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Energy of a System

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Work and Energy

- □ Work Done by a Constant Force and Some Applications
- □ Work Done by a Varying Force and Some Applications
- Potential Energy of a System
- □ Conservative and Non-conservative Forces
- □ Relationship Between Conservative Forces and Potential Energy
- □ Kinetic Energy and Work-Kinetic Energy Theorem
- Power



- F is the magnitude of the force \vec{F}
- Δ r (or dr) is the magnitude of the object's displacement
- θ is the angle between \vec{F} and $\Delta \vec{r}$ (or $d\vec{r}$)

Units of Work

Work = (Force) × (Displacement)

= (Newtons) × (meters) (in SI Units)

(Newton) × (meter) \rightarrow Joule (J)

or (kg. m²/s²) \rightarrow **Joule (J)** *

* Joule is named after James Prescott Joule (1818-1889) who made major contributions to the understanding of energy, heat, and electricity

Work Done by a Force

Work Done by a Constant Force: (A Scalar Quantity)

 The work, W, done by a constant force on an object is defined as the product of the component of the force along the direction of displacement and the magnitude of the displacement

 $W \equiv (F\cos\theta)\Delta r$

- F is the magnitude of the force
- Δ r is the magnitude of the object's displacement
- θ is the angle between \vec{F} and $\Delta \vec{r}$

 $\vec{F} = \text{constant}$ θ $F \cos \theta$ $F \cos \theta$

Work Done by a Force

 Positive scalar quantity⇒ when force and displacement are both in the same direction

 $W = (F\cos\theta)\Delta r \equiv +ve$

 Negative scalar quantity ⇒ when force and displacement are in opposite directions

$$W = (F\cos\theta)\Delta r \equiv -ve$$





Work Done by a Force



 $= \Delta r$

Work Done by a Force

Example

A particle moving in the *xy* plane undergoes a displacement given by $\Delta \vec{r} = (2\hat{i} + 3\hat{j})m$ as a constant force $\vec{F} = (5\hat{i} + 2\hat{j})N$ acts on the particle.

- (a) Calculate the magnitude of the force and the displacement of the particle.
- (b) Calculate the work done by the force on the particle
- (c) What is the angle between the force and displacement?

 $= \Delta r$

Work Done by a Force

Solution Given: $\Delta \vec{r} = (2\hat{i} + 3\hat{j})m$

 $\vec{F} = (5\hat{i} + 2\hat{j})N$

(a) To calculate the magnitude of the force and the displacement of the particle:

$$F = \sqrt{5^2 + 2^2} = 5.38N \qquad \Delta r = \sqrt{2^2 + 3^2} = 3.6m$$

(b) To calculate the work done by the force on the particle:

$$W = \vec{F} \bullet \Delta \vec{r} \qquad \Rightarrow W = (5\hat{i} + 2\hat{j}) \bullet (2\hat{i} + 3\hat{j}) = 10 + 6 = 16J$$

(c) To find the angle between the force and displacement:

$$\cos\theta = \frac{\vec{F} \bullet \Delta \vec{r}}{(F)(\Delta r)} = \frac{16J}{(5.38N)(3.6m)}$$
$$\cos\theta = 0.826 \qquad \theta = \cos^{-1}(0.826) = 34.3^{\circ}$$

Work Done by a Force



X

 \vec{F}

 $F_x = F \cos \theta$

 $|\Delta \vec{r}| = \Delta x$

Work Done by Several Forces

V

 F_N

Mg

 If several forces act on an object of mass M, then the total work is equal to the algebraic sum of the work done by the individual forces

 $W_{net} = \sum_{i=1}^{m} W_i \quad i = 1, 2, 3...m,$ where m is the number of forces

• In the shown figure, three forces act on the object of mass M, namely: F_N , Mg and \vec{F}

 You should remember that the work is a scalar quantity, and the algebraic sum of the work done by the three forces is:

 $W_{net} = W_g + W_N + W_F$

but, $W_g = W_N = 0$, because F_N and Mg are both perpendicular to $\Delta \vec{r}$.

Thus
$$W_{net} = W_F = (F \cos \theta) \Delta x$$

where $|\Delta \vec{r}| = \Delta x$







Satellite in a Circular Motion

Does the Earth do work on the satellite?





A 50.0-kg crate is pulled 40.0 m by a person with a constant force (F = 100 N and $\theta = 37.0^{\circ}$). A friction force $f_k = 50.0$ N is exerted on the crate. Determine the work done by each force acting on the crate. What is the net work?





Example

A 50.0-kg crate is pulled 40.0 m by a person with a constant force (F = 100 N and $\theta = 37.0^{\circ}$). A friction force $f_k = 50.0$ N is exerted on the crate. Determine the work done by each force acting on the crate. What is the net work?



A 50.0-kg crate is pulled 40.0 m by a person with a constant force (F = 100 N and $\theta = 37.0^{\circ}$). A friction force $f_k = 50.0$ N is exerted on the crate. Determine the work done by each force acting on the crate. What is the net work? Solution $W = (F\cos\theta)\Delta r$ V **180°** $W_{F_{\mathcal{N}}} = W_g = 0$ because F_N and mg are perpendicular X to d. F_N $W_F = (F\cos\theta)d$ = 100N $\Rightarrow=(100N\cos 37^{\circ})(40m)$ $\theta = 37^{\circ}$ *d*=40*m* $W_{F} = 3194.5J$ $f_k = 50N$ $F\cos\theta$ $W_f = (f \cos 180^\circ)d$ $W_f = -(50N)(40m) = -2000J$ **90**° Mg = 40N

Work Done by Several Forces



velocity



A block of mass m = 2.5 kg is pushed a distance d = 2.2 m along a frictionless horizontal table by a constant applied force of magnitude F = 16 N directed at angle $\theta = 25.0^{\circ}$ below the horizontal as shown. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, (c) the gravitational force, (d) the net force on the block, and (e) If a friction between block and table does exist and exerts a work of 11.2 J against the motion of the block find the coefficient of kinetic friction



Solution

Given: $m = 2.5 \text{ kg}, d = 2.2 \text{ m}, F = 16 \text{ N}, \theta = 25.0^{\circ}$

(a) To find the work done by the applied force:

 $W_F = (F\cos\theta)d$

 $\Rightarrow=(16N\cos 25^{\circ})(2.2m)$

 $W_{F} = 70.2J$



Work Done by Several Forces Solution

Given: m = 2.5 kg, d = 2.2 m, F = 16 N, $\theta = 25.0^{\circ}$

(b), (c) To find the work done by the normal force exerted by the table and by the gravitational force:

 $W = (F\cos\theta)\Delta r$

 $W_{F_N} = W_g = 0$, because F_N and mg are perpendicular to d.



Given: $m = 2.5 \text{ kg}, d = 2.2 \text{ m}, F = 16 \text{ N}, \theta = 25.0^{\circ}$

(d) To find the work done by the net force on the block.

 $W_{net} = W_F = (F\cos\theta)d = 70.2J$

Solution



Solution

Given: m = 2.5 kg, d = 2.2 m, F = 16 N, $\theta = 25.0^{\circ}$

(e) To find the coefficient of kinetic friction when work done by frictional force is 11.2 J when friction between table and block does exist:

 $W_f = (f \cos 180^\circ)d$ $W_f = -f_k d$ $11.2J = -f_k (2.2m)$ $f_k = -5.1N$

$$\begin{split} \varSigma F_y &= 0 \Rightarrow F_N - mg - Fsin\theta = 0\\ Or &\Rightarrow F_N = mg + Fsin\theta\\ \Rightarrow F_N &= (2.5kg)(9.8m/s^2) + (16N)sin25 = 31.26N \end{split}$$

 \Rightarrow (5.1N)= μ_k (31.26N) \Rightarrow μ_k =0.16



 \vec{F} = variable

 $F\cos\theta$

 $d\vec{r}$

Work Done by a Varying Force

Work Done by a Varying Force: (A Scalar Quantity) • Again the work, W_{F} , done by a varying force on an object is defined as the product of the component of the force along the direction of displacement and the magnitude of the displacement $W_F \equiv \int_{r}^{r_2} (F \cos \theta) dr$

- *F* is the magnitude of the force
- *dr* is the magnitude of the object's displacement
- θ is the angle between \vec{F} and $d\vec{r}$

Work Done by a Varying Force



Work Done by a Varying Force

The sum of rectangular areas under the curve can be approximated as the area of the curve for a continuously varying force when number of rectangles increases (from Figure b to c). Thus $W_F \equiv \sum F_i (\Delta x)_i$



Example

A force $\vec{F} = (4x\hat{i} + 3y\hat{j})N$, where x and y are in meters, acts on a an object as the object moves in the x- direction from the origin to x = 5 m. Find the work done by the force on the object.

Solution

$$W_F = \int_{r}^{2} \vec{F} \cdot d\vec{r}$$

Since $d\vec{r} = dx\hat{i} + dy\hat{j}^{r_1}$, and the object moves in the x-direction and there is no change in y-direction, then $dy = 0$
 $\therefore d\vec{r} = dx\hat{i}$
 $\vec{F} \cdot d\vec{r} = (4x\hat{i} + 3y\hat{j}) \cdot dx\hat{i}$
 $\Rightarrow \vec{F} \cdot d\vec{r} = 4xdx$
 $\therefore W_F = \int_{x_1}^{x_2} F_x dx \Rightarrow W_F = 4 \int_{x_1=0}^{x_2=5m} xdx \Rightarrow W_F = 50J$

Example

The force acting on a particle varies as shown. Find the work done by the force on the particle as it moves (a) from x= o to x = 8 m, (b) from x = 8 mto x = 10 m, and (c) from x = 0to x = 10 m.





Work Done by a Varying Force: Spring Force

Equilibrium position

Spring Force: It is a restoring force

$$F_s = -kx$$
 (Hooke's Law)

k is elastic (or spring) constant

Note: (1) The force exerted by spring is always opposite to the displacement from equilibrium.

(2) The applied force F_{app} has the same magnitude as the spring force but opposite in direction, namely,

$$F_{app} = kx$$

x = 0Spring in its original length Stretched spring Compressed spring

Work Done by a Varying Force: Spring Force

Work done by a Spring Force:

 $W = \int_{x_1}^{x_2} F_x dx$

$$F_s = -kx$$

$$W_s = \int_{x}^{x_2} (-kx) dx$$

$$W_{s} = -k(\frac{x^{2}}{2}) \Big|_{x_{1}}^{x_{2}}$$
$$W_{s} = -\frac{k}{2}(x_{2}^{2} - x_{1}^{2})$$

This expression can be applied for a stretched or a compressed spring



Work Done by Applied Force:

Work Done by Applied Force:

Work Done to Stretch or Compress Spring

$$W_{app} = \frac{k}{2} (x_2^2 - x_1^2)$$



Example Work Done by Several Forces

When a 4-kg object is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.5 cm. If the 4-kg object is removed, (a) how far will the spring stretch if a 1.5-kg is hung on it? (b) how much work must an external agent do to stretch the same spring 4 cm from its un-stretched position?



Solution

For $m_1 = 4 kg$, $y_1 = 2.5 cm$. (a) To find y_2 when the mass m_1 is replaced by $m_2 = 1.5 kg$



Solution

W

(b) To find the work done by an external agent to stretch the spring 4 cm from its un-stretched position. Knowing that k = 1568 N/m

$$W_{app} = -\frac{k}{2}(y^{2} - y_{o}^{2})$$
$$W_{app} = \frac{k}{2}(y^{2} - y_{o}^{2})$$

=-W

$$y = 4 cm = 0.04 m$$
, $y_0 = 0$, $k = 1568 N/m$

$$W_{app} = \frac{(1568N/m)}{2}((0.04)^2 - 0) = 1.28J$$



Types of Forces

Conservative forces

- Work and energy associated with the force can be recovered
- Examples: Gravity, Spring Force
- Nonconservative forces
 - The forces are generally dissipative and work done against it cannot easily be recovered
 - Examples: Kinetic friction, air drag forces, normal forces, tension forces, applied forces ...





- A force is conservative if the work it does on an object moving between two points is independent of the path the objects take between the points
- ✓ The work depends only upon the initial and final positions of the object (it does not depend on the movement path)
- ✓ The work done by this force in a closed path is zero
- ✓Any conservative force can have a potential energy function associated with it
- ✓ Work done by gravity
- ✓ Work done by spring force

$$W_g = PE_i - PE_f = mgy_i - mgy_f$$
$$W_s = PE_{si} - PE_{sf} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$



Example

A 4-kg object moves from the origin O to point C having coordinates x = 5 m and y = 5 m, as shown. One force on the object is the gravitational force acting in the negative y-direction. Using the relation $W = \vec{F} \cdot \Delta \vec{r}$ to calculate the work done by the gravitational force on the object as it goes from O to C along (a) the path OAC, (b) OC, and (c) OBC (d) Your results should be identical. Why?



Solution

Given: m = 4 kg C (x = 5 m, y = 5 m). To calculate the work done by the gravitational force on the object as it goes from O to C along (a) the path OAC



Solution

Given: m= 4 kg C (x = 5 m, y = 5 m). To calculate the work done by the gravitational force on the object as it goes from O to C along (b) the path OC $W = \vec{F} \cdot \Delta \vec{r}$



Solution

Given: m= 4 kg C (x = 5 m, y = 5 m). To calculate the work done by the gravitational force on the object as it goes from O to C along (c) the path OBC



(d) The answer is the same in (a), (b) and (c) because the force is conservative

Plots for Potential Energy and Force of Spring



Relationship Between Conservative Forces and Potential Energy

When the PE (or U) is a function of one coordinate, say, x (i.e. U=U(x)) and this potential energy is related to a conservative force,

Where, $W_c = -\Delta PE = -\Delta U$ $PE=(1/2)kx^{2}$ and $W_c = F_x dx$, One can write $dU = -F_x dx$, Or $F_x = - \frac{dU}{dx}$ The potential energy stored in a spring is $U = (\frac{1}{2})kx^2$ One can find the spring force using $F_y = -dU/dx$ to get $F_y = -kx$ Χ

Relationship Between Conservative Forces and Potential Energy

But if the PE (or U) is a function of two coordinates, say, x and y (i.e. U=U(x, y)) and this potential energy is related to a conservative force,

This force may have two components, namely:

$$F_x = -\partial U(x, y) / \partial x$$
 and $F_y = -\partial U(x, y) / \partial y$

Relationship Between Conservative Forces and Potential Energy Example

A potential energy function for a system in which a two dimensional force acts is of the form $U(x, y) = 3x^2y - 7x$. Find the force that acts at the point (x, y). What is the magnitude of this force when x = -1m and y = 2m?

The two components of this force, are:

$$F_x = -\partial U(x, y) / \partial x = -6xy + 7$$
 and $F_y = -\partial U(x, y) / \partial y = -3x^2$

at the point (x =-1, y=2)m, $F_x =-(6(-1)(2) - 7)=19N$ and $F_y =-3(2)^2 =-12N$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(19)^2 + (-12)^2} = 22.5N$$

Solution

Kinetic Energy and Work-Kinetic Energy Theorem

- Kinetic energy is defined as
 - **Use the equation of motion**

$$K = \frac{1}{2}mv^2$$
$$v_f^2 = v_i^2 + 2a\Delta x$$

Divide both sides of this equation by m/2 to get:

$$\frac{mv_f^2}{2} = \frac{mv_i^2}{2} + ma\Delta x$$
$$ma\Delta x = \frac{m}{2}(v_f^2 - v_i^2)$$

Or

The work-kinetic energy theorem can be written as:

$$W = K_f - K_i = \Delta K$$

Note: This equation is correct whether the forces that act on an object are constant or variable

A 3-kg object has a velocity $(6\hat{i} - 1\hat{j})m/s$.

Example

- (a)What is its kinetic energy at this moment?
- (b) what is the net work done on the object if its velocity changes to $(8\hat{i} + 4\hat{j})m/s$? [Note: Use the definition of the dot product, $v^2 = \vec{v} \bullet \vec{v}$]

Solution

Given m= 3 kg and $\vec{v}_o = (6\hat{i} - 1\hat{j})m/s$ (a)To find the kinetic energy at this moment. $\vec{v}_{o} \bullet \vec{v}_{o} = (6\hat{i} - 1\hat{j}) \bullet (6\hat{i} - 1\hat{j}) = 37m^{2} / s^{2}$ $K_o = \frac{1}{2}mv_o^2 = \frac{1}{2}(3kg)(37m^2/s^2)$ $K_{o} = 55.5J$

$$v_o^2 = \vec{v}_o \bullet \vec{v}_o$$

$$\vec{v} = (8\hat{i} + 4\hat{j})m/s$$

$$K = \frac{1}{2}mv^{2} = \frac{1}{2}(3kg)(80m^{2}/s^{2}) \qquad K = 120J$$

 $W_{not} = 120 - 55.5 = 64.5J$ $W_{net} = K - K_o = \Delta K$

Example

A 0.6-kg particle has a speed of 2 m/s at point A and kinetic energy of 7.5 J at point B, what is (a) its kinetic energy at A, (b) its speed at B, and (c) the net work done on the particle by external forces as it moves from A to B.



Solution

(b)To find the speed at B: $Given m= 0.6 \text{ kg}, v_A = 2 \text{ m/s } K_B = 7.5 \text{ J}$ (a) To find the kinetic energy at A: $\Rightarrow K_A = \frac{1}{2} (0.6 kg) (2m/s)^2 = 1.2J$ $v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{2(7.5J)}{(0.6 kg)}} = 5m/s$

(c) To find the net work done on the particle by external forces as it moves from A to B.

$$W_{net} = \Delta K = K_B - K_A$$

:
$$W_{net} = 7.5 - 1.2 = 6.3J$$

