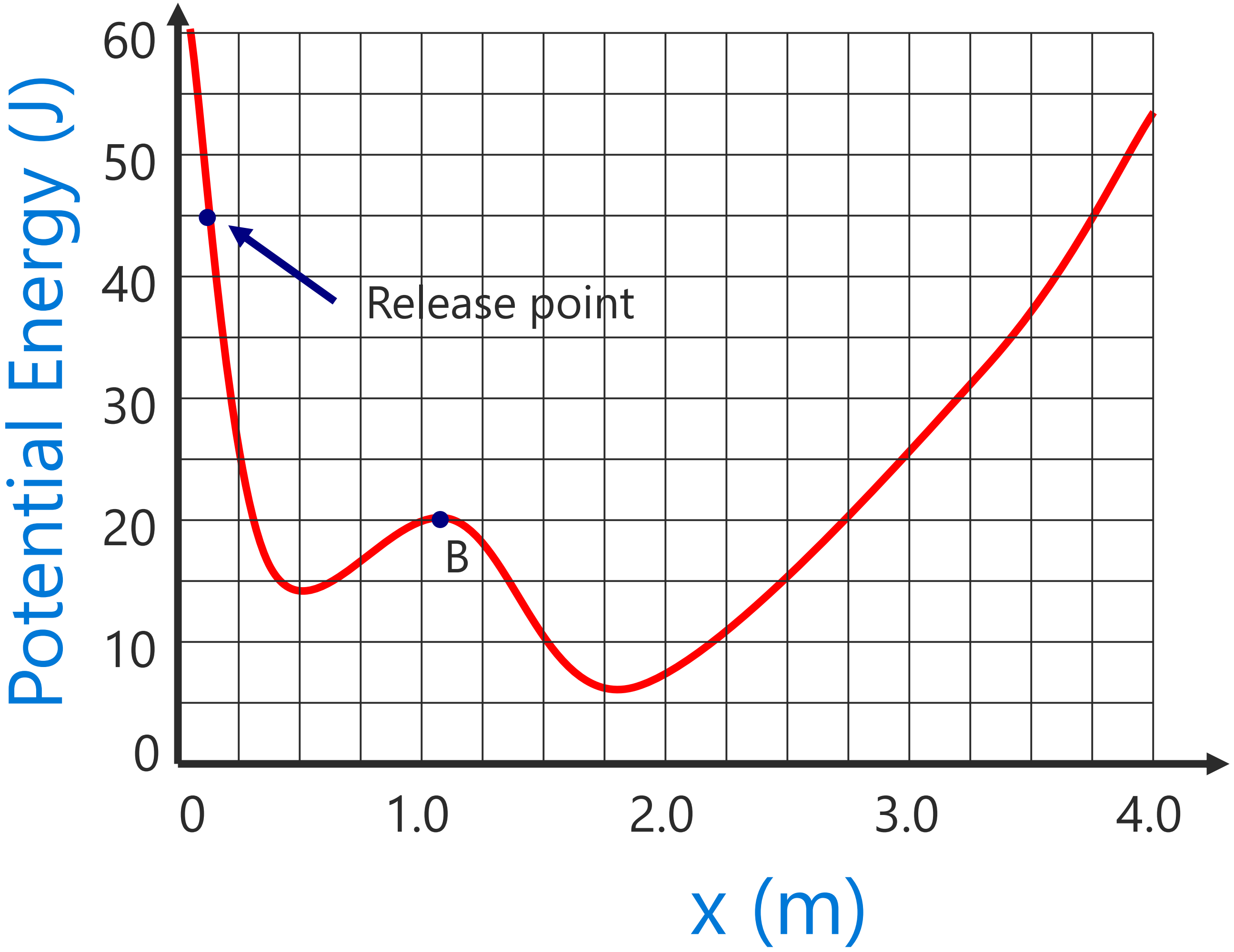
a) force

b) acceleration

c) force and acceleration

d) velocity

e) kinetic energy

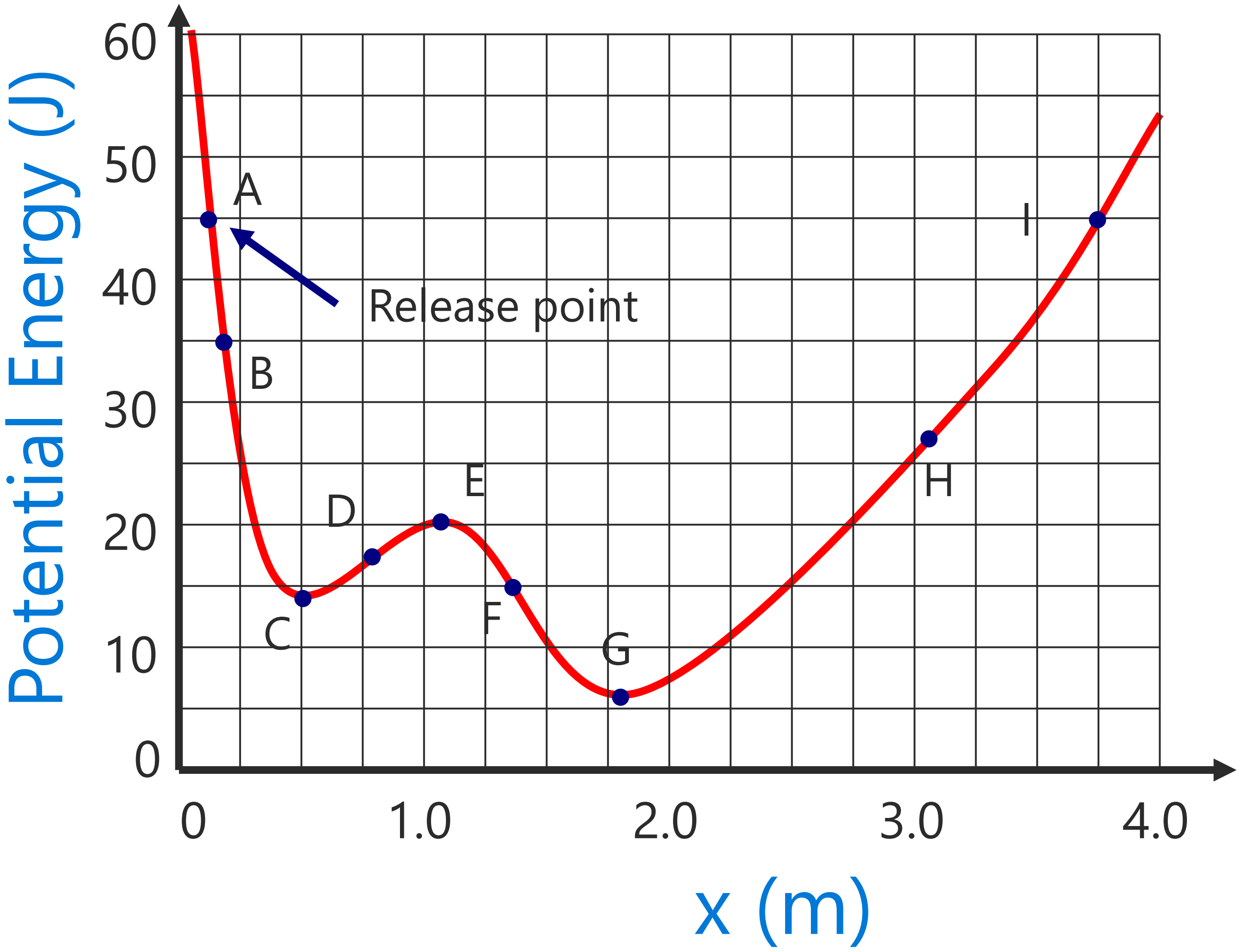


Conservation of Energy

Example

All points for which force is negative (to the left):

a) C, E and G
b) B and F
c) A and I
d) D and H
e) D, H and I



Conservation of Energy

Example

At point 'D', which are/is zero?

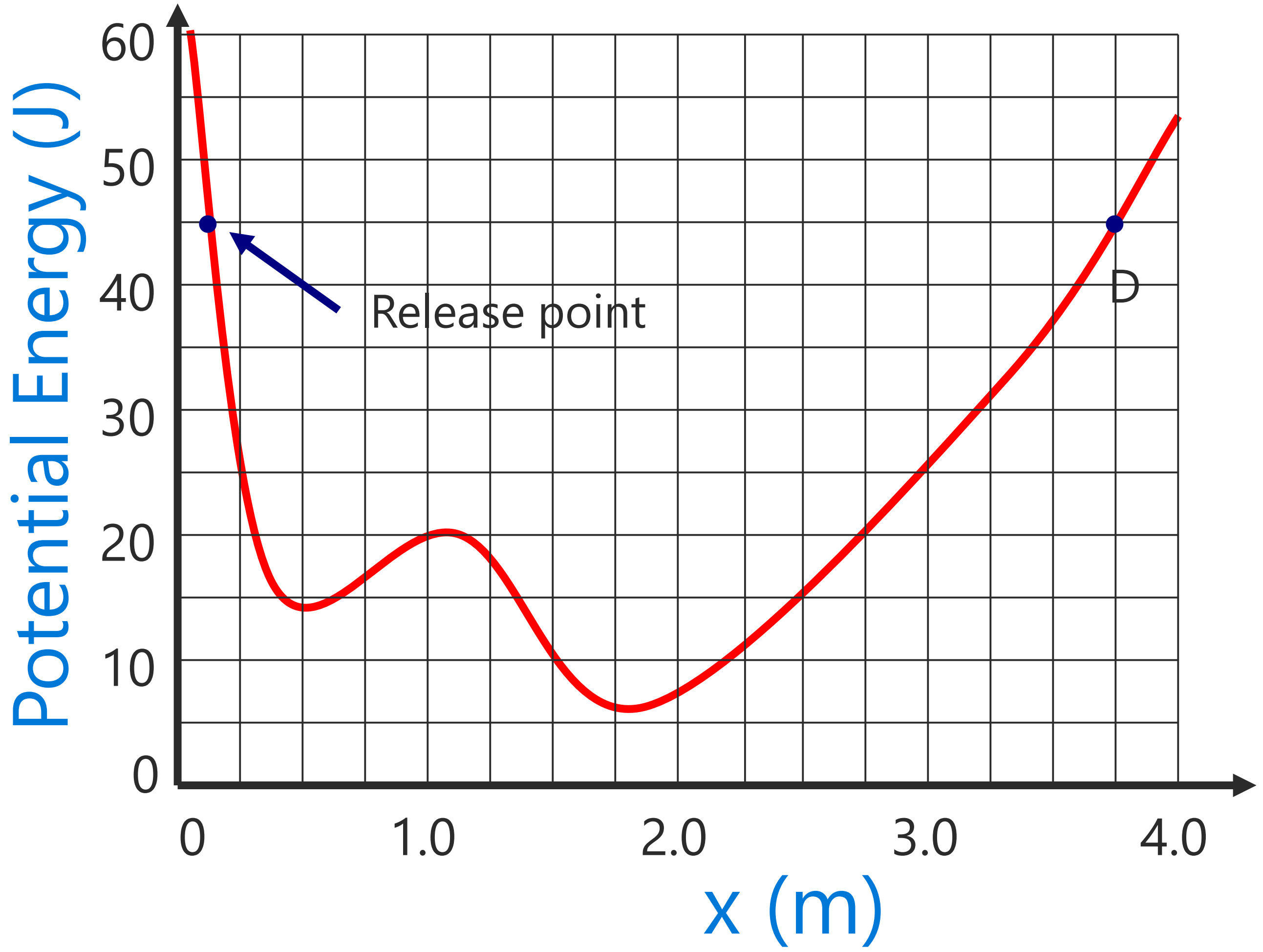
a) force

b) acceleration

c) force and acceleration

d) velocity

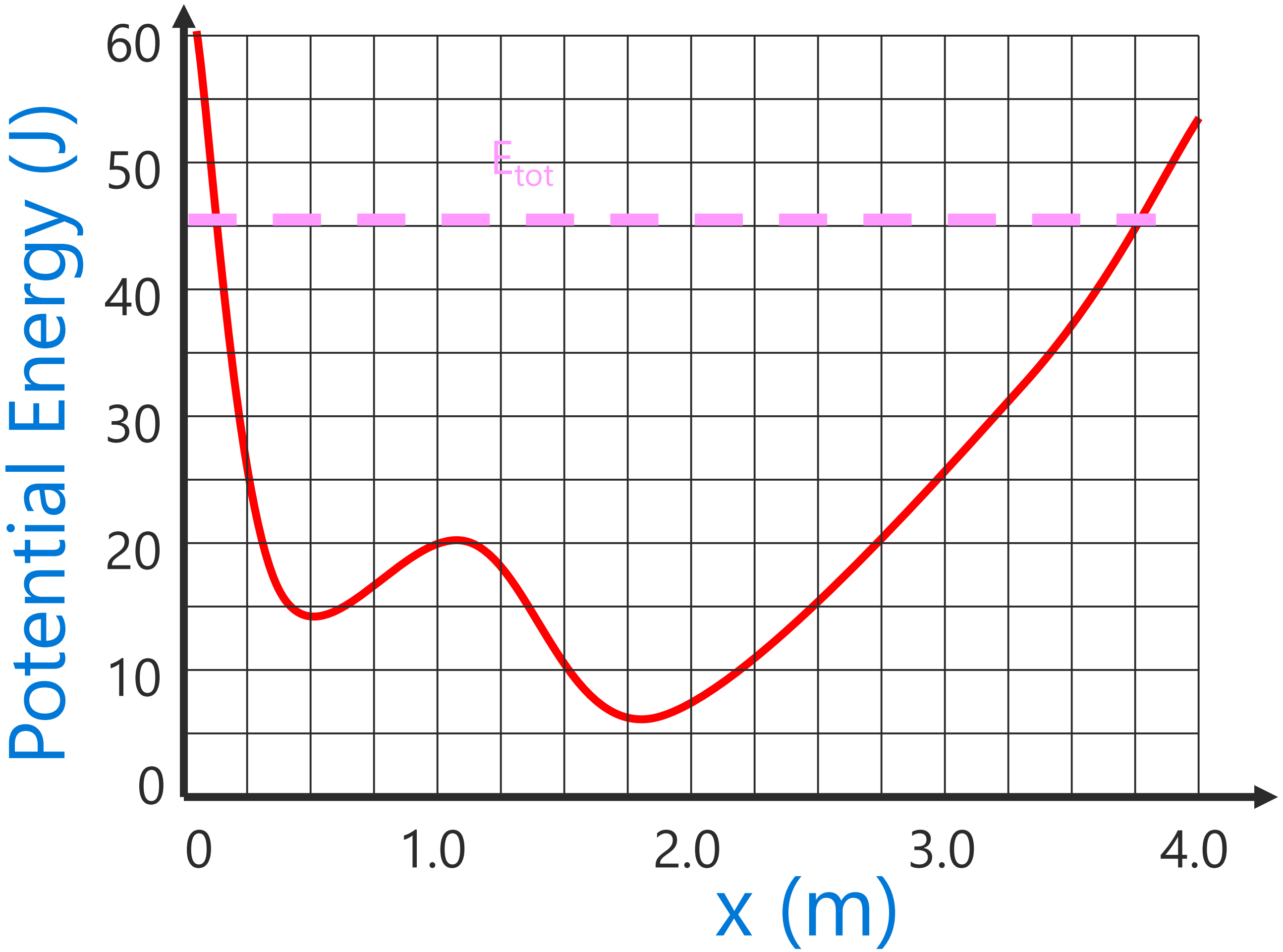
e) Velocity and kinetic energy



Conservation of Energy

Example

A particle of mass m = 0.5 kg is at a position x = 1.0 m and has a velocity of -10.0 m/s. What is the furthest points to the left and right it will reach as it oscillates back and forth?

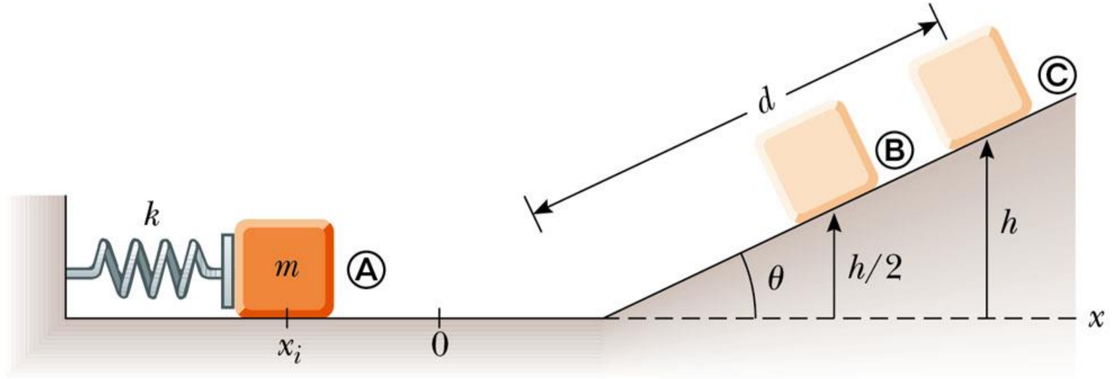


Answer: 0.125 and 3.75 m

Conservation of Energy: All types of Forces

Example

- A 0.5-kg block rests on a horizontal, frictionless surface. The block is pressed back against a spring having a constant of k = 625 N/m, compressing the spring by 10.0 cm to point A. Then the block is released.
- (a) Find the maximum distance d the block travels up the frictionless incline if θ = 30° .
- (b) How fast is the block going when halfway to its maximum height?



Conservation of Energy: All types of Forces

Solution

(a)

- Point A (initial state): $v_i = v_A = 0, y_A = 0, x_i = -10cm = -0.1m$
- Point C (final state): $v_C = 0, y_C - y_A = h = d\sin\theta, x_f = 0$

$$(\Delta U)_g + (\Delta U)_s + (\Delta K)_{A\to C} = 0 \Rightarrow$$

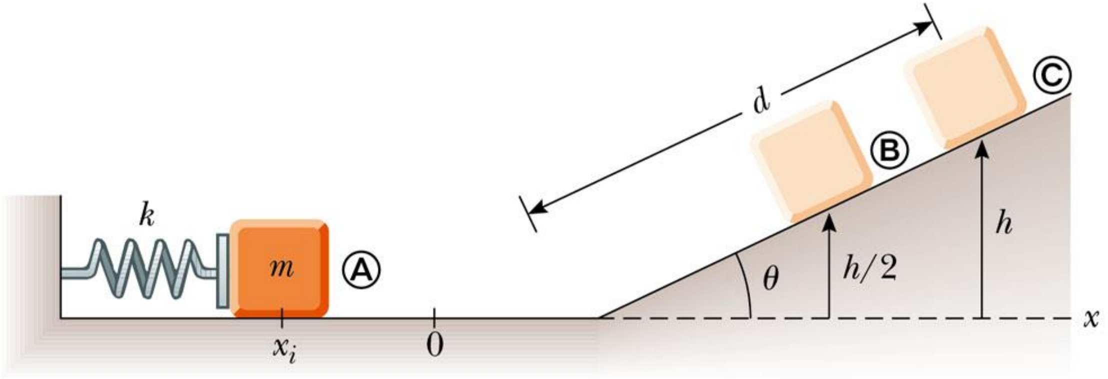
$$+mgh + (\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2) + (\frac{1}{2}mv_C^2 - \frac{1}{2}mv_A^2) = 0$$

$$\frac{1}{2}kx_i^2 = mgh = mgd\sin\theta$$

$$d = \frac{\frac{1}{2}kx_i^2}{mg\sin\theta}$$

$$= \frac{0.5(625N/m)(-0.1m)^2}{(0.5kg)(9.8m/s^2)\sin 30^\circ}$$

$$= 1.28m$$



Conservation of Energy: All types of Forces

Solution (b)

- Point A (initial state): $v_i = v_A = 0, y_A = 0, x_i = -10cm = -0.1m$
- Point B (final state): $v_B = ?, y_B - y_A = h/2 = d\sin\theta/2, x_f = 0$

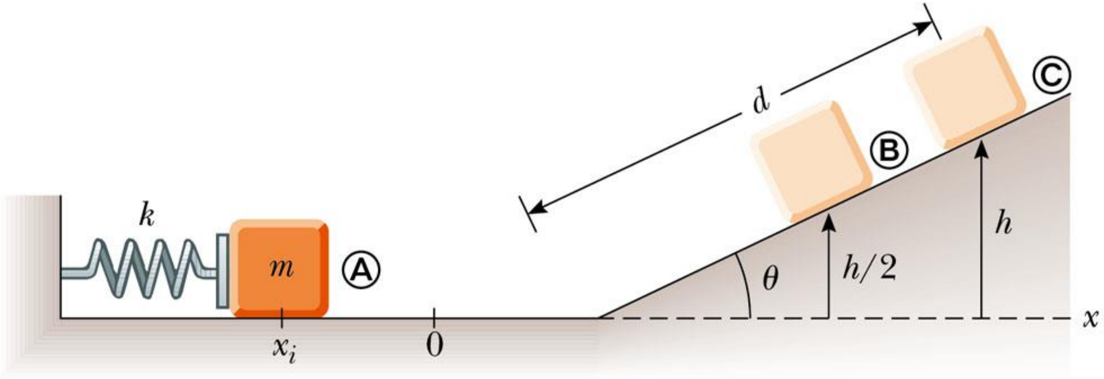
$$(\Delta U)_g + (\Delta U)_s + (\Delta K)_{A\to B} = 0$$

$$\Rightarrow +mg\frac{h}{2} + (\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2) + (\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2) = 0$$

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_B^2 + mg(\frac{h}{2})$$

$$h = d\sin\theta = (1.28m)\sin 30^\circ = 0.64m$$

$$\frac{k}{m}x_i^2 = v_B^2 + gh$$



$$v_B = \sqrt{\frac{k}{m}x_i^2 - gh}$$

$$\therefore v_B = \sqrt{\frac{(625N/m)}{0.5kg}(0.1m)^2 - (9.8m/s^2)(0.64m)} = 2.5m/s$$

Conservation of Energy: All types of Forces

- Any work done by conservative forces can be accounted for by changes in potential energy

$$W_c = U_i - U_f = -(U_f - U_i) = -\Delta U$$

$$W_{nc} = \Delta K + \Delta U = (K_f - K_i) + (U_f - U_i)$$

$$W_{nc} = (K_f - K_i) + (U_f - U_i)$$

- Mechanical energy includes kinetic and potential energies

$$E = K + U = K + U_g + U_s = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

$$W_{nc} = E_f - E_i$$

Problem Solving Strategy

- Define the system to see if it includes non-conservative forces (especially friction, drag force ...)
- Without non-conservative forces

$$(\Delta U)_g + (\Delta U)_s + \Delta K = 0 \Leftrightarrow \quad mgy_f - mgy_i + \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 + \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0$$

- With non-conservative forces

$$W_{nc} = (U_f - U_i) + (K_f - K_i)$$

$$-fd + \sum W_{otherforces} = mgy_f - mgy_i + \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 + \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

- Select the location of zero potential energy
  - Do *not* change this location while solving the problem

- Identify two points the object of interest moves between
  - One point should be where information is given
  - The other point should be where you want to find out something

Conservation of Mechanical Energy

Example

A block of mass m = 0.40 kg slides across a horizontal frictionless counter with a speed of v = 0.50 m/s. It runs into and compresses a spring of spring constant k = 750 N/m. When the block is momentarily stopped by the spring, by what distance d is the spring compressed?

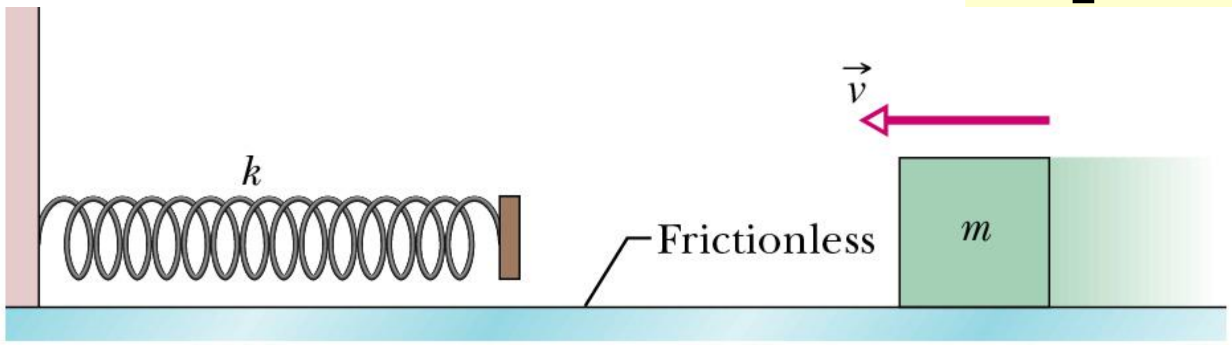
Solution

$$(\Delta U)_s + (\Delta K) = 0 \qquad or \quad (K_f - K_i) + (U_f - U_i) = 0$$

$$\Rightarrow \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 = 0$$

$x_f = d$, $x_i = 0$, $v_f = 0$, $v_i = v$,

$$(0 - \frac{1}{2}mv_i^2) + (\frac{1}{2}kd^2 - 0) = 0 \qquad or \quad \frac{1}{2}mv^2 = \frac{1}{2}kd^2$$



$$\therefore d = \sqrt{\frac{m}{k}v^2} = 1.15cm$$

Conservation of Mechanical Energy (Conservative Forces ONLY)

Problem

A 3-kg crate slides down a ramp. The ramp is 1 m in length and inclined at an angle of 30° as shown. The crate starts from rest at the top. The surface friction can be negligible. Use energy methods to determine the speed of the crate at the bottom of the ramp.

Solution

$$-f_k d + \sum W_{otherforces} = \Delta U + \Delta K$$

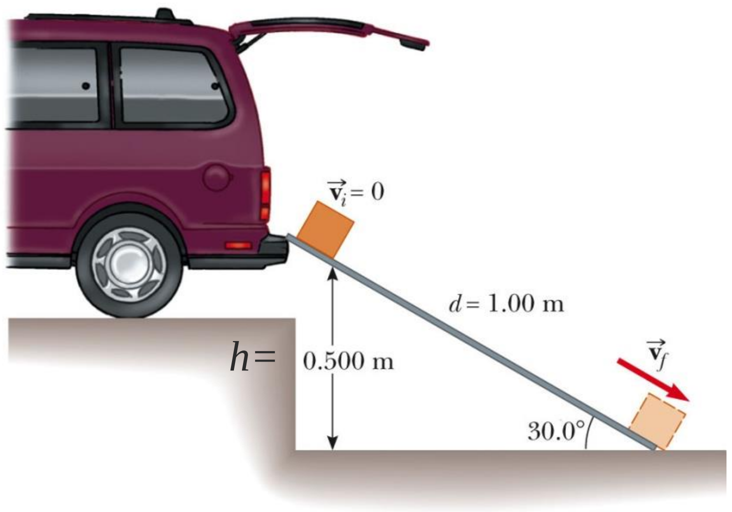
$$0 = -(mgy_f - mgy_i) + (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2)$$

$$but, \quad h = \Delta y, \; v_i = 0$$

$$\therefore mgh = \frac{1}{2}mv_f^2$$

$$d = 1m, \quad h = d\sin 30^\circ = 0.5m, \; v_f = ?$$

$$v_f = \sqrt{2gh} = 3.1m/s$$



Changes in Energy for Nonconservative Forces

Problem

A 3-kg crate slides down a ramp. The ramp is 1 m in length and inclined at an angle of 30° as shown. The crate starts from rest at the top. The surface in contact have a coefficient of kinetic friction of 0.15. Use energy methods to determine the speed of the crate at the bottom of the ramp.

Solution

$$-f_k d + \sum W_{otherforces} = \Delta U + \Delta K$$

$$-f_k d + \sum W_{otherforces} = -(mgy_f - mgy_i) + (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2)$$

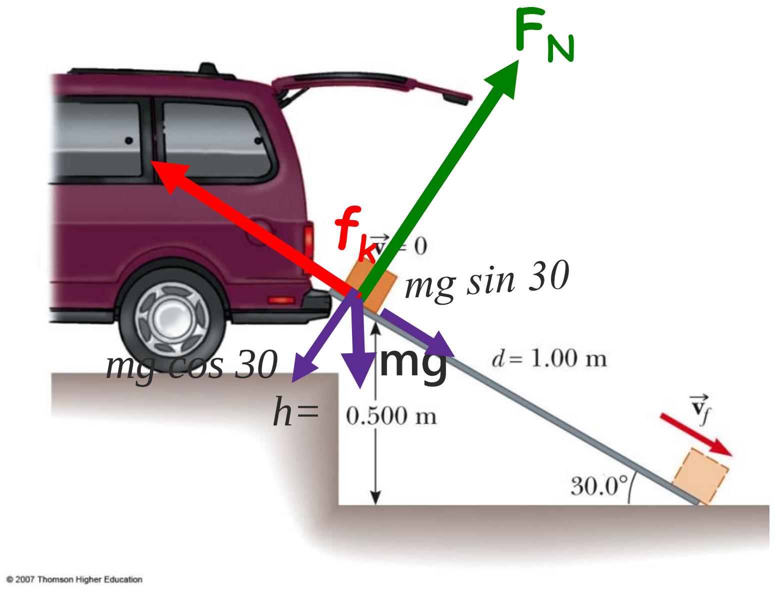
$$-\mu_k F_N d + 0 = -mgh + (\frac{1}{2}mv_f^2 - 0)$$

$$\mu_k = 0.15 \ , d = 1m, \ \ h = d\sin 30^\circ = 0.5m, \ \ F_N = ?, v_f = ?$$

$$F_N - mg\cos\theta = 0 \qquad \Rightarrow F_N = mg\cos\theta$$

$$-\mu_k dmg\cos\theta = \frac{1}{2}mv_f^2 - mgh$$

$$v_f = \sqrt{2g(h - \mu_k d\cos\theta)} = 2.7 m/s$$



Changes in Energy for Nonconservative Forces

Problem

A 3-kg crate slides down a ramp. The ramp is 1 m in length and inclined at an angle of 30° as shown. The crate starts from rest at the top. The surface in contact have a coefficient of kinetic friction of 0.15. How far does the crate slide on the horizontal floor if it continues to experience a friction force.

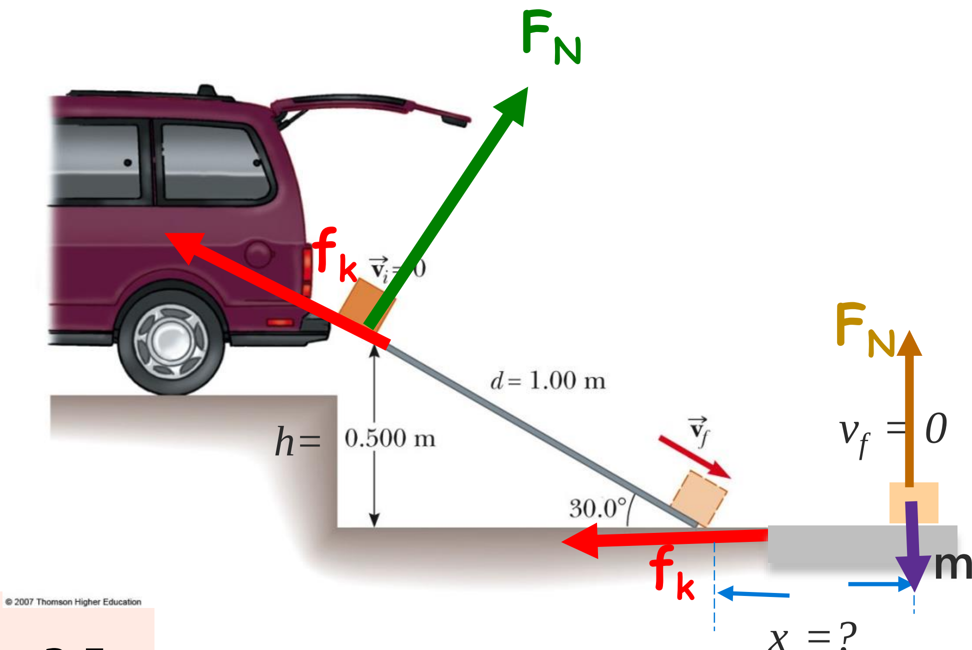
Solution

$$-f_k d + \sum W_{otherforces} = \Delta U + \Delta K$$

$$-f_k d + \sum W_{otherforces} = -(mgy_f - mgy_i) + (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2)$$

$$v_i^{horizonta} = v_f^{ramp} = 2.7 m/s, \quad h' = 0$$

$$-\mu_k F_N x + 0 = 0 + (0 - \frac{1}{2}mv_i^2)$$



On the horizontal floor, the normal force is found as:

$$F_N - mg = 0 \qquad \Rightarrow F_N = mg$$

$$\Rightarrow -\mu_k mgx = -\frac{1}{2}mv_i^2$$

$$\mu_k = 0.15, v_i = 2.7 m/s$$

$$\therefore x = \frac{v_i^2}{2\mu_k g} = 2.5m$$

Block-Spring Collision

Problem

- A block having a mass of 0.8 kg is given an initial velocity v_A = 1.2 m/s to the right and collides with a spring whose mass is negligible and whose force constant is k = 50 N/m as shown in figure. Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

Solution

$$(\Delta U)_s + (\Delta K) = 0$$

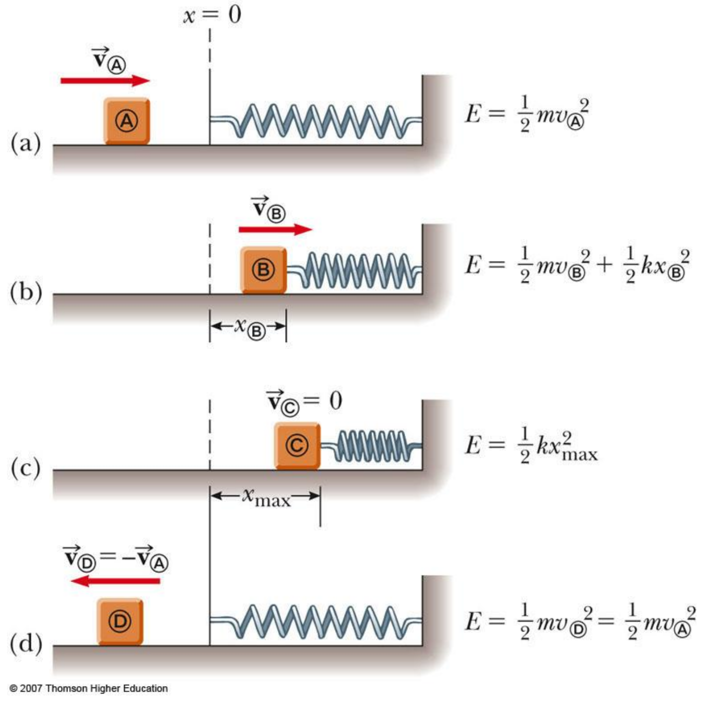
$$\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 + \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0$$

$$v_A = v_{\max}, \quad v_C = 0, \ x_C = x_{\max}$$

$$(\frac{1}{2}kx_{\max}^2 - 0) + (0 - \frac{1}{2}mv_A^2) = 0$$

$$k = 50N/m, \quad v_A = 1.2m/s, \quad m = 0.8kg$$

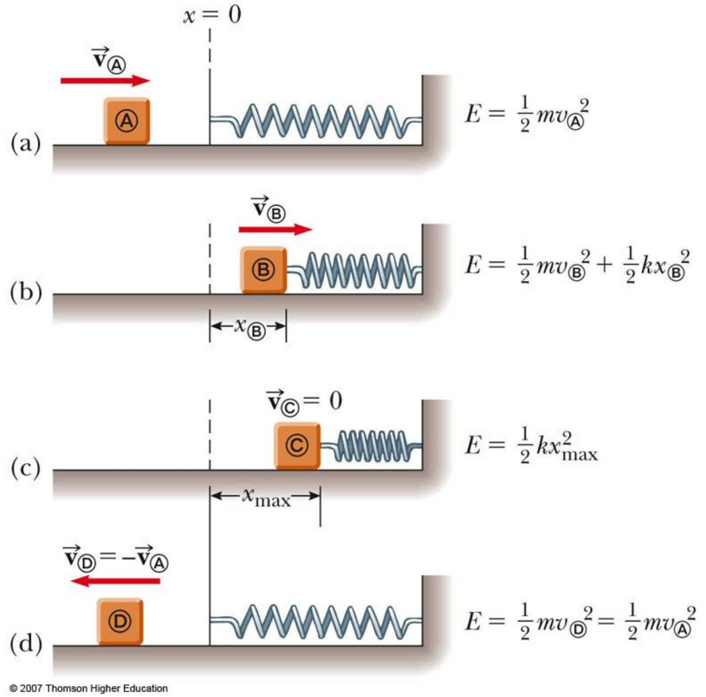
$$x_{\max} = \sqrt{\frac{m}{k}}v_A = \sqrt{\frac{0.8kg}{50N/m}}(1.2m/s) = 0.15m$$



Block-Spring Collision

Problem

- A block having a mass of 0.8 kg is given an initial velocity v_A = 1.2 m/s to the right and collides with a spring whose mass is negligible and whose force constant is k = 50 N/m as shown in figure. Suppose a constant force of kinetic friction acts between the block and the surface, with μ_k = 0.5, what is the maximum compression x_c in the spring.



Solution

$$-f_k d + \sum W_{otherforces} = \Delta U + \Delta K$$

$$-f_k d + \sum W_{otherforces} = (\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2) + (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2)$$

$$v_f = v_C = 0, \quad , v_i = v_A, \quad , x_i = 0 \text{ and } x_f = x_c = d$$

$$-\mu_k F_N d + 0 = (\frac{1}{2}kx_c^2 - 0) + (0 - \frac{1}{2}mv_A^2)$$

$$F_N = mg$$

$$\frac{1}{2}kx_c^2 - \frac{1}{2}mv_A^2 = -\mu_k mgx_c$$

$$25x_c^2 + 3.9x_c - 0.58 = 0 \qquad x_c = 0.093m$$

problem

Connected Blocks in Motion

- Two blocks are connected by a light string that passes over a frictionless pulley. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

Solution

$$-f_k d + \sum W_{otherforces} = \Delta U + \Delta K$$

$$-f_k d + 0 = (\Delta U)_s + (\Delta U)_{g_2} + (\Delta K)_1 + (\Delta K)_2$$

$$-f_k d = (\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2) + (-m_2 gh) + (\frac{1}{2}m_1 v_f^2 - \frac{1}{2}m_1 v_i^2)_1 + (\frac{1}{2}m_2 v_f^2 - \frac{1}{2}m_2 v_i^2)_2$$

$$v_{i1} = v_{i2} = 0, \quad v_{f1} = v_{f2} = 0, \quad x_i = 0 \quad \text{and} \quad x_f = x$$

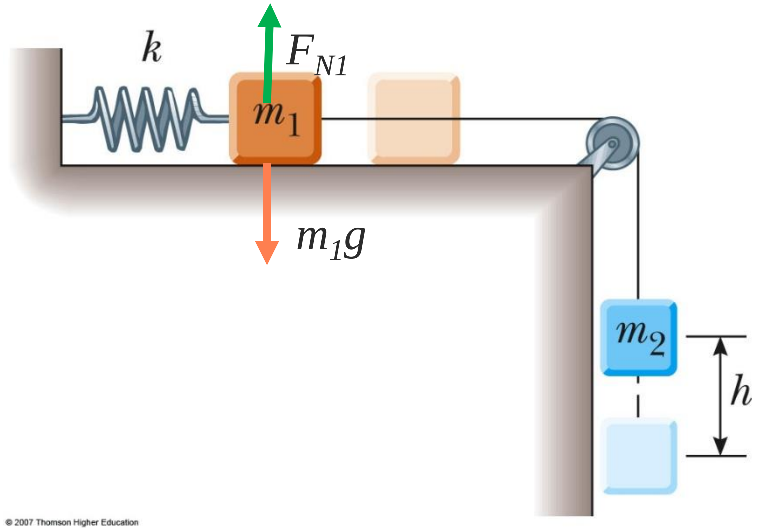
$$-f_k d = -m_2 gh + (\frac{1}{2}kx^2 - 0) + (0 + 0)$$

$$-\mu_k F_{N1} x = -m_2 gh + \frac{1}{2}kx^2$$

$$F_{N1} = m_1 g \quad \text{and} \quad x = h$$

$$\Rightarrow -\mu_k m_1 gh = -m_2 gh + \frac{1}{2}kh^2$$

$$\therefore \mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

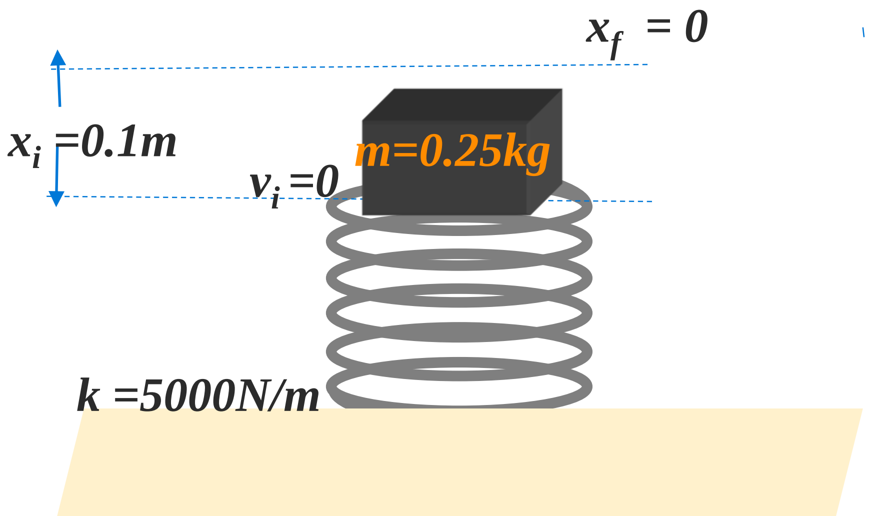


Conservation of Mechanical Energy

Problem

- A block of mass m= 0.25 kg is placed on top of a light vertical spring of force constant k = 5000 N/m and pushed downward so that the spring is compressed by 0.1 m. After the block is released, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise.

Solution



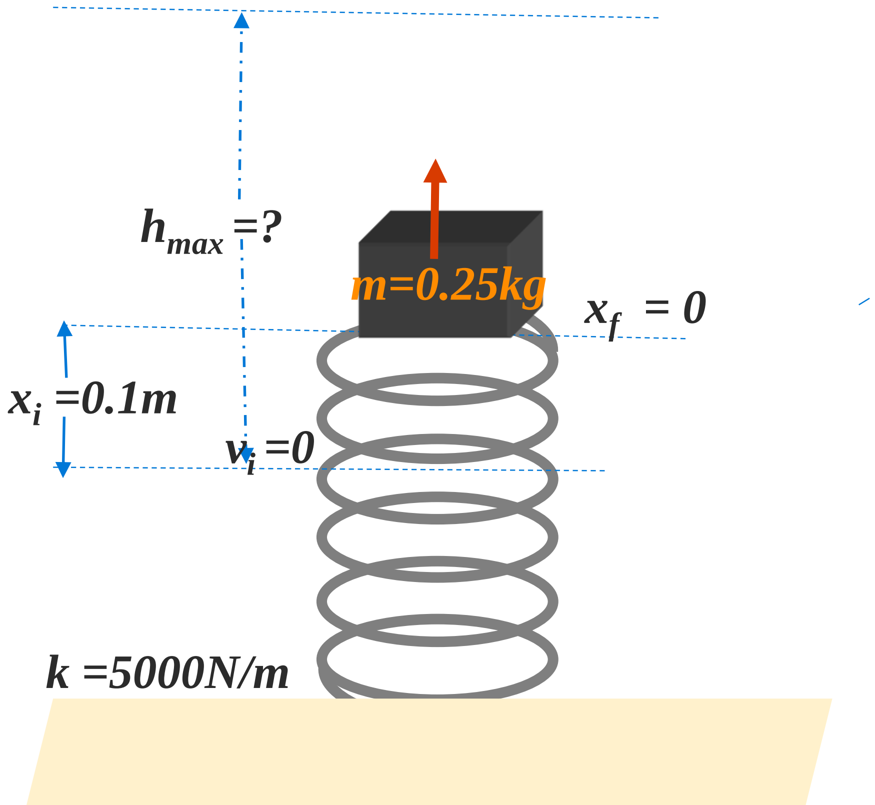
Conservation of Mechanical Energy

Problem

- A block of mass m= 0.25 kg is placed on top of a light vertical spring of force constant k = 5000 N/m and pushed downward so that the spring is compressed by 0.1 m. After the block is released, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise.

Solution

$$(\Delta U)_g + (\Delta U)_s + (\Delta K) = 0$$



Conservation of Mechanical Energy

Problem

- A block of mass m= 0.25 kg is placed on top of a light vertical spring of force constant k = 5000 N/m and pushed downward so that the spring is compressed by 0.1 m. After the block is released, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise.

Solution

$$(\Delta U)_g + (\Delta U)_s + (\Delta K) = 0$$

$$\Delta K = 0$$

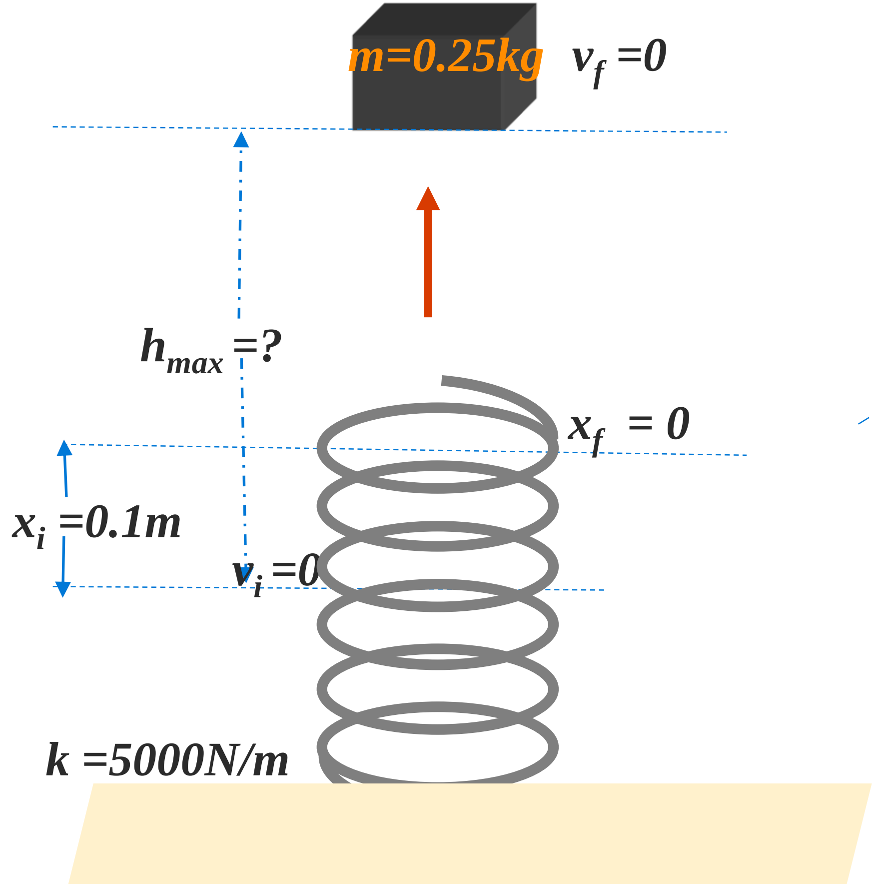
$$(\Delta U)_g = mgh_{\max}$$

$$\Delta U_s = \frac{1}{2}k(x_f^2 - x_i^2) = -\frac{1}{2}kx_i^2$$

$$mgh_{\max} = \frac{1}{2}kx_i^2$$

$$(0.25kg)(9.8m/s^2)h_{\max} = 25J$$

$$h_{\max} = 10.2m$$



Conservation of Mechanical Energy

Problem

- A block of mass m= 0.25 kg is placed on top of a light vertical spring of force constant k = 5000 N/m and pushed downward so that the spring is compressed by 0.1 m. After the block is released, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise.

Solution

$$(\Delta U)_g + (\Delta U)_s + (\Delta K) = 0$$

$$\Delta K = 0$$

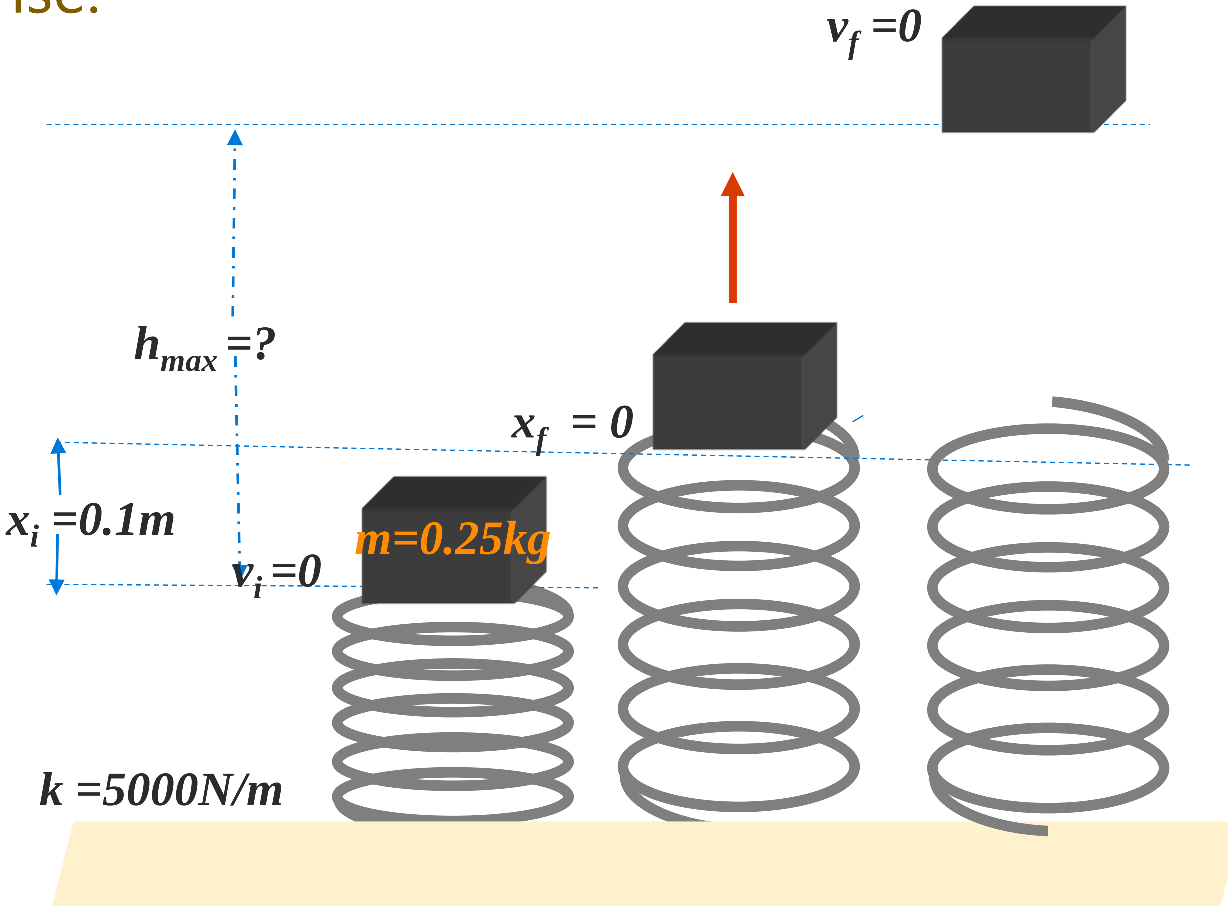
$$(\Delta U)_g = mgh_{\max}$$

$$\Delta U_s = \frac{1}{2}k(x_f^2 - x_i^2) = -\frac{1}{2}kx_i^2$$

$$mgh_{\max} = \frac{1}{2}kx_i^2$$

$$(0.25kg)(9.8m/s^2)h_{\max} = 25J$$

$$h_{\max} = 10.2m$$



Conservation of Mechanical Energy

Problem

- A bead slides without friction around a loop-the-loop. The bead is released from rest at a height h = 3.5 R. (a) What is its speed at point A? (b) How large is the normal force on the bead at point A if its mass is 5 g?

Solution

(a)

$$(\Delta U)_g + (\Delta K) = 0$$

$$-mgH + (\frac{1}{2}mv_A^2 - 0) = 0$$

H = 3.5 R – 2R =1.5 R

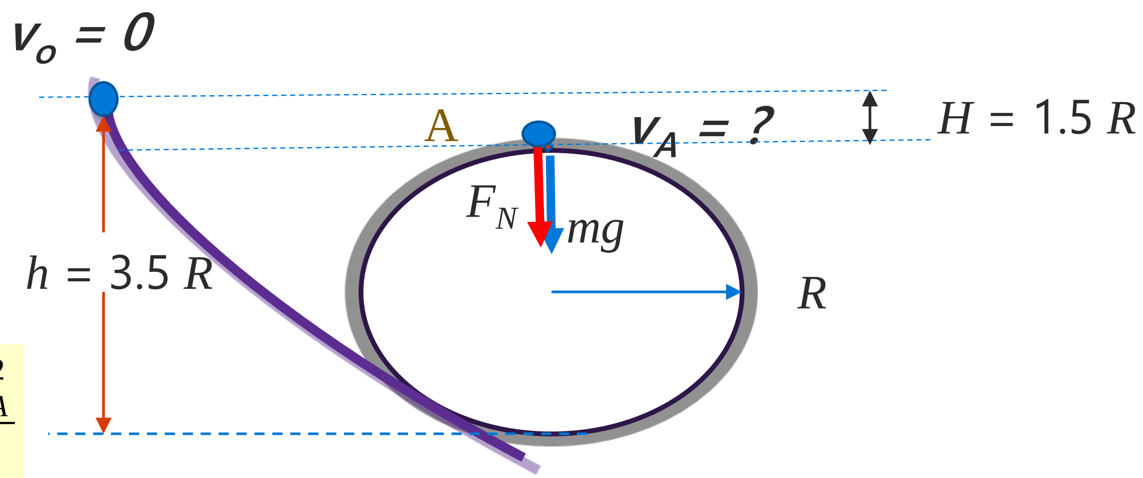
$$\Rightarrow v_A = \sqrt{2gH} = \sqrt{29.4R} m/s$$

(b)

$$\sum F_r = ma_r \quad \Rightarrow -F_N - mg = -\frac{mv_A^2}{R}$$

$$F_N = \frac{mv_A^2}{R} - mg = m(\frac{29.4R}{R} - g) = (0.005)(19.6) = 0.098N$$

down



Conservation of Mechanical Energy

Problem

- Two blocks are connected by a light string passing over a light, frictionless pulley as shown. The object of mass $m_1 = 5\ kg$ is released from rest at a height $h = 4\ m$ above the table. Using the isolated system model, (a) determine the speed of the object of mass $m_2 = 3\ kg$ just as the $5\ kg$ object hits the table and (b) find the maximum height above the table to which the $3\ kg$ object rises.

Solution

(a)

$$(\Delta U)_{g1} + (\Delta U)_{g2} + (\Delta K)_1 + (\Delta K)_2 = 0$$

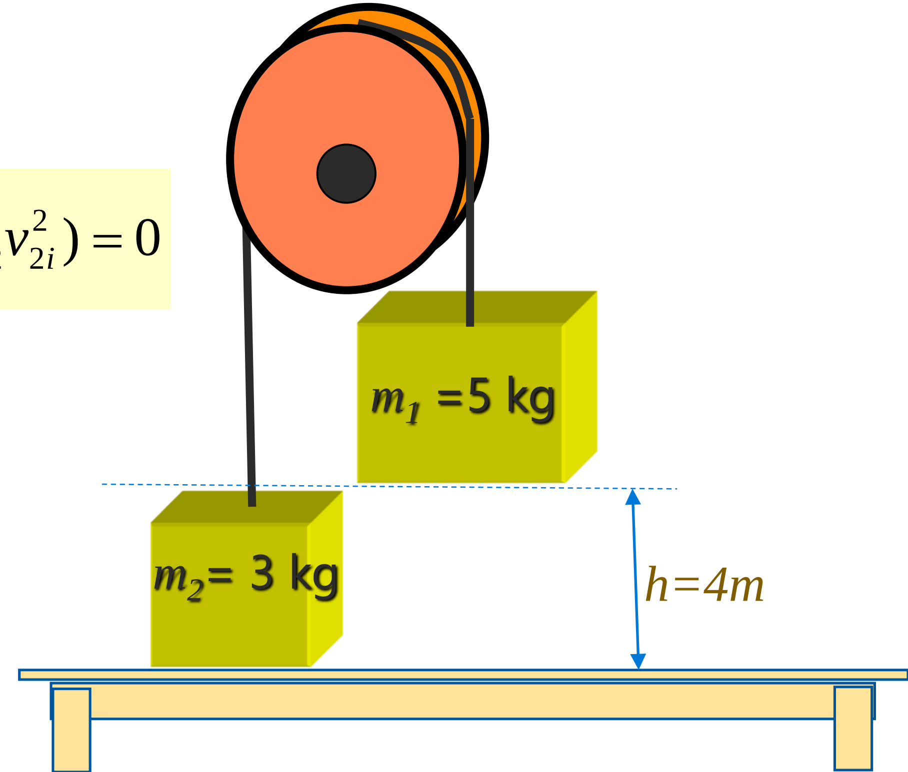
$$-m_1 gh + m_2 gh + (\frac{1}{2} m_1 v_{1f}^2 - \frac{1}{2} m_1 v_{1i}^2) + (\frac{1}{2} m_2 v_{2f}^2 - \frac{1}{2} m_2 v_{2i}^2) = 0$$

$$v_{2f} = v_{1f} = v, v_{1i} = v_{2i} = 0$$

$$(m_2 - m_1) gh + \frac{1}{2}(m_1 + m_2) v^2 = 0$$

$$v = \sqrt{\frac{2gh(m_1 - m_2)}{m_1 + m_2}}$$

$$\Rightarrow v = \sqrt{\frac{2(9.8)(4)(5-3)}{5+3}} = 4.4 m/s$$



Conservation of Mechanical Energy

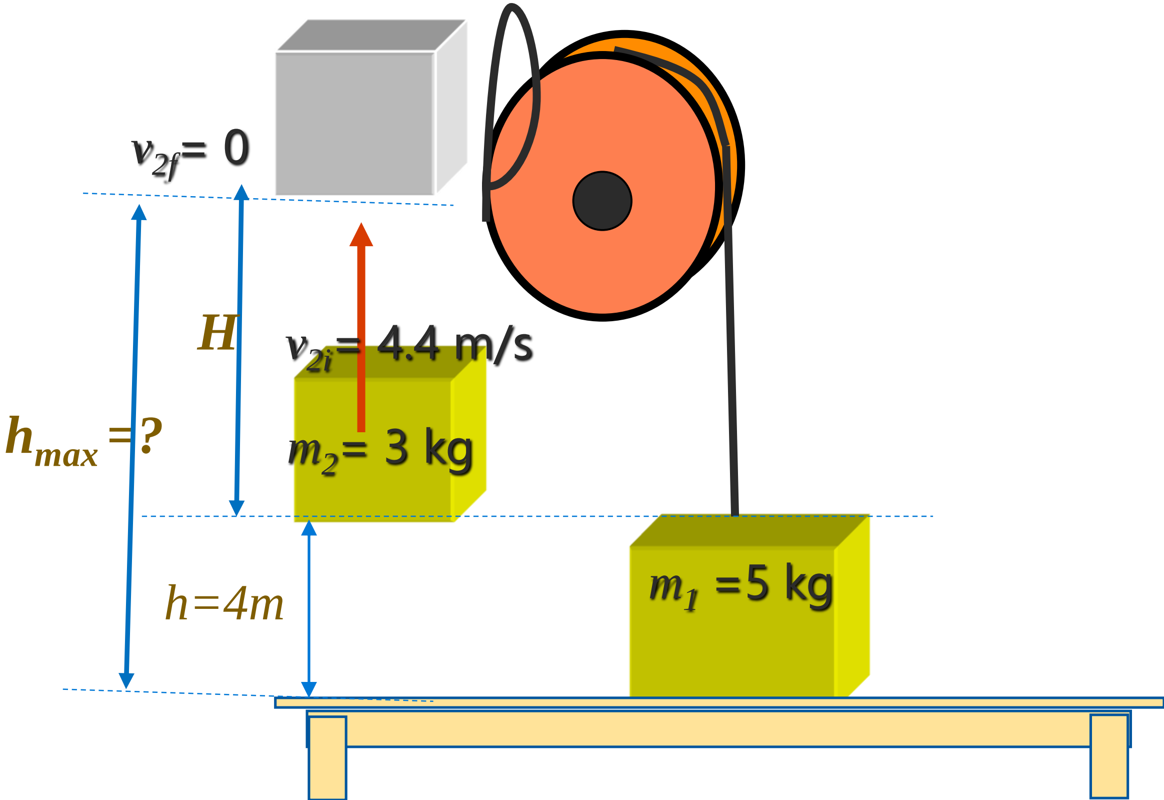
Problem

- Two blocks are connected by a light string passing over a light, frictionless pulley as shown. The object of mass $m_1 = 5\ kg$ is released from rest at a height $h = 4\ m$ above the table. Using the isolated system model, (a) determine the speed of the object of mass $m_2 = 3\ kg$ just as the $5\ kg$ object hits the table and (b) find the maximum height above the table to which the $3\ kg$ object rises.

Solution

(b) $v_{2i} = 4.4 m/s,$ $v_{2f} = 0$

$$(\Delta U)_{g2} + (\Delta K)_2 = 0$$



Conservation of Mechanical Energy

Problem

- Two blocks are connected by a light string passing over a light, frictionless pulley as shown. The object of mass $m_1 = 5\ kg$ is released from rest at a height $h = 4\ m$ above the table. Using the isolated system model, (a) determine the speed of the object of mass $m_2 = 3\ kg$ just as the $5\ kg$ object hits the table and (b) find the maximum height above the table to which the $3\ kg$ object rises.

Solution

(b) $v_{2f} = 0, \quad v_{2i} = 4.4 m/s$

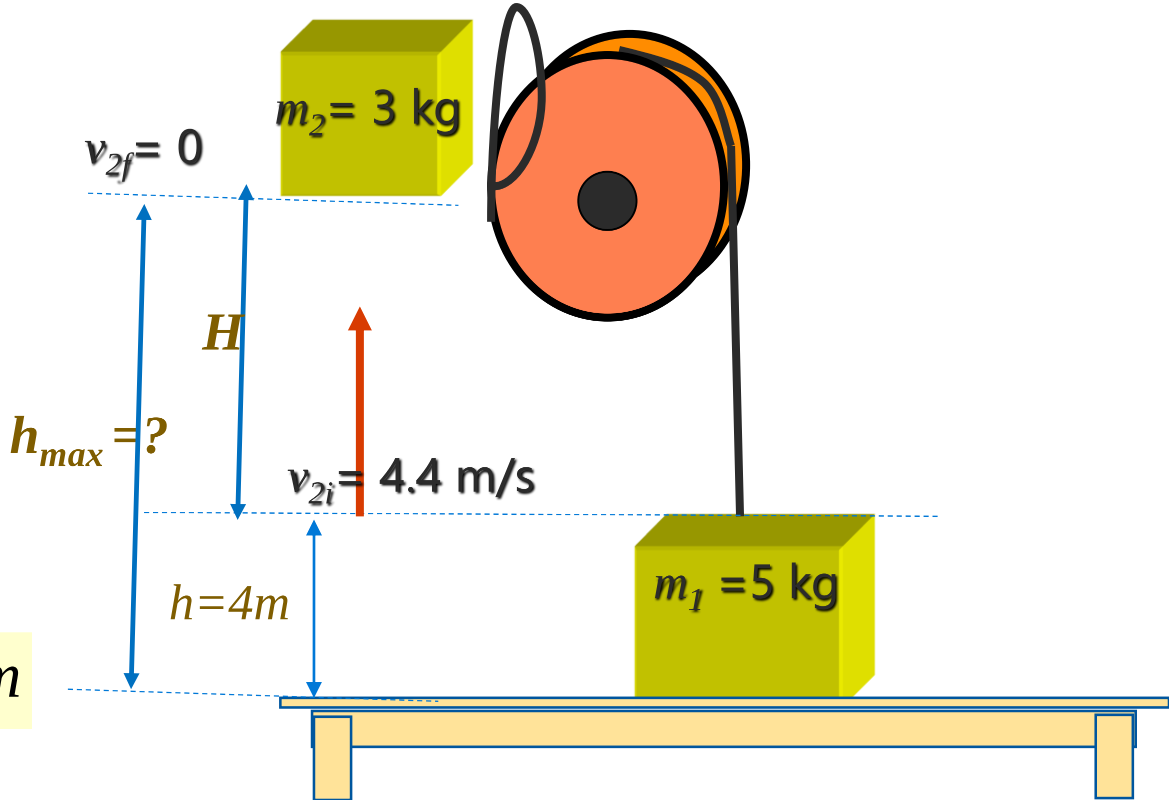
$$(\Delta U)_{g2} + (\Delta K)_2 = 0$$

$$m_2 gH + (\frac{1}{2} m_2 v_{2f}^2 - \frac{1}{2} m_2 v_{2i}^2) = 0$$

$$m_2 gH = \frac{1}{2} m_2 v_{2i}^2$$

$$\Rightarrow H = \frac{v_{2i}^2}{2g} = \frac{(4.4)^2}{2(9.8)} = 0.98m$$

$$h_{max} = H + 4 = 4.98m$$



Conservation of Mechanical Energy

Problem

- A crate of mass *m = 10 kg* is pulled up a rough incline with an initial speed of 1.5 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20° with the horizontal. The coefficient of kinetic friction $\mu_k = 0.4$, *and the crate is pulled 5 m. (a) How muck work is done by the gravitational force on the crate? (b) Determine the increase in internal energy of the crate-incline system owing to friction. (c) How muck work is done by the 100-N force on the crate? (d) What is the change in the kinetic energy of the crate? What is the speed of the crate after being pulled 5 m?*

Solution

(a)

$$W_g = F_g d\cos\phi = -mgd\sin\theta$$

$$F_g = mg\sin\theta$$

$$\cos\phi = \cos 180^o = -1$$

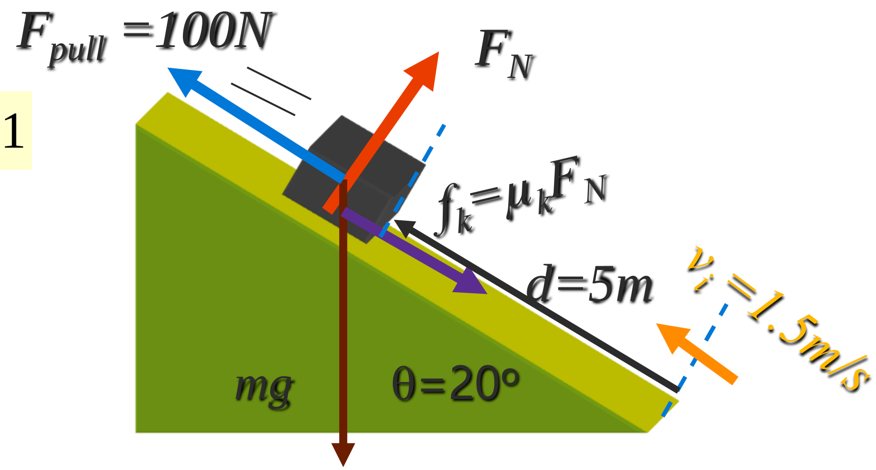
$$W_g = -(10kg)(9.8m/s^2)(5m)\sin 20^o = -167.6N$$

(b)

$$W_f = f_k d\cos 180 = -f_k d = -\mu_k F_N d$$

$$F_N = mg\cos\theta = (10)(9.8)\cos 20 = 92.1N$$

$$W_f = -(0.4)(92.1)(5) = -184.18J$$



Conservation of Mechanical Energy

Problem

- A crate of mass $m = 10\ kg$ is pulled up a rough incline with an initial speed of 1.5 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20° with the horizontal. The coefficient of kinetic friction $\mu_k = 0.4$, *and the crate is pulled 5 m. (a) How muck work is done by the gravitational force on the crate? (b) Determine the increase in internal energy of the crate-incline system owing to friction. (c) How muck work is done by the 100-N force on the crate? (d) What is the change in the kinetic energy of the crate?(e) What is the speed of the crate after being pulled 5 m?*

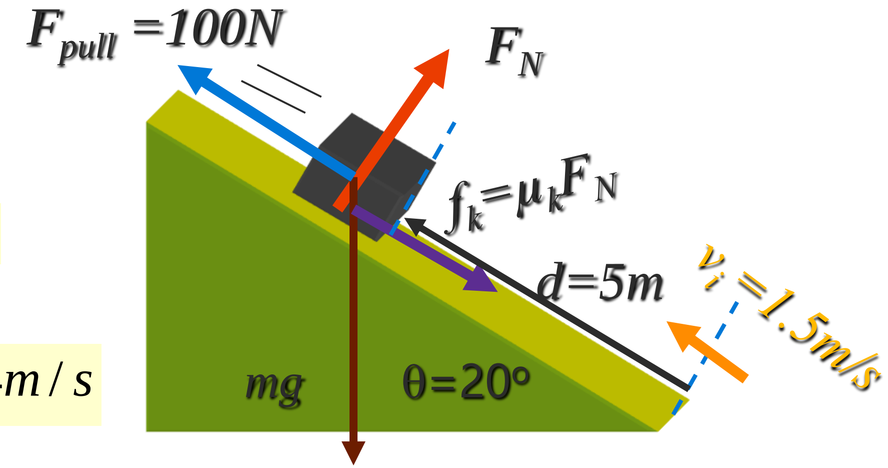
Solution

(c) $W_F = Fd\cos\varphi$

$W_F = (100)(5)\cos 0 = 500J$

(d) $W_{net} = W_g + W_F + W_f = \Delta K$ $\Rightarrow \Delta K = -167.6 + 500 - 184.18 = 148.2J$

(e) $\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(10)(v_f^2 - (1.5)^2) = 148.2J$ $\Rightarrow v_f = 5.44m/s$



Conservation of Mechanical Energy

Problem

- A $5\ kg$ block is set into motion up an inclined plane with an initial speed of $v_i = 8\ m/s$. The block comes to rest after travelling $d = 3\ m$ along the plane, which is inclined at an angle of $\theta = 30^o$ to the horizontal. For this motion, determine (a) the change in the block's kinetic energy, (b) the change in the potential energy of the block-Earth system, and (c) the friction force exerted on the block (assumed to be constant). (d) What is the coefficient of kinetic friction?

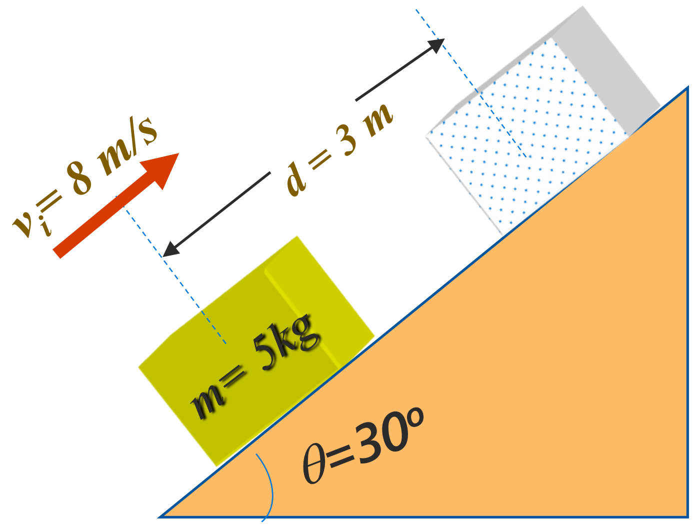
Solution

(a) $$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(5)(0-8)^2) = -160\,J$$

(b) $$(\Delta U)_g = m\ gh$$

$$h = d\sin\theta = (3m)\sin 30^\circ = 1.5m$$

$$(\Delta U)_g = (5kg)(9.8m/s^2)(1.5m) = 73.5J$$



Change in Mechanical Energy for Nonconservative Forces

Problem

- A $5\ kg$ block is set into motion up an inclined plane with an initial speed of $v_i = 8\ m/s$. The block comes to rest after travelling $d = 3\ m$ along the plane, which is inclined at an angle of $\theta = 30^o$ to the horizontal. For this motion, determine (a) the change in the block's kinetic energy, (b) the change in the potential energy of the block-Earth system, and (c) the friction force exerted on the block (assumed to be constant). (d) What is the coefficient of kinetic friction?

Solution

(c) $$W_f = (\Delta U)_g + \Delta K$$

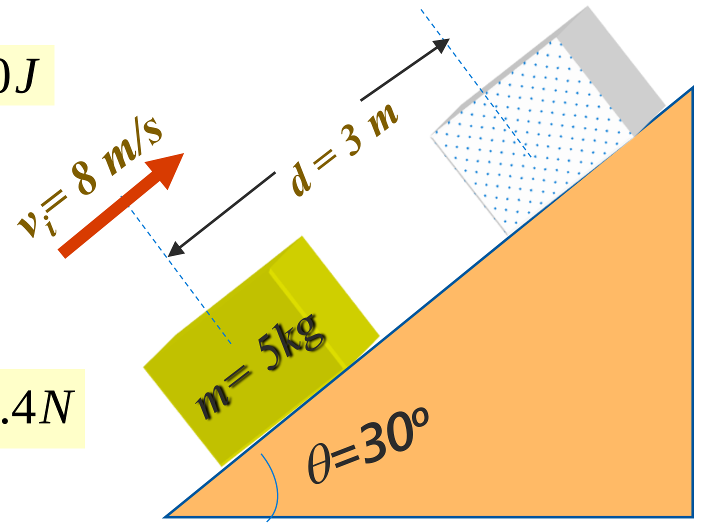
$$W_f = -f_k d \qquad (\Delta U)_g = 73.5J \qquad \Delta K = -160J$$

$$\therefore -f_k(3m) = 73.5J - 160J \qquad \Rightarrow f_k = 28.8N$$

(d) $$f_k = \mu_k F_N$$

$$\sum F_y = F_N - mg\cos\theta = 0 \qquad \Rightarrow F_N = mg\cos\theta = (5kg)(9.8m/s^2)\cos 30 = 42.4N$$

$$28.8N = \mu_k(42.4N) \qquad \Rightarrow \mu_k = 0.67$$



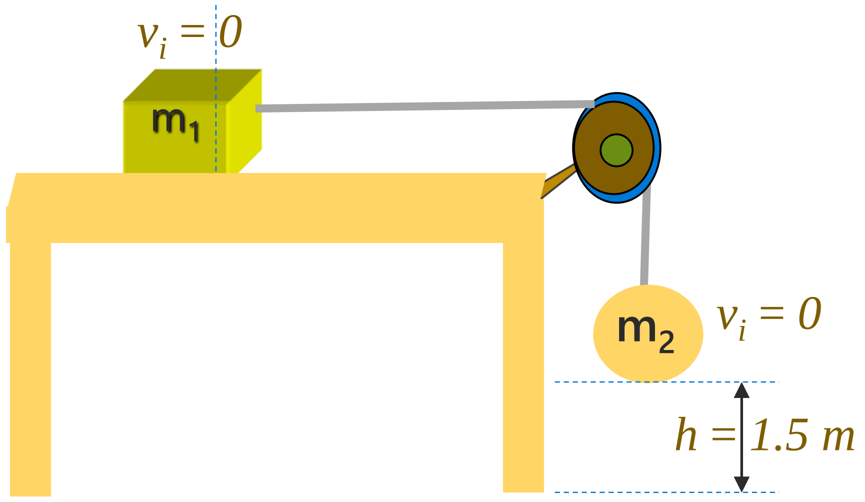
Change in Mechanical Energy for Nonconservative Forces

Problem

- The coefficient of friction between the block of mass $m_1 = 3\ kg$ and the surface is $\mu_k = 0.4$. The system starts from rest. What is the speed of the ball of mass $m_2 = 5\ kg$ when it has fallen a distance $h = 1.5\ m$?

Solution

$$W_f = (\Delta U)_{g2} + (\Delta K)_1 + (\Delta K)_2$$



Change in Mechanical Energy for Nonconservative Forces

Problem

- The coefficient of friction between the block of mass $m_1 = 3\ kg$ and the surface is $\mu_k = 0.4$. The system starts from rest. What is the speed of the ball of mass $m_2 = 5\ kg$ when it has fallen a distance $h = 1.5\ m$?

Solution

$$W_f = (\Delta U)_{g2} + (\Delta K)_1 + (\Delta K)_2$$

$$W_f = -f_k h = -\mu_k F_{N_1} h = -\mu_k m_1 gh$$

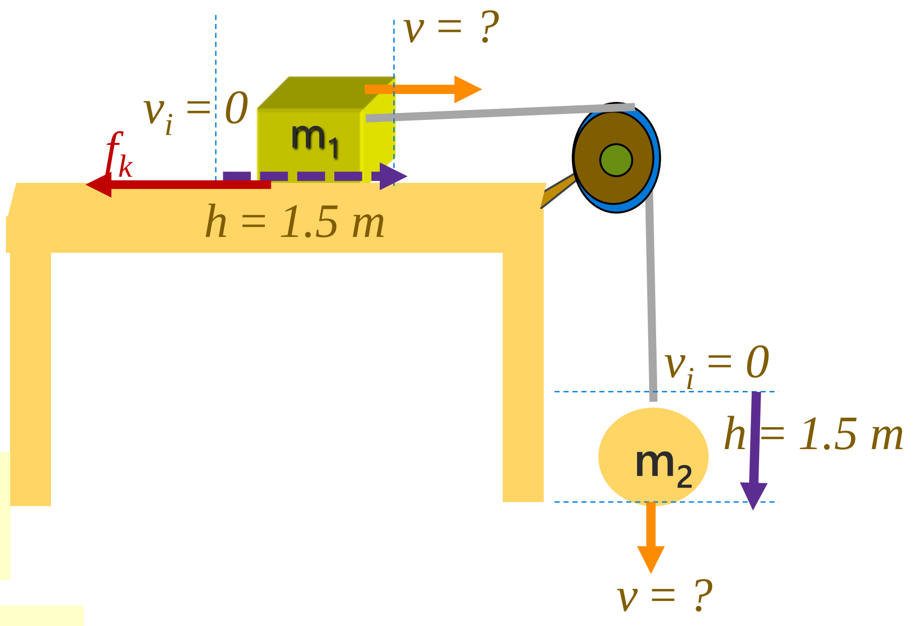
$$(\Delta U)_{g2} = -m_2 gh$$

$$(\Delta K)_1 = \frac{1}{2} m_1 v^2 \qquad (\Delta K)_2 = \frac{1}{2} m_2 v^2$$

$$-\mu_k m_1 gh = -m_2 gh + (\frac{1}{2} m_1 v_f^2) + (\frac{1}{2} m_2 v_f^2)$$

$$-\mu_k m_1 gh = -m_2 gh + \frac{1}{2}(m_1 + m_2) v_f^2 \qquad \Rightarrow (m_2 - \mu_k m_1) gh = \frac{1}{2}(m_1 + m_2) v_f^2$$

$$\Rightarrow v_f = \sqrt{\frac{2(m_2 - \mu_k m_1) gh}{(m_1 + m_2)}} \qquad \Rightarrow v_f = \sqrt{\frac{2[5-(0.4)(3)](9.8)(1.5)}{(3+5)}} = 3.7 m/s$$



Power

- Work does not depend on time interval
- The rate at which energy is transferred is important in the design and use of practical device
- The time rate of energy transfer is called power
- The average power is given by

$$P_{avg} = \frac{\Delta W}{\Delta t}$$

  - when the method of energy transfer is work

Instantaneous Power

- Power is the time rate of energy transfer. Power is valid for any means of energy transfer

- Other expression $$P_{avg} = \frac{W}{\Delta t} = \frac{F\Delta x}{\Delta t} = Fv_{avg}$$

- A more general definition of instantaneous power $$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

Units of Power

- The SI unit of power is called the watt
  - 1 watt = 1 joule / second = 1 kg · m^2 / s^3
- A unit of power in the US Customary system is horsepower
  - 1 hp = 550 ft · lb/s = 746 W
- Units of power can also be used to express units of work or energy
  - 1 kWh = (1000 W)(3600 s) = 3.6×10^6 J

Power

Problem

- *A 650-kg elevator starts from rest. It moves upward for 3 s with constant acceleration until it reaches its cruising speed of 1.75 m/s.* (a) What is the average power of the elevator motor during this time interval? (b) How does this power compare with the motor power when the elevator moves at its cruising speed?

Solution

(a) $W_{motor} = (\Delta U)_g + \Delta K$

$$W_{motor} = mgh + +(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2)$$

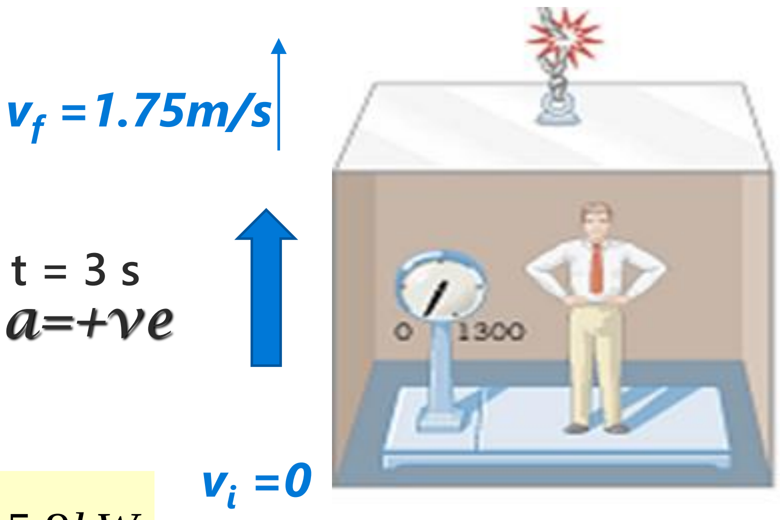
$v_i = 0$ $\quad v_f = 1.75 m/s$

$$h = (\frac{v_f + v_i}{2})t = (\frac{1.75 + 0}{2})(3) = 2.625m$$

$$\therefore W_{motor} = (650kg)(9.8m/s^2)(2.625m) + [\frac{1}{2}(650kg)(1.75m/s)^2 - 0]$$

$$\Rightarrow W_{motor} = 17716.56J = 17.716kJ$$

$$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{17716.56}{3} = 5905.5W = 5.9kW$$



Power

Problem

- A 650-kg *elevator starts from rest. It moves upward for 3 s with constant acceleration until it reaches its cruising speed of 1.75 m/s.* (a) What is the average power of the elevator motor during this time interval? (b) How does this power compare with the motor power when the elevator moves at its cruising speed?

Solution

(a) $W_{motor} = (\Delta U)_g + \Delta K$

$$W_{motor} = mgh + +(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2)$$

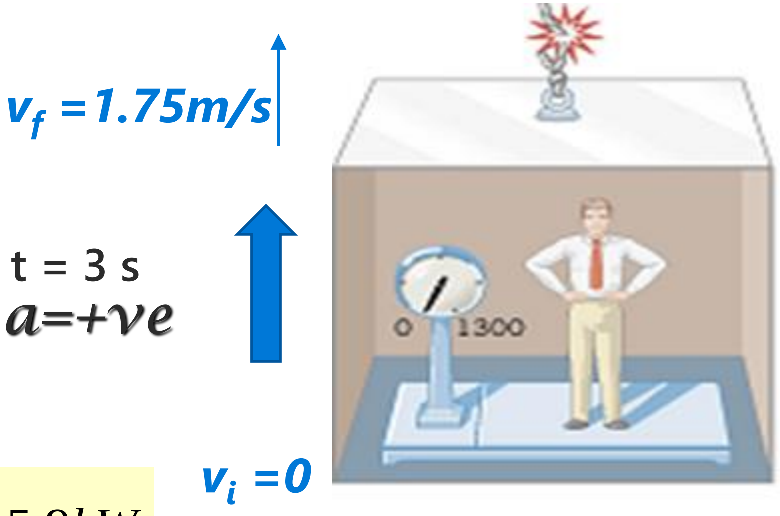
$v_i = 0$ $v_f = 1.75 m/s$

$$h = (\frac{v_f + v_i}{2})t = (\frac{1.75+0}{2})(3) = 2.625m$$

$$\therefore W_{motor} = (650kg)(9.8m/s^2)(2.625m) + [\frac{1}{2}(650kg)(1.75m/s)^2 - 0]$$

$$\Rightarrow W_{motor} = 17716.56J = 17.716kJ$$

$$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{17716.56}{3} = 5905.5W = 5.9kW$$



Power

Problem

- A 650-kg *elevator starts from rest. It moves upward for 3 s with constant acceleration until it reaches its cruising speed of 1.75 m/s.* (a) What is the average power of the elevator motor during this time interval? (b) How does this power compare with the motor power when the elevator moves at its cruising speed?

Solution

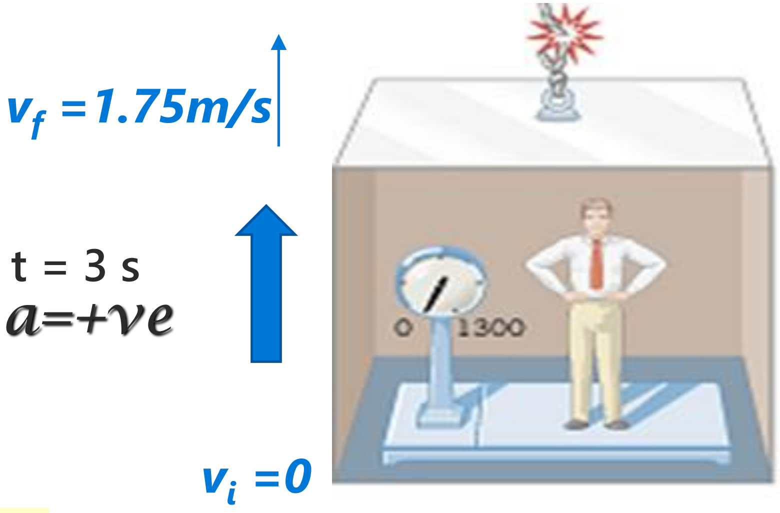
(b) $P = Fv\cos\theta = Fv$

When the elevator moves with constant speed (v = 1.75 m/s), the net force acting on it must be zero

$$\sum F_y = F_{motor} - mg = 0 \qquad \Rightarrow F_{motor} = mg$$

$$P_{motor} = F_{motor} v = mgv$$

$$\Rightarrow P_{motor} = (650kg)(9.8m/s^2)(1.75m/s) = 11147.5W = 11.147kW$$



Power Delivered by an Elevator Motor

Exercise

A 1000-kg elevator carries a maximum load of 800 kg. A constant frictional force of 4000 N retards its motion upward. What minimum power must the motor deliver to lift the fully loaded elevator at a constant speed of 3 m/s?

Solution

$$F_{net,y} = ma_y$$

$$T - f - Mg = 0$$

$$T = f + Mg = 2.16\times10^4\,N$$

$$P = Fv = (2.16\times10^4\,N)(3m/s)$$

$$= 6.48\times10^4 W$$

$$P = 64.8kW = 86.9hp$$