General

Physics 101

The Laws of Motion (Newton's Laws)

Prepared By

Prof. Rashad Badran

By Prof. Rashad Badran

Forces and Newton’s Laws of Motion

- ❑ Forces and Equilibrium
- ❑ Newton’s Three Laws of Motion
- ❑ Types of Forces
- ❑ Forces of Friction
- ❑ Application of Newton’s Second Law
- ❑ Apparent Weight

Newton's First Law

Newton's First Law: An object at rest or an object in motion at constant speed will remain at rest or at constant speed in the absence of a resultant force.(The law of Inertia)

Newton's First Law

If the object is at rest, it remains at rest (velocity = 0).
If the object is in motion (v= constant), it continues to move in a straight line with the same velocity.

No force is required to keep a body in straight line motion when effects such as friction are negligible.

An object is in **translational equilibrium** if the net force on it is zero:

$$\vec{F}_{net} = \sum_{i=1}^{n} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots + \vec{F}_n = 0$$

Translational equilibrium

- **Static Equilibrium** $\Leftrightarrow \vec{v} = 0, \Rightarrow \vec{a} = 0$
- **Dynamic Equilibrium** $\Leftrightarrow \vec{v} = const, \Rightarrow \vec{a} = 0$

$$\vec{F}_{net} = 0$$

Newton's First Law

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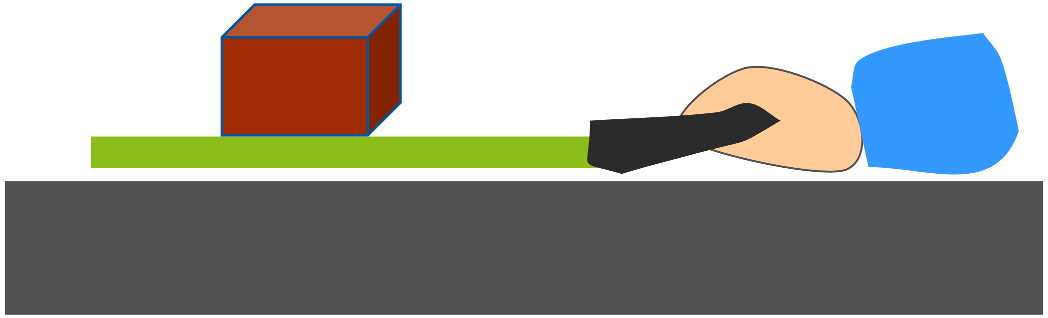

A block is placed on a sheet and the sheet is pulled quickly to the right. The block tends to remain at rest while the sheet is removed.

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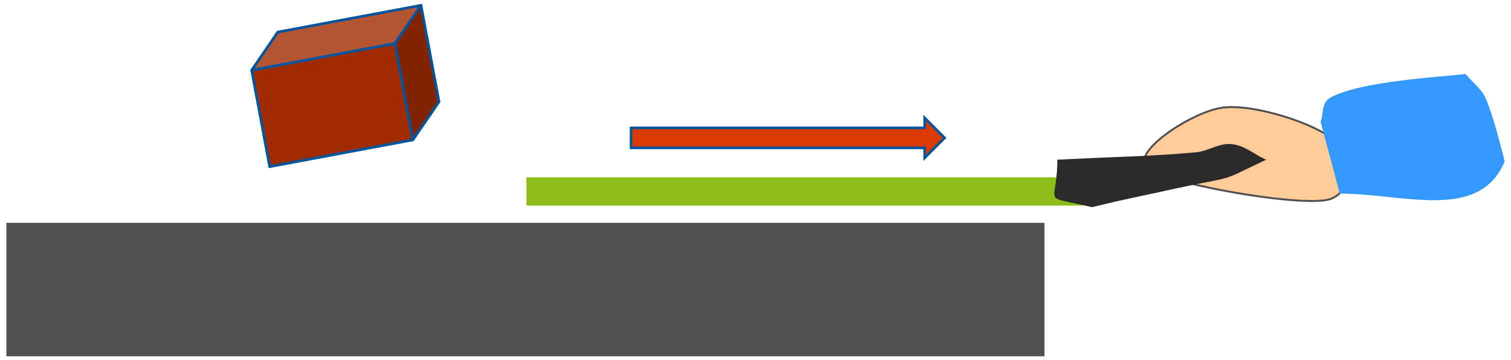

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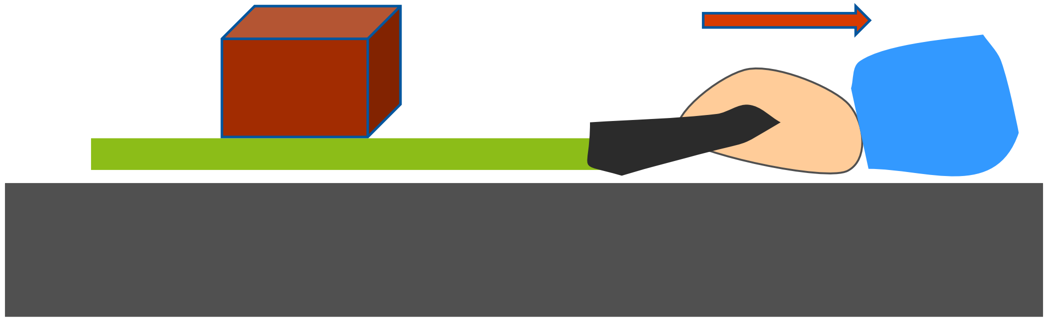

Assume block and sheet move together at constant speed. If the sheet stops suddenly, the block tends to maintain its constant speed.

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Newton's First Law

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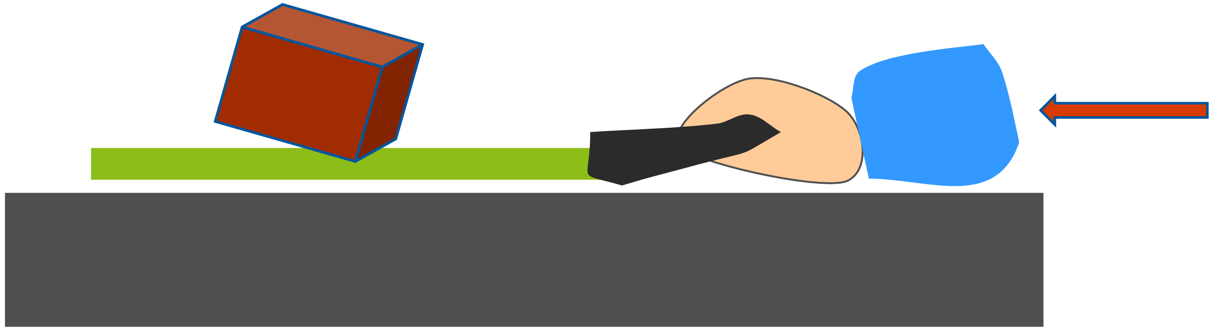

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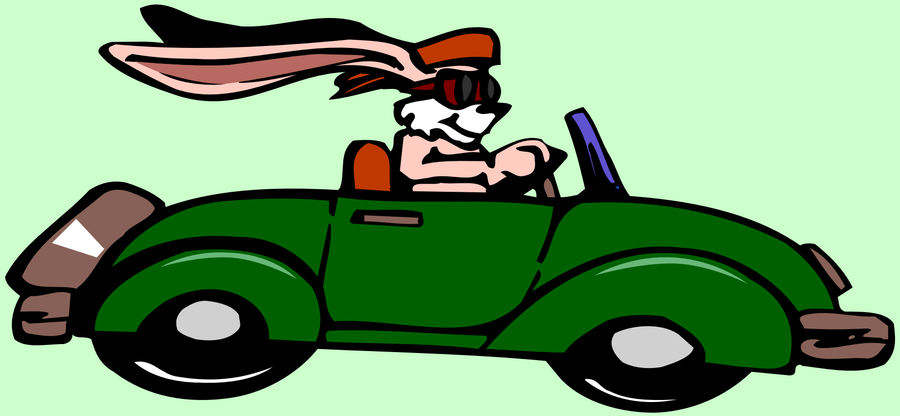
Understanding the First Law:

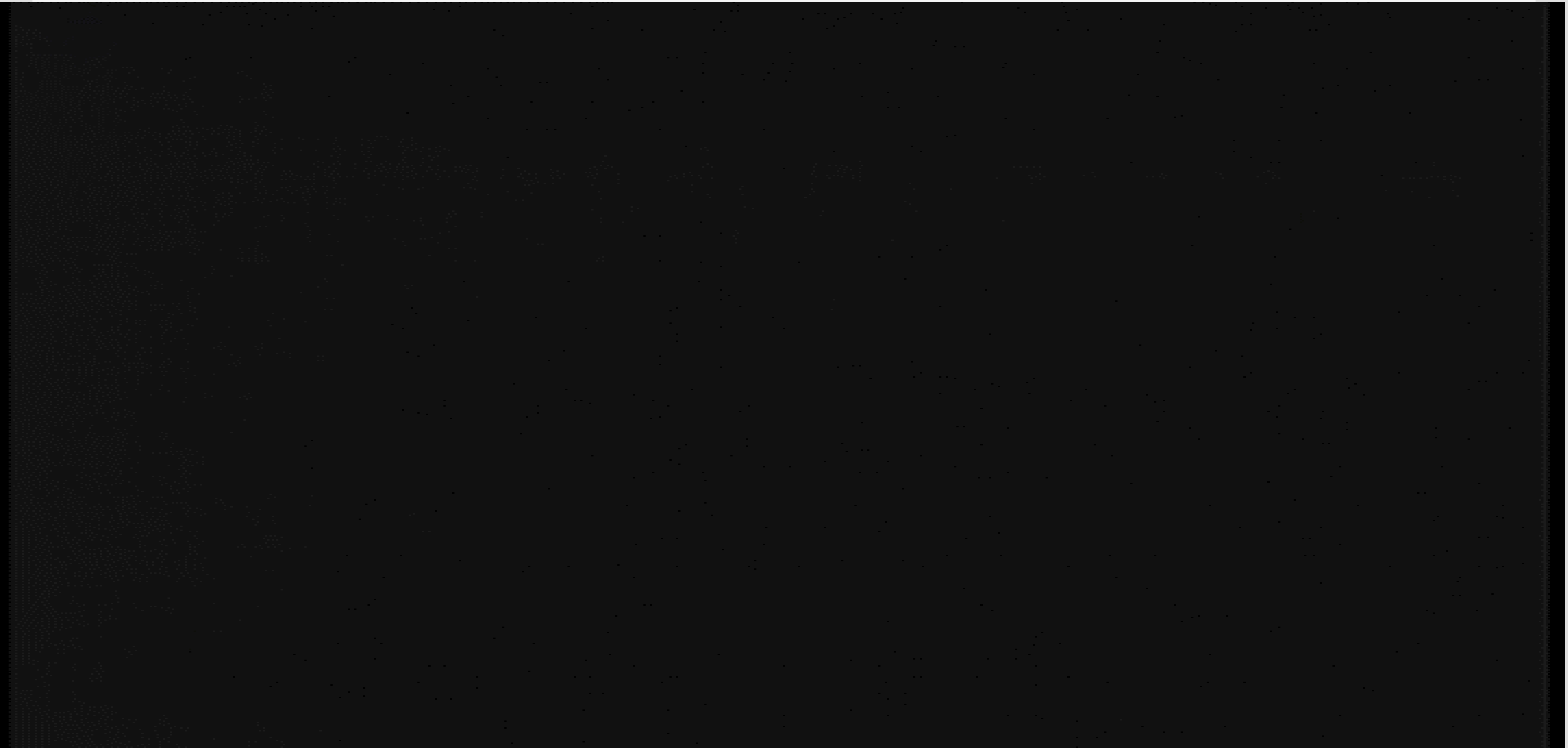


Discuss what the driver experiences when a car accelerates from rest and then applies the brakes.

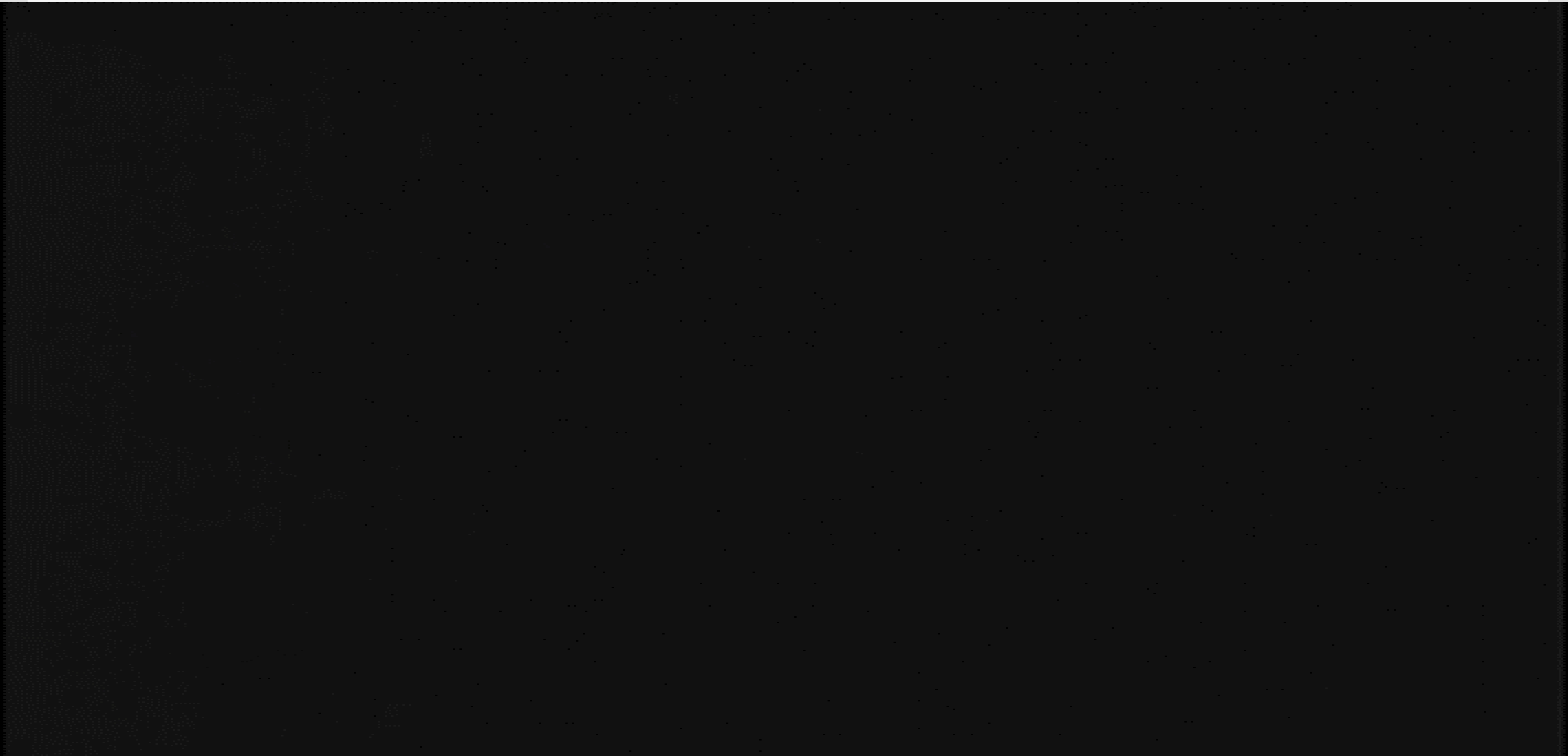
(a) The driver is forced to move forward. An object at rest tends to remain at rest.

(b) Driver must resist the forward motion as brakes are applied. A moving object tends to remain in motion.

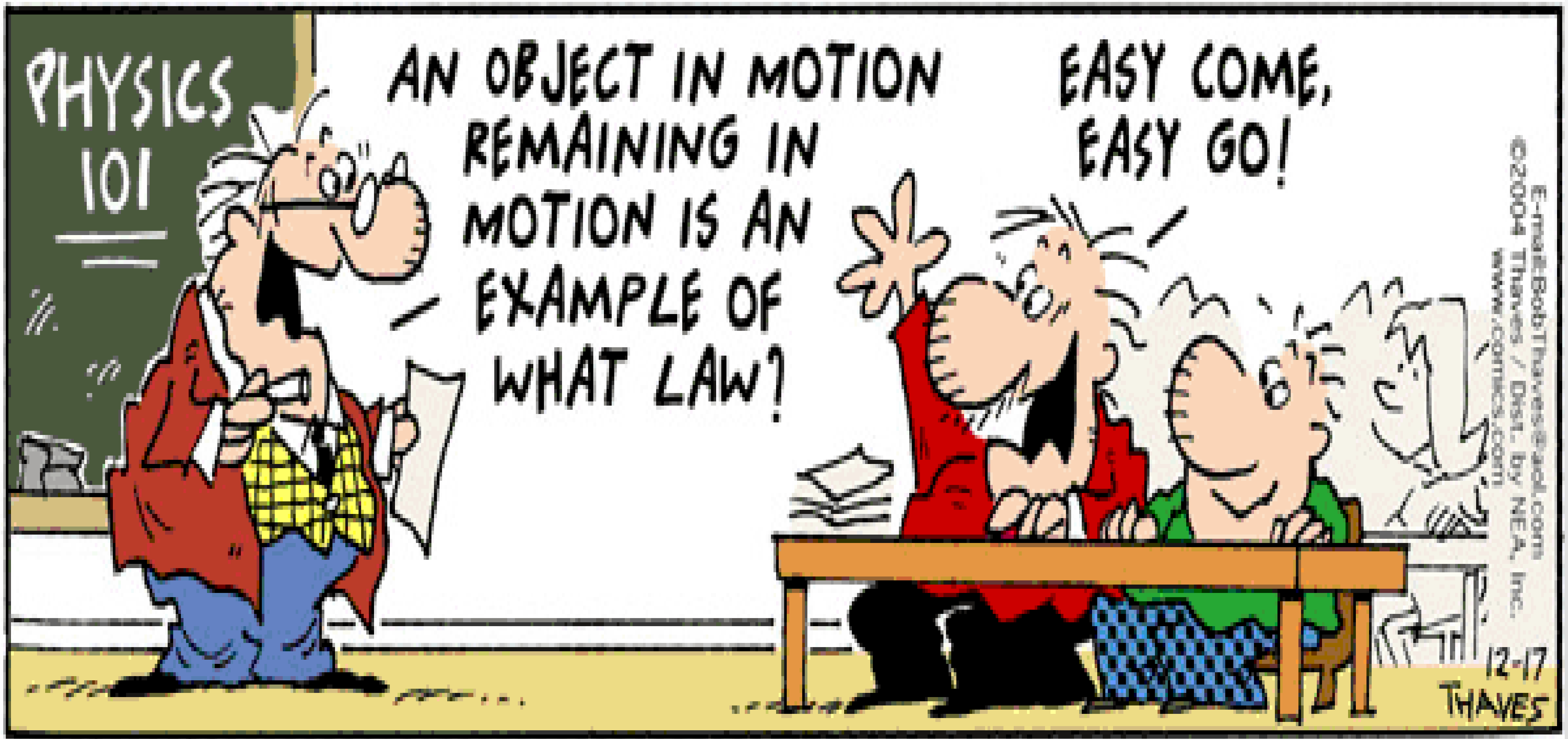
Newton’s First Law



Newton’s First Law



What Do You Learn?



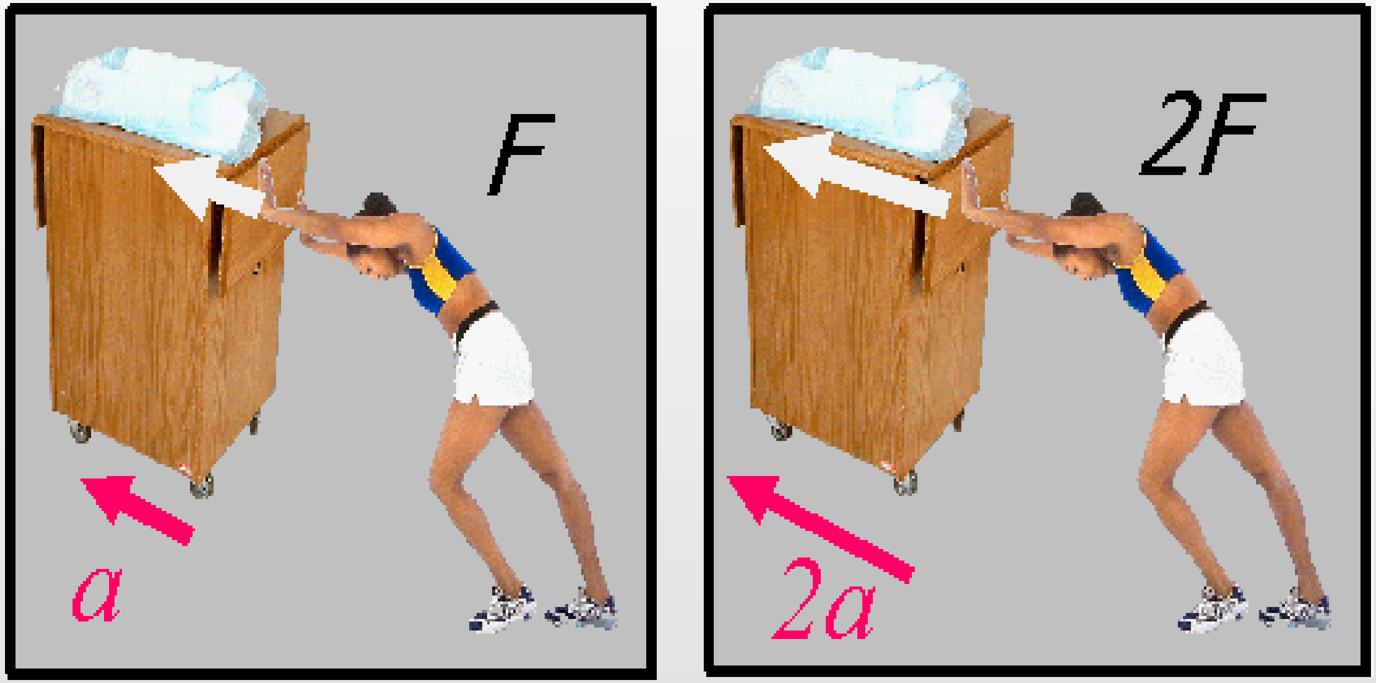
Newton's Second Law: **Dynamics**

- **Newton's Second Law:** Whenever a resultant force acts on an object, it produces an acceleration: an acceleration that is directly proportional to the force and inversely proportional to the mass.

$$\vec{F}_{net} = \sum_{i=1}^{n} \vec{F}_1 + .\vec{F}_2 + \vec{F}_3 + \ldots.\vec{F}_n = m\vec{a}$$

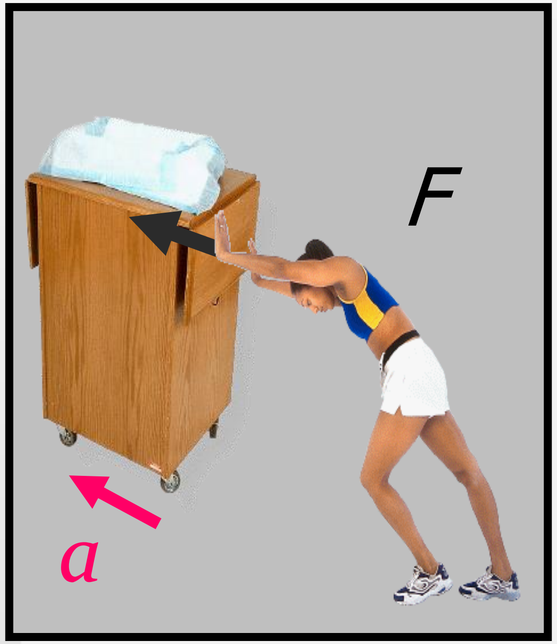
or $$\vec{a} = \frac{\vec{F}_{net}}{m}$$

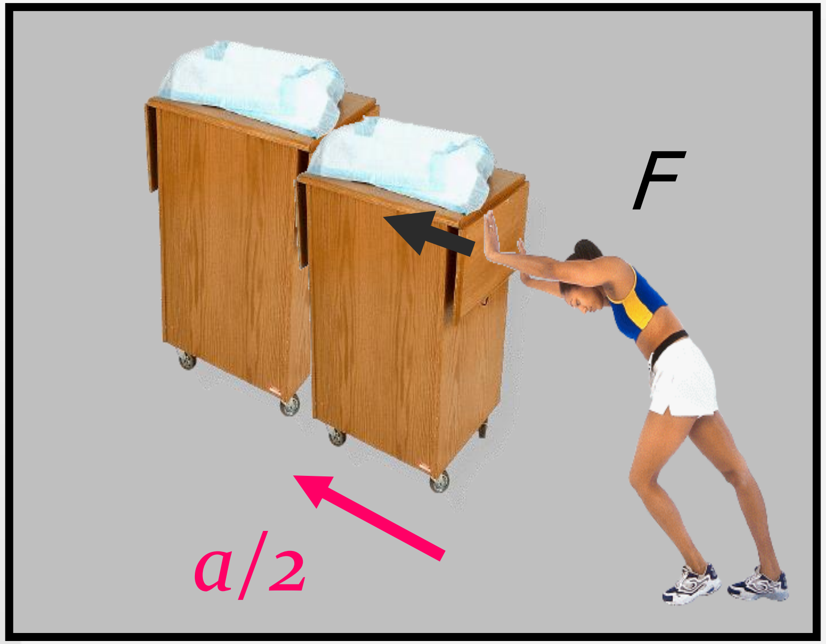
Acceleration and Force With Absence of Friction Forces



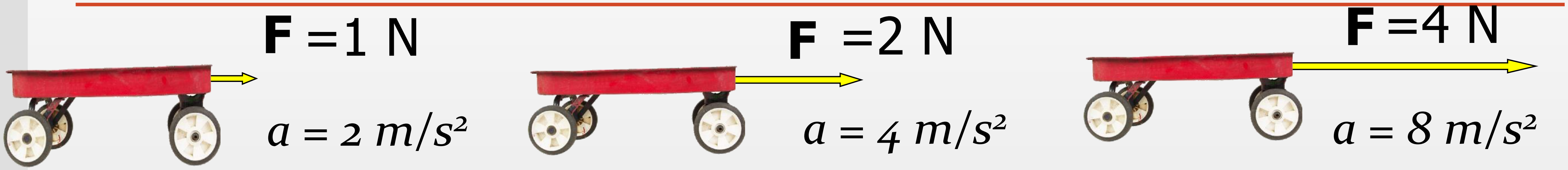
Pushing the cart with twice the force produces twice the acceleration. Three times the force triples the acceleration.

Acceleration and Force With Absence of Friction Forces





Pushing two carts with same force F produces one-half the acceleration. The acceleration varies inversely with the amount of material (the mass).



Acceleration a is directly proportional to force F and is in the direction of the force. Friction forces are ignored in this experiment.

Systems of Units

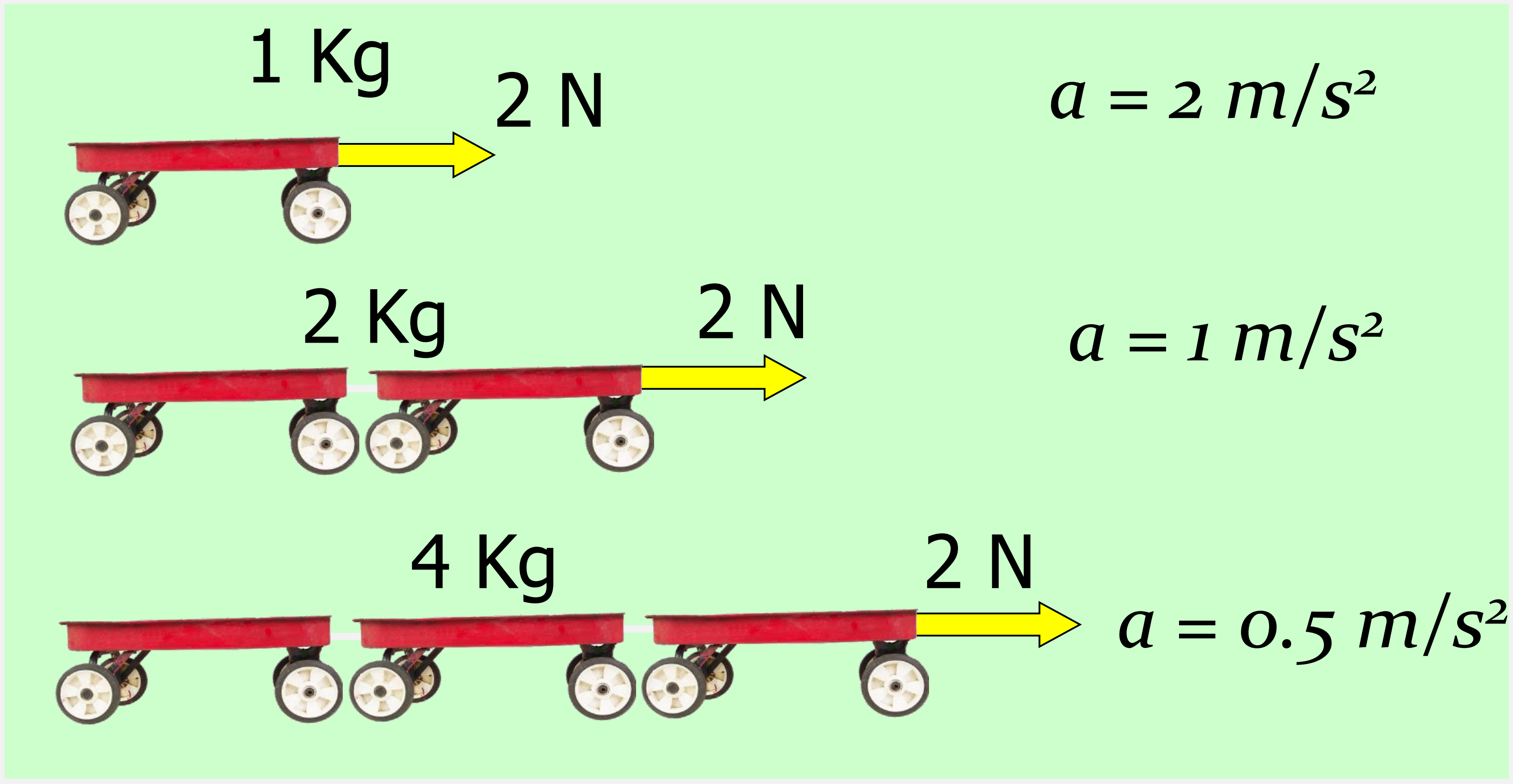
SI system: If *kg* is unit of mass, *m* is unit of length, and *s* is unit of time, one can derive unit of force, called *Newton* (N) as follows:

$$F \text{ (N)} = m \text{ (kg)} \, a \text{ (m/s}^2\text{)}$$

British system: If *lb* is unit of force, ft is unit of length, and *s* is unit of time, then on can get unit of mass, called slug *as follows*

$$m \text{ (slugs)} = F \text{ (}lb\text{)} / a \text{ (ft/s}^2\text{)}$$

Mass and Inertia



The mass is a measure of inertia. Applying a constant force on different masses shows the inertia in each case (ability of each mass to resist motion)

Mass and Inertia

Inertia is a measure of an object's resistance to changes in its motion. It is represented by the **inertial mass**.

Applications On Newton's Laws: **Dynamics**

Problem

A force $\vec{F}$ applied to an object of mass m_1 produces an acceleration of 3 m/s^2. The same force applied to a second object of mass m_2 produces an acceleration of 1 m/s^2.(a) What is the value of the ratio m_1/m_2?

(b) If m_1 and m_2 are combined into one object, find its acceleration under the action of the force $\vec{F}$.

Applications On Newton's Laws: **Dynamics**

Solution

(a)

m_1 ⟷ $a_1 = 3\ m/s^2$. m_2 ⟷ $a_2 = 1\ m/s^2$.

To find the value of the ratio m_1/m_2? .

$F = m_1 a_1 = 3\ m_1$ ------------------- (1)

$F = m_2 a_2 = m_2$ --------------------(2)

Divide equation (1) by equation (2) to get

$m_1 / m_2 = 1/3$

Applications On Newton's Laws: **Dynamics**

Solution

(b)

The combined mass $m_1 + m_2$ is under the action of the same force F

To find the acceleration: .

$$F = (m_1 + m_2)a \text{ --------------------- (3)}$$

But we have $m_2 = 3m_1$, when substituting this into equation (3) one can get:

$$F = (m_1 + 3\,m_1)a = (4m_1)a \text{ ------ (4)}$$

From the equation $F = m_1\,a_1 = 3\,m_1$ and equation (4), one may have

$$a = 0.75\ m/s^2$$

Type of Forces

- Contact forces: forces that arise due to an interaction between the atoms in the surfaces of the bodies in contact.
  e.g. Pushing or Pulling Force, Normal Force & Friction

- Tension: force transmitted through a “rope” from one end to the other.

  An **ideal** cord has zero mass, does not stretch, and the tension is the same throughout the cord.

- Gravitational Force

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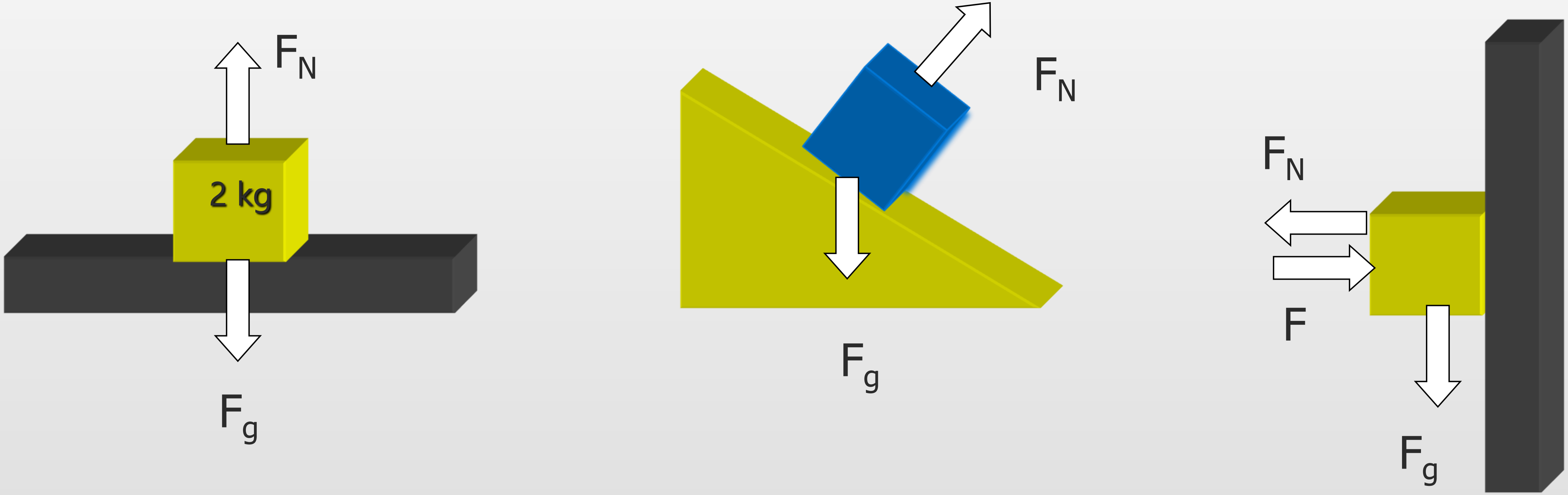
Normal Forces



Normal force acts in the direction perpendicular to the contact surface.

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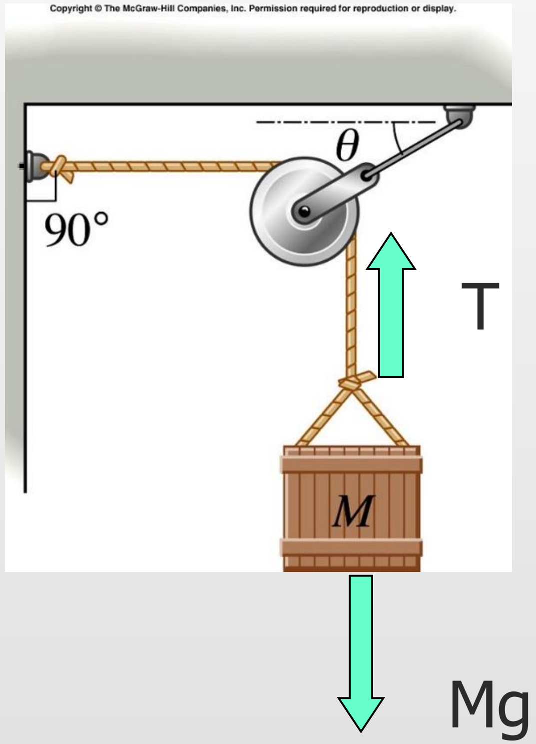
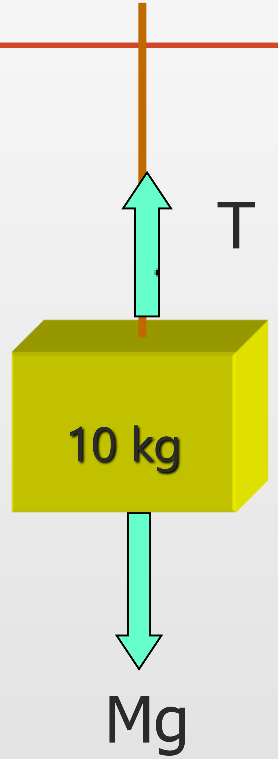
Tension





Tension

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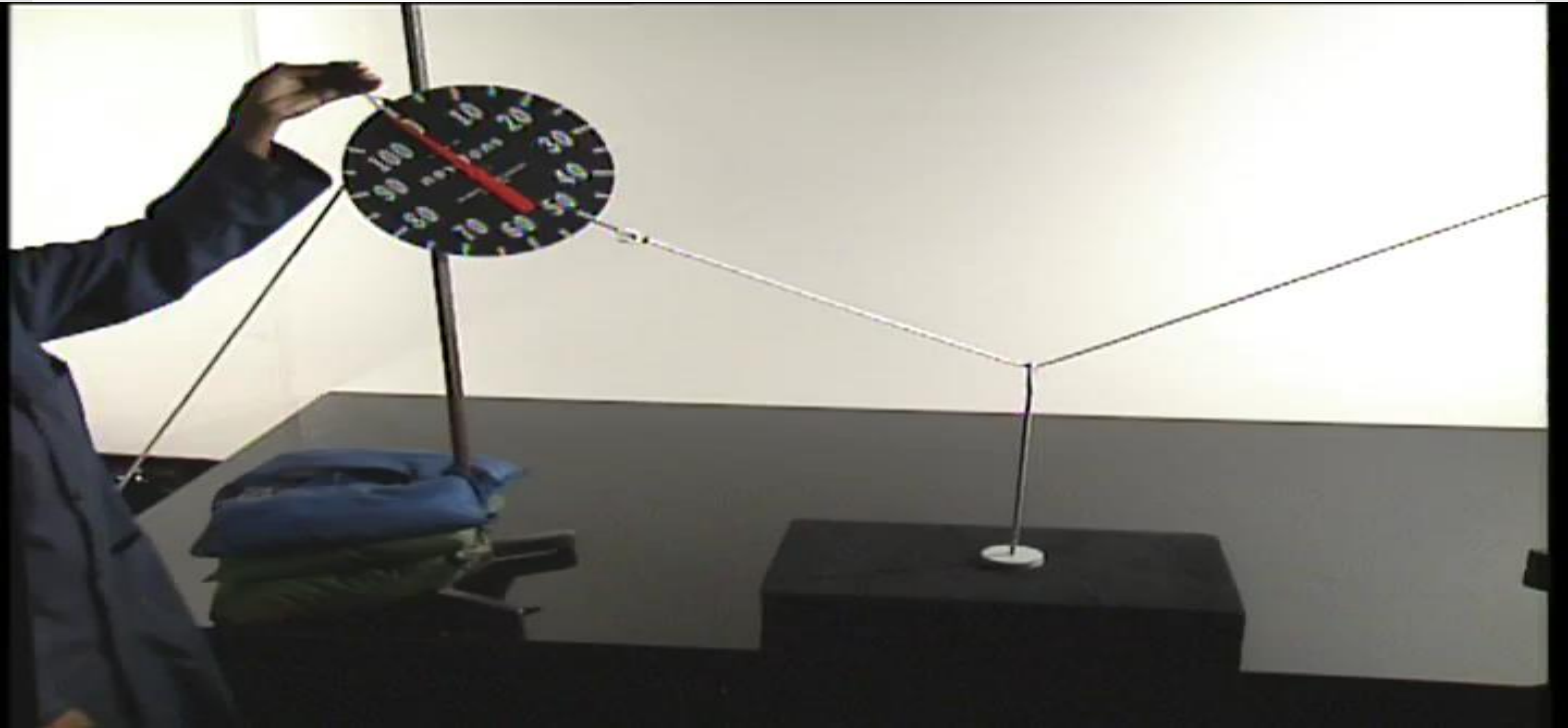

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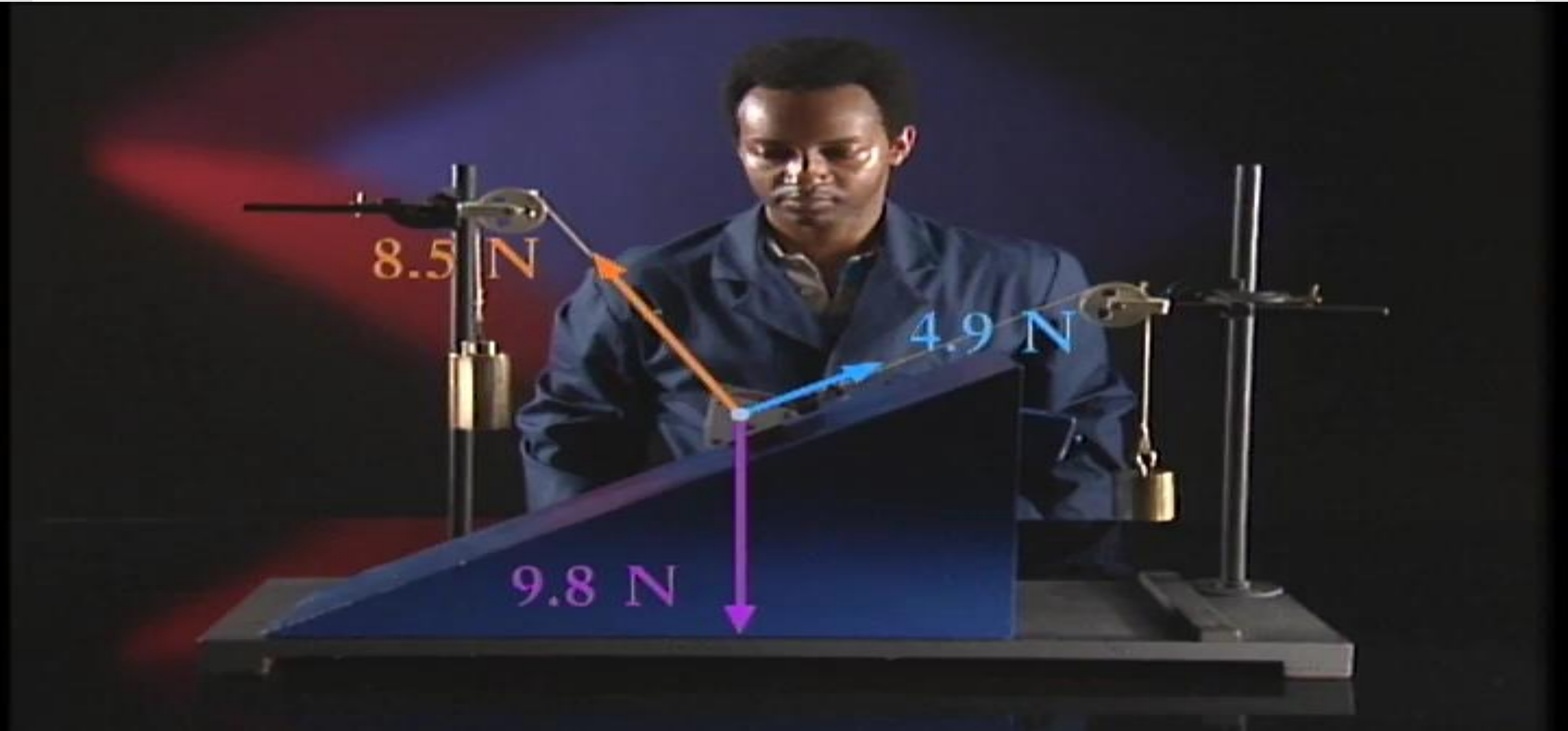

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Tension

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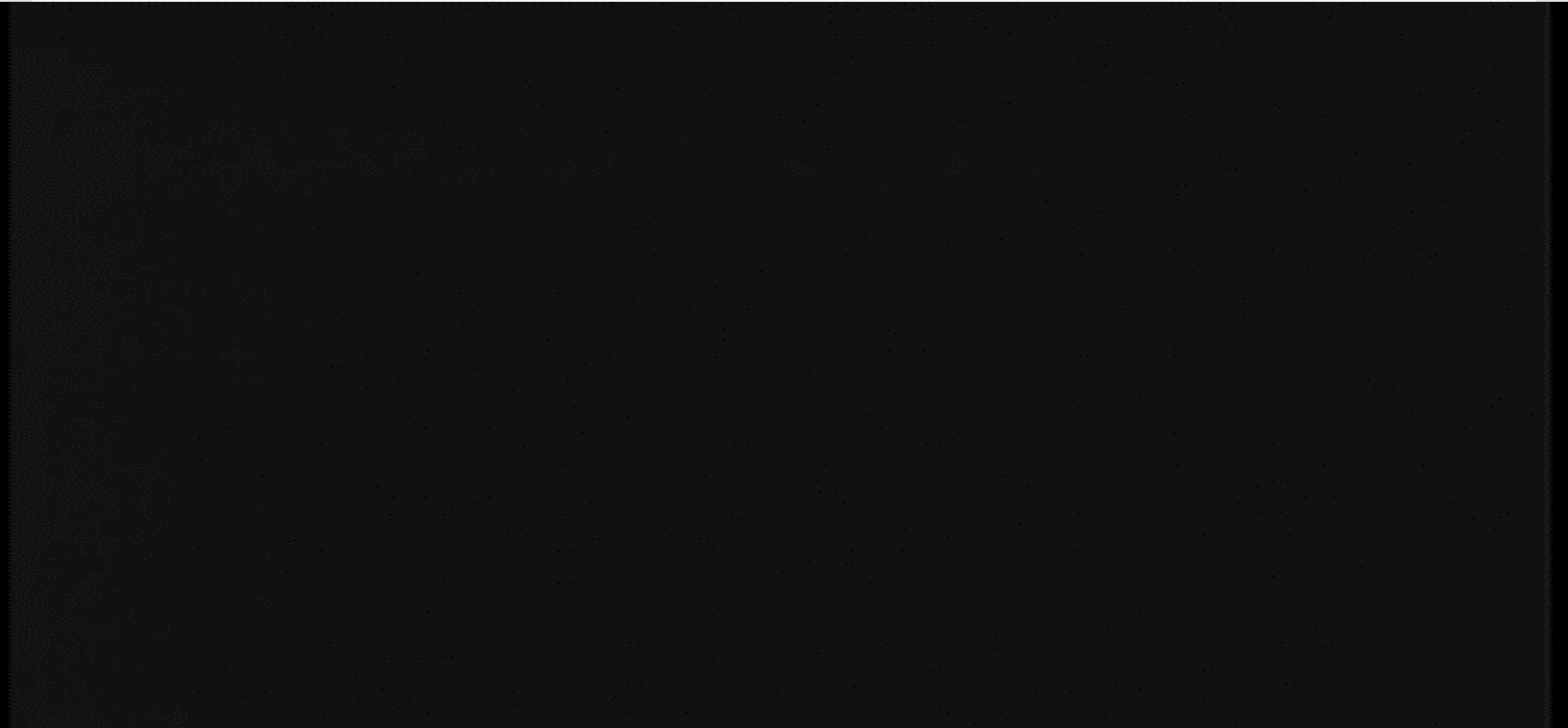
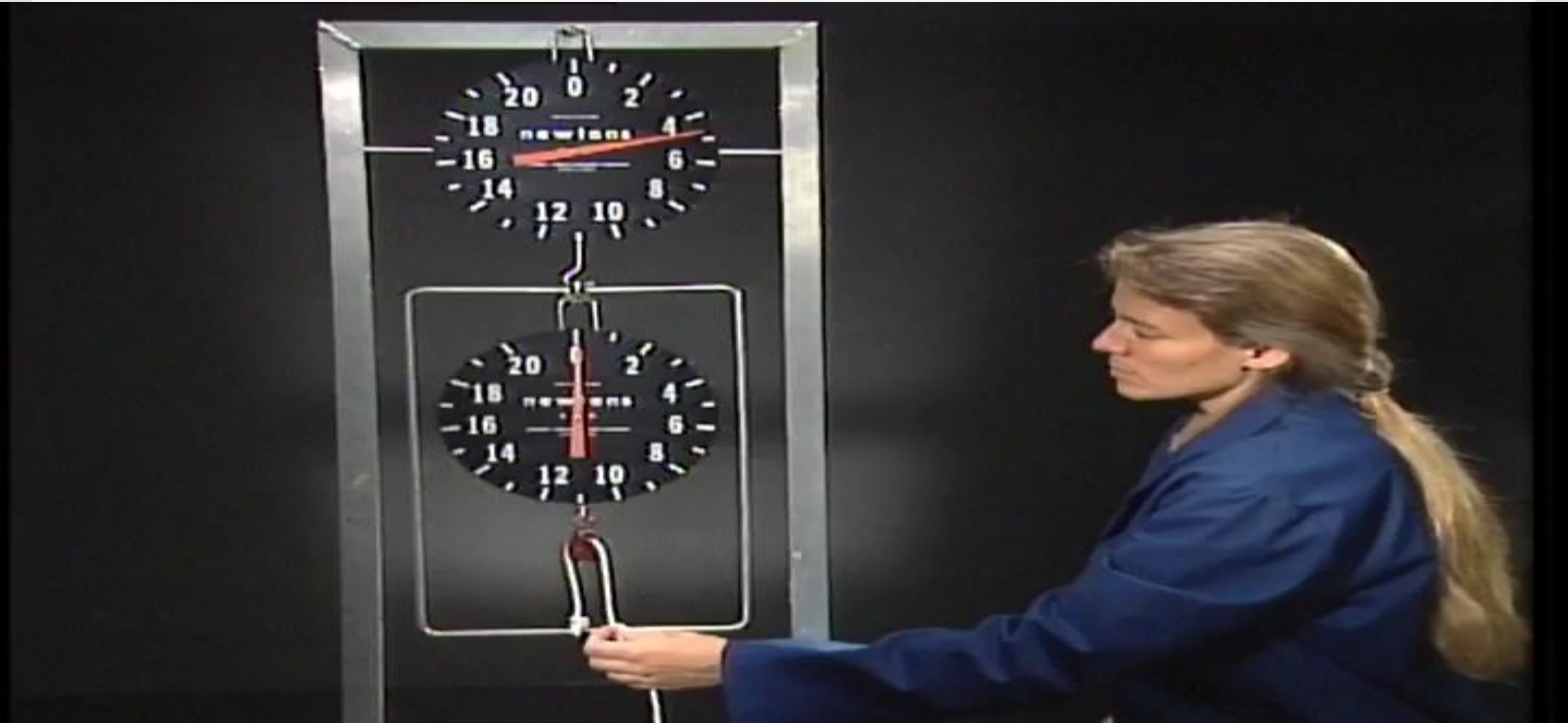
Tension



Tension



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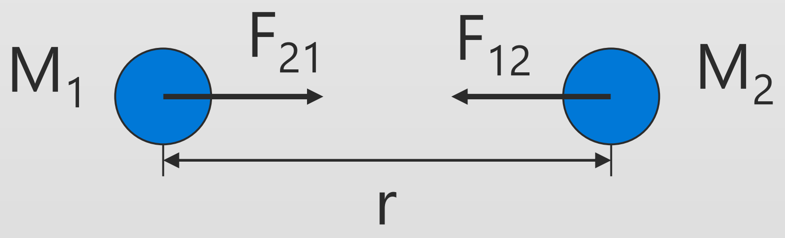
Gravitational Force

Gravity is the force between two masses. Gravity is a long-range force. No contact is needed between the bodies. The force of gravity is always attractive!

$$F = \frac{GM_1M_2}{r^2}$$

r is the distance between the two masses

M_1 and M_2 and $G = 6.67\times10^{-11}\ \mathrm{Nm^2/kg^2}$.



Gravitational Force

Let $M_1 = M_E$ = mass of the Earth.

$$F = \left(\frac{GM_E}{r^2}\right)M_2$$

Here F = the force the Earth exerts on mass M_2. This is the force known as weight, w.

$$w = \left(\frac{GM_E}{r_E^2}\right)M_2 = gM_2.$$

$$M_\mathrm{E} = 5.98\times10^{24}\ \mathrm{kg}$$
$$r_\mathrm{E} = 6400\ \mathrm{km}$$

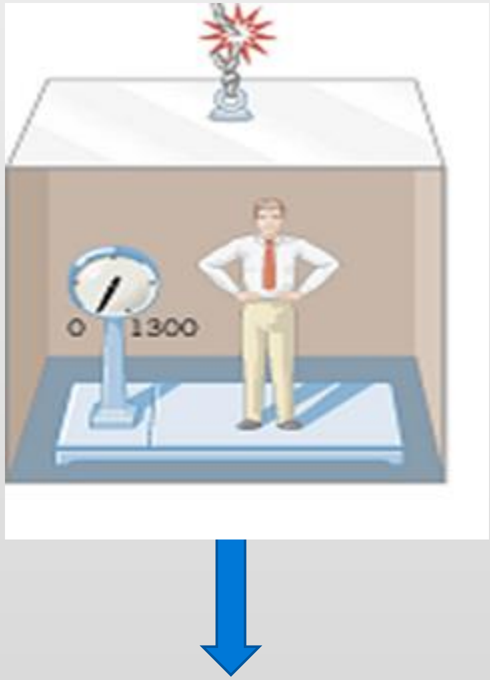
$$\text{where } g = \frac{GM_E}{r_E^2} = 9.8\ \mathrm{N/kg} = 9.8\ \mathrm{m/s^2}$$

Near the surface of the Earth

Gravitational Force

Note that $$g = \frac{F_g}{m}$$

is the gravitational force per unit mass. This is also referred to as the *acceleration due to gravity*.



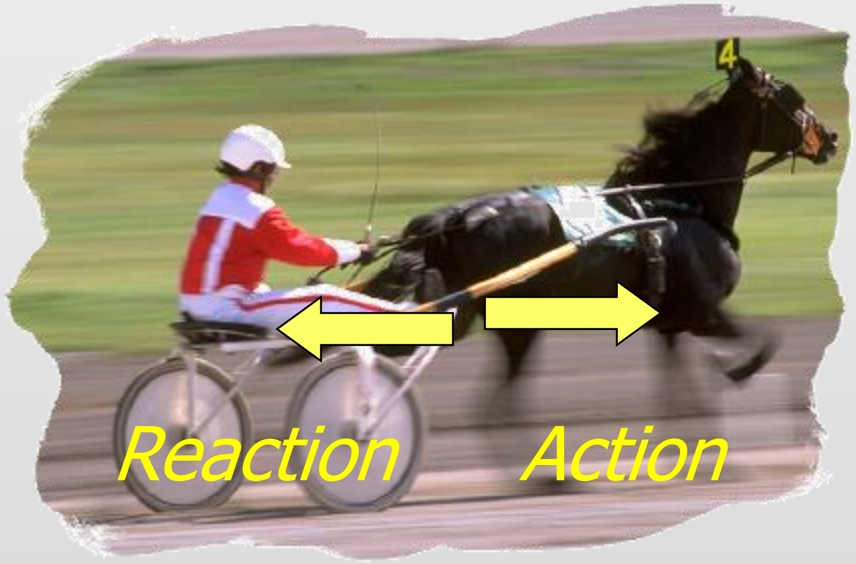
What is the direction of **g**?

What is the direction of **w**?

Answer: Downward

Newton's Third Law

- Third Law: For every action force, there must be an equal and opposite reaction force. Forces occur in pairs.



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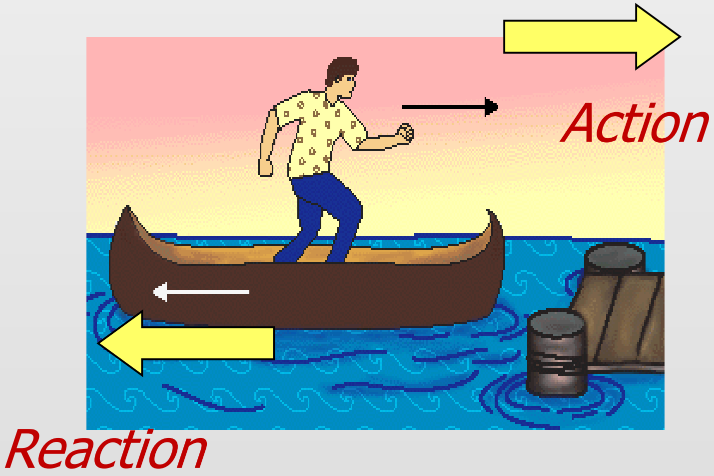
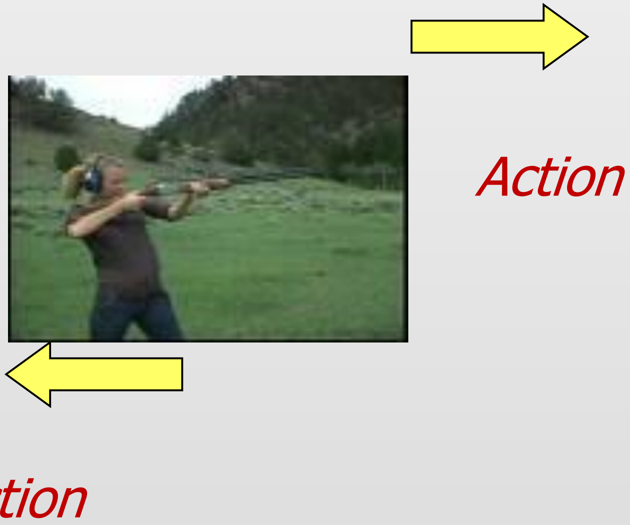
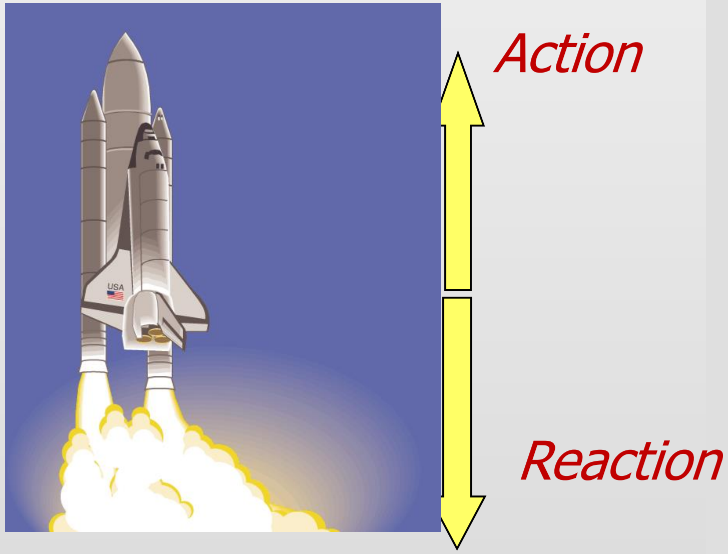


Jumping out of a boat onto the dock

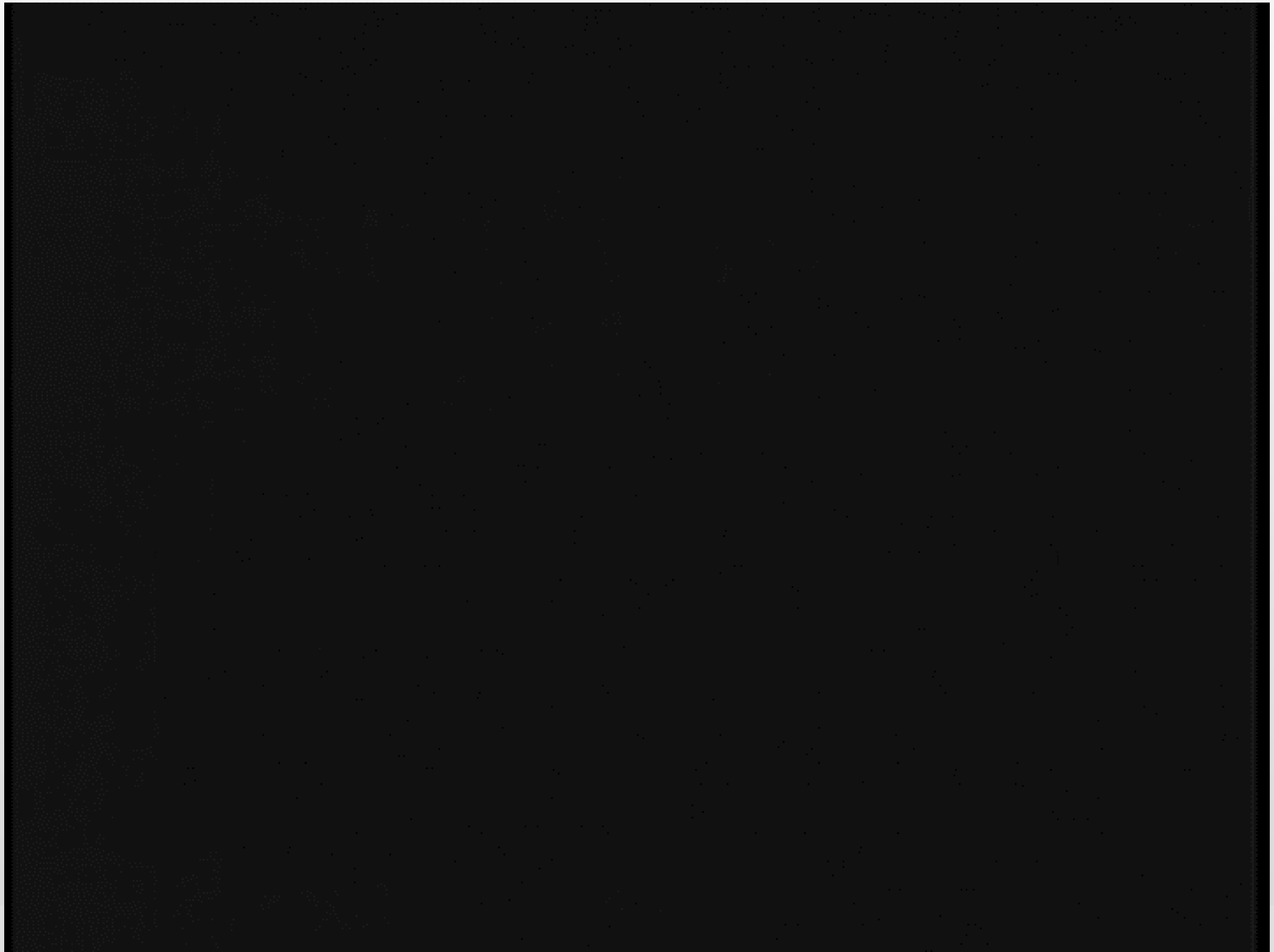


Gun being fired



Rocket leaving earth

Newton's Third Law



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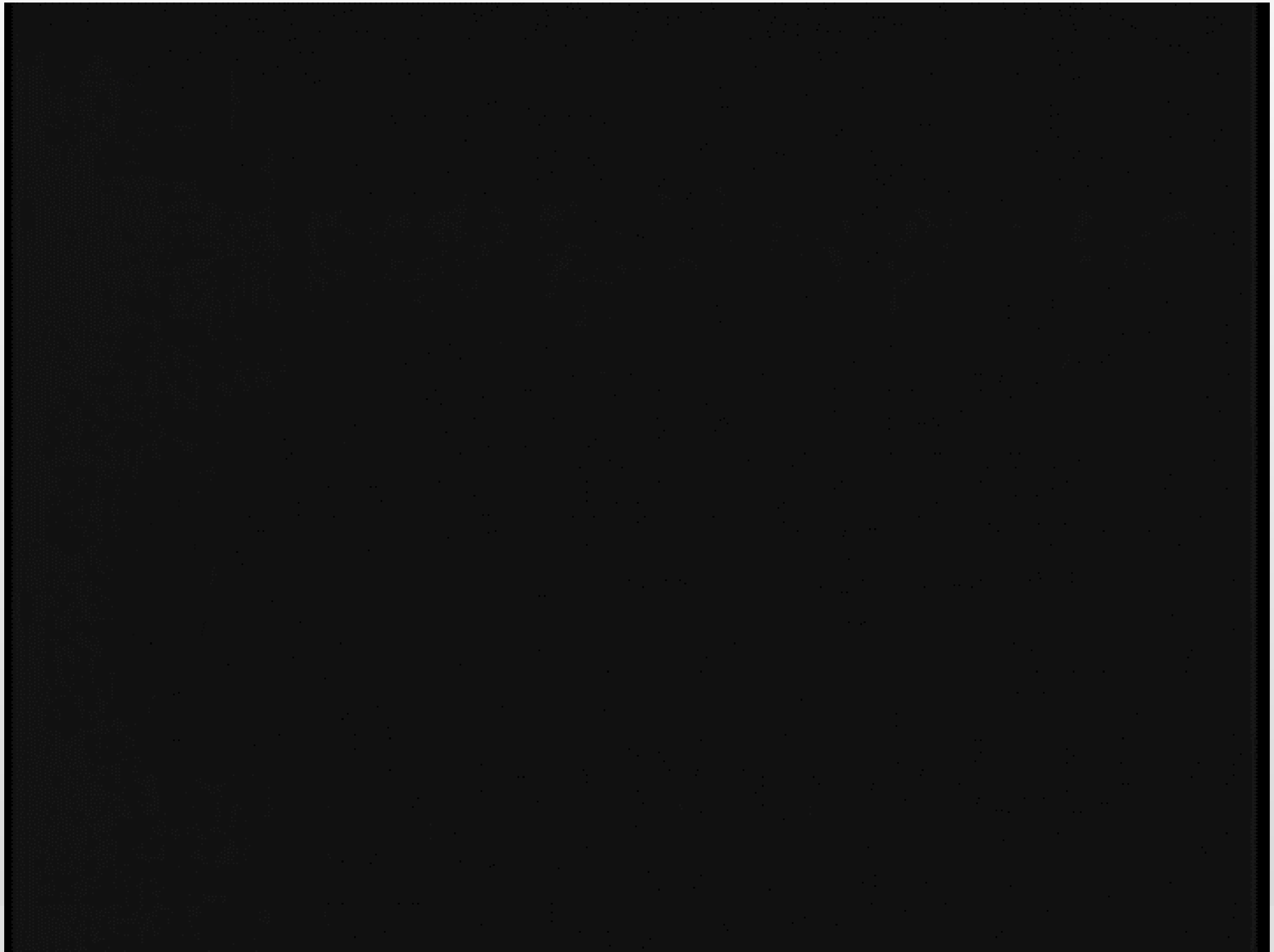

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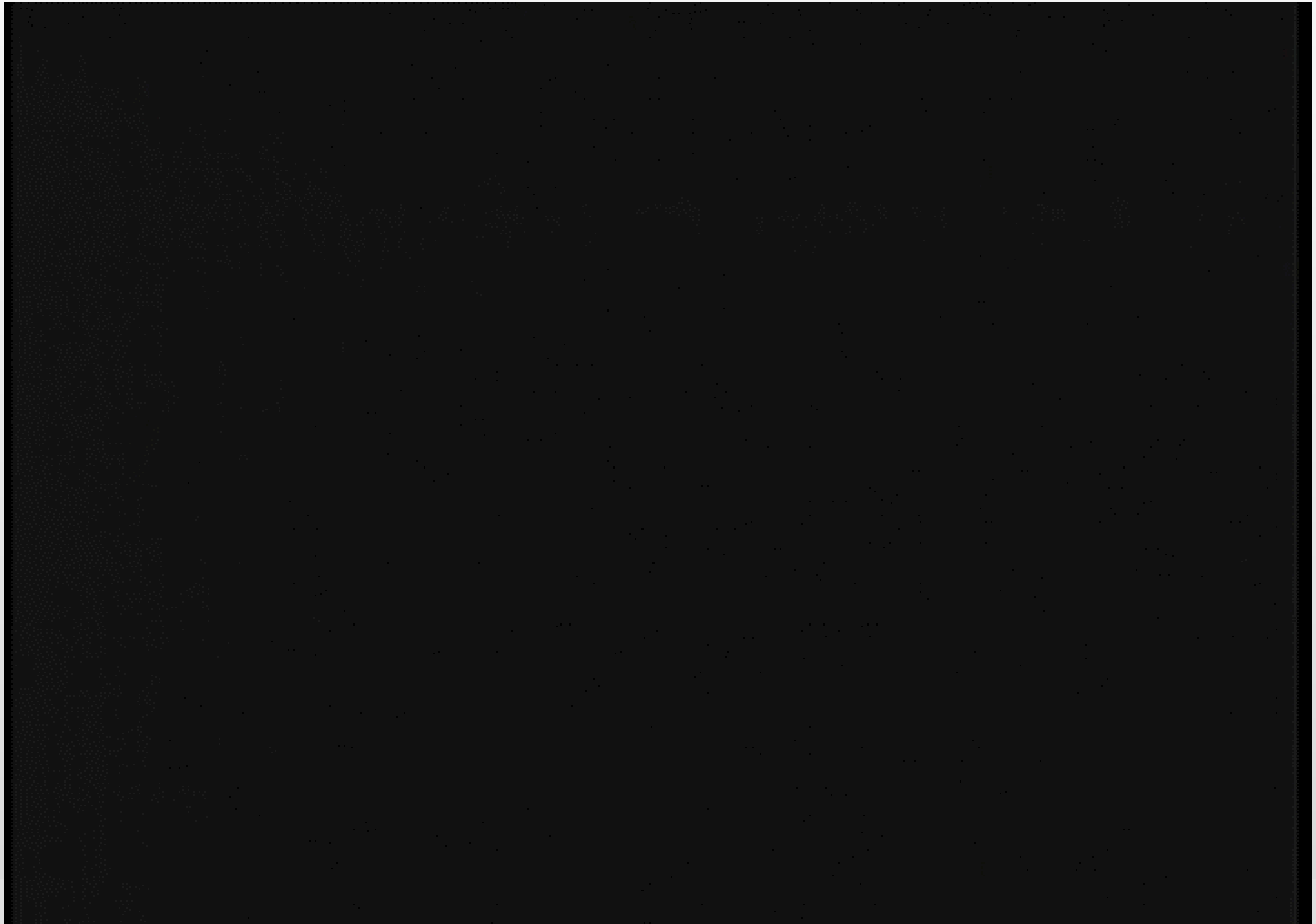

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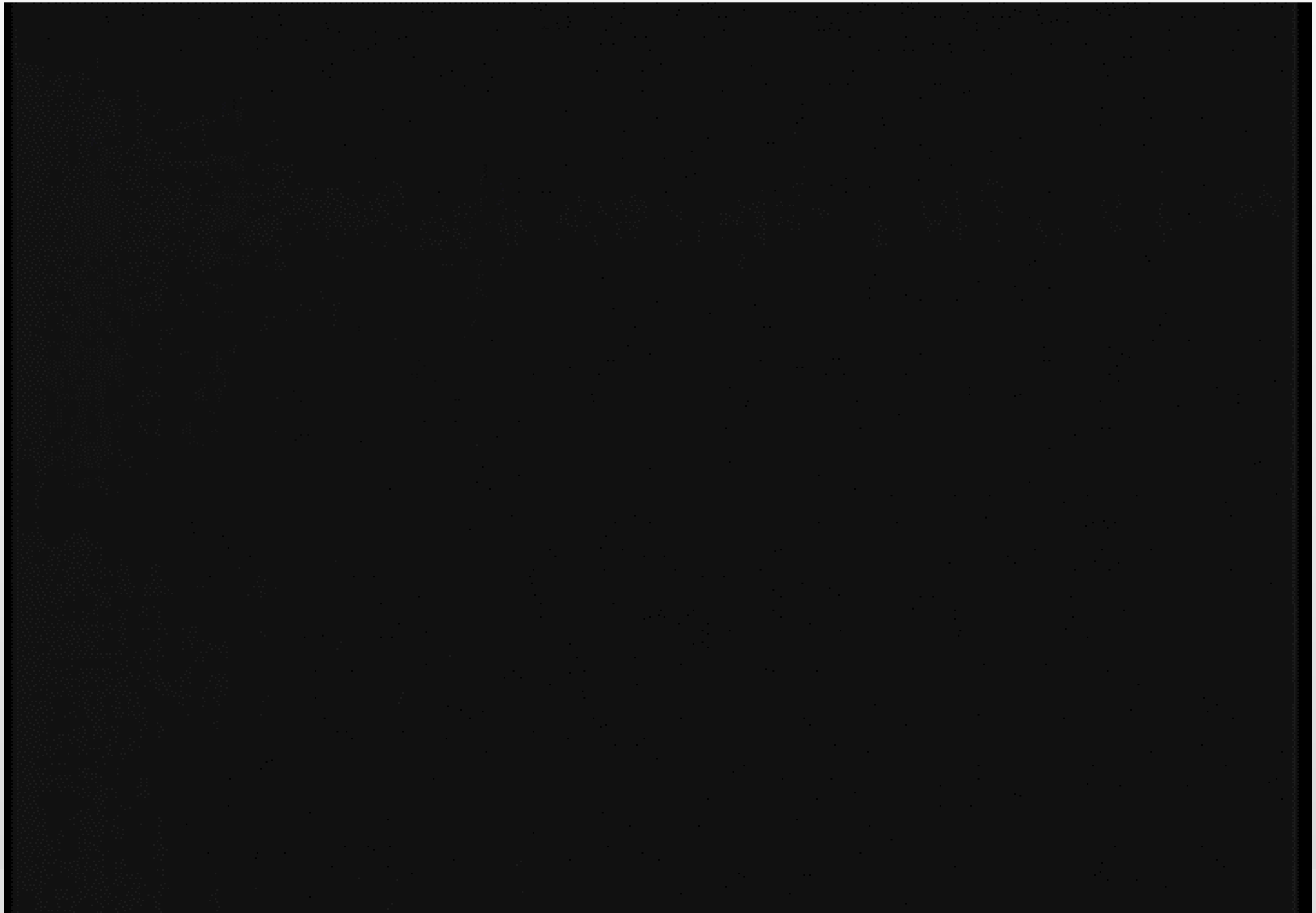


Mission in Space

The space shuttle *Endeavor* lifts off for an 11-day mission in space. All of Newton's laws of motion - the law of inertia, action-reaction, and the acceleration produced by a resultant force -are exhibited during this lift-off.

Credit: NASA Marshall Space Flight Center (NASA-MSFC).

Solving Problem Strategy for Newton Laws

- **1. Specify the forces acting on each body**
- **2. Draw a free body diagram for each body**
- **3. Break vectors into components if needed**
- **4. Find the NET force by adding and subtracting forces that are on the same axis as the direction of motion.**
- **5. Write Newton's law for both axes:**

$$\Sigma F_x = m\, a_x \;,\quad \Sigma F_y = m\, a_y$$

- **6. Solve for the unknown quantities**

NOTE: To avoid negative numbers, always subtract the smaller forces from the larger one. Be sure to remember which direction is larger.

Forces of Friction: The Case of Static Friction

Force of Friction: It represents the resistance to the motion of a body on a rough surface because of the interaction of the body with the surface

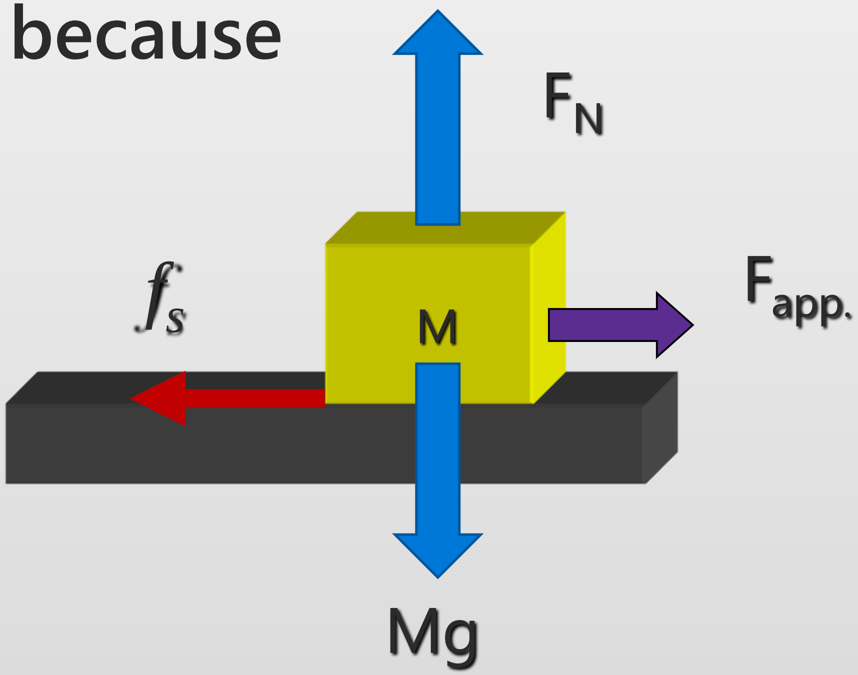
The block of mass M is in static equilibrium because

$$\Sigma F_x = 0 \Rightarrow F_{app.} - f_s = 0$$

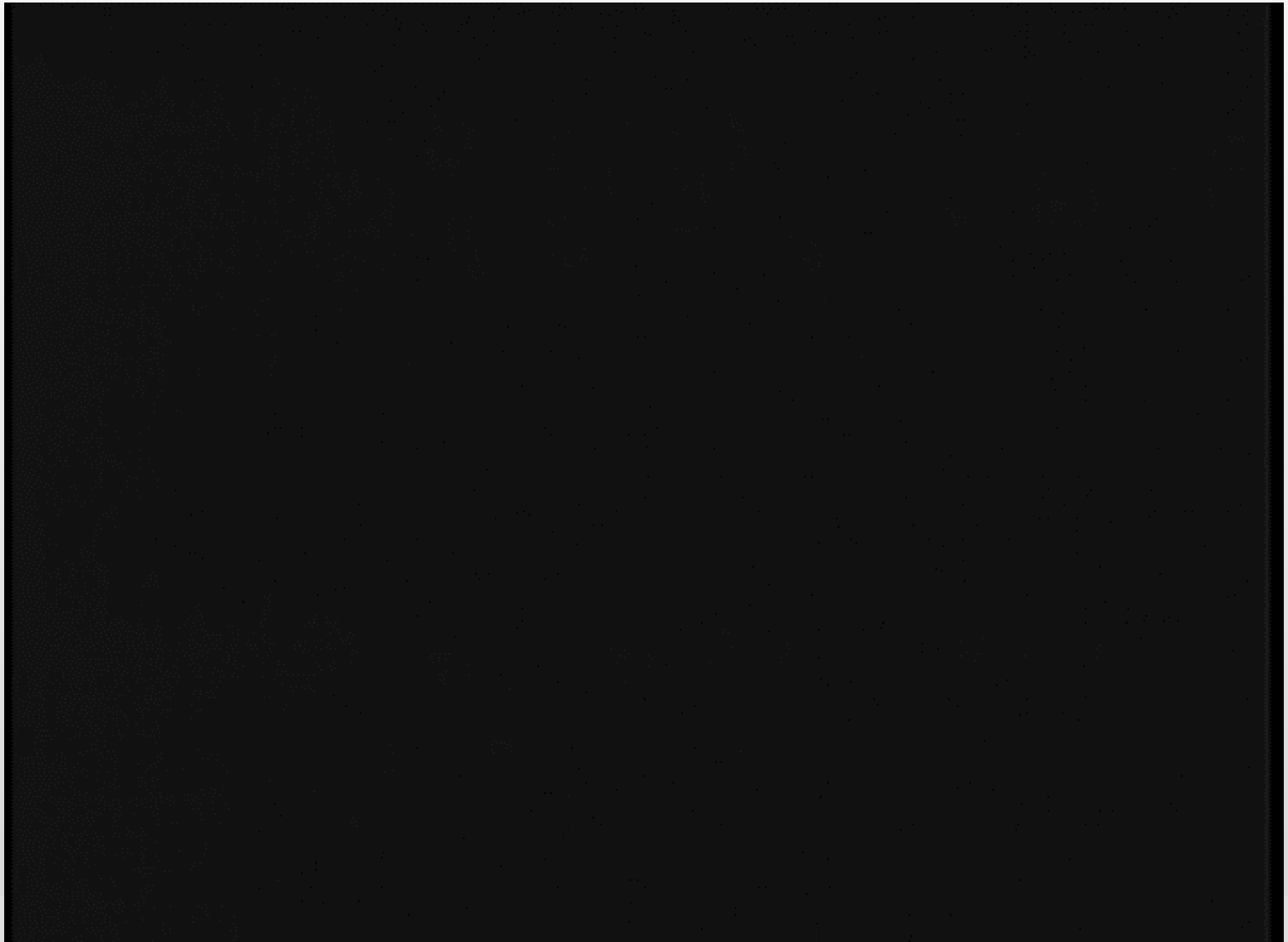
Or $\Rightarrow f_s = F_{app.}$

$$\Sigma F_y = 0 \Rightarrow F_N - Mg = 0$$

Or $\Rightarrow F_N = Mg$



Forces of Friction



Forces of Friction: The case of Static Friction

When the applied force is changed until it causes the block of mass M to be on the verge of slipping but still in static equilibrium, the retarding force that resist this applied force is called the maximum force of static friction $f_{s,\,max}=\mu_s F_N$

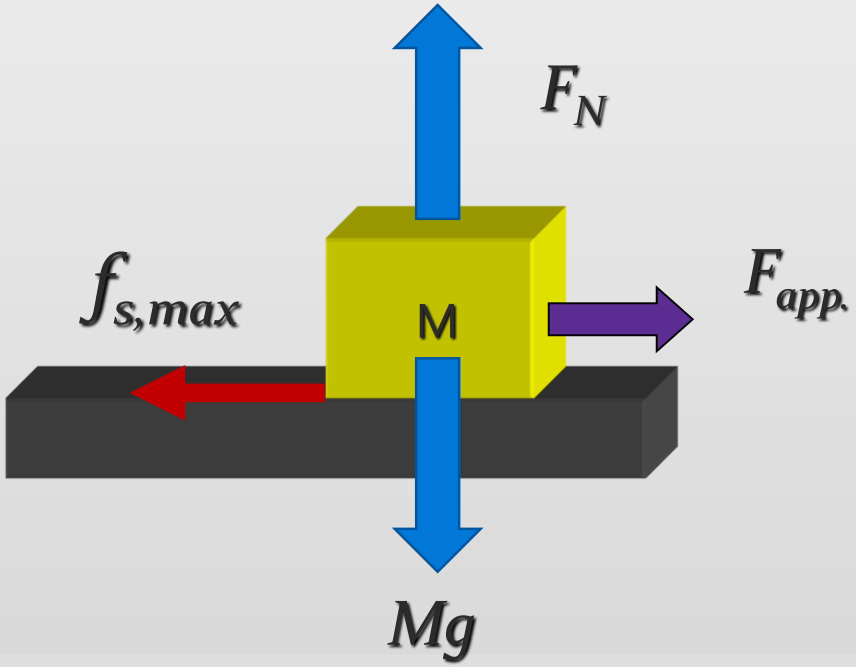
$$\Sigma F_y = 0 \Rightarrow F_N - Mg = 0$$

Or $\Rightarrow F_N = Mg$

$$\Sigma F_x = 0 \Rightarrow F_{app.} - f_{s,max} = 0$$

Or $\Rightarrow \mu_s F_N = F_{app.}$

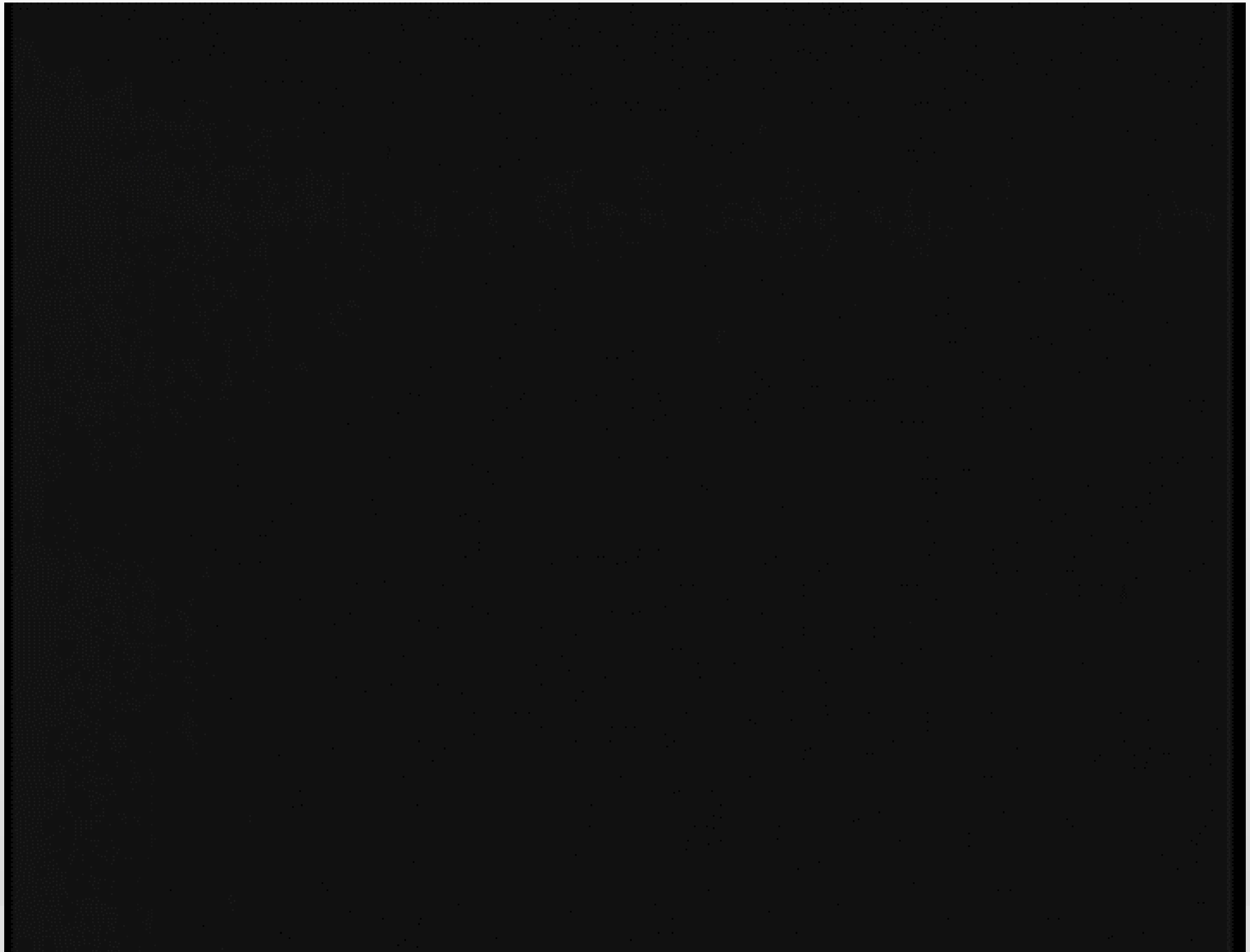
$$\Rightarrow \mu_s = F_{app.} / F_N = F_{app.} / Mg$$



μ_s ***is the coefficient of static friction***

Forces of Friction:

The maximum force of static friction $f_{s,\,max} = \mu_s F_N$



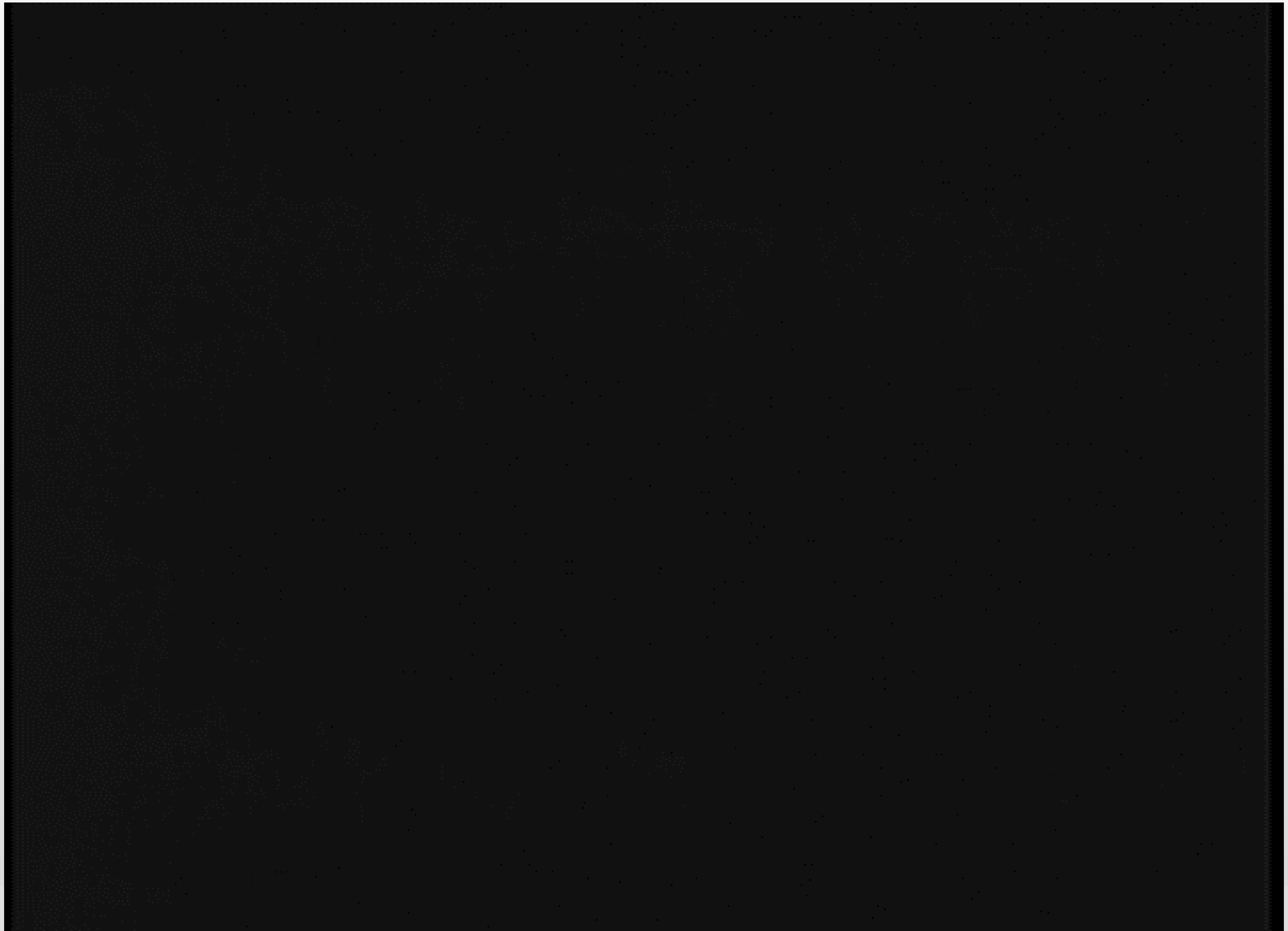
Forces of Friction:

The maximum force of static friction $f_{s,\,max}=\mu_s\,F_N$

The force of static friction $f_{s,\ max} = \mu_s\ F_N$ *does not depend on the area of contacted surfaces*

Forces of Friction

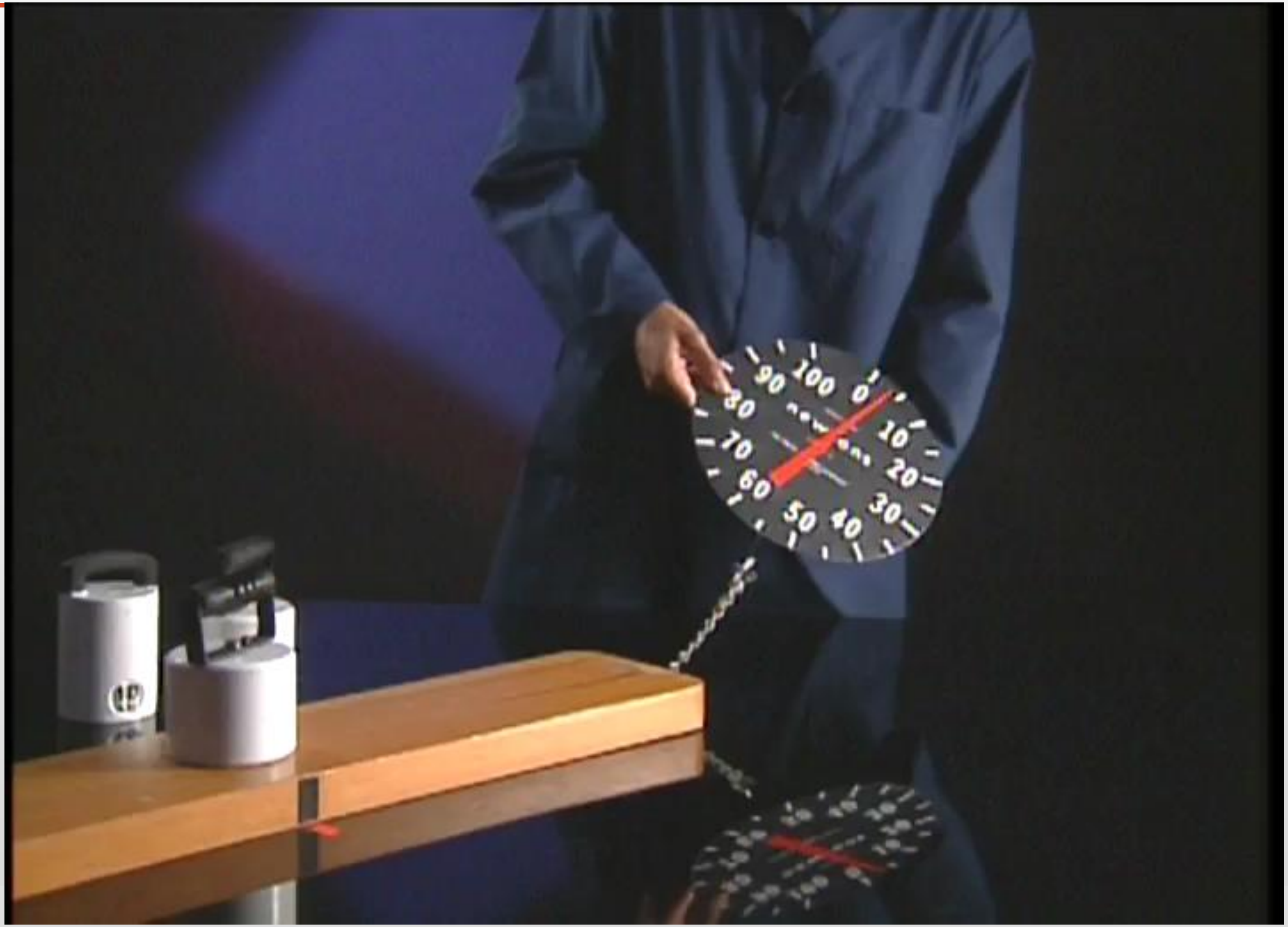
The force of static friction *$f_{s,\ max} = \mu_s F_N$ depends on the material of contact surfaces*



Forces of Friction

Forces of Friction

The force of static friction $f_{s,\ max} = \mu_s F_N$ *depends on the normal force which is equal, here, to the weight of board and masses on board*



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Forces of Friction: The case of Kinetic Friction

When the block of mass M is moving with a constant speed under the action of an applied force F_C the force that resists this motion and keeps the block moving with its constant speed where it is in dynamic equilibrium, is called the force of kinetic friction $f_k = \mu_k F_N$

$$\Sigma F_y = 0 \Rightarrow F_N - Mg = 0$$

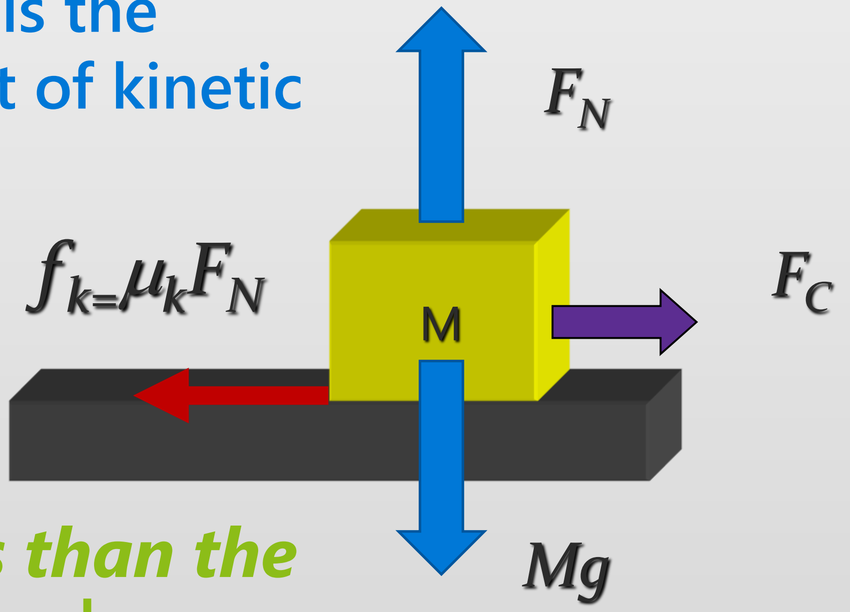
Or $\Rightarrow F_N = Mg$

$$\Sigma F_x = 0 \Rightarrow F_C - f_k = 0$$

Or $\Rightarrow \mu_k F_N = F_C$

$$\Rightarrow \mu_k = F_C / F_N = F_C / Mg$$

where μ_k is the coefficient of kinetic friction



Note: *The force of kinetic friction is always less than the maximum force of static friction (**i.e.** $f_k < f_{s,max}$)* where $\mu_k < \mu_s$

Forces of Friction

The force of kinetic friction is always less than the maximum force of static friction (**i.e.** $f_k < f_{s,max}$) where $\mu_k < \mu_s$

Forces of Friction

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Forces of Friction

Example

A skater of mass 60 kg has an initial velocity of 12 m/s. He slides on ice where the frictional force is 36 N. How far will the skater slide before he stops?

Answer:120m

Forces of Friction

Solution

A skater of mass 60 kg has an initial velocity of 12 m/s. He slides on ice where the frictional force is 36 N. How far will the skater slide before he stops?

$$\Sigma F_x = ma_x \Rightarrow -f_k = ma_x$$

Or $\Rightarrow -f_k/m = a_x$

$$\Rightarrow a_x = -36/60 = -0.6\ m/s^2$$

For a given $v_o = 12\ m/s$ and $v = 0$

Use the equation $v^2 = v_o^2 + 2a_x(x-x_o)$

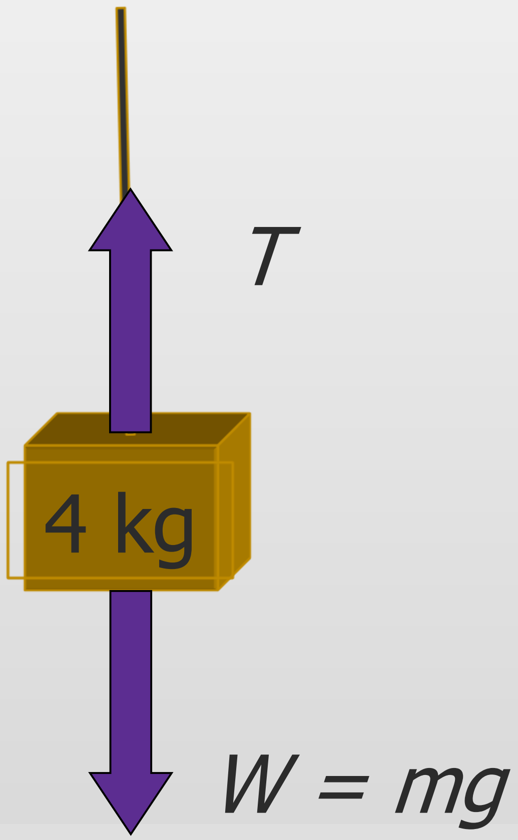
$$\Rightarrow (x-x_o) = -v_o^2/(2a_x) = -(12m/s)^2/(2(-0.6m/s^2))$$

Answer:120m

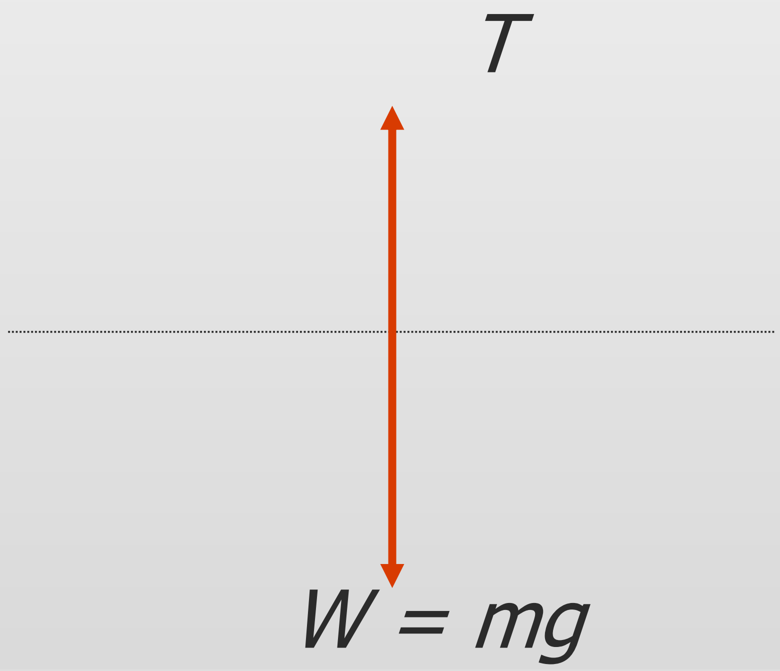
$$\Rightarrow (x-x_o) = 120m$$

Applications on Newton's Laws

Example



Free-Body Diagram



Applications on Newton's Laws

Solution

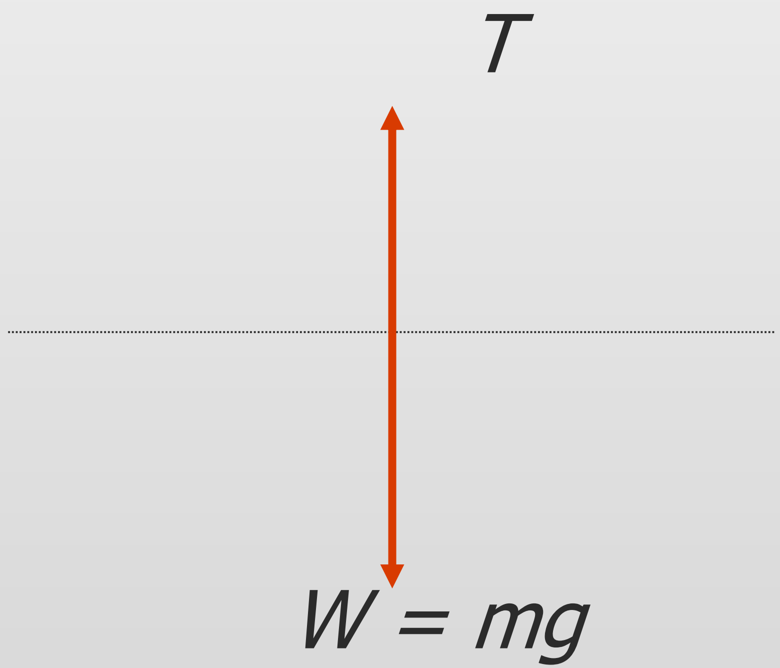
The mass m = 4kg is at static equilibrium

$$\sum F_y = 0$$

$$T - mg = 0$$

$$T = 39.2N$$

Free-Body Diagram



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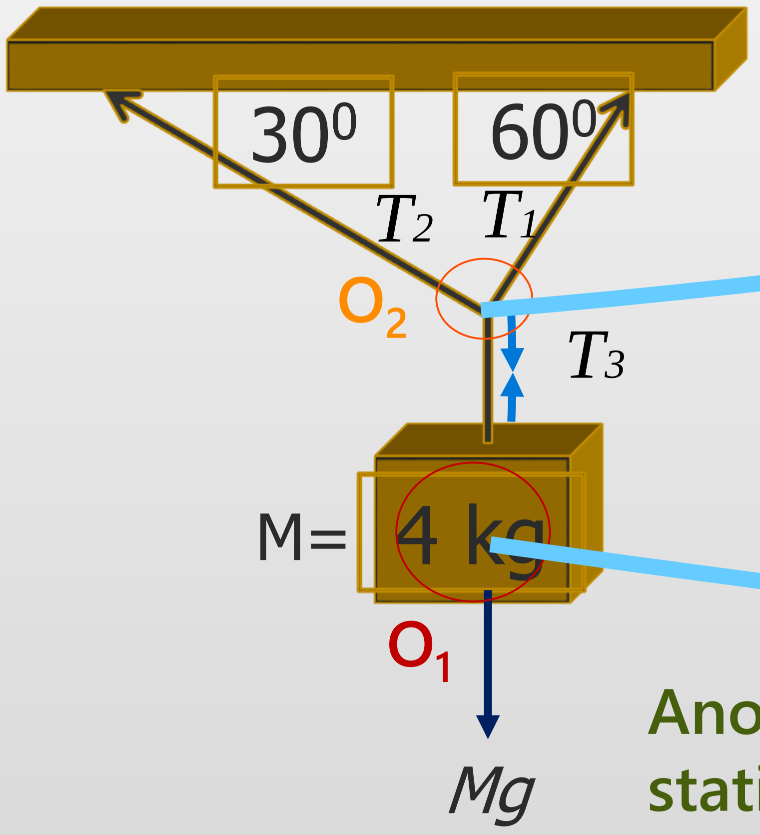
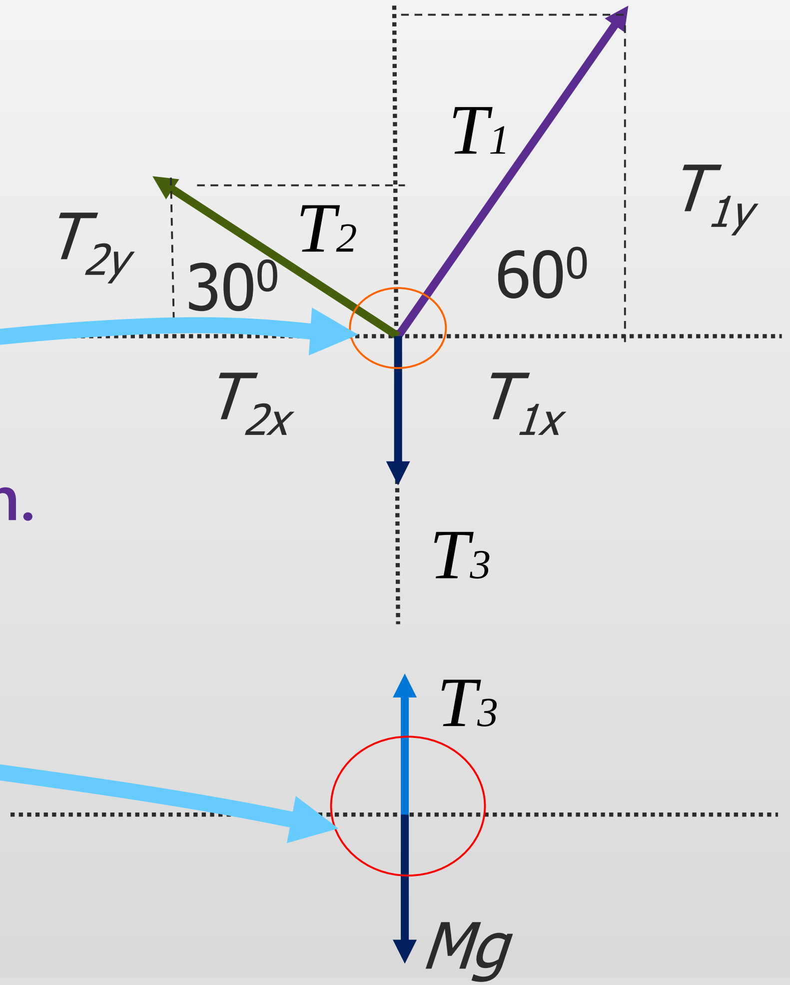
Applications on Newton's Laws

Example If the system is in equilibrium, find the tension in the ropes



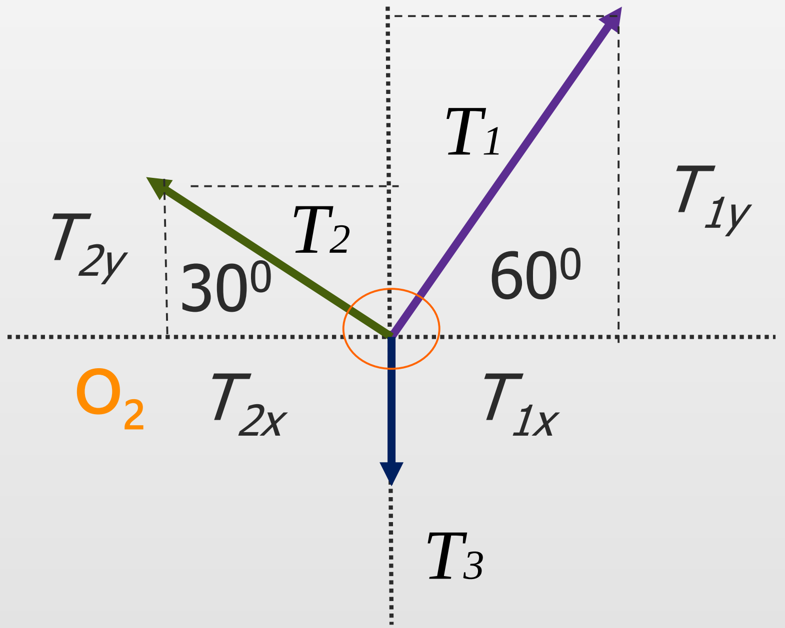
The point O_2 is at static equilibrium.



Another point O_1 is at static equilibrium.

Applications on Newton's Laws

Solution



At point O_1 : $\sum F_y = 0$

$T_3 - Mg = 0$ $\qquad$ $T_3 = Mg = 39.2N$

At point O_2 : $\sum F_x = 0,$

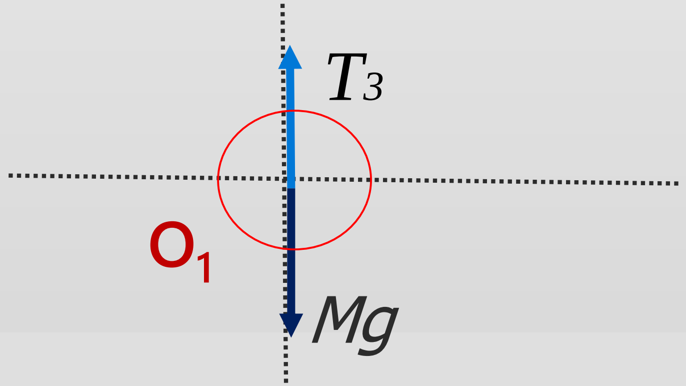
$T_{1x} - T_{2x} = 0 \Rightarrow T_1 \cos 60^o - T_2 \cos 30^o = 0$

$\Rightarrow T_1 = (1.73) T_2$

$\sum F_y = 0 \Rightarrow T_{1y} + T_{2y} - T_3 = 0$

$T_1 \sin 60 + T_2 \sin 30 - T_3 = 0 \Rightarrow 0.86 T_1 + 0.5 T_2 = 39.2$

$T_1 = 34.1N$ and $T_2 = 19.7N$



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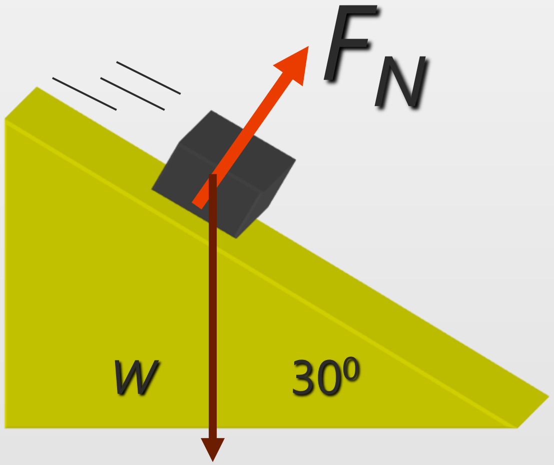
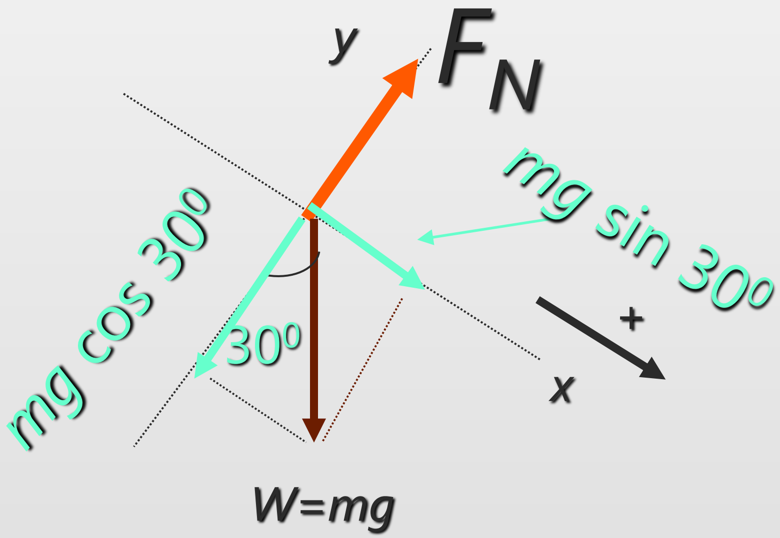
Applications on Newton's Laws: Dynamics

Example

In the absence of friction, what is the acceleration down the 30^0 incline?





$$\sum F_x = ma_x$$

$$mg\sin\theta = ma_x$$

$$a_x = g\sin\theta$$

$$a_x = (9.8\,\text{m/s}^2)\sin 30^o = 4.9\,\text{m/s}^2$$

$$\sum F_y = 0$$

$$F_N - mg\cos\theta = 0$$

$$F_N - mg\cos 30^0 = 0$$

NOT INTERESTED

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Applications on Newton's Laws: **Dynamics**

Exercise

In the presence of friction ($\mu_k = 0.4$), what is the acceleration down the 30^0 incline?

$$\sum F_x = ma_x$$

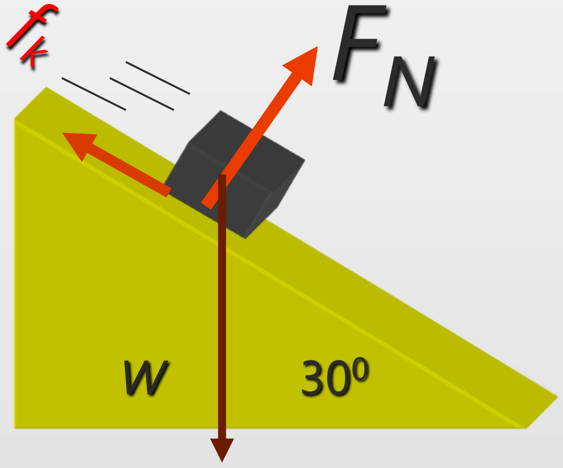
$$mg\ sin\theta - f_k = ma_x$$

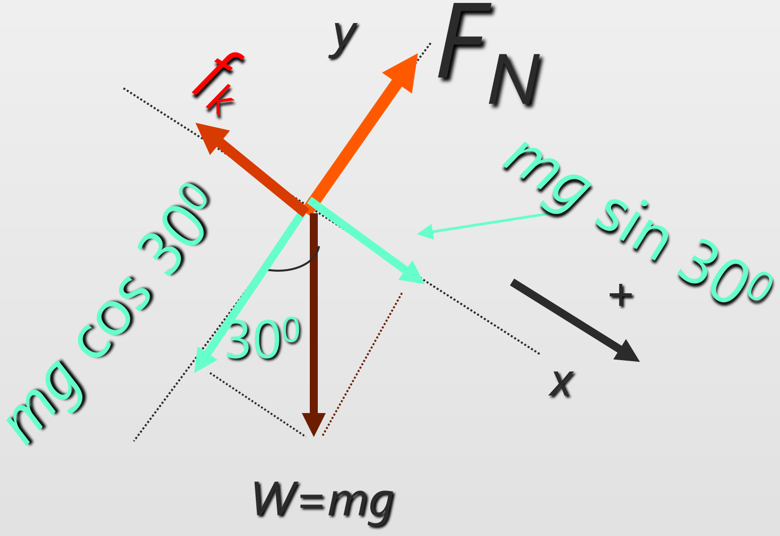
$$mg\ sin\theta - \mu_k F_N = ma_x$$

$$mg\ sin\theta - \mu_k mg \cos\theta = ma_x$$

$$a_x = (9.8\,\text{m/s}^2)(\sin 30^o - (0.4)\cos 30^o)$$

$$a_x = 1.5\ m/s^2$$





$$\sum F_y = 0$$

$$F_N - mg\cos\theta = 0$$

$$F_N = mg\cos 30^0$$

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Applications on Newton's Laws: **Dynamics**

Example

Find the critical angle of incline θ_c that allows the block to be on the verge of skidding, when the coefficient of static friction between the block and incline is μ_s

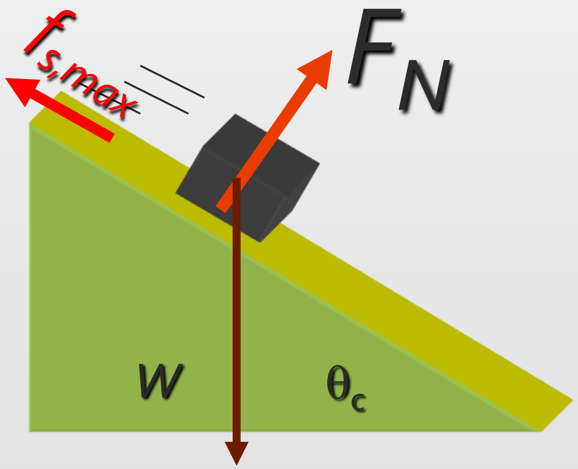
$$\sum F_x = 0$$

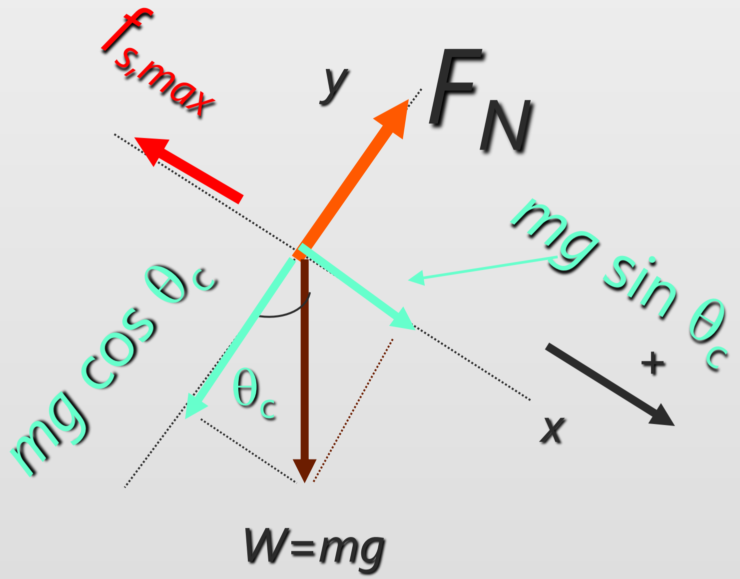
$$mg\sin\theta_c - \mu_s F_N = 0 \text{----(1)}$$

$$\sum F_y = 0$$

$$F_N - mg\cos\theta_c = 0$$

$$F_N = mg\cos\theta_c \text{-----(2)}$$





From equations (1) and (2), one can get

$$mg\sin\theta_c - \mu_s mg\cos\theta_c = 0$$

$$\Rightarrow \mu_s = \tan\theta_c \quad \Rightarrow \quad \theta_c = \tan^{-1}(\mu_s)$$

Applications on Newton's Laws: **Dynamics**

Exercise

Find the critical angle of incline θ_{ck} that allows the block to move down the incline with constant speed, when the coefficient of kinetic friction between the block and incline is μ_k

$$\sum F_x = 0$$

$$mg\sin\theta_{ck} - \mu_k F_N = 0 \text{-----(1)}$$

$$\sum F_y = 0$$

$$F_N - mg\cos\theta_{ck} = 0$$

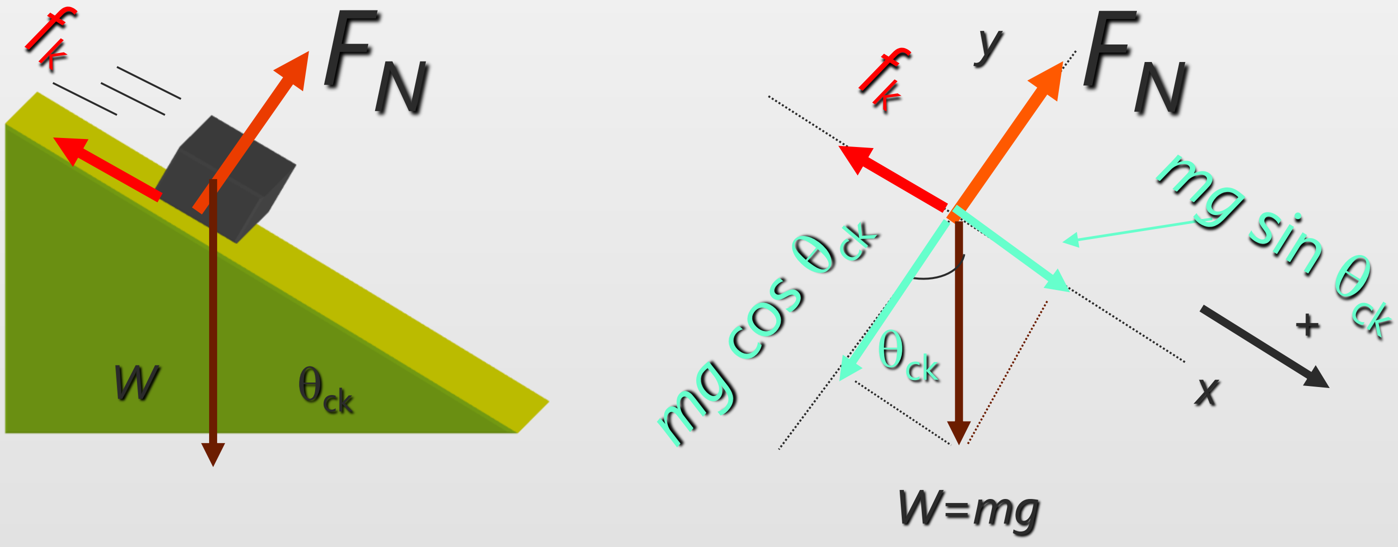
$$F_N = mg\cos\theta_{ck} \text{------(2)}$$

From equations (1) and (2), one gets

$$mg\sin\theta_{ck} - \mu_k mg\cos\theta_{ck} = 0$$

$$\mu_k = \tan\theta_{ck}$$

$$\theta_{ck} = \tan^{-1}(\mu_k)$$



Applications on Newton's Laws: **Dynamics**

Problem

Two forces $\vec{F}_1 = (-6\hat{i} - 4\hat{j})N$ and $\vec{F}_2 = (-3\hat{i} + 7\hat{j})N$, act on a particle of mass 2 kg that is initially at rest at coordinates (- 2 m, + 4 m).

(a) What are the components of the particle's velocity at t = 10 s?

(b) In what direction is the particle moving at t = 10 s?

(c) What displacement does the particle undergo during the first 10 s?

(d) What are the coordinates of the particle at t = 10 s?

Applications on Newton's Laws: Dynamics

Solution

(a)

m=2kg $\vec{F}_1 = (-6\hat{i} - 4\hat{j})N$ $\vec{F}_2 = (-3\hat{i} + 7\hat{j})N$

$\vec{v}_o = 0$ $\vec{r}_o = (-2\hat{i} + 4\hat{j})m$

$$\vec{F}_{net} = \sum \vec{F}_i = m\vec{a}$$

$$\vec{F}_1 + \vec{F}_2 = m\vec{a}$$

$$(-6\hat{i} - 4\hat{j}) + (-3\hat{i} + 7\hat{j}) = 2\vec{a}$$

$$(-9\hat{i} + 3\hat{j}) = 2\vec{a} \Rightarrow \quad \vec{a} = (-4.5\hat{i} + 1.5\hat{j})m/s^2$$

The particle's velocity at t = 10 s can be obtained using the equation of motion

$$\vec{v} = \vec{v}_o + \vec{a}t \longrightarrow \because \vec{v}_o = 0 \longrightarrow \vec{v} = (-4.5\hat{i} + 1.5\hat{j})m/s^2(10s) = (-45\hat{i} + 15\hat{j})m/s$$

v_x = -45 m/s and v_y = 15 m/s

Applications on Newton's Laws: **Dynamics**

Solution

(b) To find the direction of the particle's motion at t = 10 s?

$v_x = -45\ m/s$ and $v_y = 15\ m/s$

$$\theta_v = \arctan\frac{v_y}{v_x} = \arctan\frac{15}{-45} = -18^o = 180^o - 18^o = 162^o$$

Note: The direction is at angle – 18° which is 18° from negative x-axis (clockwise direction). Alternatively, the direction is at angle of 162° with respect to the positive x-axis (counterclockwise direction)

Applications on Newton's Laws: Dynamics

Solution

(c) To find the displacement of the particle during the first 10 s, use the equation of motion

$$\vec{r} - \vec{r}_o = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

$$\vec{a} = (-4.5\hat{i} + 1.5\hat{j})m/s^2 \qquad \vec{v}_o = 0$$

$$\vec{r} - \vec{r}_o = (0.5)(-4.5\hat{i} + 1.5\hat{j})m/s^2(10s)^2$$

$$\Rightarrow \vec{r} - \vec{r}_o = (-225\hat{i} + 75\hat{j})m$$

Applications on Newton's Laws: Dynamics

Solution

(d) To find the coordinates of the particle at t = 10 s, use the equation of motion

$$\vec{r} - \vec{r}_o = (-225\hat{i} + 75\hat{j})m$$

$$\vec{r}_o = (-2\hat{i} + 4\hat{j})m$$

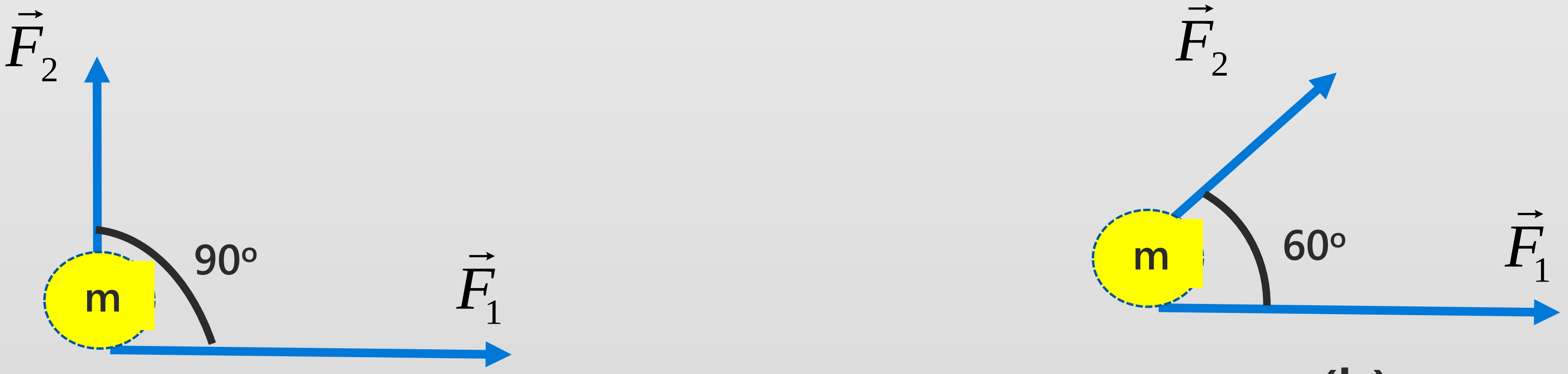
$$\vec{r} = (-227\hat{i} + 79\hat{j})m$$

x = -227 m and y = 79 m

Applications on Newton's Laws: **Dynamics**

Problem

Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a 5-kg object. Taking F_1 = 20 N and F_2 = 15 N, find the acceleration of the object for the configurations of shown parts (a) and (b) of the figure.



(a) (b)

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Applications on Newton's Laws: Dynamics

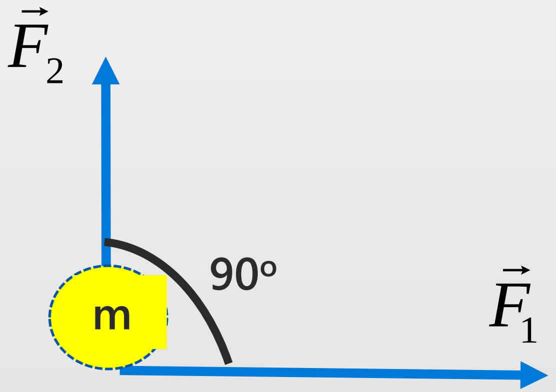
Solution

(a) For forces in Figure (a): m = 5kg $\vec{F}_1 = 20\hat{i}$ $\vec{F}_2 = 15\hat{j}$

$$\vec{F}_1 + \vec{F}_2 = m\vec{a}$$

$$20\hat{i} + 15\hat{j} = 5\vec{a}$$

$$\Rightarrow \vec{a} = (4\hat{i} + 3\hat{j})m/s^2$$



(a)

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Solution

(b) For forces in Figure (b): m = 5kg $\vec{F}_1 = 20\hat{i}N$ $\vec{F}_2 = F_{2_x}\hat{i} + F_{2_y}\hat{j}$

$$F_{2_x} = F_2\cos 60^o = (15)(0.5) = 7.5N$$

$$F_{2_y} = F_2\sin 60^o = (15)(0.86) = 13N$$

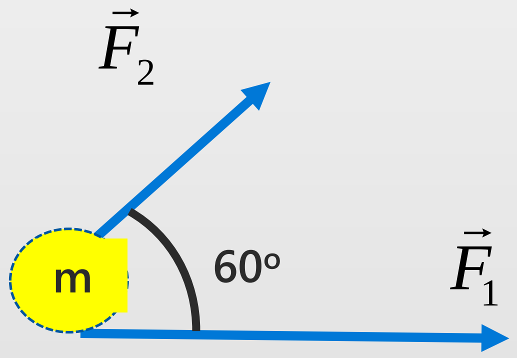
$$\vec{F}_2 = (7.5\hat{i} + 13\hat{j})N$$

$$\vec{F}_1 + \vec{F}_2 = m\vec{a}$$

$$20\hat{i} + (7.5\hat{i} + 13\hat{j}) = m\vec{a}$$

$$\Rightarrow (27.5\hat{i} + 13\hat{j}) = 5\vec{a}$$

$$\Rightarrow \vec{a} = (5.5\hat{i} + 2.6\hat{j})m/s^2$$



(b)

Applications on Newton's Laws: Dynamics

Problem

A 3-kg object is moving in a plane, with its x and y coordinates given by $x = 5t^2 - 1$ and $y = 3t^2 + 2$, where x and y are in meters and t is in seconds. Find the magnitude of the net force acting on the object at t = 2 s.

Solution

To find the magnitude of the net force acting on the object at t = 2 s, one must find first the acceleration using $x = 5t^2 - 1$, $y = 3t^2 + 2$ and

$$a_x \equiv \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

$$a_y \equiv \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$$

$$a_x = 10m/s^2$$

$$a_y = 6m/s^2$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} \quad \longrightarrow \quad \vec{a} = (10\hat{i} + 6\hat{j})m/s^2$$

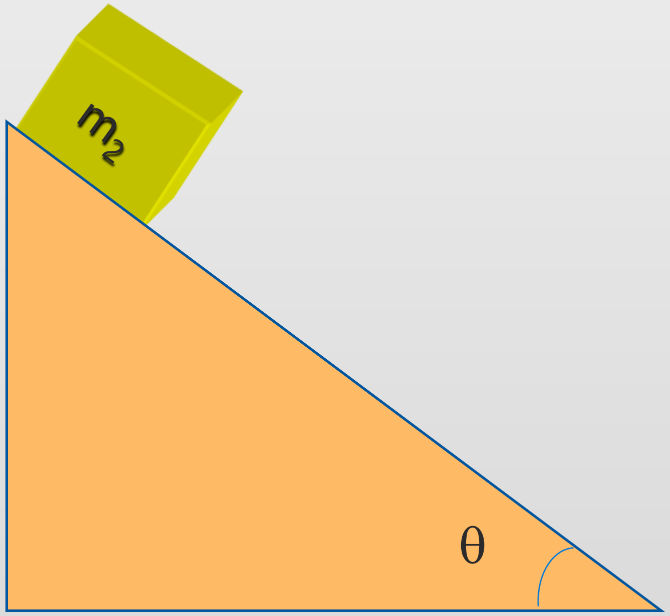
$$\vec{F} = m\vec{a} \quad \longrightarrow \quad \vec{F} = (3kg)(10\hat{i} + 6\hat{j})m/s^2 = (30\hat{i} + 18\hat{j})N$$

Note: It is obvious that both the acceleration and force are not function of time so they are constant at any time

Applications on Newton's Laws: **Dynamics**

Exercise

A 3-kg block starts from rest at the top of a 30° incline plane and slides a distance of 2 m down the incline in 1.5 s. Find (a) the magnitude of the acceleration of the block, (b) the coefficient of kinetic friction between the block and the plane, and (c) the friction force acting on the block, and (d) the speed of the block after it has slid 2 m.



Answer: (a) a = 1.77 m/s^2

(b) μ_k = 0.368

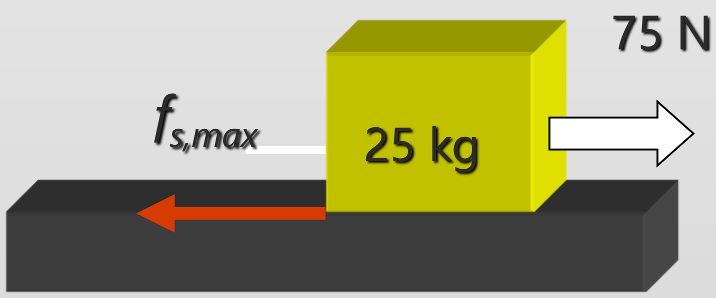
(c) f_k = - 9.37 N

(d) v = 2.66 m/s

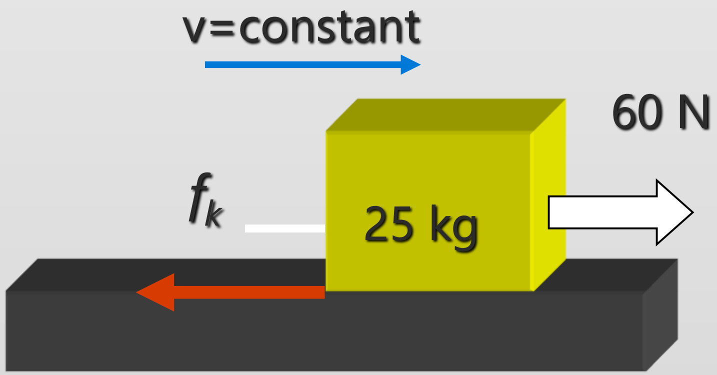
Applications on Newton's Laws: **Dynamics**

Problem

A 25-kg block is initially at rest on a horizontal surface. A horizontal force of 75 N is required to set the block in motion, after which a horizontal force of 60 N is required to keep the block moving with constant speed. Find: (a) the coefficient of static friction and, (b) The coefficient of kinetic friction between the block and the surface.



(a)



(b)

Applications on Newton's Laws: **Dynamics**

Solution

(a) To find the coefficient of static friction μ_s= ? m= 25 kg, v_o= 0, F_{app}= 75N

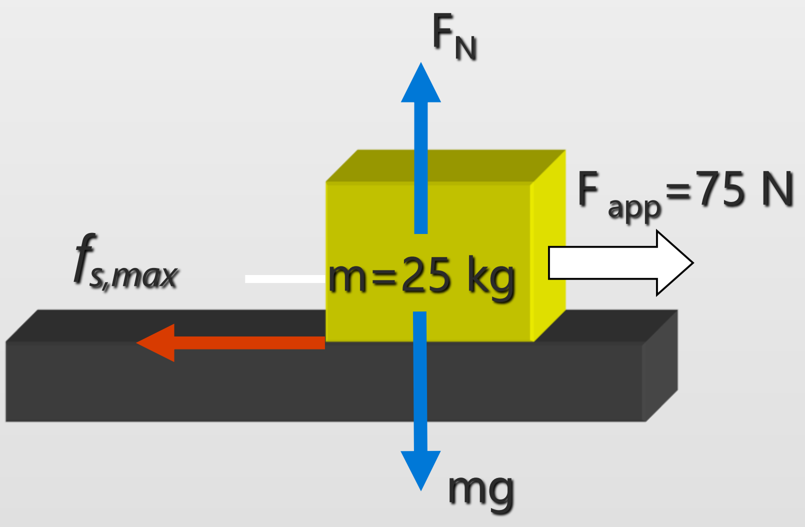
$$\Sigma F_x = 0 \Rightarrow F_{app.} - f_{s,max} = 0$$

Or $\Rightarrow \mu_s F_N = F_{app.}$

$$\Rightarrow \mu_s = F_{app.} / F_N = F_{app.} / mg$$

$$mg = (25)(9.8) = 245\ N$$

$$\Rightarrow \mu_s = 75N/245N = 0.3$$



(a)

Applications on Newton's Laws: **Dynamics**

Solution

(a) To find the coefficient of kinetic friction μ_k= ? m= 25 kg, v_o= constant, F_{app}= 60N

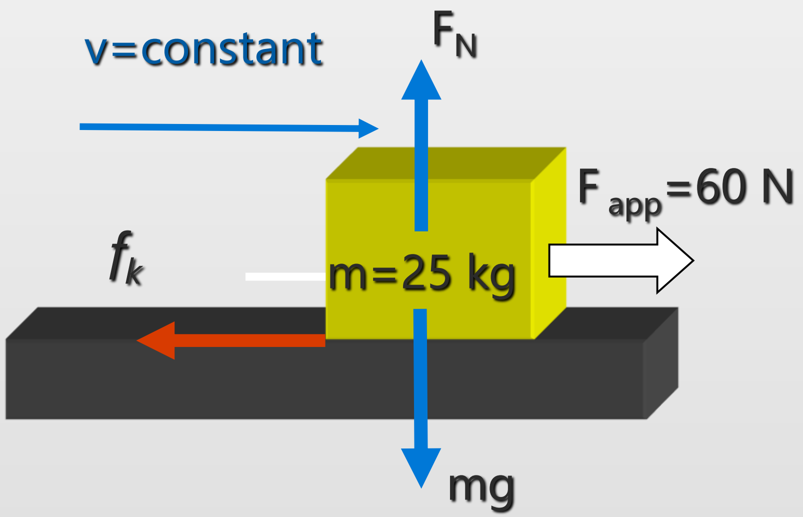
$$\Sigma F_x = 0 \Rightarrow F_{app.} - f_k = 0$$

Or $$\Rightarrow \mu_k F_N = F_{app.}$$

$$\Rightarrow \mu_k = F_{app.}/F_N = F_{app.}/ mg$$

mg = (25)(9.8) = 245 N

$$\Rightarrow \mu_k = 60N/245N = 0.24$$



(b)

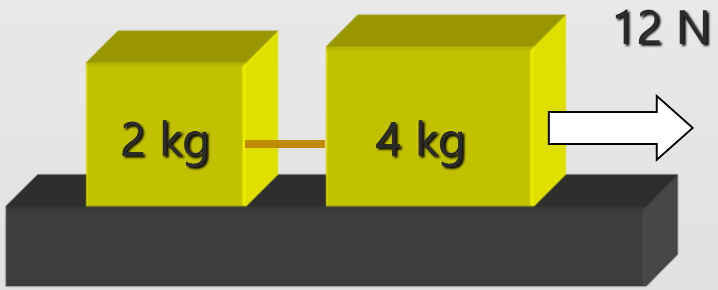
Applications on Newton's Laws: **Dynamics**

Example

Two-Body Problem:

(a) Find tension in the connecting rope if there is no friction on the surfaces.

(b) Find acceleration of system .



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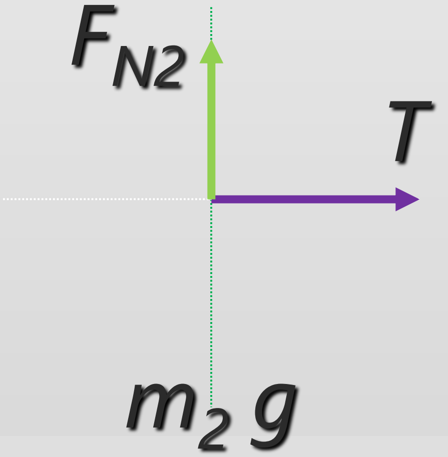
Applications on Newton's Laws: Dynamics

Solution

Two-Body Problem: (a) Find tension in the connecting rope if there is no friction on the surfaces.
(b) Find acceleration of system.

Now find tension T in connecting cord. Focus on mass 2 kg only



$$\Sigma F_x = m_2 a$$

$$T = (2\ \text{kg})a \text{------------} (1)$$

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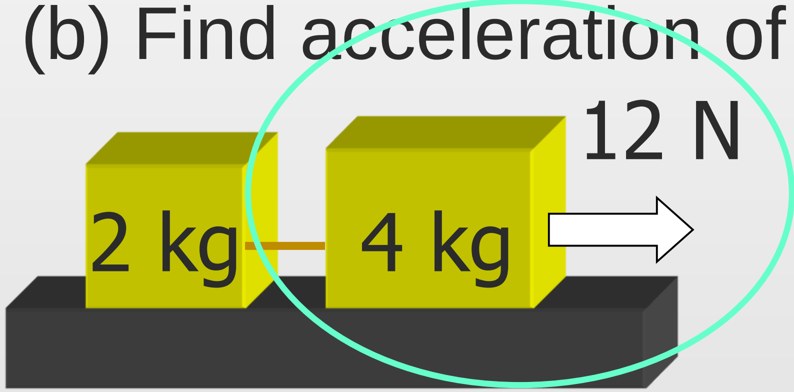
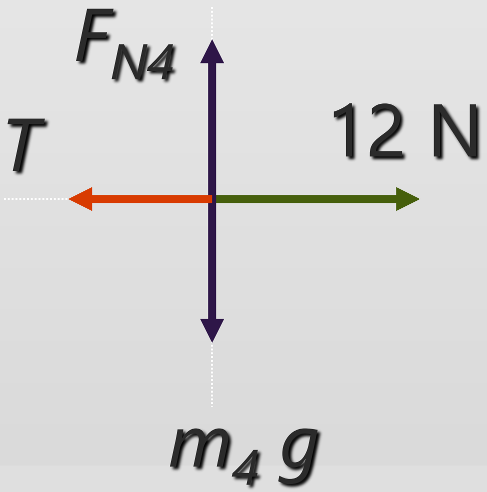
Applications on Newton's Laws: **Dynamics**

Solution

Two-Body Problem: (a) Find tension in the connecting rope if there is no friction on the surfaces.

(b) Find acceleration of system.





Focusing on 4-kg by itself one can find:

$$\Sigma F_x = m_4 a$$

$$12 \text{ N} - T = (4 \text{ kg})\, a \text{ ------(2)}$$

From equations (1) and (2) T is

$\Rightarrow \quad T = 4$ N

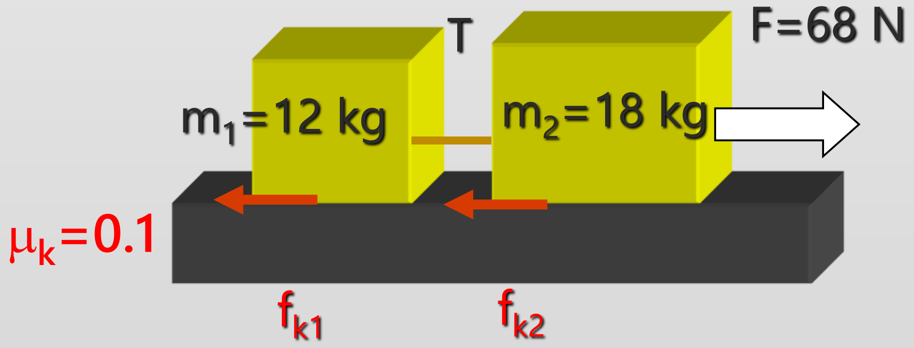
Substitute T into equation (2) to get

$\Rightarrow a = 2 \text{ m/s}^2$

Applications on Newton's Laws: **Dynamics**

Problem

Two blocks connected by a rope of negligible mass are being dragged by a horizontal force. Suppose F = 68 N, m_1 = 12 kg, m_2 = 18 kg and the coefficient of kinetic friction between each block and the surface is 0.1 (a) Draw free-body diagram for each block. Determine (b) the acceleration of the system and (c) the tension in the rope.



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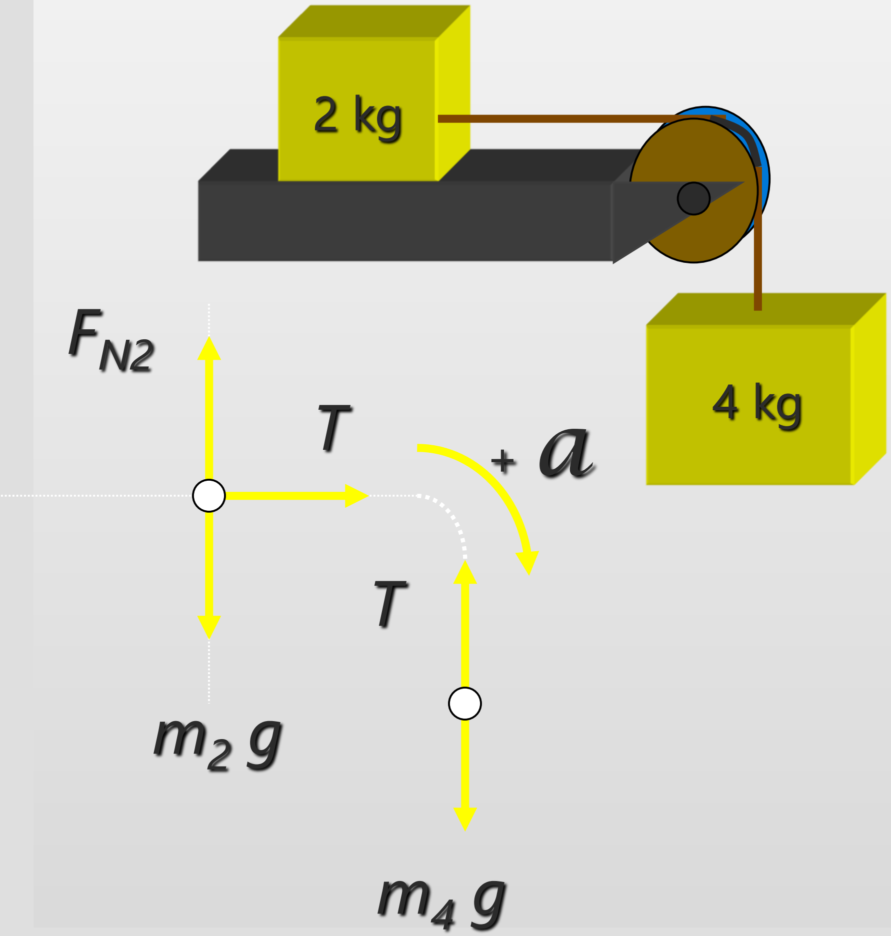
Applications on Newton's Laws: **Dynamics**

Problem

Find acceleration of system and tension in cord for the arrangement shown.



Solution

One can apply $F = m\,a$ to entire system along the line of motion.

$$\Sigma F = (m_2 + m_4)\,a$$

Note $m_2 g$ is balanced by F_{N2} because $\Sigma F_y = 0$

$$m_4 g = (m_2 + m_4)\,a$$

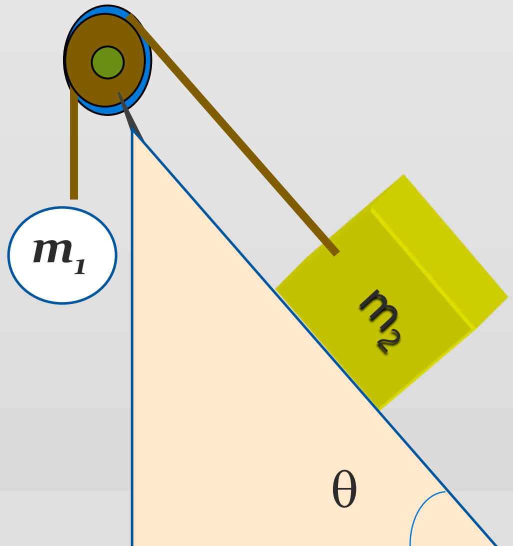
$$a = \frac{m_4 g}{m_2 + m_4} = \frac{(4\text{ kg})(9.8\text{ m/s}^2)}{2\text{ kg} + 4\text{ kg}}$$

$$a = 6.53\text{ m/s}^2$$

Applications on Newton's Laws: **Dynamics**

Problem

Two objects are connected by a light string that passes over a frictionless pulley as shown in the figure. Assume the incline is frictionless and take $m_1 = 2$ kg , $m_2 = 6$ kg and $\theta = 35^o$. (a) Draw free-body diagram of both objects. Find (b) the magnitude of the acceleration of the objects, (c) the tension in the string, and (d) the speed of each object 2 s after it is released from rest



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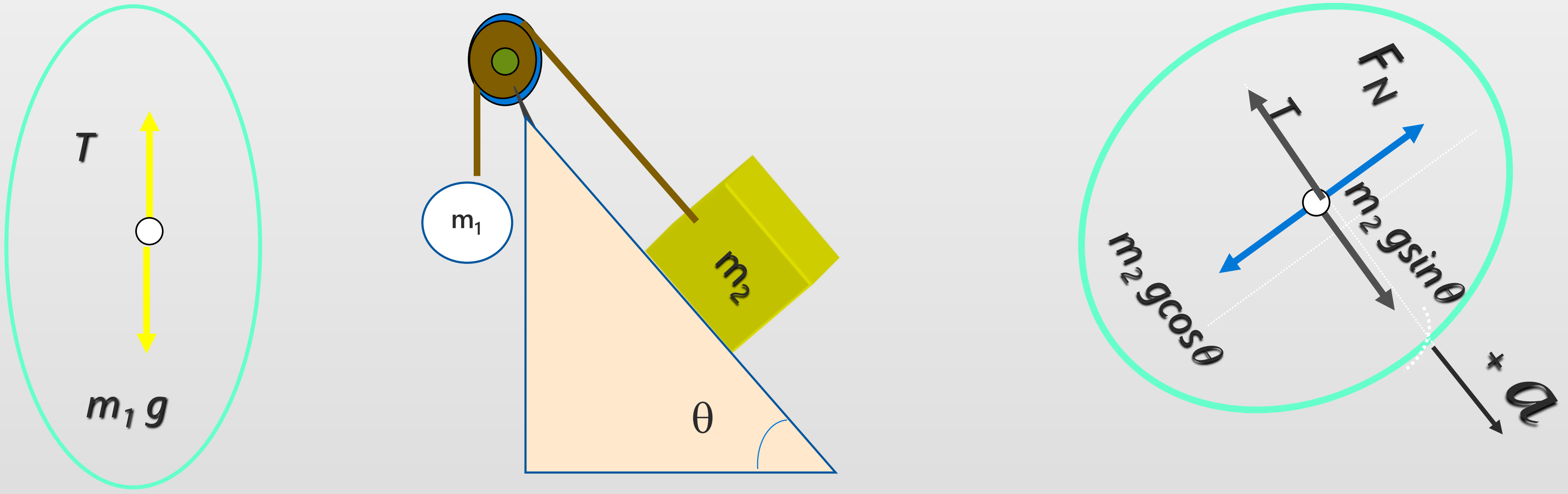
Applications on Newton's Laws: Dynamics

Solution

(a) The free-body diagram of both objects are shown as follows



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Applications on Newton's Laws: **Dynamics**

Solution

(Cont.) To find the magnitude of the acceleration: $m_1 = 2$ kg, $m_2 = 6$ kg, $\theta = 35^o$

For mass m_2: apply $\Sigma F_x = m_2 a$ to just the **6 kg** mass, ignoring 2 kg.

$$m_2 g \sin\theta - T = m_2 a$$

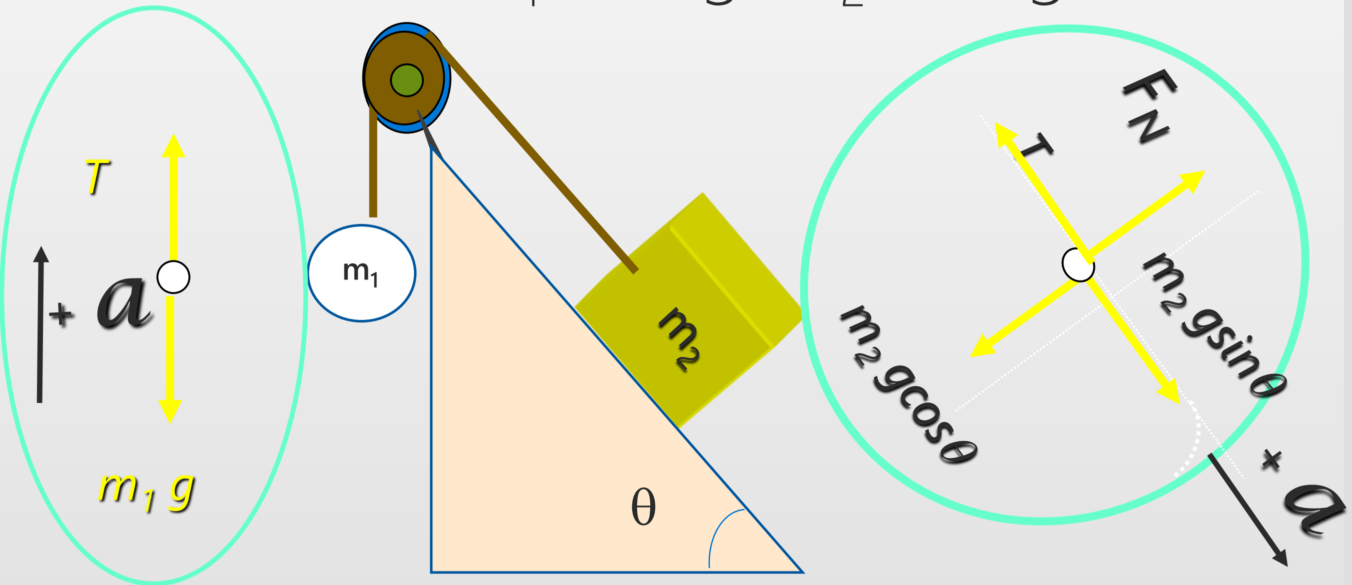
$$(6)(9.8)\sin 35^o - T = (6)a$$

$$33.7 - T = (6)(a) \text{-------}(1)$$

For mass m_1: apply $\Sigma F_y = m_1 a$ to just the **2 kg** mass, ignoring 6 kg.

$$T - m_1 g = m_1 a$$

$$T - (2)(9.8) = (2)(a) \text{------}(2)$$



From equations (1) and (2) one can find a as $\Rightarrow$

$$a = 1.76 \text{m/s}^2$$

Applications on Newton's Laws: **Dynamics**

Problem

Find the tension in the rope shown below. (The Atwood machine.)

Solution

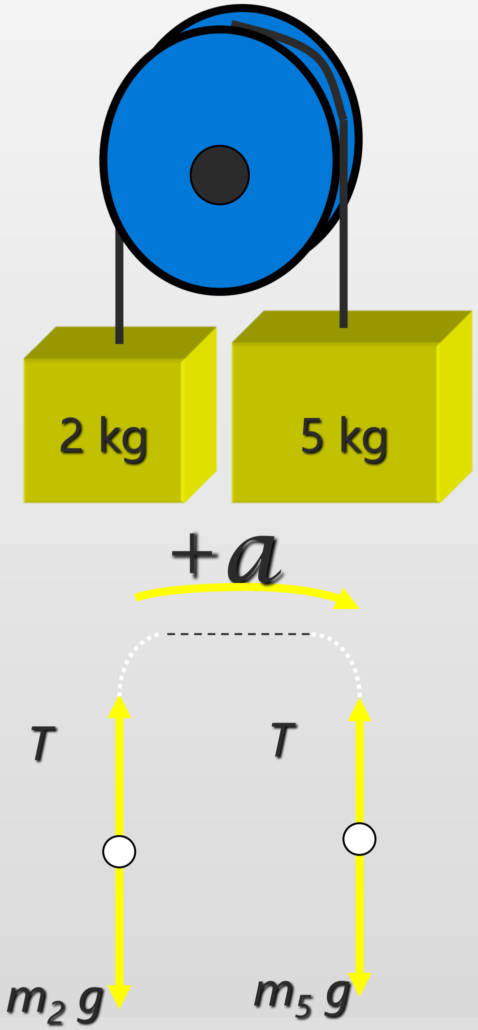
Apply $F = ma$ to mass $m_2 = 2kg$ along the line of motion (upward).

$$\Sigma F_y = m_2 a$$

$$T - m_2 g = m_2 a$$

$$T - (2)(9.8) = 2a$$

$$T - 19.6 = 2a \text{ ------- } (1)$$



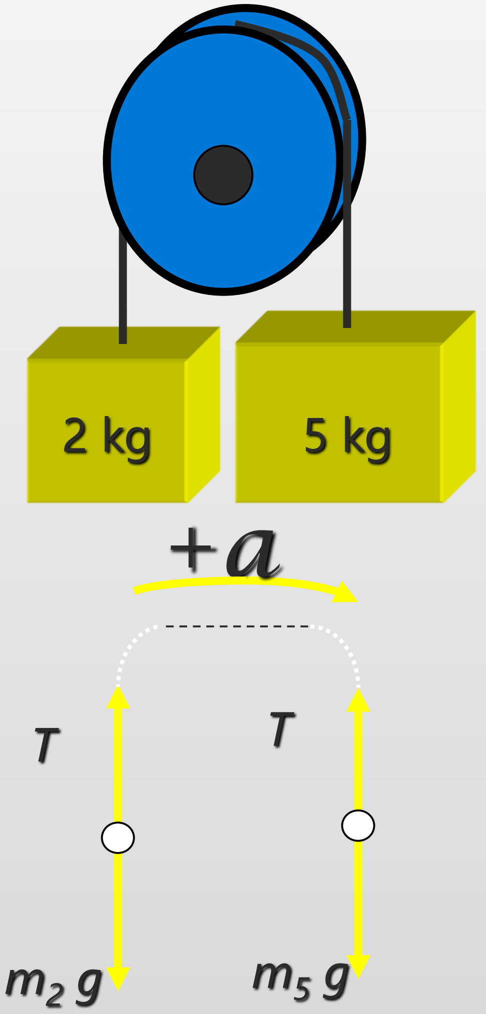
Applications on Newton's Laws: Dynamics

Problem

Find the tension in the rope shown below. (The Atwood machine.)

Solution

Apply F = ma to mass m_5 = 5kg along the line of motion (downward).



$$\Sigma F_y = m_5\, a$$

$$m_5 g - T = m_5 a$$

$$(5)(9.8) - T = 5a$$

$$49 - T = 5a \text{ --------- } (2)$$

From equations (1) and (2) one can find a as ⇒ a = 4.20 m/s²

Substitute a = 4.20 m/s² into equation (2) to get T

T = 28 N

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Applications on Newton's Laws: Dynamics

Problem

Find the acceleration of the system shown below. (The Atwood machine.)

Solution

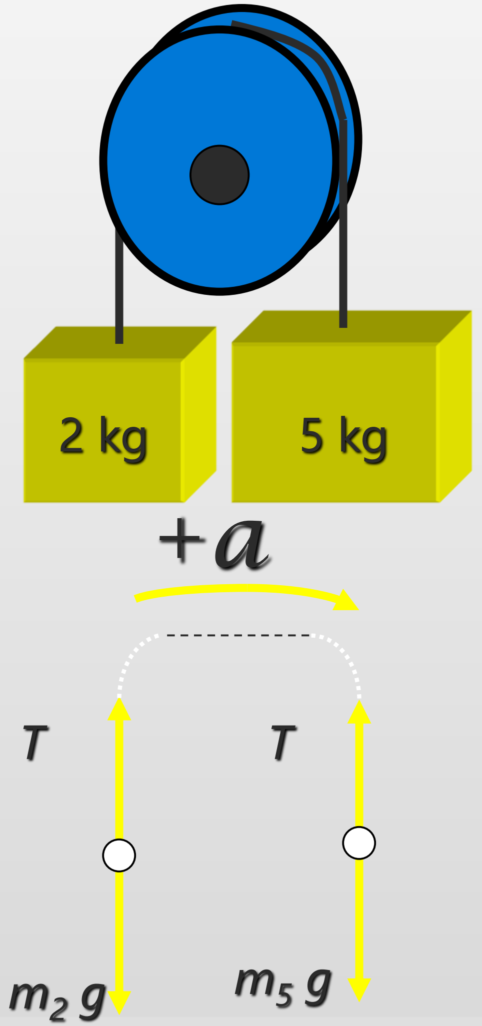
Alternatively, one can find the acceleration by applying F = ma to entire system along the line of motion.

$$\Sigma F_y = (m_2 + m_5)\, a$$

$$m_5 g - m_2 g = (m_2 + m_5) a$$

$$a = \frac{m_5 g - m_2 g}{m_2 + m_5} = \frac{(5\text{ kg} - 2\text{ kg})(9.8\text{ m/s}^2)}{2\text{ kg} + 5\text{ kg}}$$

$$a = 4.20\text{ m/s}^2$$

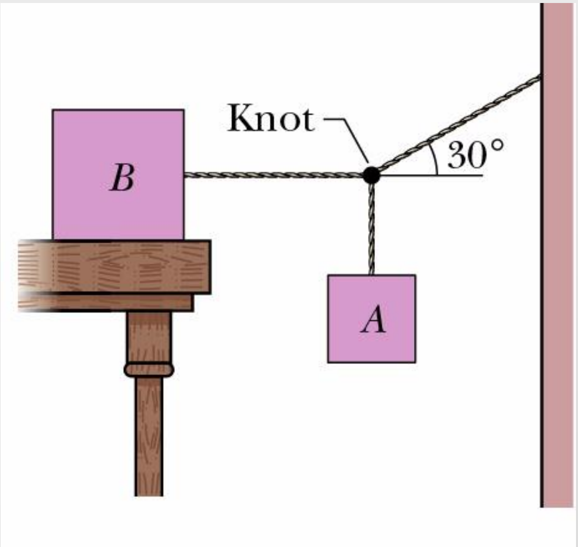


Applications on Newton's Laws

Exercise

Block B weighs **711N**. The coefficient of static friction between the block and the table is **0.25**; assume that the cord between **B** and the knot is horizontal. Find the maximum weight of block **A** for which the system will be stationary.

Answer: 103.0 N

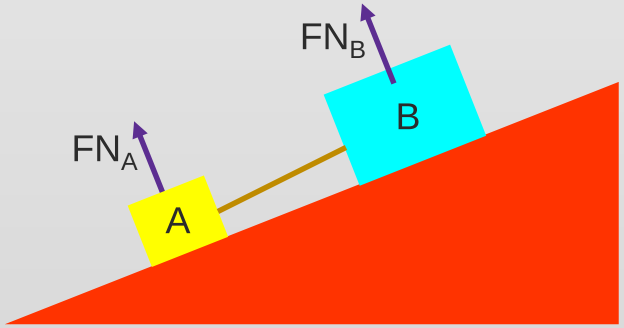


Applications on Newton's Laws

Exercise

Two blocks of weights **3.6N** and **7.2N**, are connected by a massless string and slide down a **30º** inclined plane. The coefficient of kinetic friction between the lighter block and the plane is **0.10**; that between the heavier block and the plane is **0.20**. Assuming that the lighter block leads, find (a) the magnitude of the acceleration of the blocks and (b) the tension in the string. (c) Describe the motion if, instead, the heavier block leads.

Answer: (a) a = 3.5 m/s^2
(b) T = 0.2 N

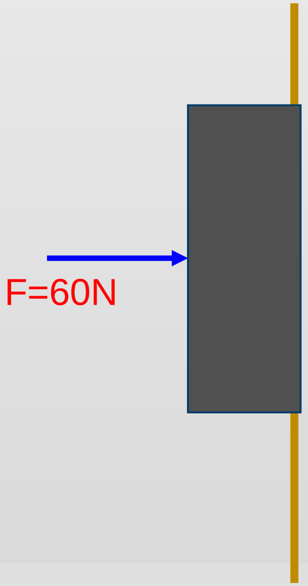


Light block **A** leads

Applications on Newton's Laws

Exercise

A block weighing **22N** is held against a vertical wall by a horizontal force **F** of magnitude **60N**. The coefficient of static friction between the wall and the block is **0.55** and the coefficient of kinetic friction between them is **0.38**. A second force **P** acting parallel to the wall is applied to the block. For the following magnitudes and directions of **P**, determine whether the block moves, the direction of motion, and the magnitude and direction of the frictional force acting on the block:
(a) **34N** up, (b) **12N** up, (c) **48N** up, (d) 62N up, (e) 10N down,
(f) **18N** down.

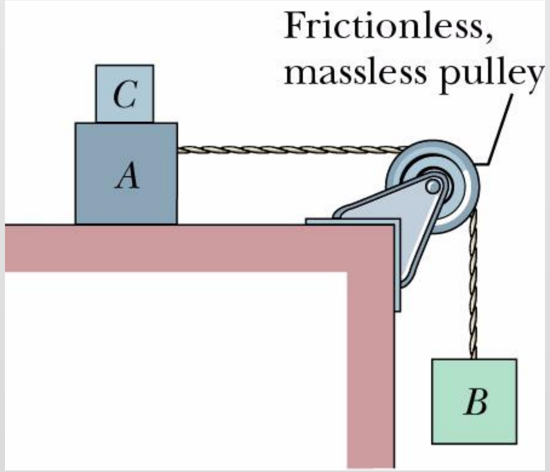


Answer: (a) Block does not move
(b) Block does not move
(c) Block does not move
(d) a =7.6 m/s^2 UP
(e) Block does not move
(f) a =7.6 m/s^2 DOWN

Applications on Newton's Laws

Exercise

Blocks A and B have weights of 44N and 22N, respectively. (a) Determine the minimum weight of block C to keep A from sliding if μ_s between A and the table is 0.2. (b) Block C suddenly is lifted of A. What is the acceleration of block A if μ_k between A and the table is 0.15?

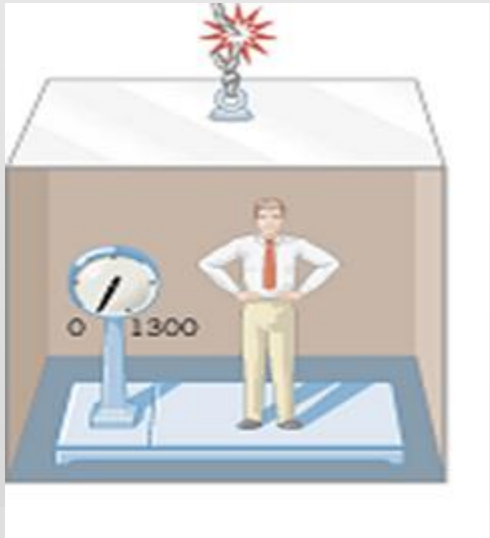


Answer: (a) W_C = 66 N
(b) a = 2.28 m/s²

Apparent Weight

Example

A person stands on a scale in an elevator. As the elevator starts, the scale has a constant reading of **591 N**. As the elevator later stops, the scale reading is **391 N**. Assume the magnitude of the acceleration is the same during starting and stopping, and determine (a) the weight of the person, (b) the person's mass, and (c) the acceleration of the elevator.



Apparent Weight

Solution

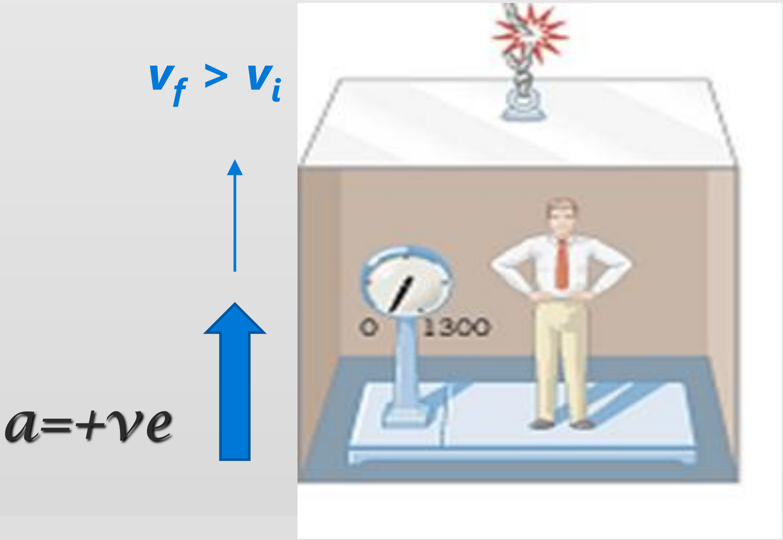
A person stands on a scale in an elevator. As the elevator starts, the scale has a constant reading of **591 N**. As the elevator later stops, the scale reading is **391 N**. Assume the magnitude of the acceleration is the same during starting and stopping, and determine (a) the weight of the person, (b) the person's mass, and (c) the acceleration of the elevator.

$S_1 = 591\ N$

$\Sigma F_y = m\,a$

$S_1 - mg = m\,a$

$591 - mg = m\,a$ -----(1)



Apparent Weight

Solution

A person stands on a scale in an elevator. As the elevator starts, the scale has a constant reading of **591 N**. As the elevator later stops, the scale reading is **391 N**. Assume the magnitude of the acceleration is the same during starting and stopping, and determine (a) the weight of the person, (b) the person's mass, and (c) the acceleration of the elevator.

$S_2 = 391\ N$

$\Sigma F_y = m\,a$

$S_2 - mg = -m\,a$

$391 - mg = -m\,a$ -----(2)

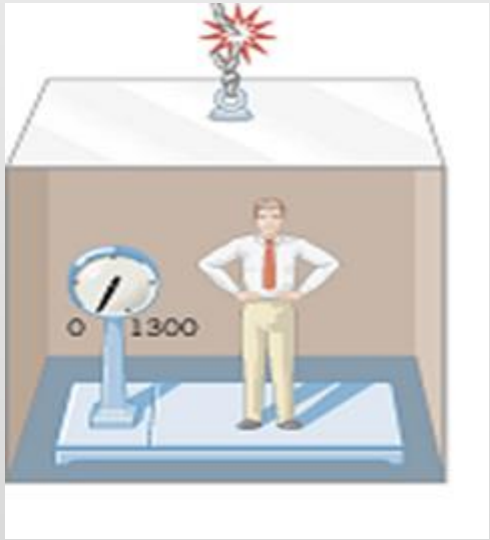
Add equations (1) and (2) to get

$982 = 2mg \Rightarrow$ $mg = 491\ N \Rightarrow$ $m = 50\ kg$

$a = 2\ m/s^2$

$v_f = 0$

$a = -ve$



What Do You Learn?