General

Physics 101

Circular Motion and Other Applications of Newton's Laws

Prepared By

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Circular Motion and Other Applications of Newton's Laws of Motion

- ❑ Uniform Circular Motion and Centripetal Acceleration
- ❑ Applications on Uniform Circular Motion
- ❑ Non-Uniform Circular Motion (Centripetal and Tangential Forces)
- ❑ Applications on Non-Uniform Circular Motion

Applications of Newton's Laws

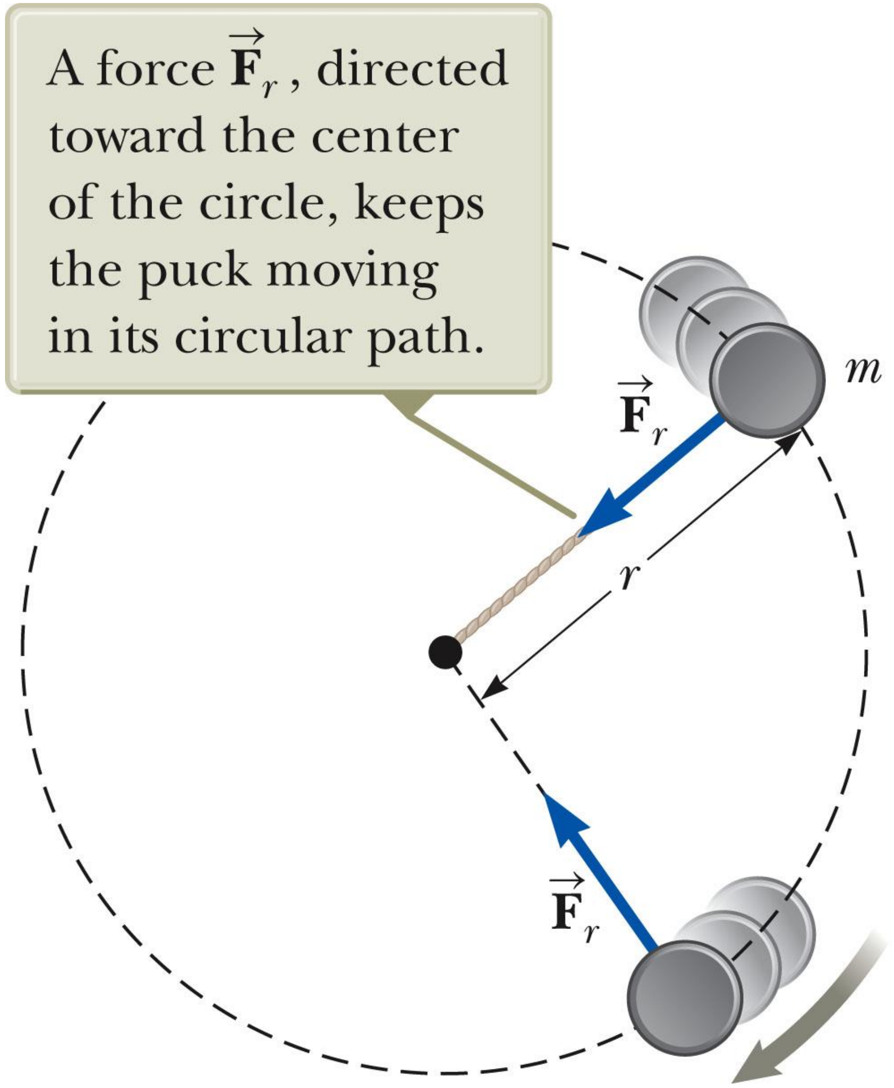
- **Newton's Laws have been applied to particles travelling in linear motion, and projectile motion**

- **Newton's Laws can be applied to:**
  - **Particles travelling in Circular Paths (Circular Motion)**
  - **Particles observed from an accelerating frame of reference (Relative Motion)**

Uniform Circular Motion-Dynamics

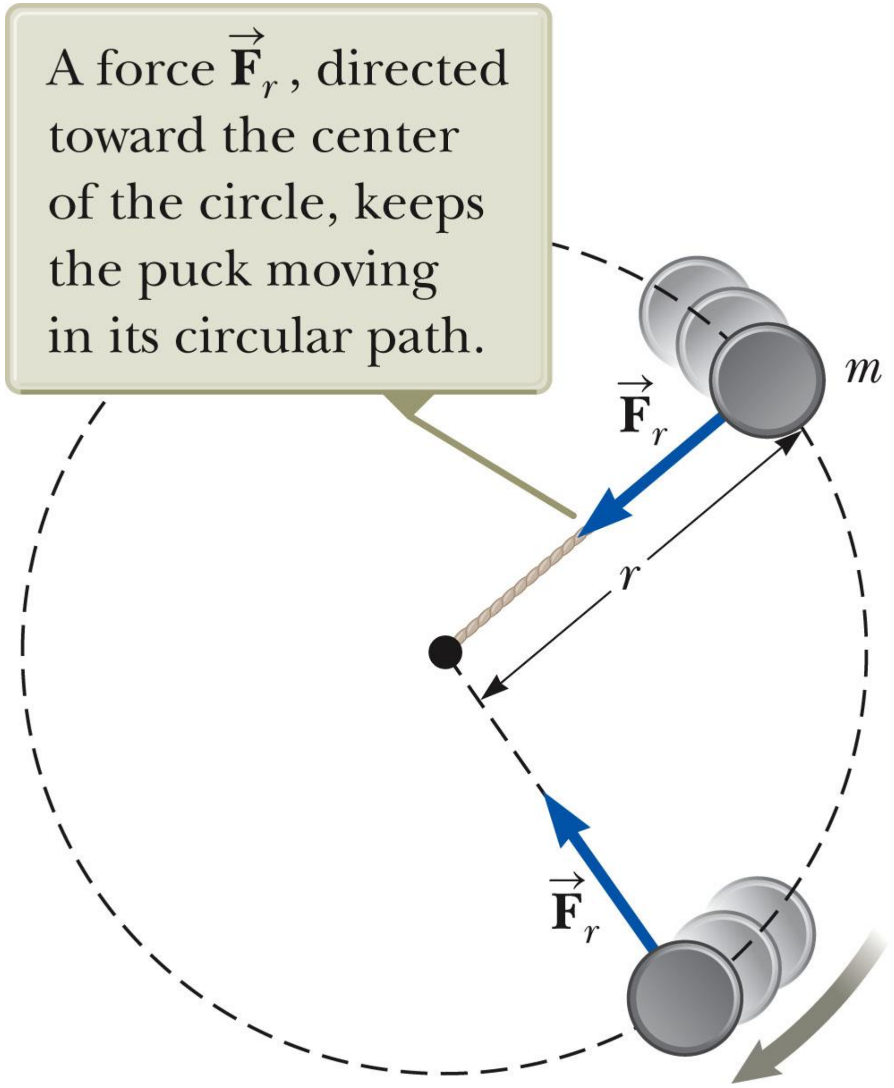
- A force, $\vec{\mathbf{F}}_r$, (called centripetal force) is associated with the centripetal acceleration.
- The force is also directed toward the center of the circle.
- Applying Newton's second law along the radial direction gives

$$\sum F_r = ma_r = \frac{mv^2}{r}\text{, where}\qquad a_r = \frac{v^2}{r}$$



Uniform Circular Motion-Dynamics

- A force, $\vec{F}_r$, directed towards the center of the circular path, has a magnitude of F_r. It is also called radial force which corresponds to radial acceleration a_r
- The force causes a change in the direction of the velocity
- The force is also keeping the puck moving with a constant speed v and a fixed radius r.

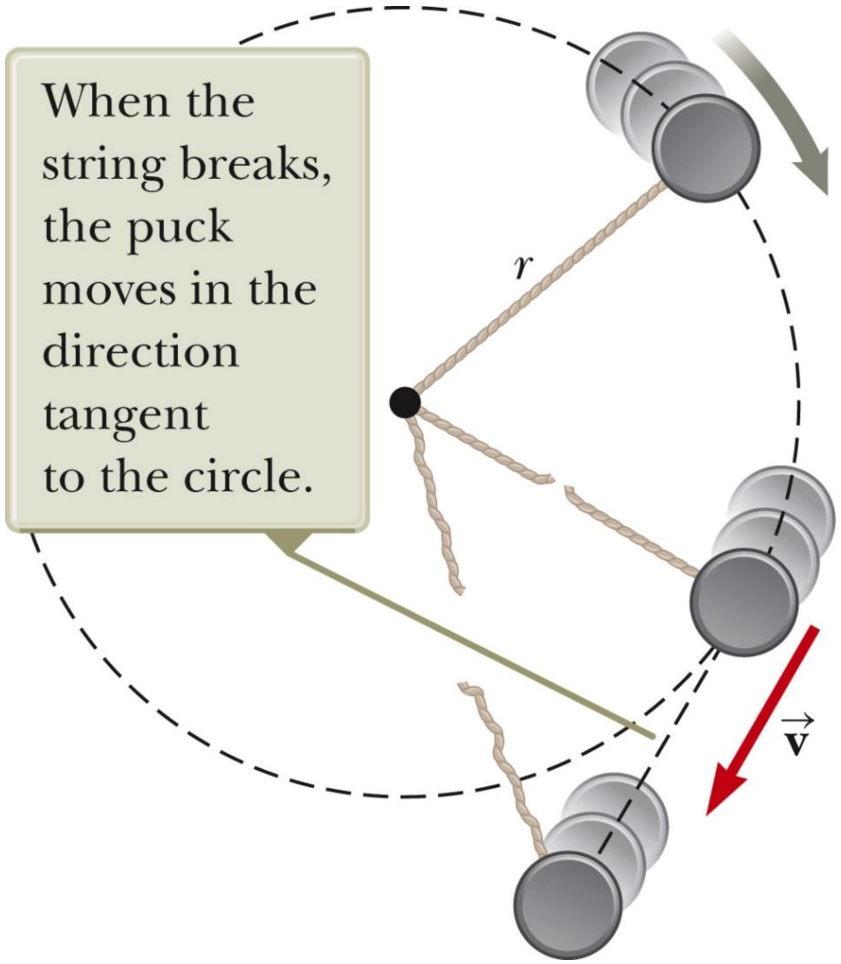


Uniform Circular Motion-Dynamics

- What will happen if the string breaks up (i.e. Centripetal force vanishes) ?

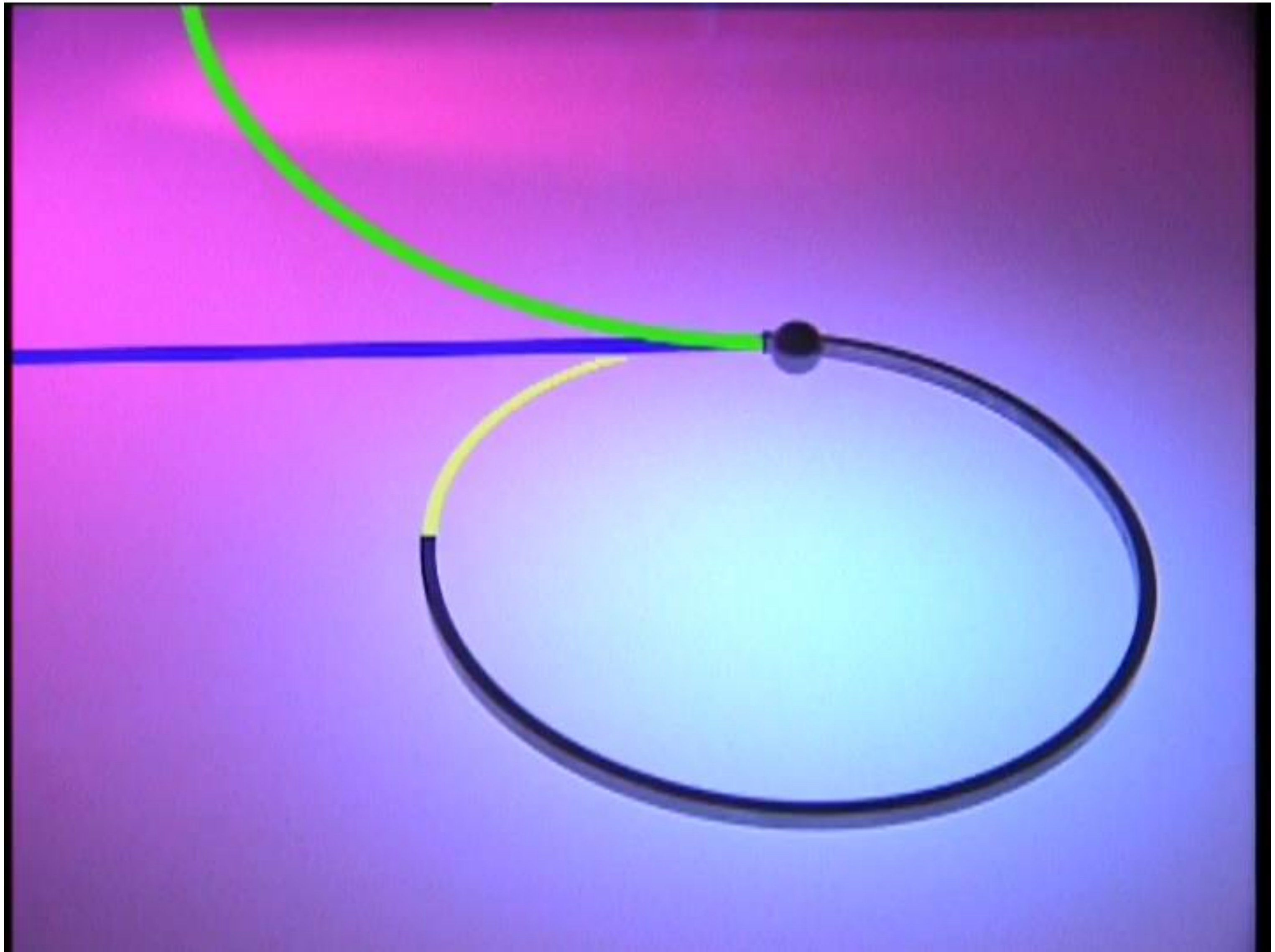
Answer:

✓ The puck will move in a straight-line path ***tangent*** to the circle.

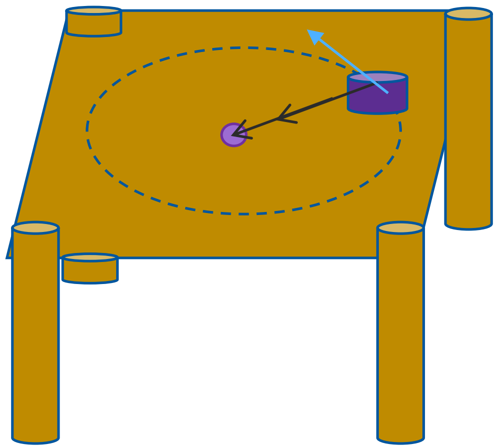


Uniform Circular Motion-Dynamics

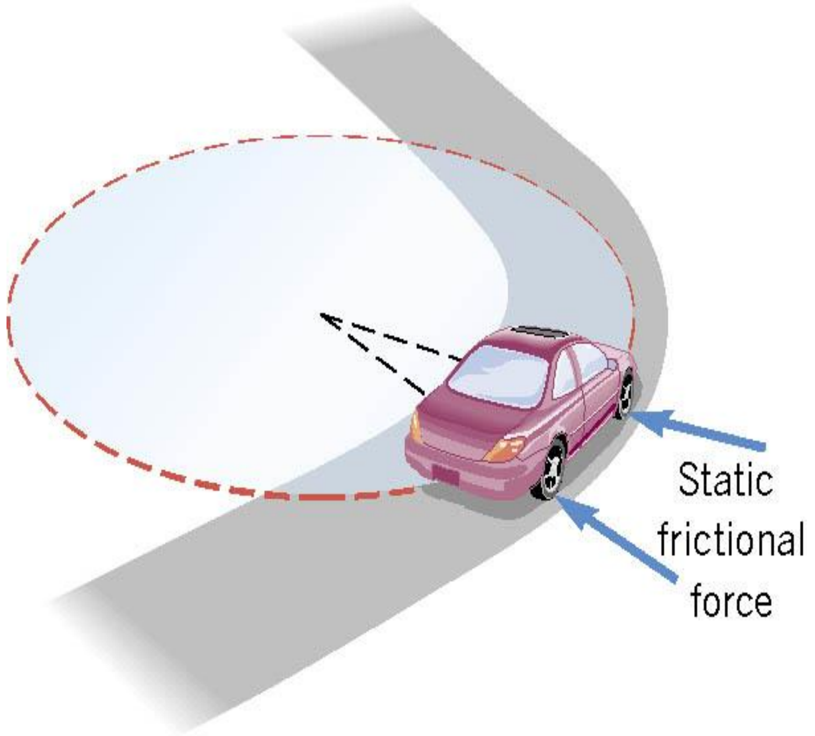
Uniform Circular Motion-Dynamics



Examples on Centripetal Force



❖**Mass-String (horizontal plane):**
- ***Centripetal Force*** ≡ ***Tension***



❖**A car moving in flat circular track:**
- ***Centripetal Force*** ≡ ***Friction***

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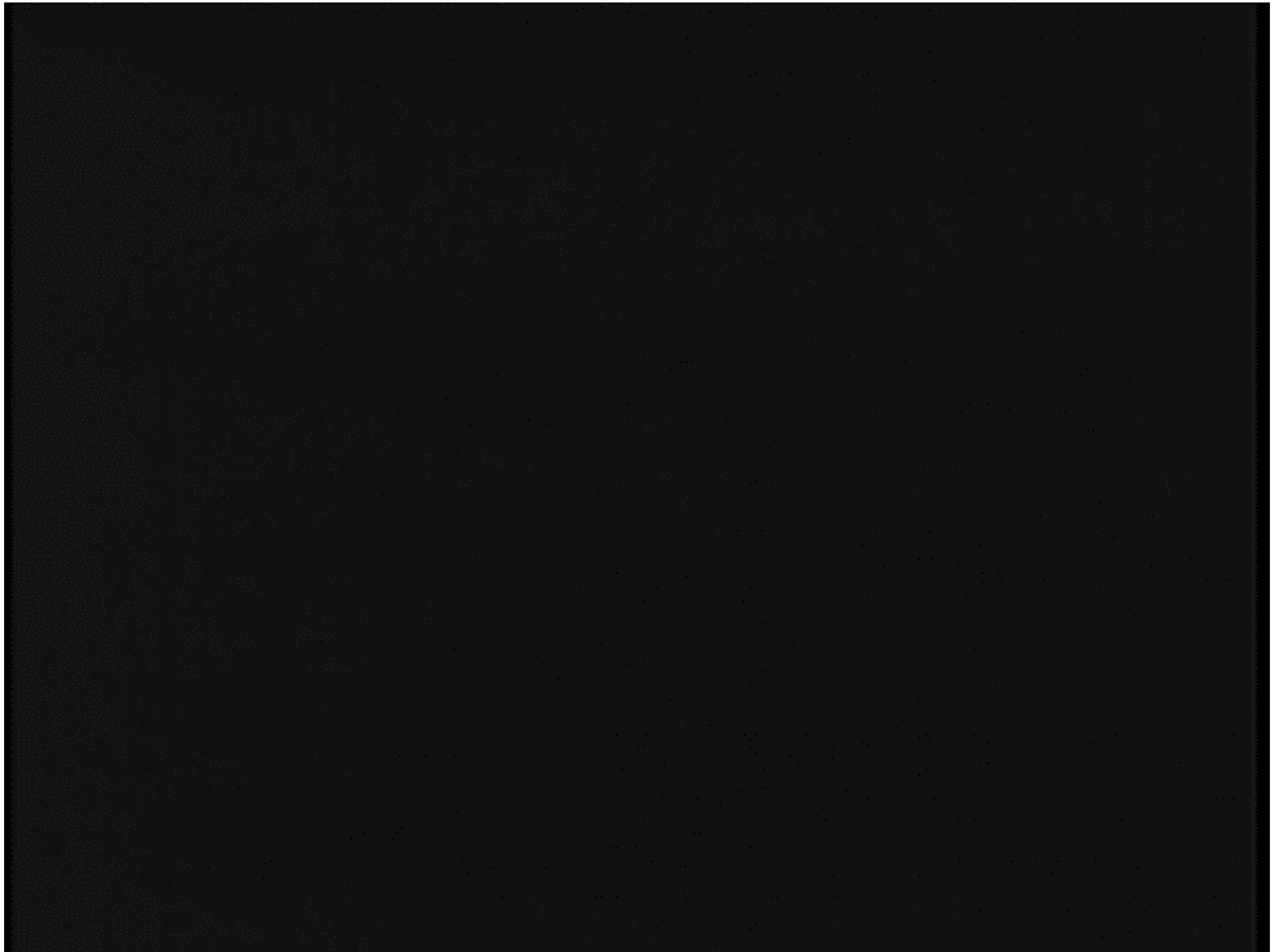
Uniform Circular Motion-Dynamics

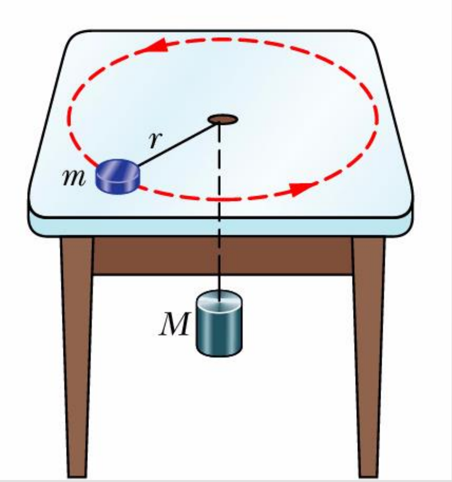


Uniform Circular Motion-Dynamics

Uniform Circular Motion-Dynamics

Example:

A puck of mass m slides on a frictionless table while attached to a hanging cylinder of mass M by a cord through a hole in the table. What speed keeps the cylinder at rest?



A puck of mass m slides on a frictionless table while attached to a hanging cylinder of mass M by a cord through a hole in the table. What speed keeps the cylinder at rest?

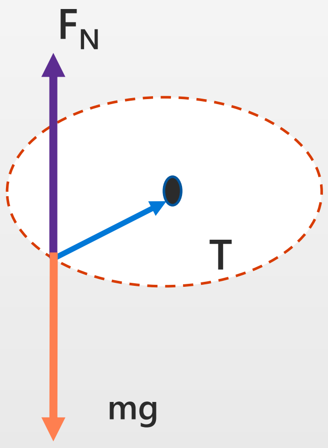
Solution:

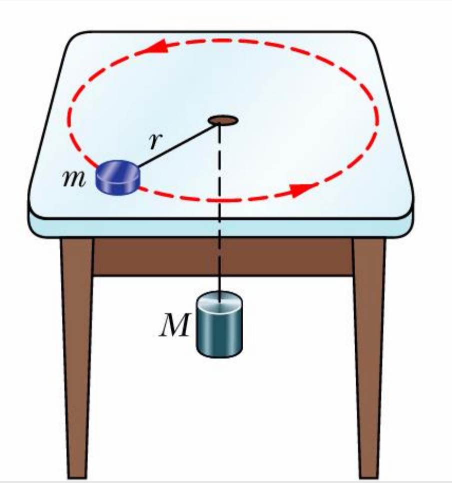
For the puck of mass m:

$\Sigma F_r = m\, a_r$

$-T = -mv^2/r$

Or $T = mv^2/r$ ---------(1)





The two forces F_N and mg are perpendicular to the circle. They balance each other

For the cylinder of mass M:

$\Sigma F_y = 0 \Rightarrow$ $T - Mg = 0$ Or $T = Mg$--------(2)

From equations (1) and (2), one can get $Mg = mv^2/r$ Or $v = (Mg\, r/m)^{1/2}$

Uniform Circular Motion-Dynamics

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Exercise:

A 1500-kg car moving on a flat, horizontal road negotiate a curve as shown in the ~~figure. If the radius of the curve is 35 m and the coefficient of static friction is μ_s =~~ 0.523, find the maximum speed the car can have and still make the turn successfully.

Solution:

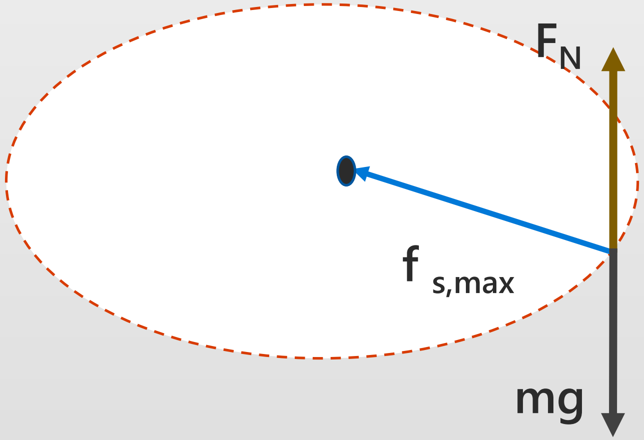
$$\Sigma F_r = m\, a_r = mv^2/r$$

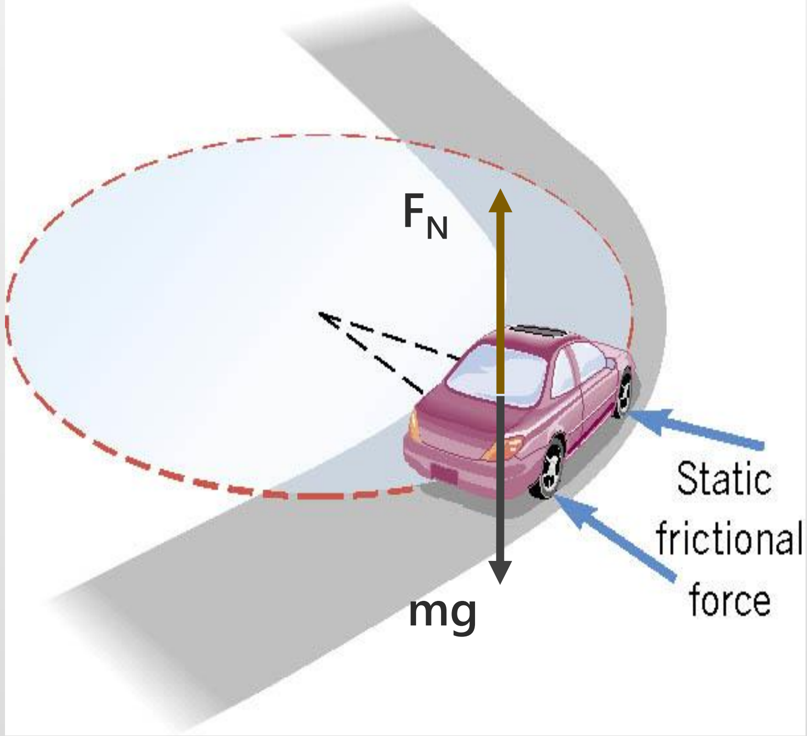
$f_s \rightarrow f_{s,max}$ this implies $v \rightarrow v_{max}$

$$-f_{s,max} = -mv_{max}^2/r$$

$$\Rightarrow f_{s,max} = mv_{max}^2/r$$

Or $\quad \mu_s F_N = mv_{max}^2/r \quad$ ---(1)





$$\Sigma F_{Normal} = 0 \Rightarrow F_N - mg = 0$$

Or $\quad F_N = mg \quad$ ----------(2)

From equations (1) and (2), one can get

$\mu_s\, mg = mv_{max}^2/r \qquad or\ v_{max} = (\mu_s\, g r)^{0.5} \quad \Rightarrow \quad \boxed{v_{max} = 13.4\ m/s}$

Exercise:

If the coefficient of static friction between your coffee cup and the horizontal dashboard of your car is **μ_s = 0.800**, how fast can you drive on a horizontal roadway around a right turn of radius **30.0 m** before the cup starts to slide? If you go too fast, in what direction will the cup slide relative to the dashboard?

Answer: $v = 15.3\ m/s$

Exercise:

A coin placed 30 cm from the center of a rotating horizontal turntable slips when its speed is 50 cm/s.

(a) What force causes the centripetal acceleration when the coin is stationary relative to the turntable?

(b) What is the coefficient of static friction between coin and turntable?

Answer: (a) The force of static friction causes the centripetal acceleration
(b) $\mu = 0.083$

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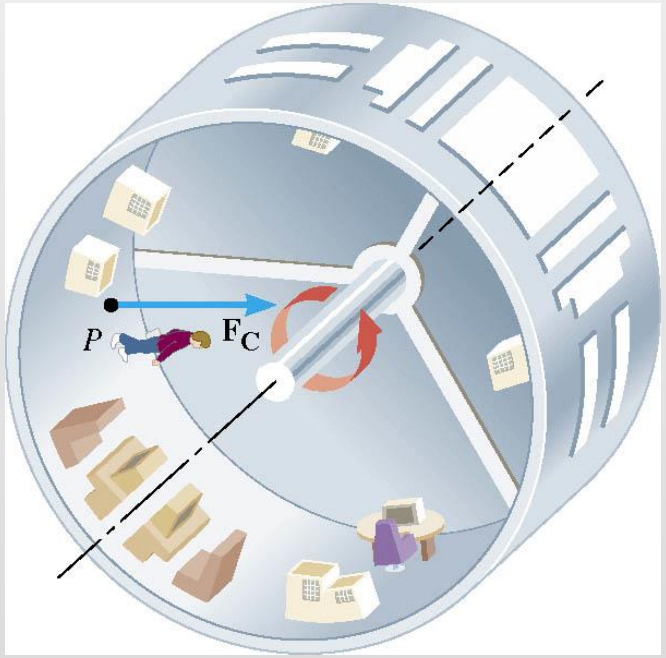
Circular Motion

Example: Artificial Gravity

At what speed must the surface of the space station (r=1700m) move in the figure, so that the astronaut at point P experiences a push on his feet that equals his earth weight?



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Circular Motion

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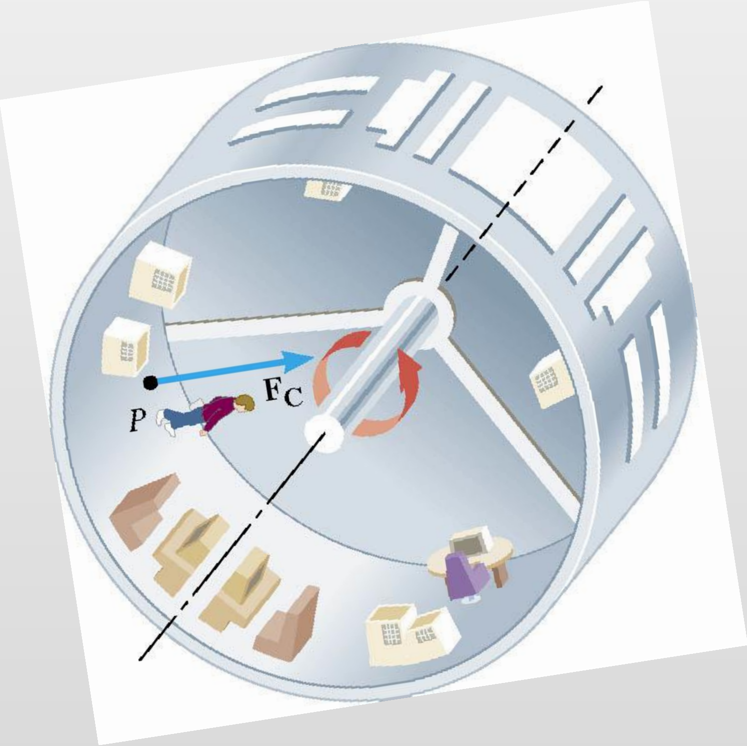

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Circular Motion

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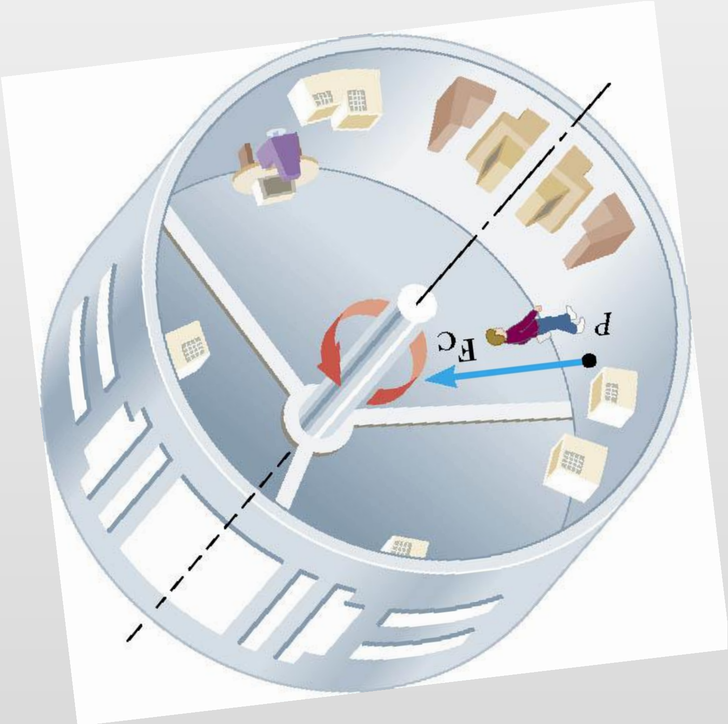


Circular Motion Example: Artificial Gravity

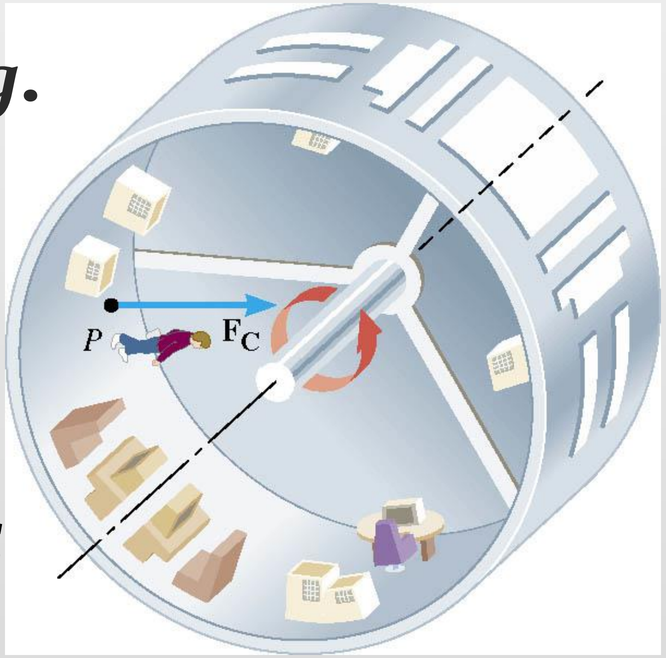
~~At what speed must the surface of the space station (r=1700m) move~~ in the figure, so that the astronaut at point P experiences a push on his feet that equals his earth weight?

$$F_r = mv^2/r$$

Earth weight of the astronaut (mass=m) is mg.

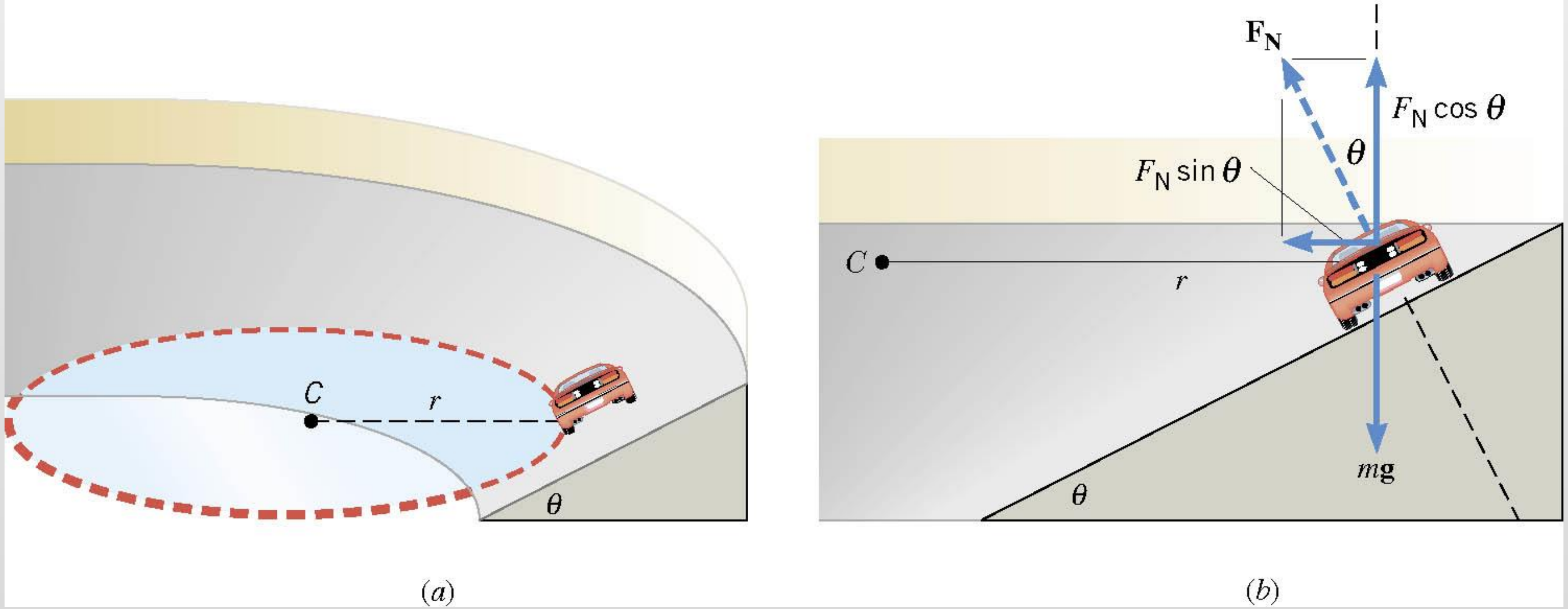
$$F_r = mg = mv^2/r$$

$$v = \sqrt{rg} = \sqrt{(1700m)(9.80m/s^2)} = 130m/s$$



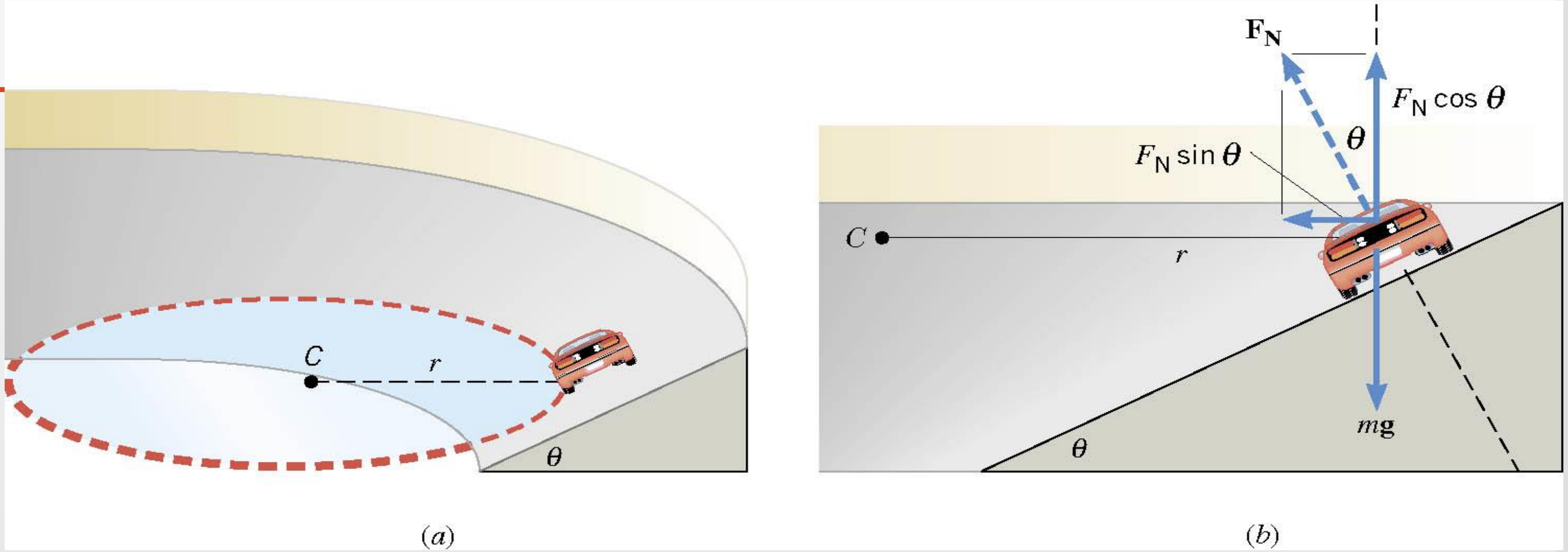
Examples on Centripetal Force

A civil engineer wishes to design a banked curved runway in such a way that a car will not have to rely on friction to round the curve without skidding. In otherwords, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a road is usually banked, which means that the roadway is tilted toward the inside of the curve as shown. Suppose the designated speed for the road is to be 13.4 m/s and the radius is 35 m. At what angle should the curve be banked.



Examples on Centripetal Force

Solution:



$\Sigma F_r = m\, a_r$

$-F_N \sin\theta = -mv^2/r$

Or $F_N \sin\theta = mv^2/r$ -----(1)

$\Sigma F_\perp = 0 \quad \Rightarrow$

$F_N \cos\theta = mg$ ----------(2)

Dividing equation (1) by (2), one can get

$\tan\theta = v^2/gr$

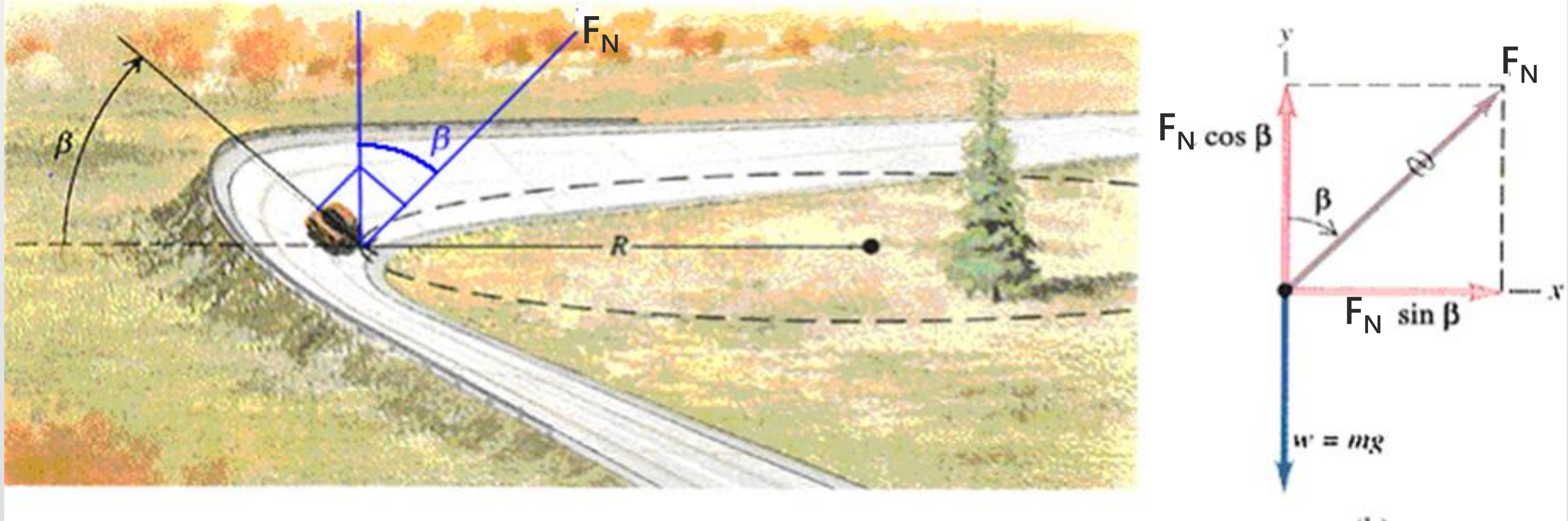
$\theta = \tan^{-1}(\, v^2/gr\,)$

$\theta = 27.6^o$

Exercise:

For a car traveling with speed v around a curve of radius $R=28\ m$, determine the speed of the car at which the road should be banked with angle of $\beta = 30^o$ so no friction is required.

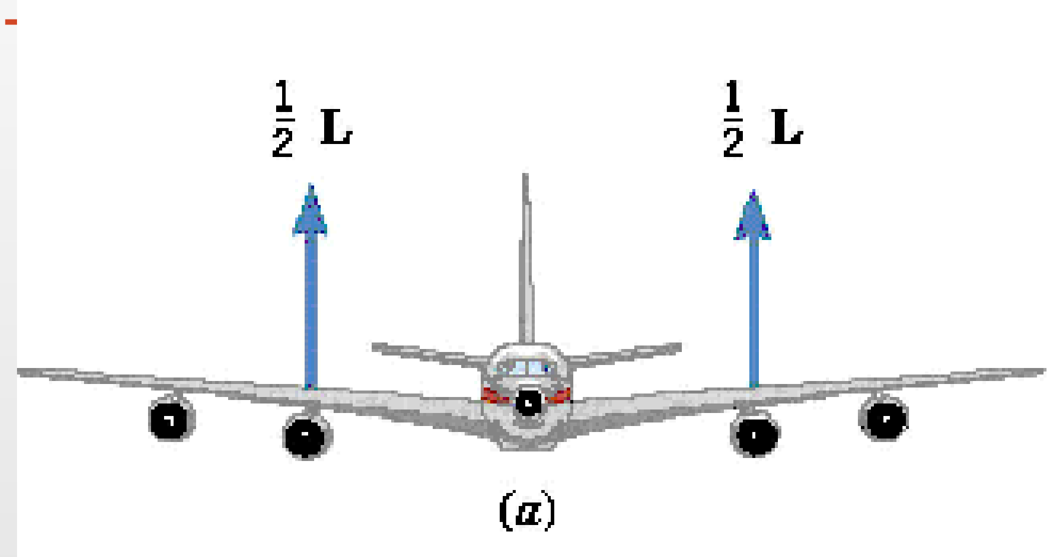
Answer: $v = 12.6\ m/s$

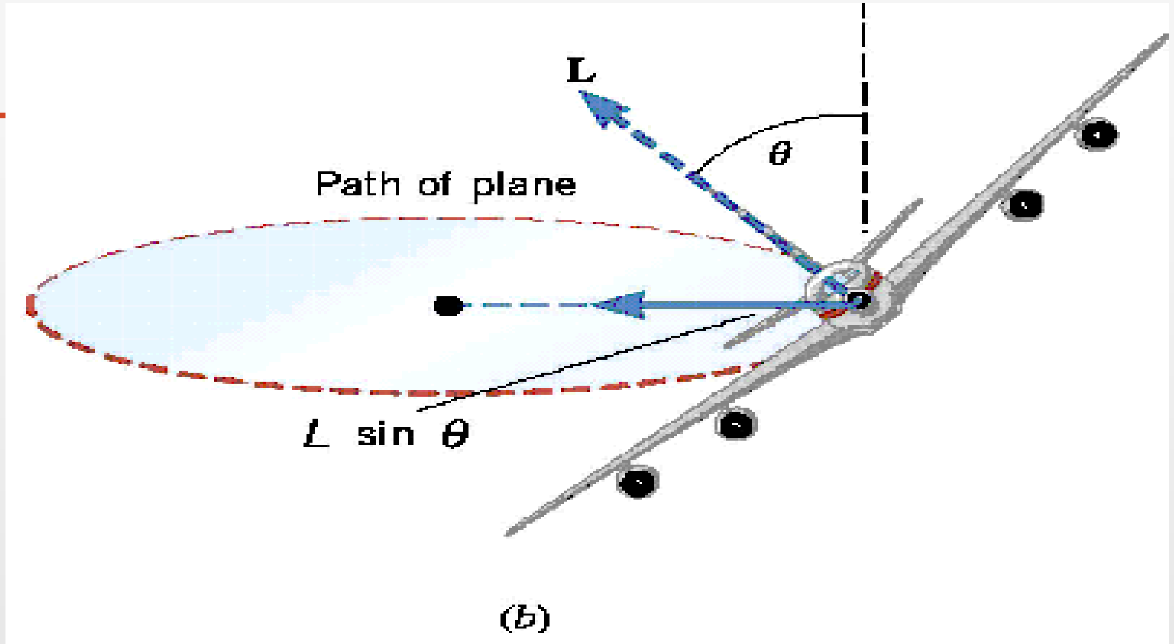


(a)

(b)

Examples on Centripetal Force



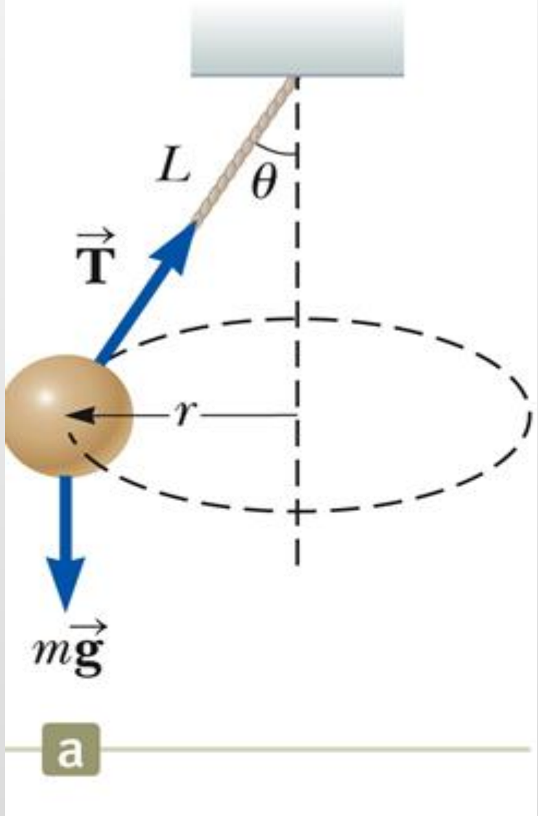


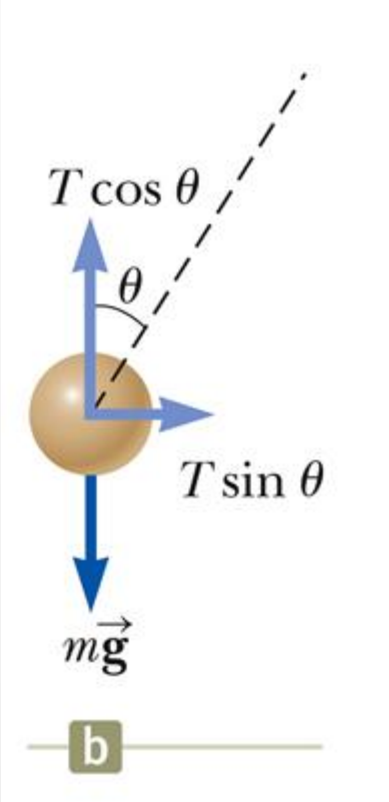
Upward on the wing surfaces with a net lifting force *L*, the plane is banked at an angle θ, a component *L* sin θ of the lifting force is directed toward the center of the turn.

Greater speeds and/or tighter turns require greater centripetal forces. As it was shown previously that banking into a turn also has an application in the construction of high-speed roadways.

Examples on Centripetal Force

- **Conical Pendulum (horizontal plane):**
  - ***Centripetal Force* ≡ *Component of Tension***





Example:

Conical Pendulum

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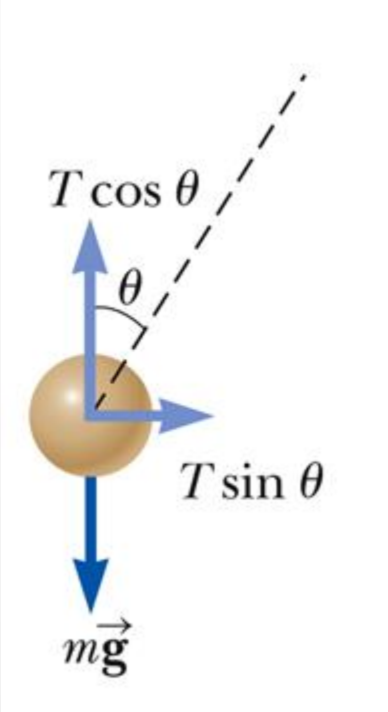
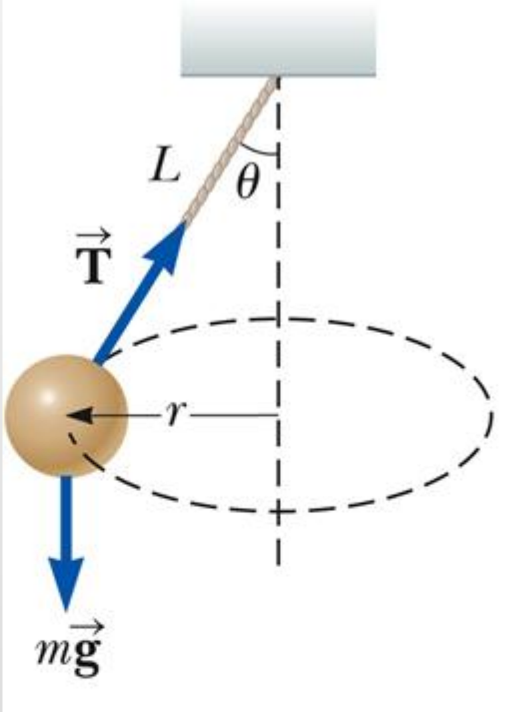
A bob of mass m is attached to a string of length L and moving in a horizontal circle of radius r with speed v, as shown. The string makes an angle θ with the vertical. Find the speed of the bob in terms of L and θ and show that the speed is independent of m.





Solution

Conical Pendulum

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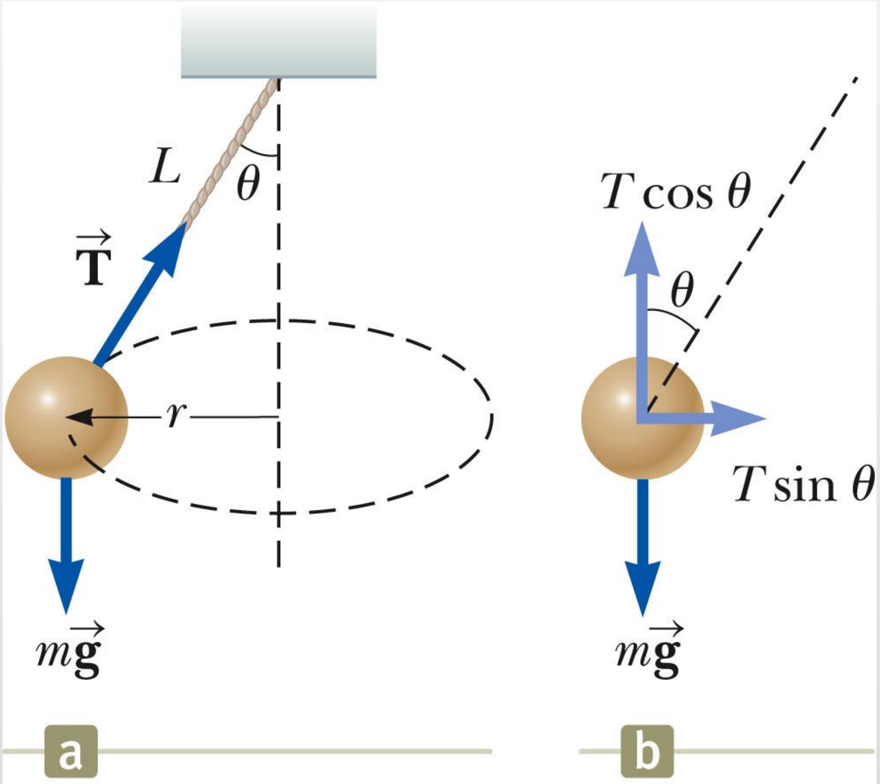
- The object is in equilibrium in the vertical direction and undergoes uniform circular motion in the horizontal direction

$\sum F_{\perp} = 0$ ⟹ $T\cos\theta = mg$ ---------(1)

$\Sigma F_r = m\,a_r$ ⟹ $T\sin\theta = mv^2/r$--------(2)



- Dividing (2) by (1) and using $\sin\theta/\cos\theta = \tan\theta$
- we eliminate T and find:

$$\tan\theta = \frac{v^2}{rg} \quad \Rightarrow \quad v = \sqrt{rg\tan\theta}$$

Using r = L sinθ

$$\boxed{v = \sqrt{Lg\sin\theta\tan\theta}}$$

v is independent of m

Problem

Conical Pendulum

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Consider a conical pendulum with an **8.0-kg** bob on a **10.0-m** cord making an angle of **5.00°** with the vertical. Determine (a) the horizontal and vertical components of the force exerted by the cord on the pendulum and (b) the radial acceleration of the bob.

Solution

(a) The vertical component of the tension can be found from perpendicular forces as follows:

$$\sum F_{\perp} = 0$$

$$T\cos\theta = mg = (8)(9.8) = 78.4\text{ N}$$

To find the horizontal component of the tension one may first need to get T from previous equation

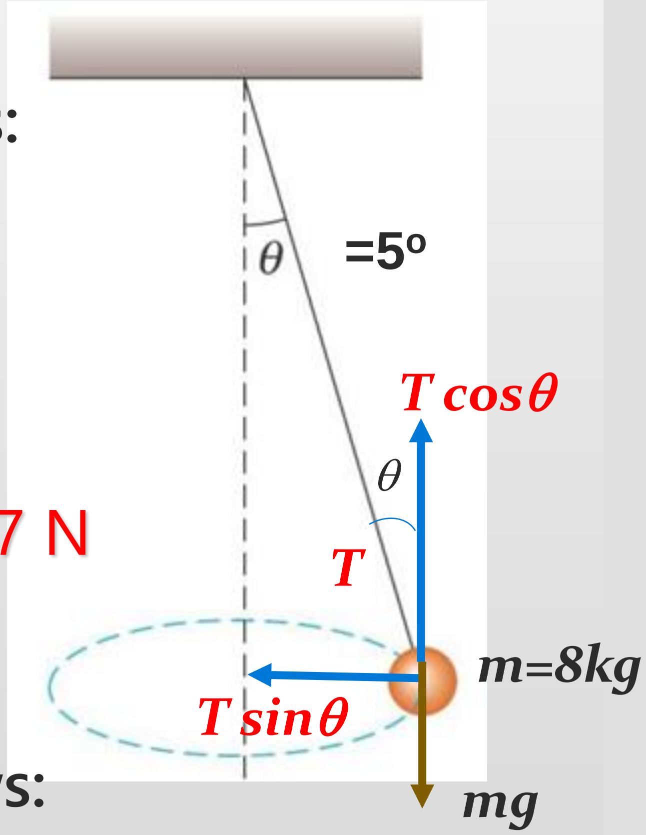
$$T = 78.4/\cos\theta = 78.4/\cos 5 = 78.7\text{ N}$$

Thus the horizontal component of the tension is

$$T\sin\theta = (78.7)(\sin 5) = 6.86\text{ N}$$

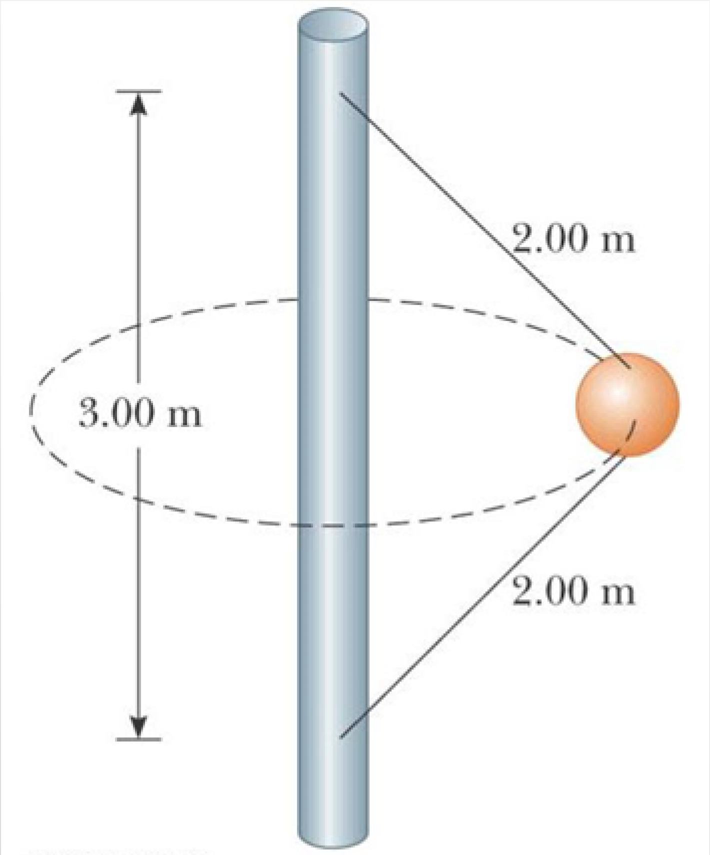
(b) Thus the radial acceleration of the bob can be found as follows:

$$T\sin\theta = 6.86\text{ N} = m\,a_r \qquad \Rightarrow a_r = 6.86/8 \qquad \Rightarrow a_r = 0.86\text{ m/s}^2$$



Exercise

A **4.00-kg** object is attached to a vertical rod by two strings, as in Figure. The object rotates in a horizontal circle at constant speed **6.00 m/s**. Find the tension in (a) the upper string and (b) the lower string.



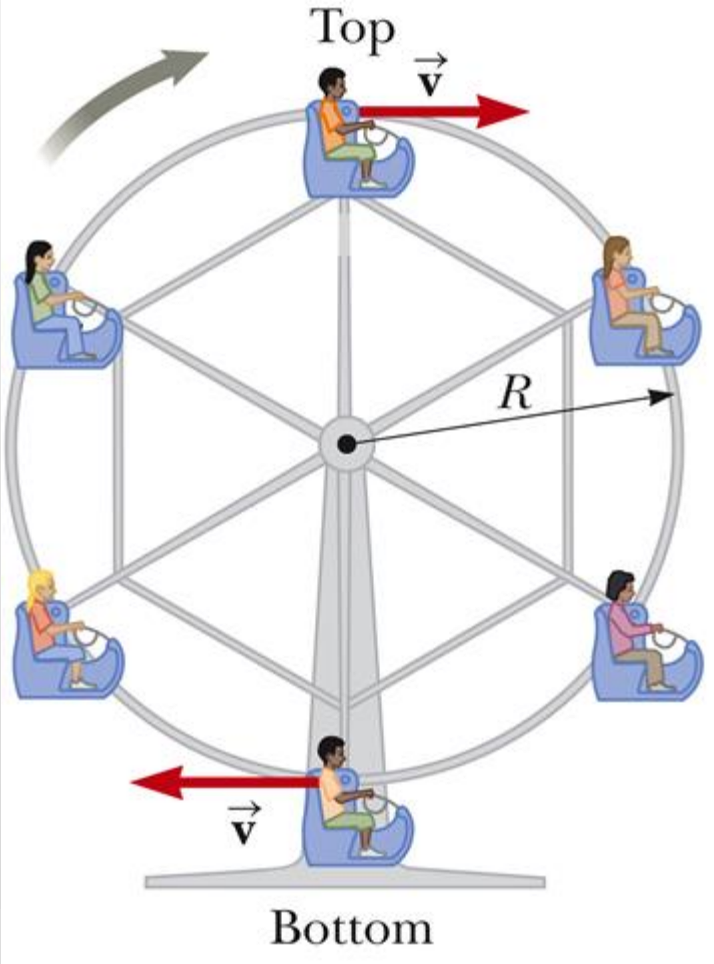
Answer: (a) $T_{upper} = 109.1$ N
(b) $T_{lower} = 57.2$ N

Examples on Uniform Circular Motion

A child of mass m rides on a Ferris wheel as shown in the figure. The child moves in a vertical circle of radius 10 m at a constant speed of 3 m/s.

(a) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child.

(b) Determine the force exerted by the seat on the child at the top of the ride.



Note: $v_{Bot.} = v_{Top} = v$, *for a uniform circular motion*

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Examples on Uniform Circular Motion

~~(a) To find the normal force at the bottom of the ride~~

$$\Sigma F_r = m\, a_r$$

$$- F_{N\text{-}Bot.} + mg = m\, a_r$$

$$- F_{N\text{-}Bot.} + mg = -m v^2_{Bot}/r$$

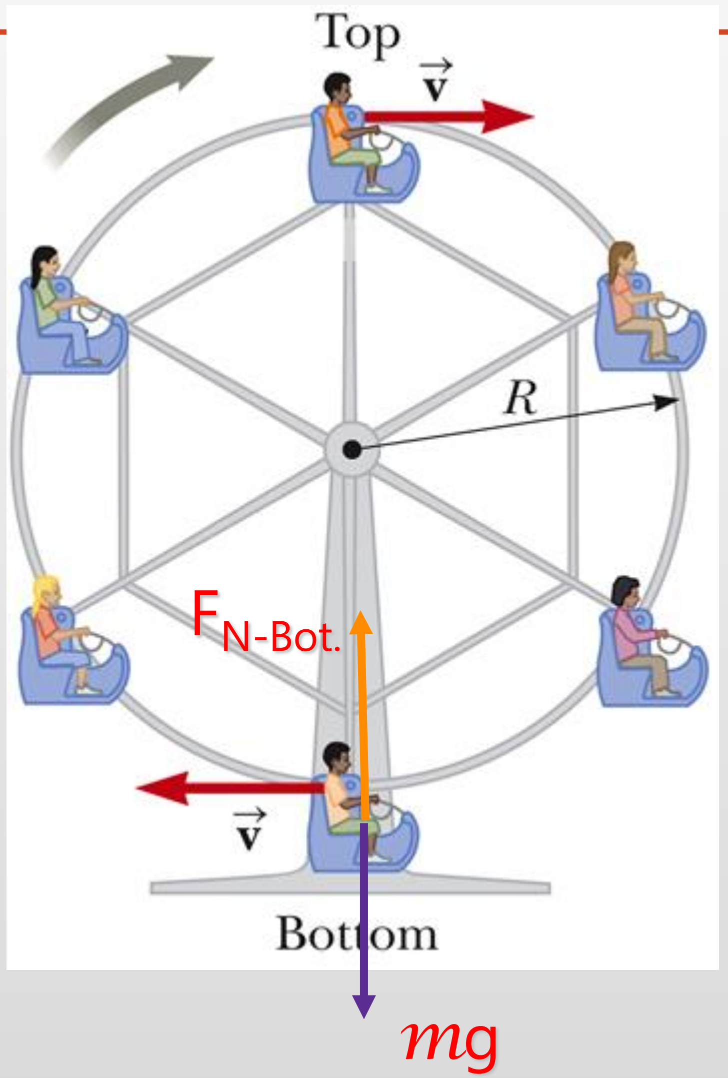
$$\Rightarrow F_{N\text{-}Bot.} - mg = m v^2_{Bot}/r$$

Or $F_{N\text{-}Bot.} = mg\,(1 + v^2_{Bot}/gr)$

Given: $v_{Bot} = 3$ m/s , $r = 10$ m, $g = 9.8$ m/s^2

$$\Rightarrow F_{N\text{-}Bot.} = mg\,(1 + (3)^2/(9.8)(10))$$

$$\therefore F_{N\text{-}Bot.} = 1.09\, mg$$



Note: $v_{Bot.} = v_{Top} = v$, *for a uniform circular motion*

Examples on Uniform Circular Motion

(b) To find the normal force at the top of the ride

$$\Sigma F_r = m\,a_r$$

$$F_{N\text{-}Top} - mg = m\,a_r$$

$$F_{N\text{-}Top} - mg = -mv^2_{Top}/r$$

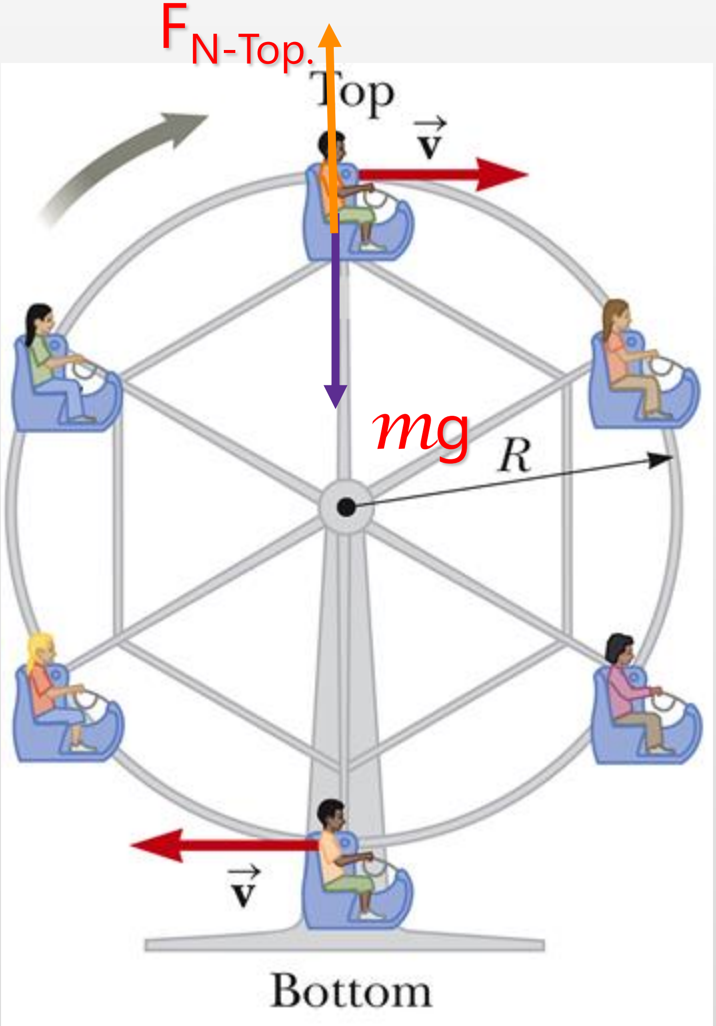
$$\Rightarrow -F_{N\text{-}Top} + mg = mv^2_{Top}/r$$

Or $F_{N\text{-}Top} = mg\,(1 - v^2_{Top}/gr)$

Given: $v_{Top} = 3$ m/s , $r = 10$ m, $g = 9.8$ m/s^2

$$\Rightarrow F_{N\text{-}Top} = mg\,(1 - (3)^2/(9.8)(10))$$

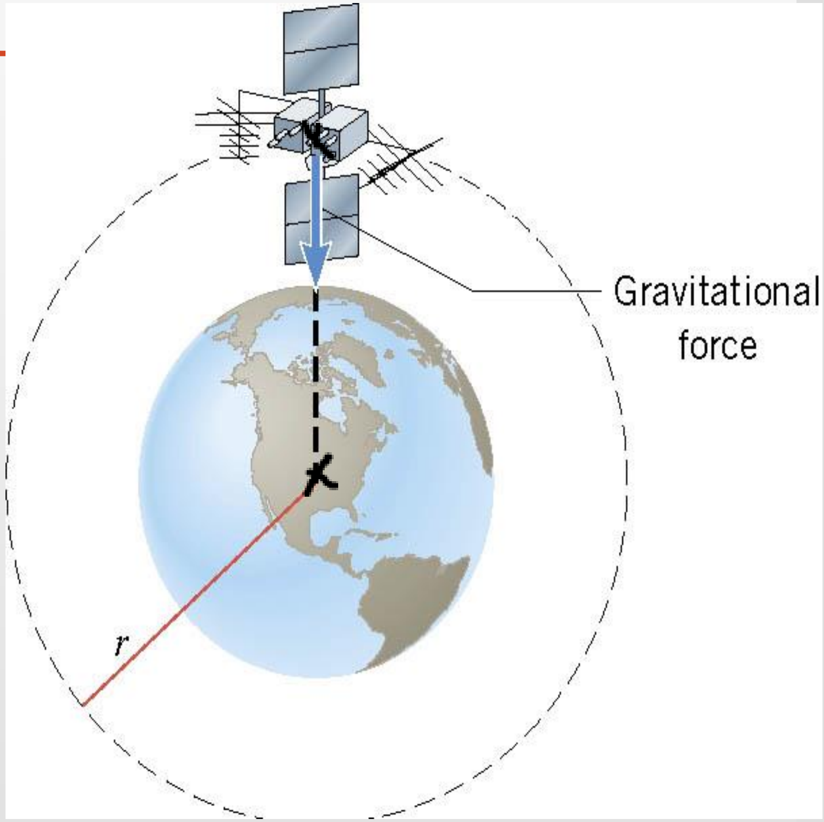
$$\therefore F_{N\text{-}Top} = 0.908\,mg$$



Note: $v_{Bot.} = v_{Top} = v$, *for a uniform circular motion*

Examples on Centripetal Force

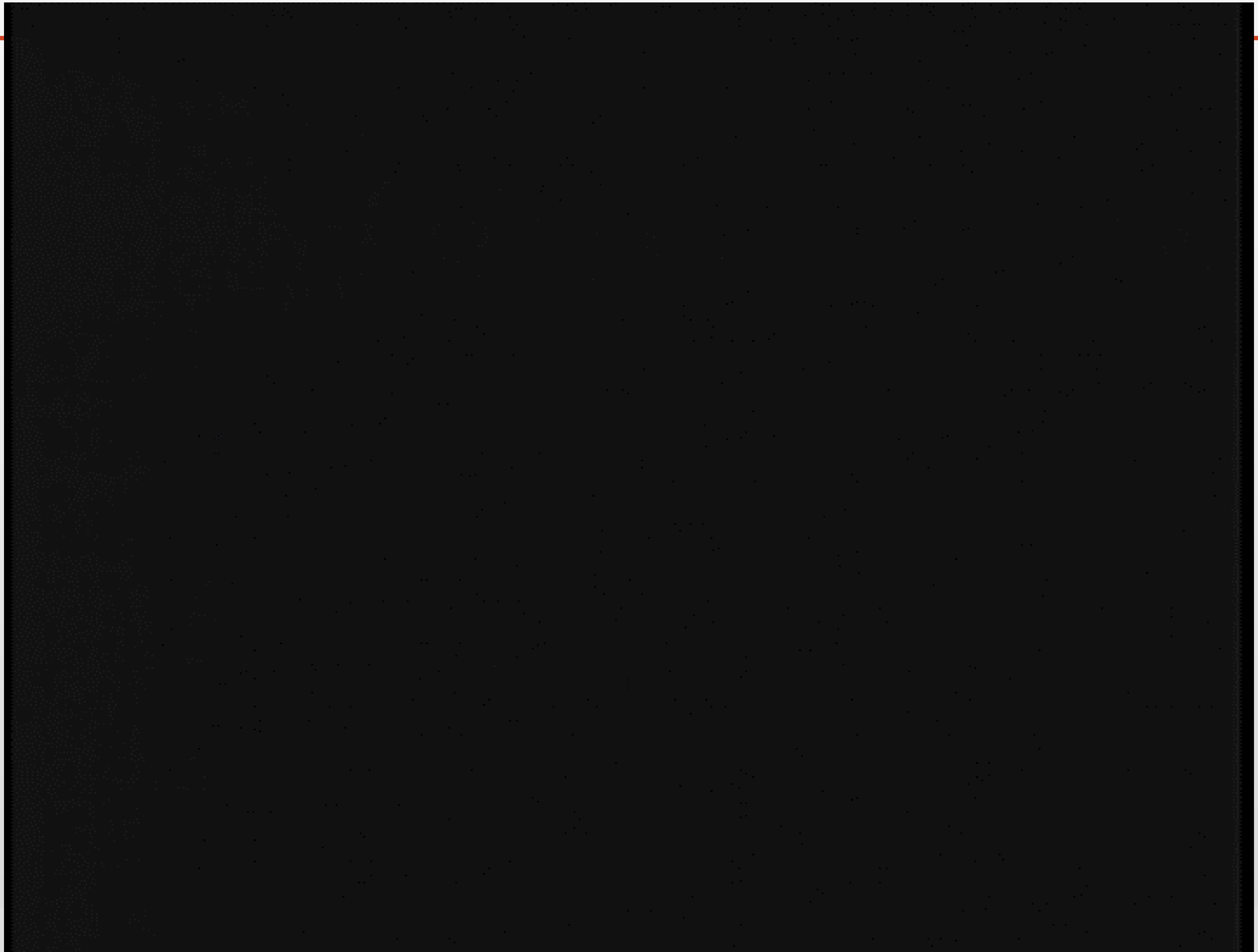
- **Satellite-Earth orbit:**
  - ***Centripetal Force*** ≡ ***Gravity***



Objects in uniform circular motion continually accelerate or "fall" toward the center of the circle, in order to remain on the circular path.

The only difference between the satellite and the elevator is that the satellite moves on a circle, so that its "falling" does not bring it closer to the earth. True weight is the gravitational force ($F=GmM_E/r^2$) that the earth exerts on an object and is not zero.

Circular Motion-Dynamics



Non-Uniform Circular Motion-Dynamics

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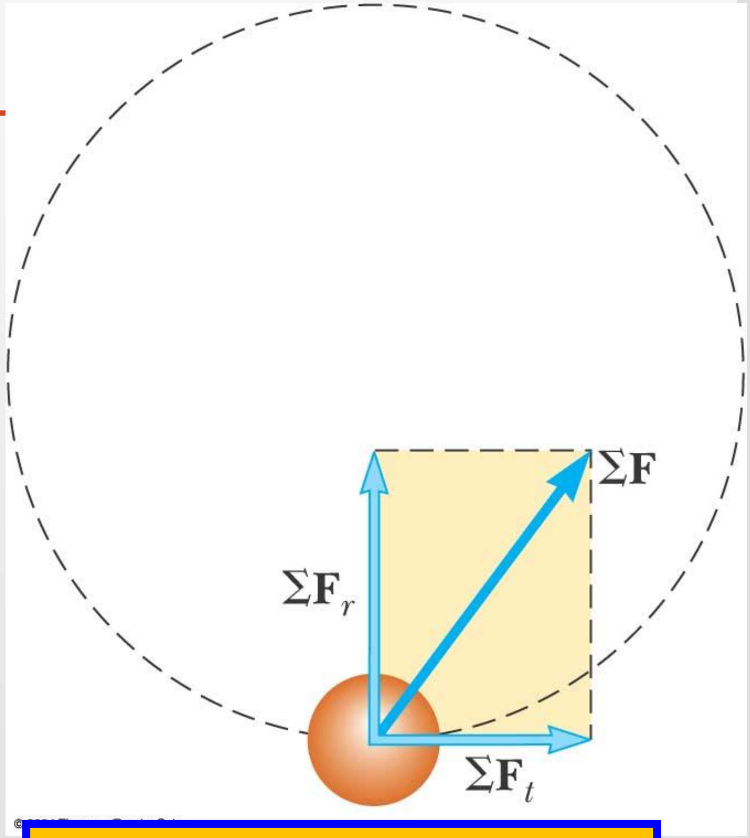
- When the force acting on a particle moving in a circular path has a tangential component $\sum \vec{F}_t$, the particles speed changes.
- The acceleration has a tangential component
- **F_t** produces the tangential acceleration.
- **F_r** produces the centripetal acceleration
- The total force is the vector sum of the radial (or centripetal) force and tangential force:

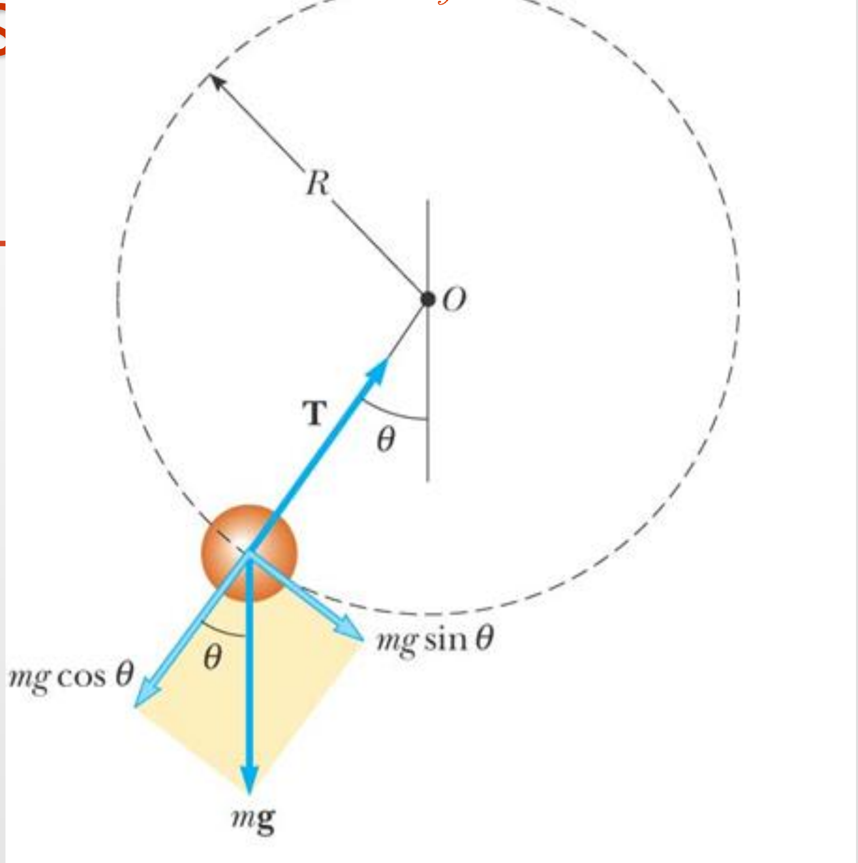


$$\boxed{\sum \vec{F} = \sum \vec{F}_r + \sum \vec{F}_t}$$

Non-Uniform Circular Motion-Dynamics

- The gravitational force exerts a tangential force on the object
  - Look at the components of $\mathbf{F_g}$

- The gravitational force resolve into a tangential component **mg sinθ** and a radial component **mg cosθ**.



$$\boxed{\sum \vec{F} = \sum \vec{F}_r + \sum \vec{F}_t}$$

Non-Uniform Circular Motion-Dynamics

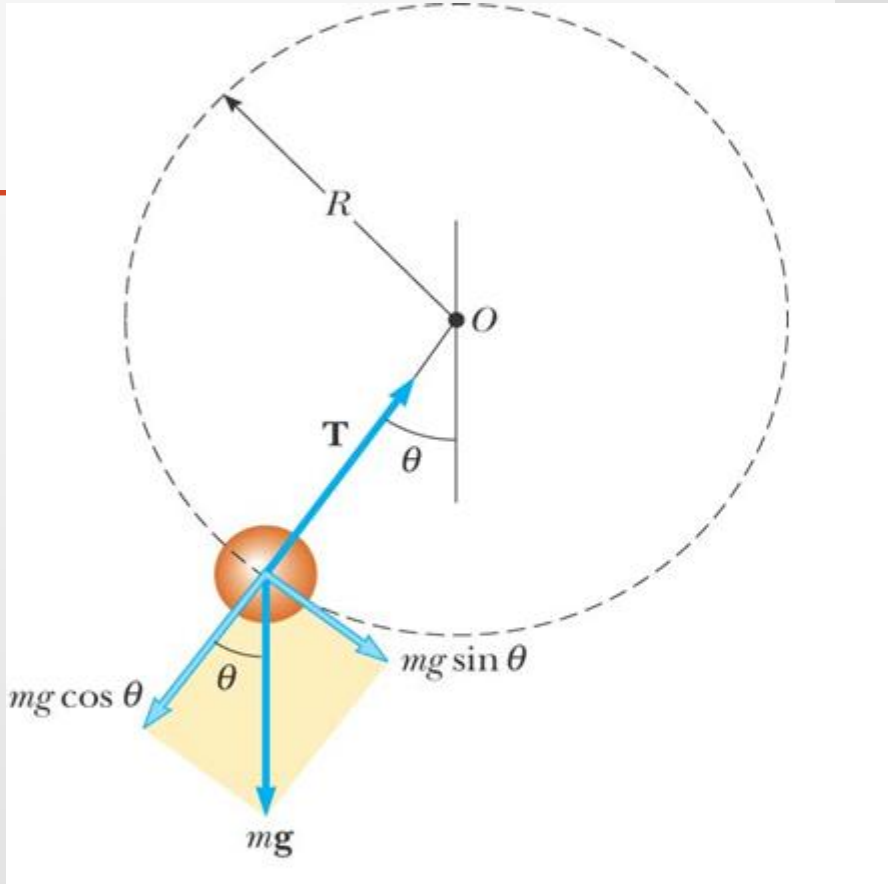
Apply Newton's second law on tangential and radial directions:

$$\sum F_t = mg\sin\theta = ma_t \qquad a_t = g\sin\theta ....(1)$$

$$.\sum F_r = T - mg\cos\theta = \frac{mv^2}{R} ....(2)$$

From equation (2) the tension at any point can be found:

$$\boxed{T = m\left(\frac{v^2}{R} + g\cos\theta\right)}$$

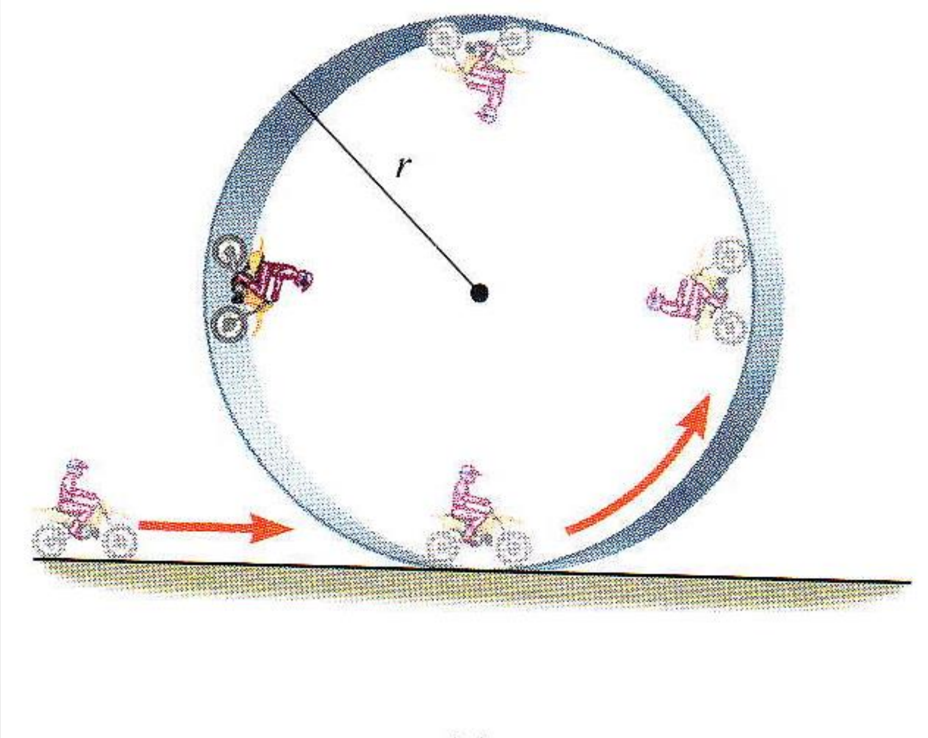


mg is resolved into two components: tangential and radial directions

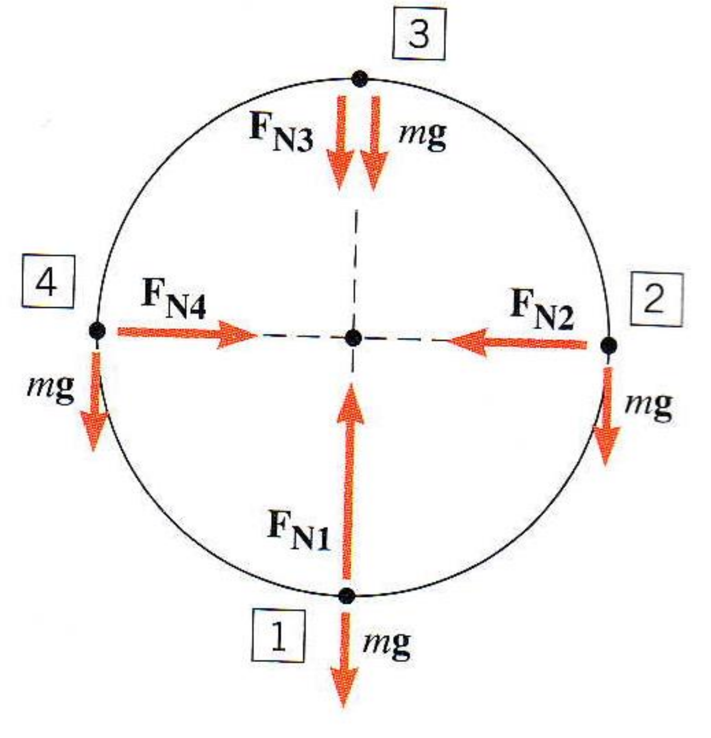
Non-Uniform Circular Motion-Dynamics

Examples on Centripetal and Tangential Forces

- **Vertical Track (highest and lowest points):**
  - ***Centripetal Force ≡ Normal – Gravity***



(a)

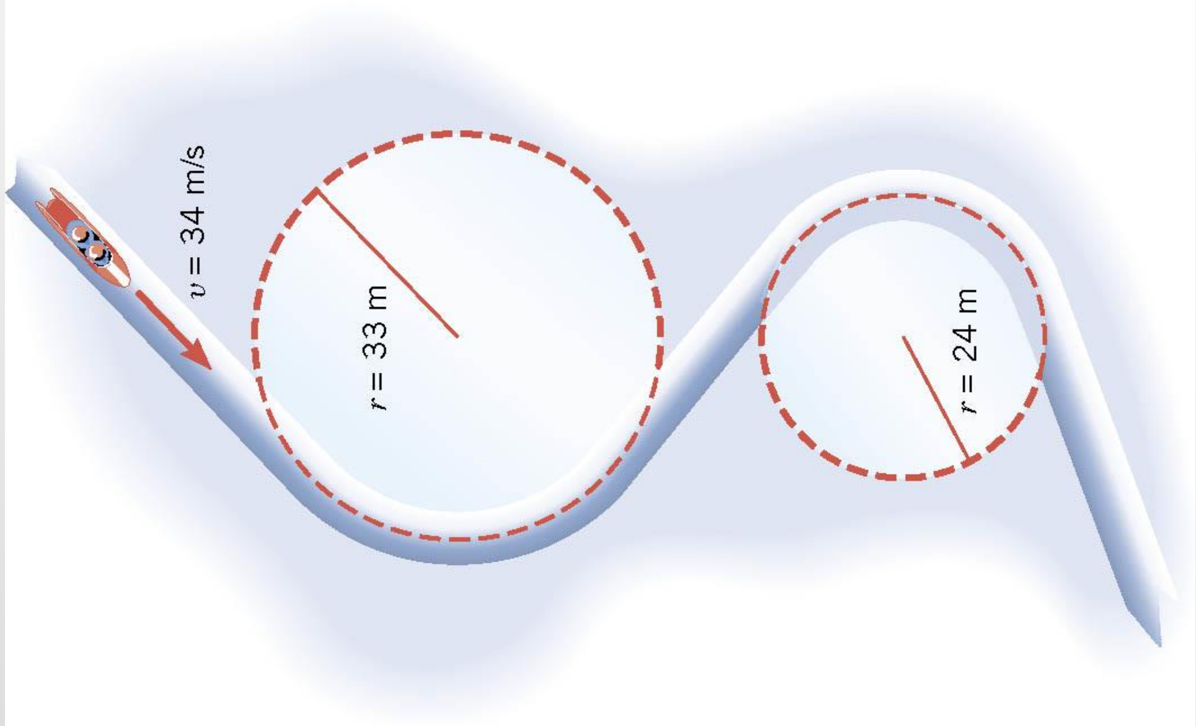


(b)

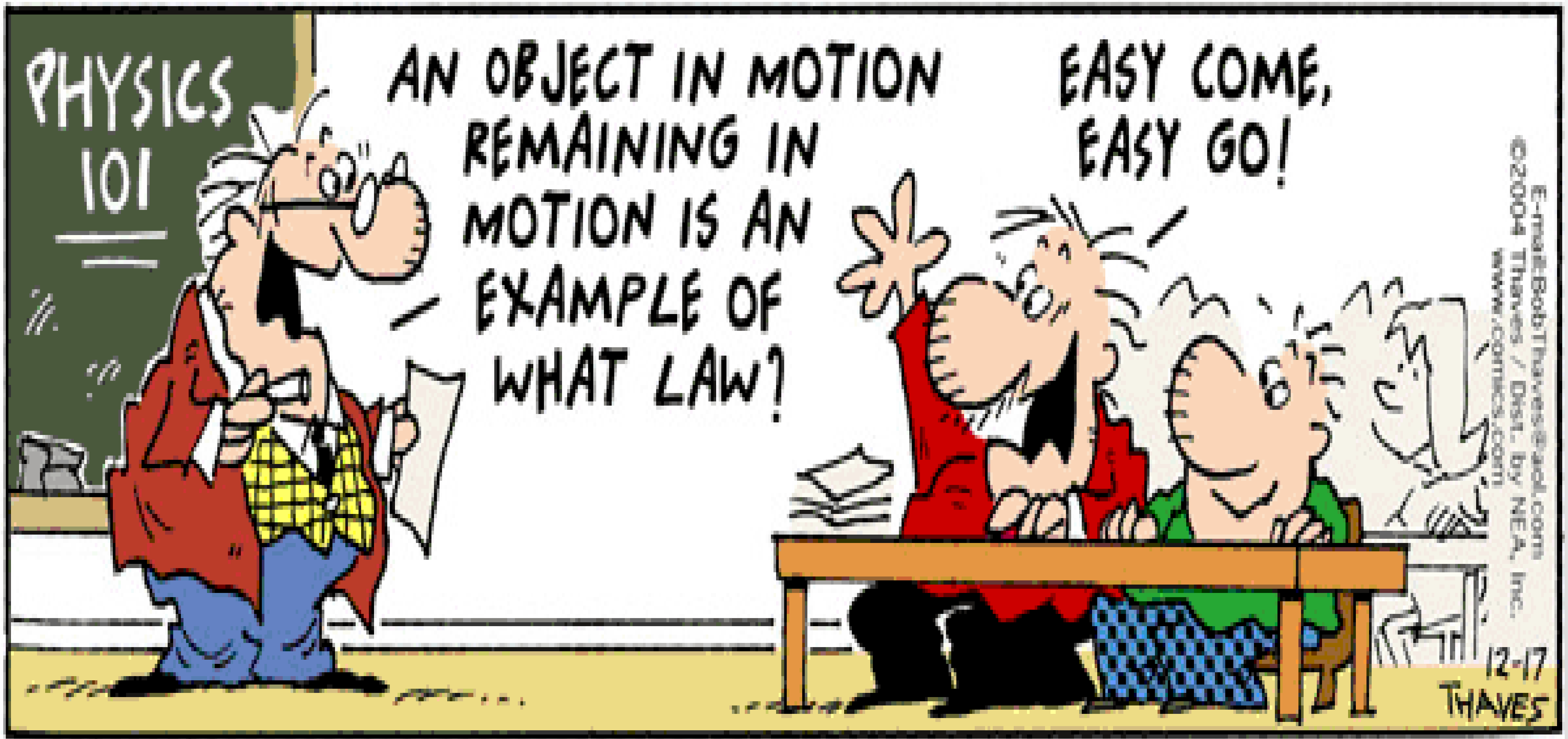
Examples on Centripetal and Tangential Forces

❖ **Vertical Track (lowest and highest points):**

- ***Centripetal Force*** ≡ ***Normal – Gravity***



What Do You Learn?



By Prof. Rashad Badran

Non-Uniform Circular Motion

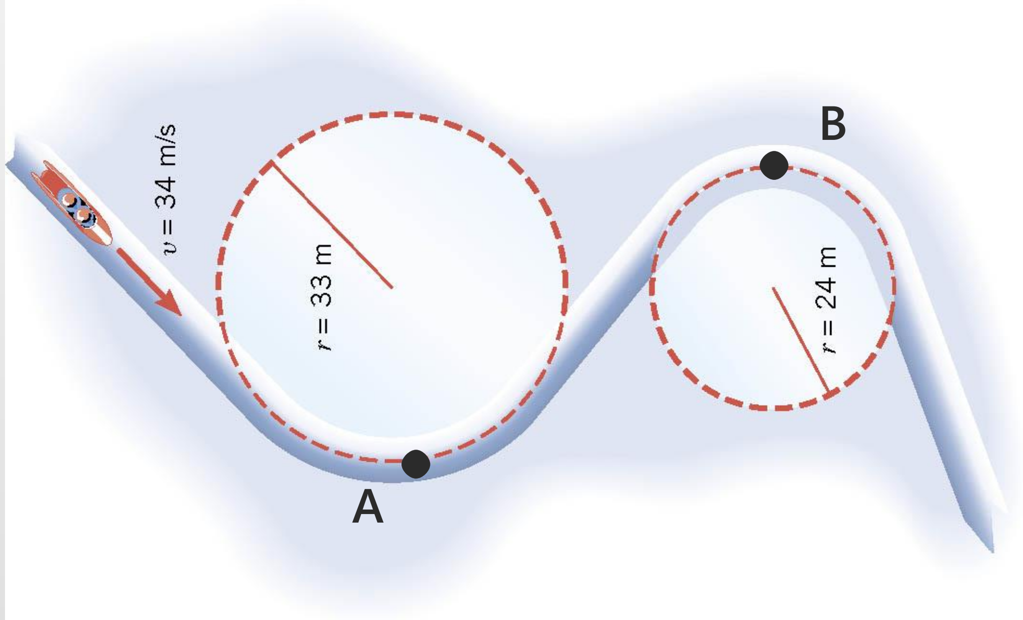
Problem:

A roller-coaster car has a mass of 500 kg when fully loaded with passengers. The path of the coaster from its initial point shown in the figure, to point B involves only up-and-down motion, with no motion to the left or right.

(a) If the car has a speed of 34 m/s at point A, what is the force exerted by the track on the car at this point?

(b) What is the maximum speed the car can have at point B and still remain on the track?

Note: Assume the roller-coaster tracks at points A and B are parts of vertical circles of radius $r_A = 33\ m$ and $r_B = 24\ m$



Non-Uniform Circular Motion

Solution

(a) To find F_N at point A, If the car has a speed of $v_A = 34\ m/s$ at point A, and $r_A = 33\ m$. Given : $m = 500\ kg$

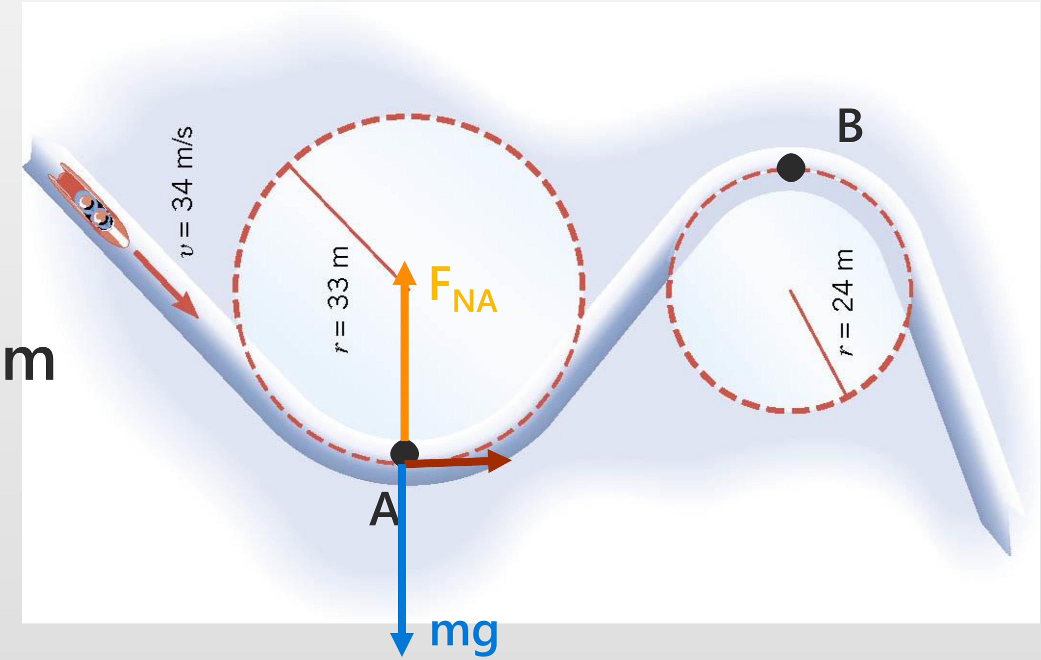
$\Sigma F_r = m\, a_r$

$-F_{NA} + mg = -m v_A^2 / r_A$

Or $F_{NA} = mg + m v_A^2 / r_A$

$F_{NA} = (500\ kg)(9.8 m/s^2) + (500kg)\ (34m/s)^2/33m$

$F_{NA} = 22415\ N$



Non-Uniform Circular Motion

Solution

(b) To find maximum seed at point B and car still remain on track (i.e. $v^{max}{}_B$ =? at point B), and $r_B = 24\ m$. Given : $m = 500\ kg$

$$\Sigma F_r = m\,a_r$$

$$F_{NB} - mg = -mv^2{}_B/r_B$$

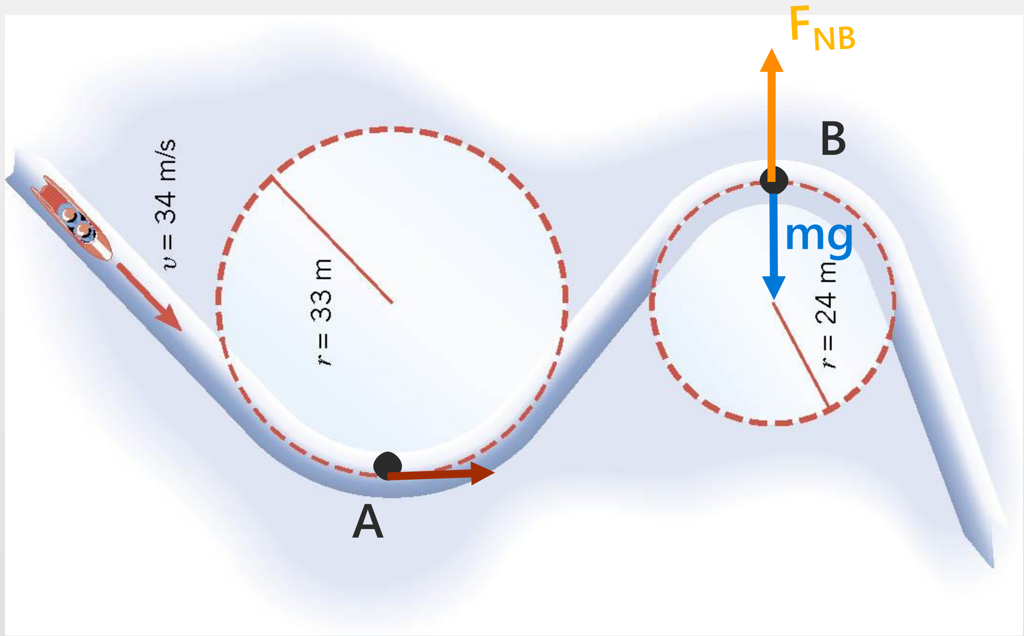
$$mg - F_{NB} = mv^2{}_B/r_B$$

The maximum speed of car at B, while car still remain on track, occurs when $F_{NB} = 0$

Thus: $$mg = mv^2{}_B/r_B$$

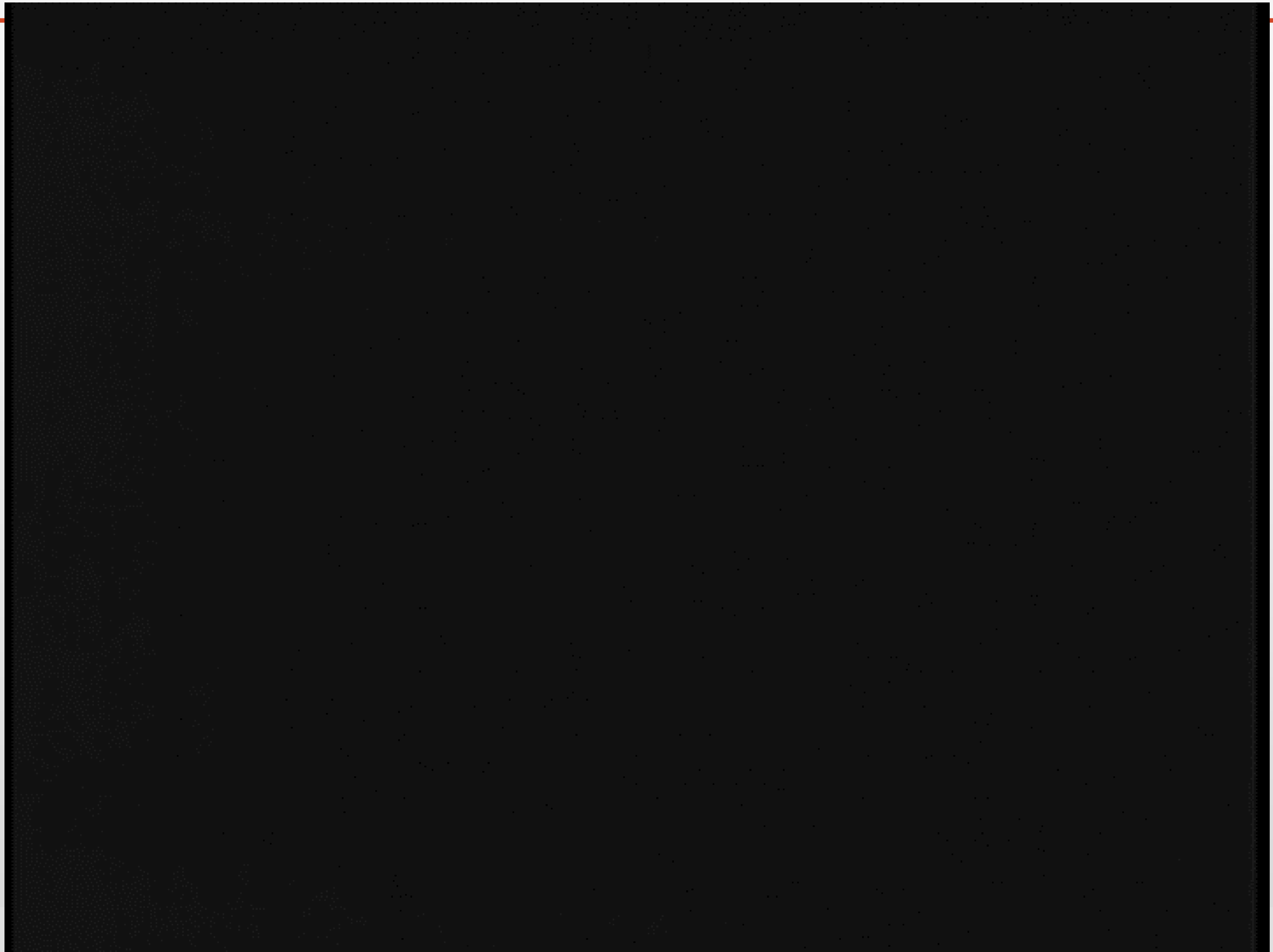
At point B: $$v_{\max} = \sqrt{g r_B}$$

$$v_{max} = 15.3\ m/s$$



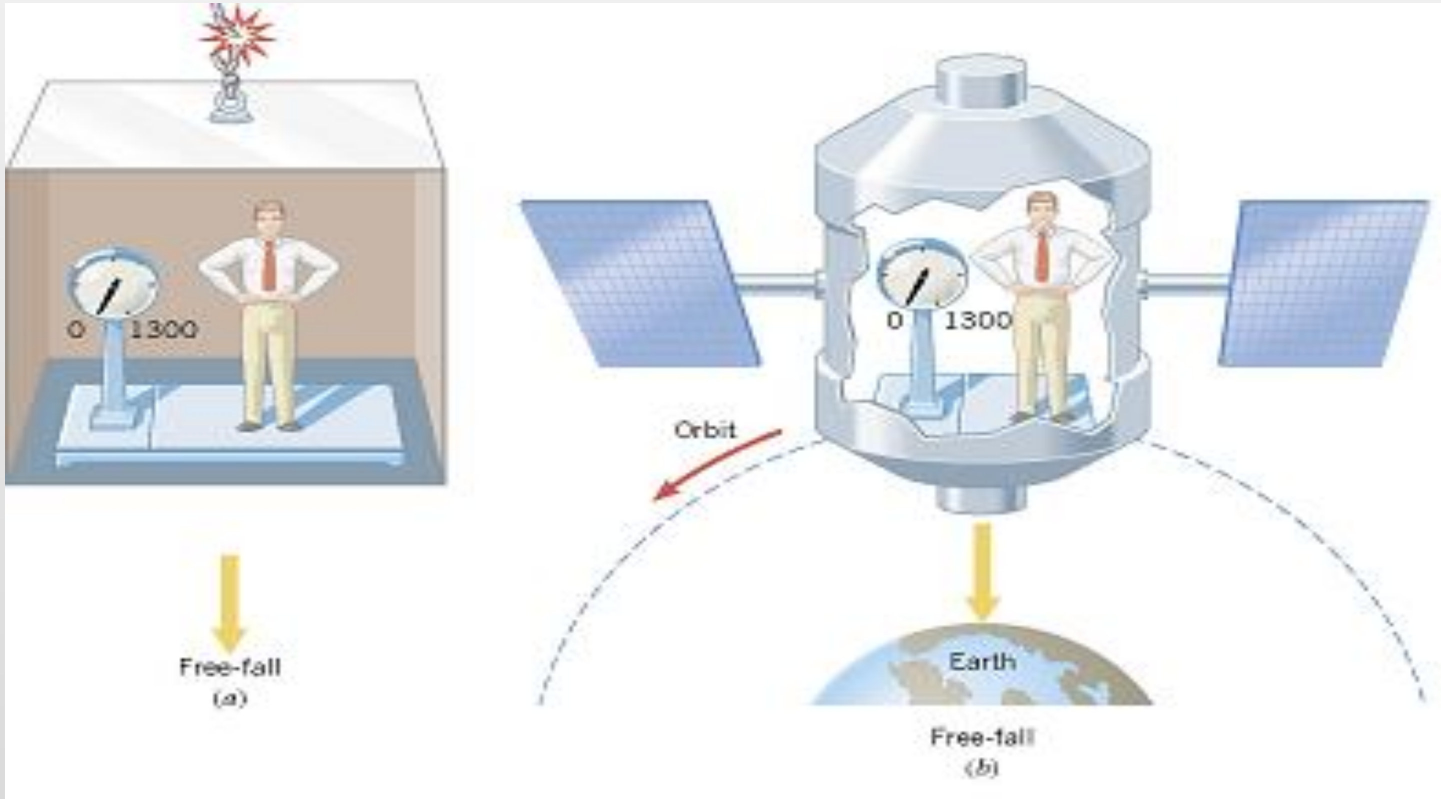
By Prof. Rashad Badran

Circular Motion-Dynamics



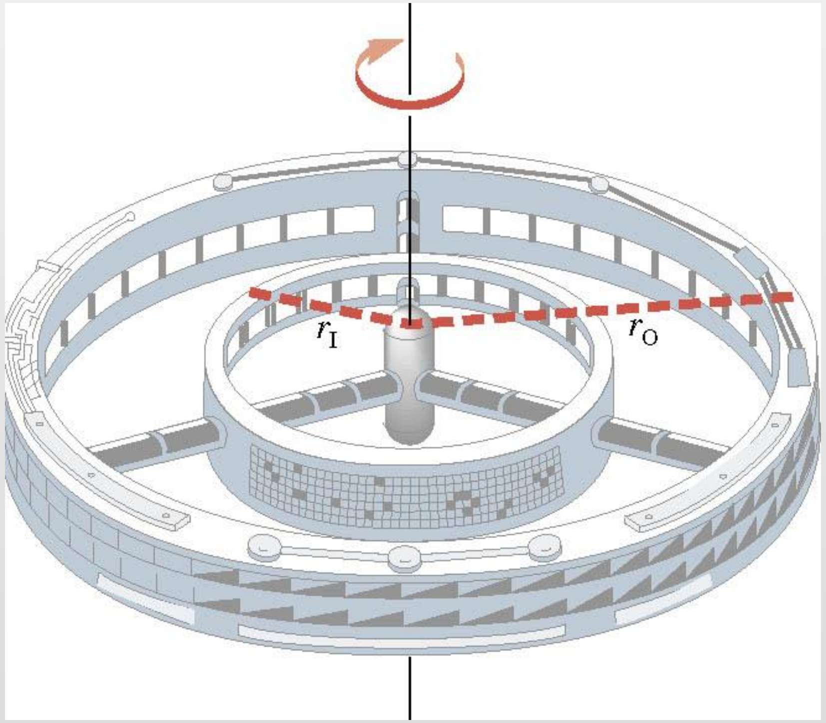
Apparent Weight: Linear and circular Motion

The idea of life on board an orbiting satellite conjures up visions of astronauts floating around in a state of "weightlessness"



Circular Motion Example: A Rotating Space Laboratory

A space laboratory is rotating to create artificial gravity. Its period of ~~rotation is chosen so the outer ring (r_o=2150m) simulates the acceleration~~ due to gravity on earth (9.80 m/s^2). What should be the radius r_1 of the inner ring, so it simulates the acceleration due to gravity on the surface of Mars (3.72 m/s^2)?



The outer ring ($radius$=r_o) simulates gravity on earth, while the inner ring ($radius$=r_1) simulates gravity on Mars

Circular Motion Example: A Rotating Space Laboratory

Centripetal acceleration: $a_r = v^2/r$,

speed v and radius r:

T is the period of the motion.

The laboratory is rigid. All points on a rigid object make one revolution in the same time.

Both rings have the same period.

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

By Prof. Rashad Badran

Circular Motion Example: A Rotating Space Laboratory

$$\underbrace{9.80m/s^2 = \frac{4\pi^2(2150m)}{T^2}}_{\text{Outer ring}} \qquad \underbrace{3.72m/s^2 = \frac{4\pi^2 r_1}{T^2}}_{\text{Inner ring}}$$

Dividing the inner ring expression by the outer ring expression,

$$\frac{3.72m/s^2}{9.80m/s^2} = \frac{r_1}{2150m}$$

$$r_1 = 816\ m$$