General

Physics 101

Motion in Two Dimensions

Prepared By

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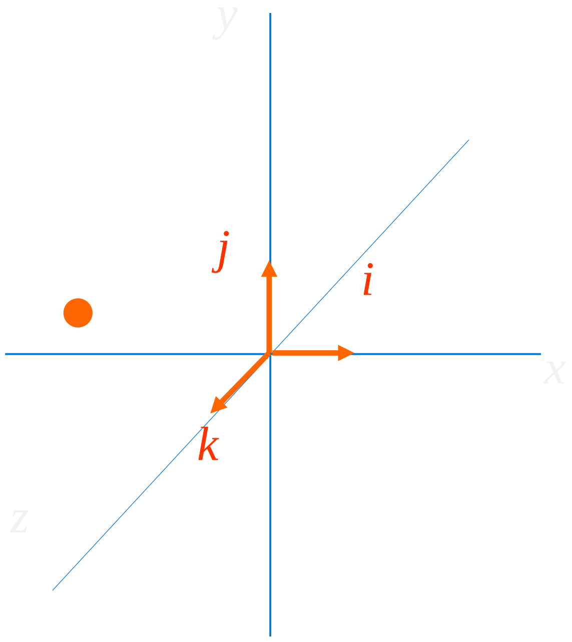
Motion in Two Dimensions- Kinematics

- ❑ Displacement and position in 2-D
- ❑ Average and instantaneous velocity in 2-D
- ❑ Average and instantaneous acceleration in 2-D
- ❑ Two dimensional motion with a constant acceleration
- ❑ Projectile motion
- ❑ Circular motion
- ❑ Relative velocity*

Motion in Two Dimensions - Kinematics

- Kinematic variables in one dimension
  - Position: $x(t)$ m
  - Velocity: $v(t)$ m/s
  - Acceleration: $a(t)$ m/s²

- Kinematic variables in two dimensions
  - Position: $\vec{r}(t) = x\hat{i} + y\hat{j}$ m
  - Velocity: $\vec{v}(t) = v_x\hat{i} + v_y\hat{j}$ m/s
  - Acceleration: $\vec{a}(t) = a_x\hat{i} + a_y\hat{j}$ m/s²
- Kinematic variables in three dimensions
  - Position: $\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$ m
  - Velocity: $\vec{v}(t) = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$ m/s
  - Acceleration: $\vec{a}(t) = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ m/s²
- All are vectors: have directions and magnitudes

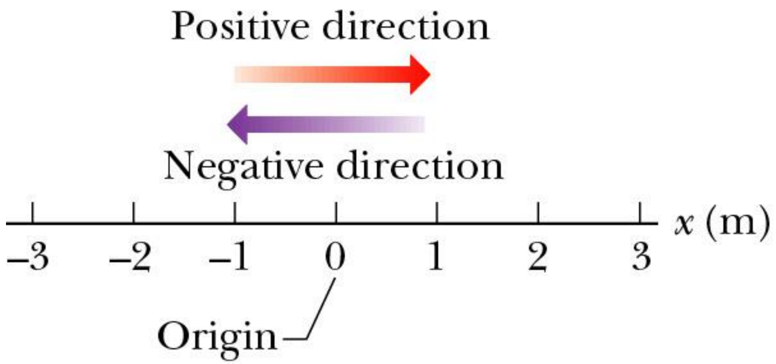


Position and Displacement

- In one dimension

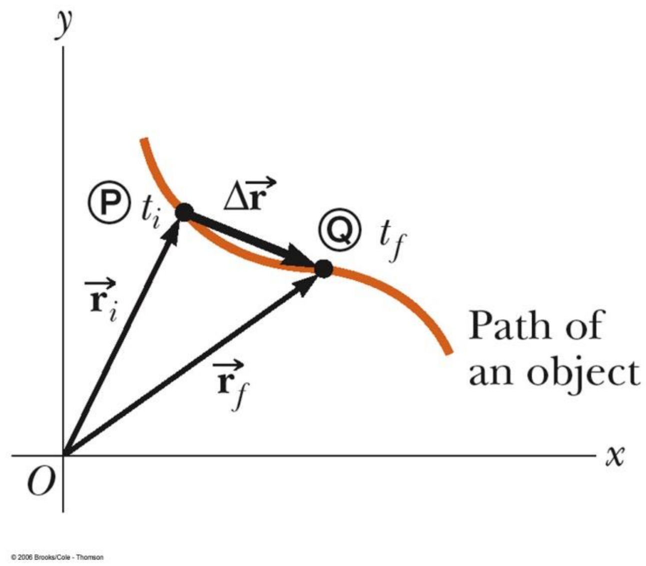
$$\Delta x = x_2(t_2) - x_1(t_1)$$

$x_1\,(t_1)$ = - 3.0 m, $x_2\,(t_2)$ = + 1.0 m
Δx = +1.0 m + 3.0 m = +4.0 m



- In two dimensions
  - Position: the position of an object is described by its position vector $\vec{r}(t)$ --always points to particle from origin.
  - Displacement: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$$\Delta\vec{r} = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$
$$= \Delta x\hat{i} + \Delta y\hat{j}$$



Average & Instantaneous Velocity

- Average velocity

$$\vec{v}_{avg} \equiv \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$$

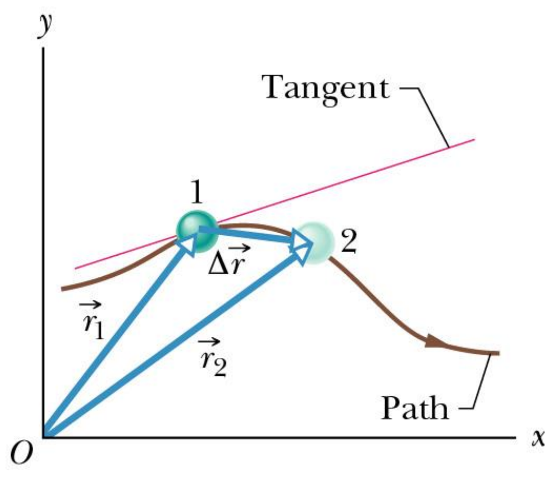
- Instantaneous velocity

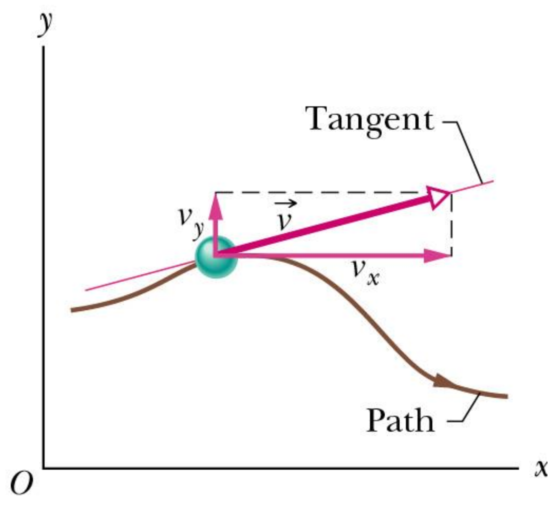
$$\vec{v} \equiv \lim_{t\to 0}\vec{v}_{avg} = \lim_{t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

where $\vec{r}(t) = x\hat{i} + y\hat{j}$

- v is tangent to the path in x-y graph;





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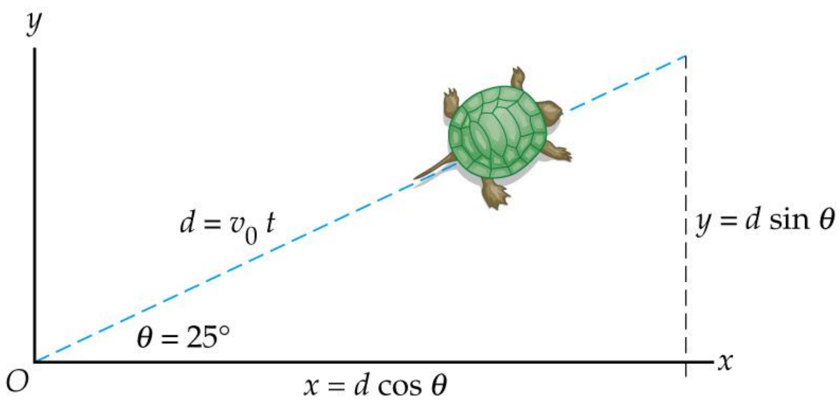
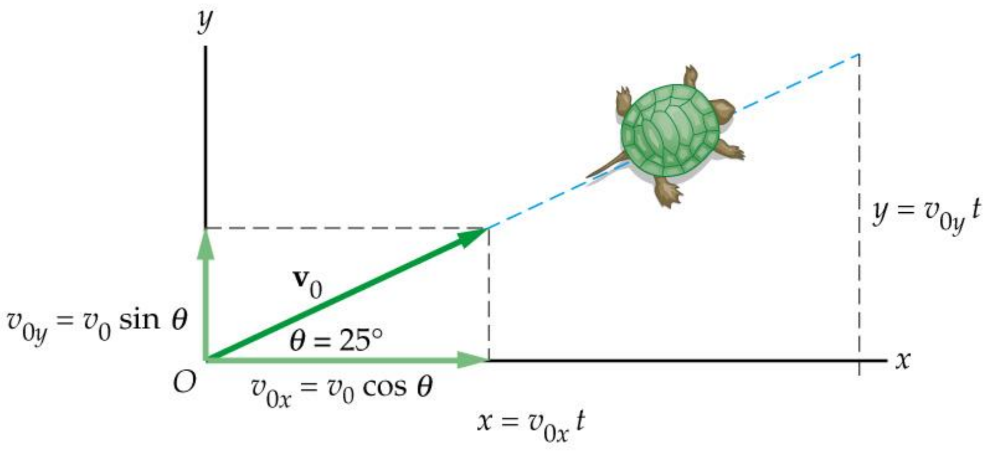
Motion of a Turtle



(a)



(b)

A turtle starts at the origin and moves with the speed of v_0=10 cm/s in the direction of 25° to the horizontal.

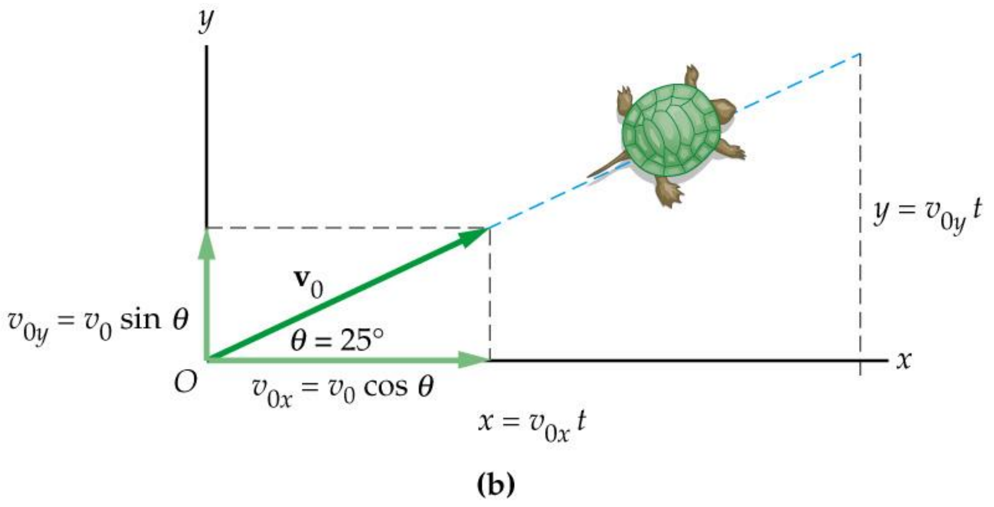
(a) Find the coordinates of a turtle 10 seconds later.

(b) How far did the turtle walk in 10 seconds?

Motion of a Turtle

Notice, you can solve the equations independently for the horizontal (x) and vertical (y) components of motion and then combine them!

$$\vec{v}_0 = \vec{v}_{ox} + \vec{v}_{oy}$$



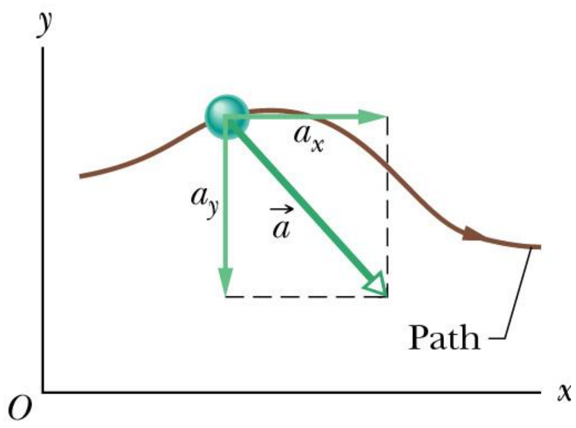
- x components: $v_{0x} = v_0 \cos 25^\circ = 9.06$ cm/s ➡ $\Delta x = v_{0x} t = 90.6$ cm
- y components: $v_{0y} = v_0 \sin 25^\circ = 4.23$ cm/s ➡ $\Delta y = v_{0y} t = 42.3$ cm
- Distance from the origin: $d = \sqrt{\Delta x^2 + \Delta y^2} = 100.0$ cm

Average & Instantaneous Acceleration

- Average acceleration $$\vec{a}_{avg} \equiv \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} = a_{avg,x}\hat{i} + a_{avg,y}\hat{j}$$

where $$\vec{v}(t) = v_x\hat{i} + v_y\hat{j}$$



- Instantaneous acceleration

$$\vec{a} \equiv \lim_{t\to 0}\vec{a}_{avg} = \lim_{t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

- The magnitude of the velocity (the speed) can change
- The direction of the velocity can change, even though the magnitude is constant
- Both the magnitude and the direction can change

Summary for Motion in Two Dimensions

- Position

$$\vec{r}(t) = x\hat{i} + y\hat{j}$$

- Average velocity

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$$

- Instantaneous velocity

$$v_x \equiv \frac{dx}{dt} \qquad v_y \equiv \frac{dy}{dt}$$

$$\vec{v}(t) = \lim_{t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

- Acceleration

$$a_x \equiv \frac{dv_x}{dt} = \frac{d^2x}{dt^2} \qquad a_y \equiv \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$$

$$\vec{a}(t) = \lim_{t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

- $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ are not necessarily same direction.

Two dimensional motion

Example:

A particle is moving in x-y plane where its x- motion is $x = (18\ t)$ m and its y- motions is $y = (4\ t - 4.9\ t^2)$ m. (a) Write a vector position expression as a function of time using unit vector notation. (b) Find the average velocity in the time interval t = 0 to t= 2 s. (c) Write a vector velocity expression as a function of time. (d) Find the average acceleration from t=1 s to t = 3s. (e) What is the acceleration of the particle.

Two dimensional motion

Solution:

(**a**) Substitute **x =(18 t) m** and **y= (4 t - 4.9 t²)m** into the position vector expression $\vec{r}(t) = x\hat{i} + y\hat{j}$ to get:

$$\vec{r} = (18t)\hat{i} + (4t - 4.9t^2)\hat{j}$$

(**b**) $$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$$

At t_i = 0 the initial position is : $\vec{r}_i = 0$

At t_f = 2 s, the final position is $\vec{r}_f = [(18)(2)]\hat{i} + [4(2) - 4.9(2)^2]\hat{j}$

or, $\vec{r}_f = 36\hat{i} - 11.6\hat{j}$

Thus Δx = 36 m and Δy = -11.6 m

Thus, the average velocity vector is

$$\vec{v}_{avg} = \frac{36m}{2s}\hat{i} - \frac{11.6m}{2s}\hat{j} = (18\hat{i} - 5.8\hat{j})m/s$$

Two dimensional motion

Solution:

(c) Use the definition of the velocity vector :

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j}$$

Where, $v_x \equiv \frac{dx}{dt}$ $v_y \equiv \frac{dy}{dt}$ and, $\vec{r} = (18t)\hat{i} + (4t - 4.9t^2)\hat{j}$

The derivative of the x- and y components give: $v_x = 18$ and $v_y = 4 - 9.8\,t$

Thus, the velocity vector is $\vec{v}(t) = 18\hat{i} + (4 - 9.8t)\hat{j}$

(d) Use the definition of the average acceleration: $\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} = a_{avg,x}\hat{i} + a_{avg,y}\hat{j}$

At $t_i = 1$ s the initial velocity is : $\vec{v}_i = 18\hat{i} - 5.8\hat{j}$

At $t_f = 3$ s the final velocity is : $\vec{v}_f = 18\hat{i} - 25.4\hat{j}$

Thus $\Delta v_x = 0$ and $\Delta v_y = -19.6$ m/s

$$\vec{a}_{avg} = 0\hat{i} + \frac{(-19.6m/s)}{2}\hat{j} \longrightarrow \vec{a}_{avg} = (-9.8\hat{j})m/s^2$$

Two dimensional motion

Solution:

(e) Use the definition of the acceleration vector **:** $\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$

Where, $a_x \equiv \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$, $a_y \equiv \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$ and, $\vec{v}(t) = 18\hat{i} + (4 - 9.8t)\hat{j}$

The derivative of the x- and y components give: $a_x = 0$ and $a_y = -9.8\ m/s^2$

Thus, the acceleration vector is $\vec{a} = (-9.8\hat{j})m/s^2$

Note: The average acceleration is equal to the instantenous acceleration

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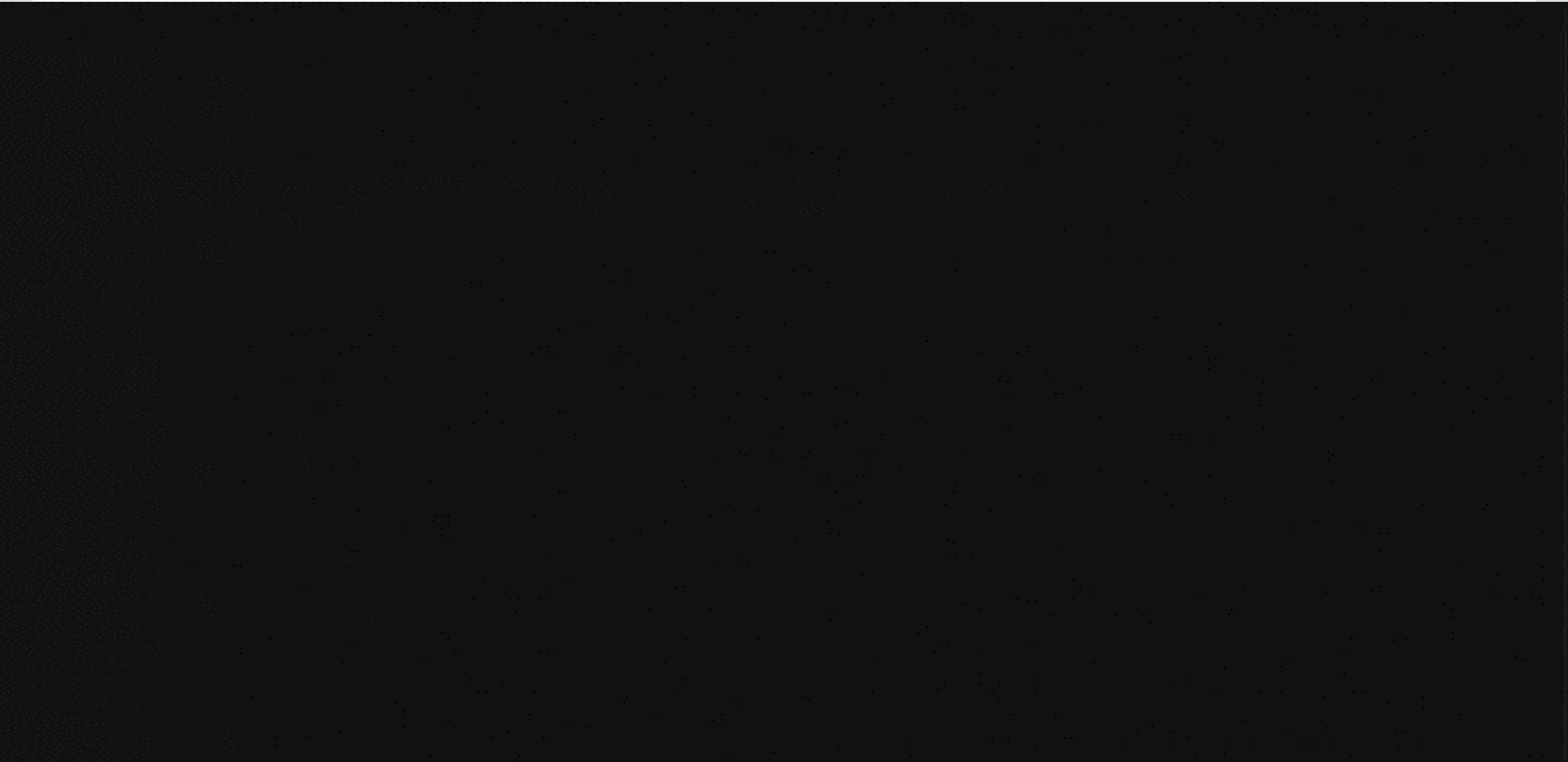
How do two different one dimensional motions form a 2-D motion?



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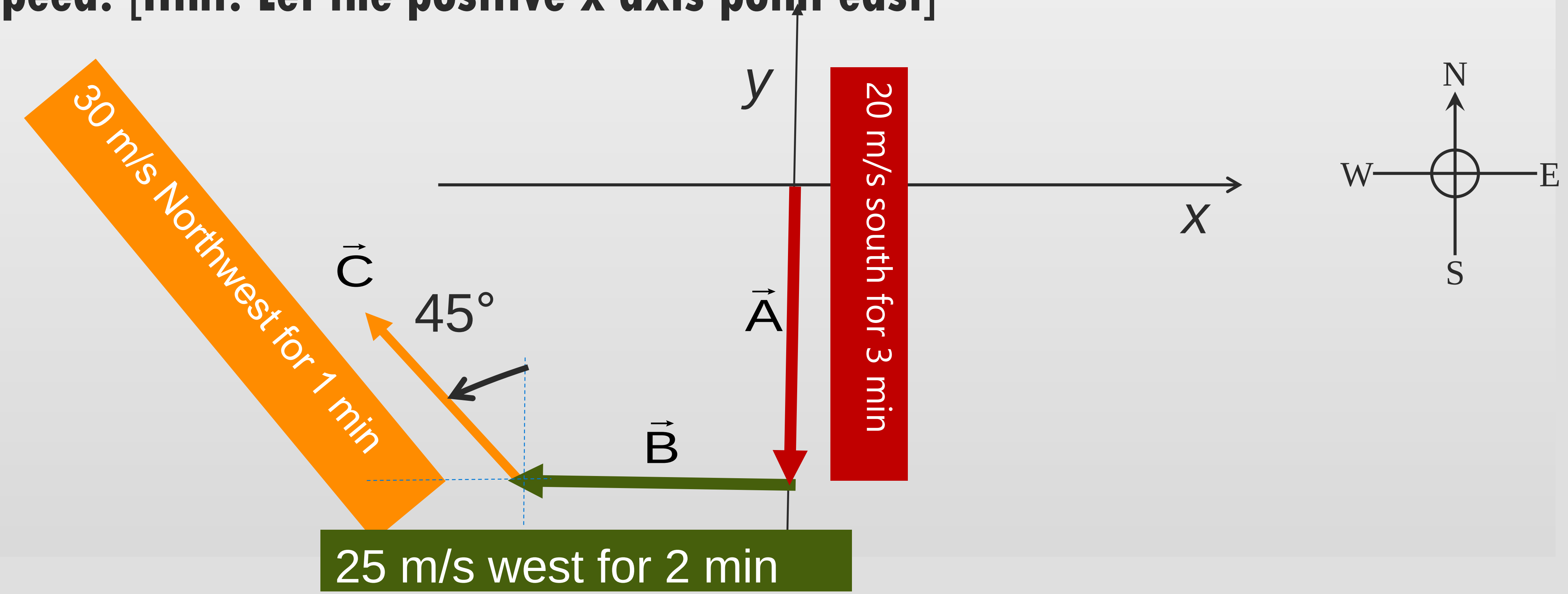
Two dimensional motion

Example:

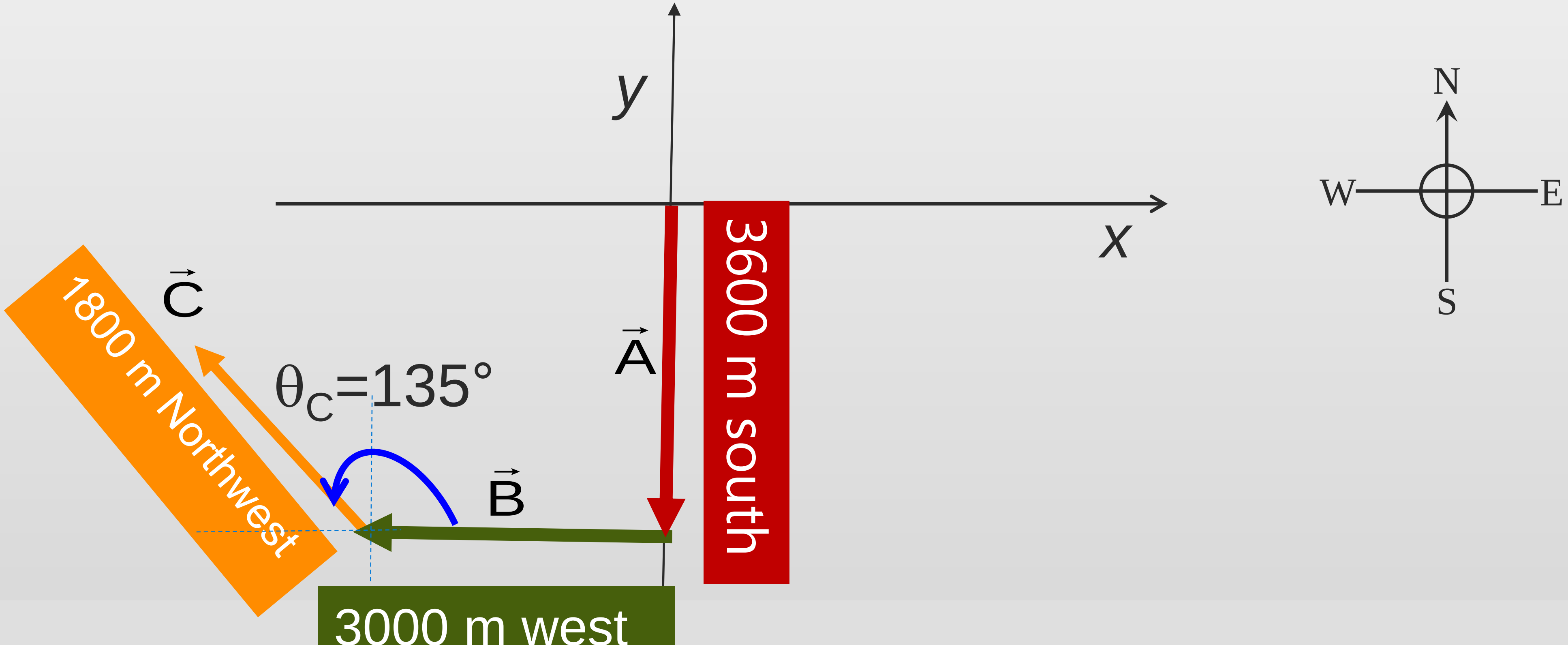
A person drives his car **south** at **20 m/s** for **3 min**, then turns **west** and travels at **25 m/s** for **2 min**, and finally travels **northwest** at **30 m/s** for **one min**. For this **6-min trip**, find (a) the total vector displacement, (b) the average velocity, and (c) the average speed. [Hint: Let the positive x axis point east]



Two dimensional motion

Solution:

Magnitude of first displacement A= (20 m/s)×(3min) × 60 s/min = 3600 m
Magnitude of second displacement B= (25 m/s) × (2 min) × 60s/min = 3000 m
Magnitude of third displacement C= (30 m/s) × (1min) × 60 s/min = 1800 m.

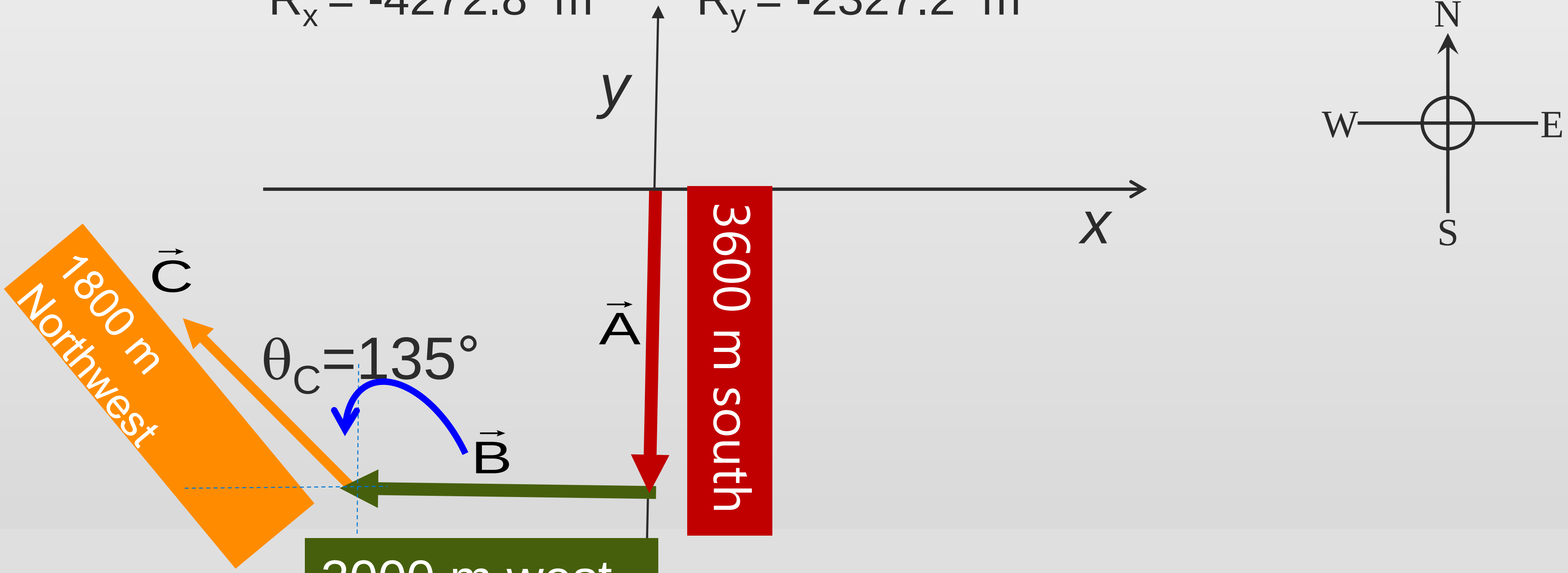


(a)

Two dimensional motion

Solution:

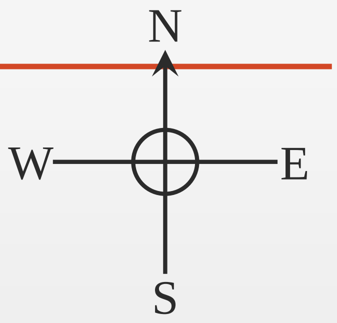
Distance	Angle	x-component	y-component
A = 3600 m	270°	00.00 m	-3600 m
B = 3000 m	180°	-3000 m	00.00 m
C = 1800 m	135°	-1272.8 m	1272.8 m
		R_x = -4272.8 m	R_y = -2327.2 m



Two dimensional motion

(a)

Solution:



The total displacement is:

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

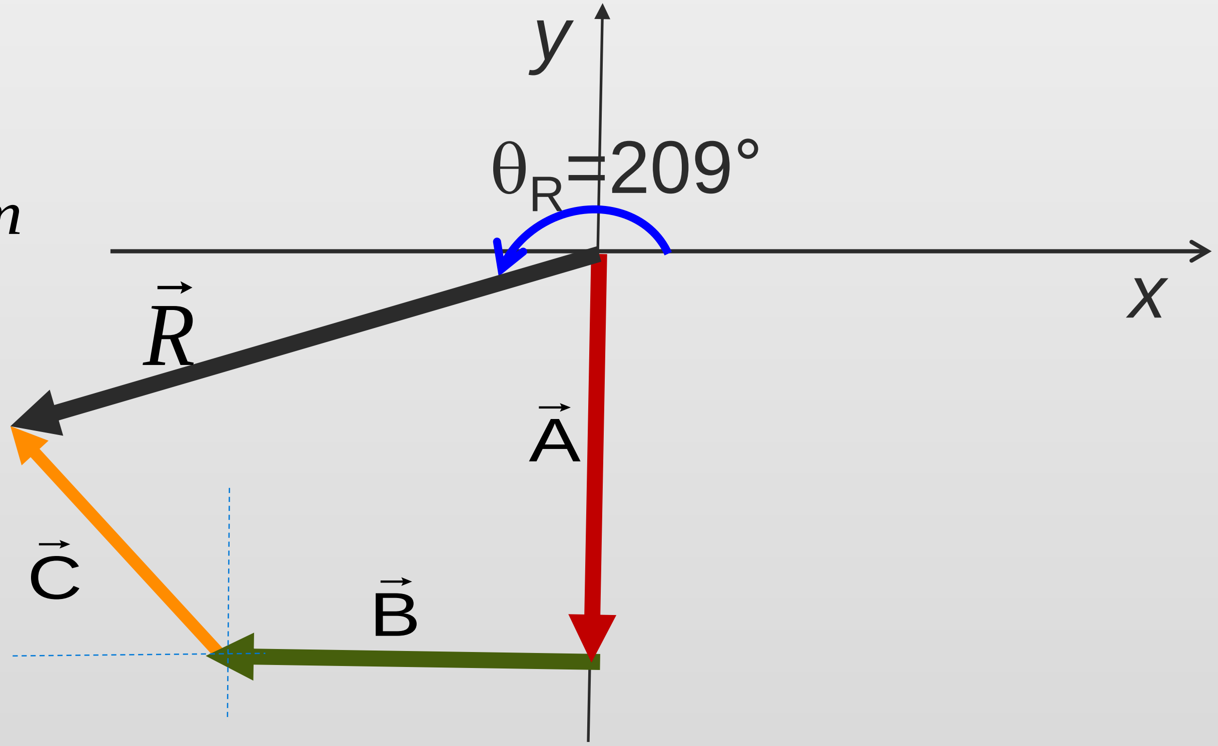
$$\vec{R} = \text{-}4272..8\hat{i} \text{ - } 2327.2\,\hat{j}$$

$$\left|\vec{R}\right| = \sqrt{R_x^2 + R_y^2}\ ,$$

$$\left|\vec{R}\right| = \sqrt{(\text{-}4272.8)^2 + (\text{-}2327.2)^2} = 4870\,m$$

$$\theta_R = \arctan\frac{R_y}{R_x}$$

$$\theta_R = \arctan\frac{-2327.2}{-4272.8} = 209^o$$



Two dimensional motion

Solution:

(b)

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{R}}{6\min}$$

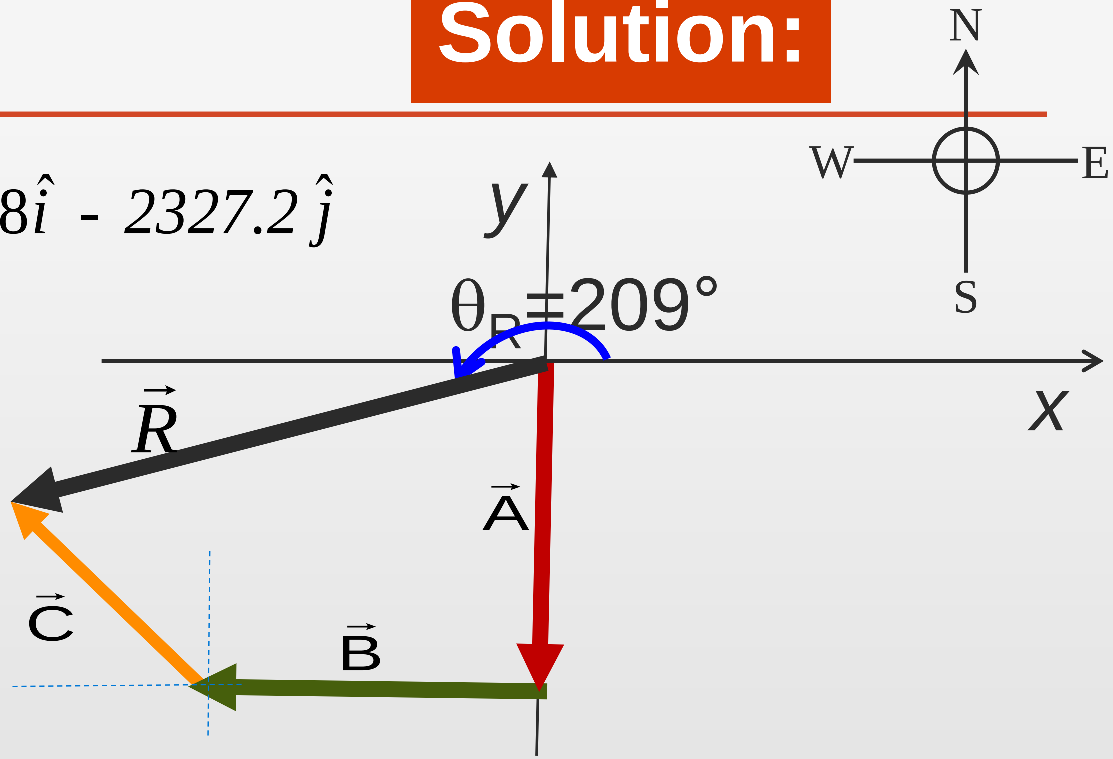
where $\vec{R} = -4272..8\hat{i} - 2327.2\hat{j}$

$$\vec{v}_{avg} = \frac{-4272.8\hat{i} - 2327.2\hat{j}}{(6\min)(60s/\min)}$$

$$\vec{v}_{avg} = (-11.8\hat{i} - 6.5\hat{j})m/s$$

$$v_{avg} = \sqrt{(-11.8)^2 + (-6.5)^2} = 13.5m/s$$

$$\theta_R = \arctan\frac{v_{avg_y}}{v_{avg_x}} \qquad \theta_R = \arctan\frac{-11.8}{-6.5} = 209^o$$



Or the magnitude of average velocity can be directly obtained as follows

$$v_{avg} = \frac{|\vec{R}|}{360s} = \frac{4870m}{360s} = 13.5m/s$$

Motion in Two Dimensions with a Constant Acceleration

- Motions in both dimensions are independent components
- Constant acceleration equations

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{r} - \vec{r}_o = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

$$\vec{v} = (v_{0x} + a_x t)\hat{i} + (v_{0y} + a_y t)\hat{j}$$

$$\vec{r} - \vec{r}_o = (v_{0x}t + \tfrac{1}{2}a_x t^2)\hat{i} + (v_{0y}t + \tfrac{1}{2}a_y t^2)\hat{j}$$

Here, $\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$ $\quad \vec{a} = a_x\hat{i} + a_y\hat{j}$ $\quad \vec{r} = x\hat{i} + y\hat{j}$ $\quad \vec{r}_0 = x_0\hat{i} + y_0\hat{j}$

- Constant acceleration equations hold in each dimension (i.e. a_x and a_y are constants

$$x - Motion$$

$$v_x = v_{0x} + a_x t$$

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$v_x^{\,2} = v_{0x}^{\,2} + 2a_x(x - x_0)$$

$$y - Motion$$

$$v_y = v_{0y} + a_y t$$

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$$

$$v_y^{\,2} = v_{0y}^{\,2} + 2a_y(y - y_0)$$

Hints for solving problems

- Define coordinate system. Make sketch showing axes, origin.
- List known quantities. Find v_{xi}, v_{yi}, a_x, a_y, etc. Show initial conditions on sketch.
- List equations of motion to see which ones to use.
- Time t is the same for x and y directions.

  $x_i = x_o = x(t = 0)$, $y_i = y_o = y(t = 0)$, $v_{xi} = v_{ox} = v_x(t = 0)$, $v_{yi} = v_{oy} = v_y(t = 0)$.
- Have an axis point along the direction of a if it is constant.

$$v_x = v_{0x} + a_x t$$

$$v_y = v_{0y} + a_y t$$

$$x - x_0 = v_{0x} t + \tfrac{1}{2} a_x t^2$$

$$y - y_0 = v_{0y} t + \tfrac{1}{2} a_y t^2$$

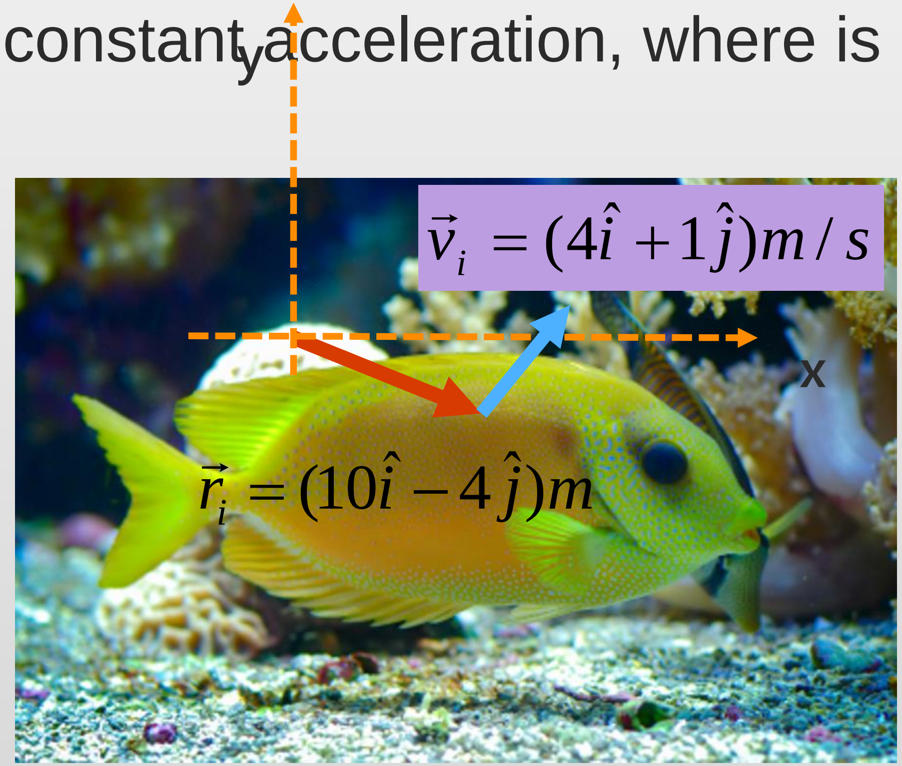
$$v_x^{\;2} = v_{0x}^{\;2} + 2a_x (x - x_0)$$

$$v_y^{\;2} = v_{0y}^{\;2} + 2a_y (y - y_0)$$

Two dimensional motion

Example:

A fish swimming in a horizontal plane has velocity $\vec{v}_i = (4\hat{i} + 1\hat{j})m/s$ at a point in the ocean where the position relative to a certain rock is $\vec{r}_i = (10\hat{i} - 4\hat{j})m$. After the fish swims constant acceleration for 20 s, its velocity is $\vec{v} = (20\hat{i} - 5\hat{j})m/s$. (a) What are the components of the acceleration of the fish? (b) What is the direction of its acceleration with respect to unit vector $\hat{i}$? (c) If the fish maintains constant acceleration, where is it at t= 25 s and in what direction is it moving?



Answer: (a) $\vec{a} = (0.8\hat{i} - 0.3\hat{j})m/s^2$

(b) 339°

(c) $\vec{r} = (360\hat{i} - 72.7\hat{j})m$, -11.4°

Two Dimensional Motion

Solution:

(a) $\vec{r}_i = (10\hat{i} - 4\hat{j})m$ $\vec{v}_i = (4\hat{i} + 1\hat{j})m/s$ $\vec{v}_f = (20\hat{i} - 5\hat{j})m/s$ at t= 20 s

$a_x = ?$ $a_y = ?$

$a_x = (v_{xf} - v_{xi})/t = (20 - 4)m/s/20\ s = 0.8\ m/s^2$

$a_y = (v_{yf} - v_{yi})/t = (-5 - 1)m/s/20\ s = -0.3\ m/s^2$

(b) $\theta_a = \arctan(a_y/a_x) = \arctan(-0.3/0.8) = 339^o$

(c) At t = 25 s $x_f - x_i = v_{xi}t + 0.5\, a_x t^2$

$\Rightarrow x_f = 10m + (4m/s)(25s) + 0.5(0.8m/s^2)(25)^2 = 360\ m$

$y_f - y_i = v_{yi}t + 0.5\, a_y t^2$

$\Rightarrow y_f = -4m + (1m/s)(25s) - 0.5(0.3m/s^2)(25)^2 = -72.7\ m$

$\theta_r = \arctan(y/x) = \arctan(-72.7/360) = -11.4^o$

Applications for Two Dimensional Motion

- Projectile Motion
- Circular Motion
- Relative Motion

Projectile Motion

- 2-D problem and define a coordinate system: x- horizontal, y- vertical (up +)
- Try to pick $x_o = 0$, $y_o = 0$ at $t = 0$
- Horizontal motion + Vertical motion
- Horizontal motion: $a_x = 0$, constant velocity
- Vertical motion: $a_y = -g = -9.8\ \mathrm{m/s^2}$, $v_{oy} = 0$
- Equations:

Horizontal

$$v_x = v_{0x} + a_x t$$

$$x - x_0 = v_{0x} t + \tfrac{1}{2} a_x t^2$$

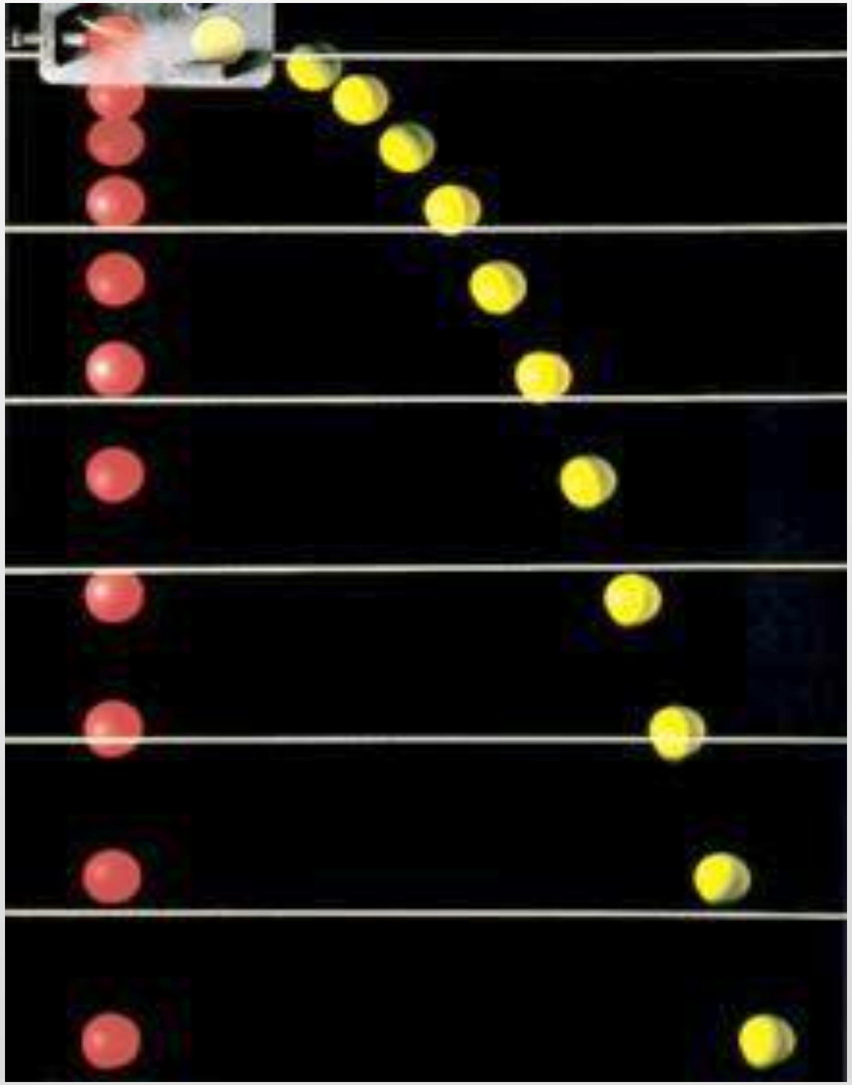
$$v_x^2 = v_{0x}^2 + 2a_x (x - x_0)$$

Vertical

$$v_y = v_{0y} + a_y t$$

$$y - y_0 = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$v_y^2 = v_{0y}^2 + 2a_y (y - y_0)$$



Projectile Motion

- x and y motions happen independently, so we can treat them separately

Horizontal

$$v_x = v_{0x}$$

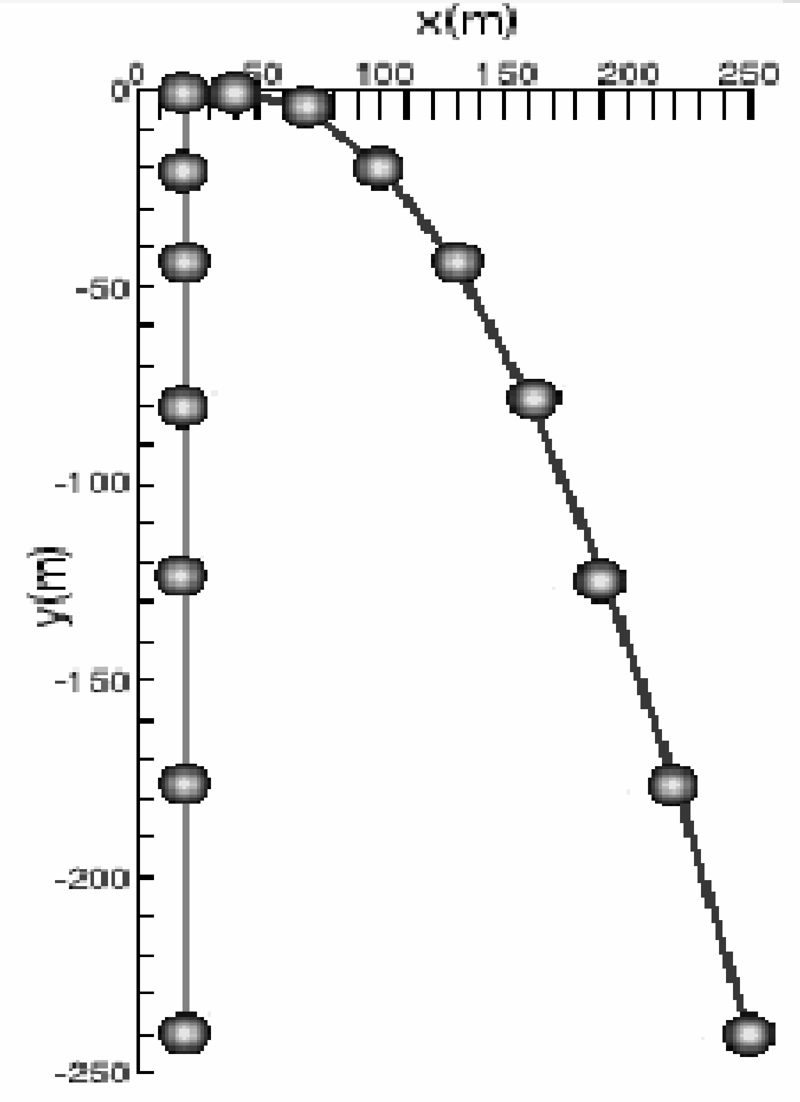
$$x = x_0 + v_{0x}t$$

Vertical

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

- Try to pick $x_o = 0$, $y_o = 0$ at t = 0
- Horizontal motion + Vertical motion
- Horizontal: $a_x = 0$, constant velocity motion
- Vertical : $a_y = -g = -9.8 \text{ m/s}^2$
- x and y are connected by time t
- $y(x)$ is a parabola



Projectile Motion

- 2-D problem and define a coordinate system.
- Horizontal: $a_x = 0$ and vertical: : $a_y = -g$.
- Try to pick $x_o = 0$, $y_o = 0$ at $t = 0$.
- Velocity initial conditions:
  - v_o can have x, y components.
  - v_{ox} is constant usually.
  - v_{oy} changes continuously.
- Equations:

Horizontal

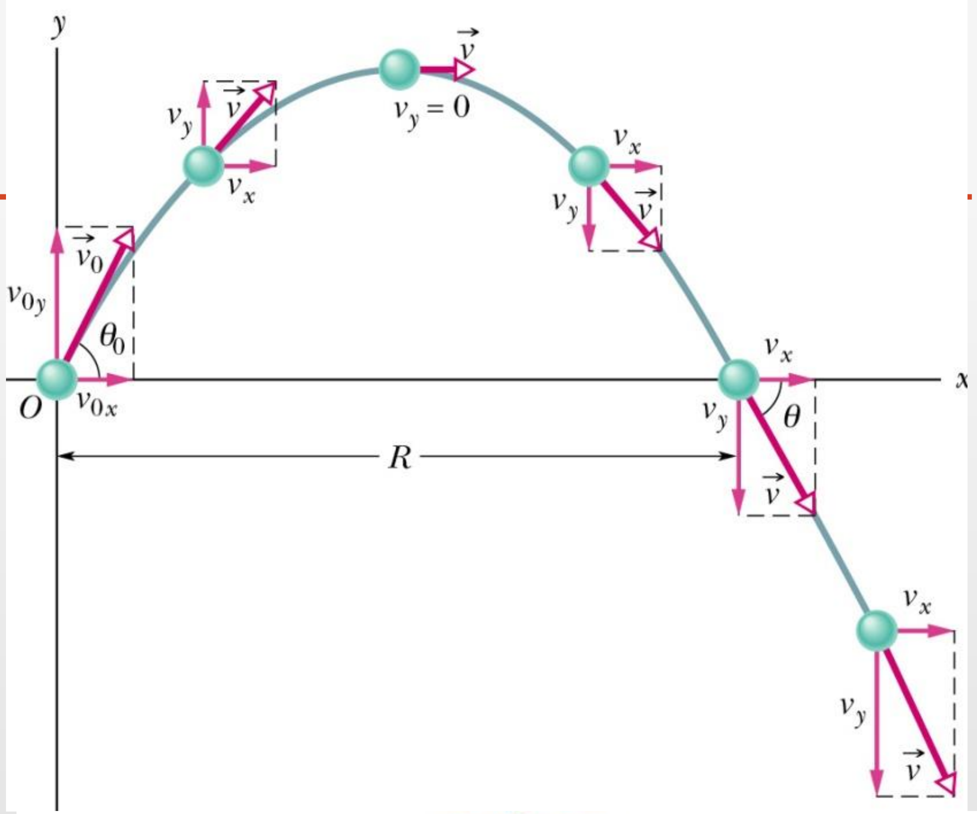
$$v_x = v_{0x}$$

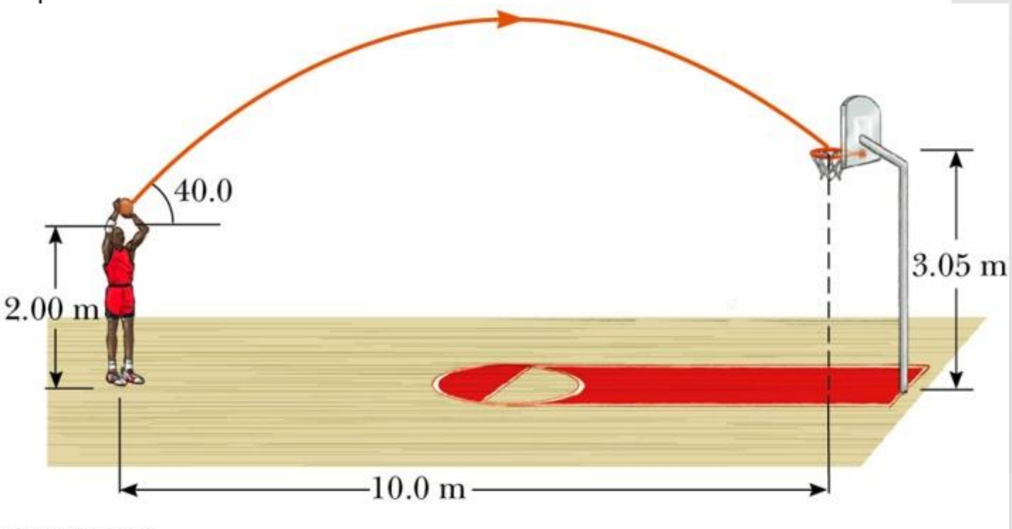
$$x = x_0 + v_{0x}t$$

Vertical

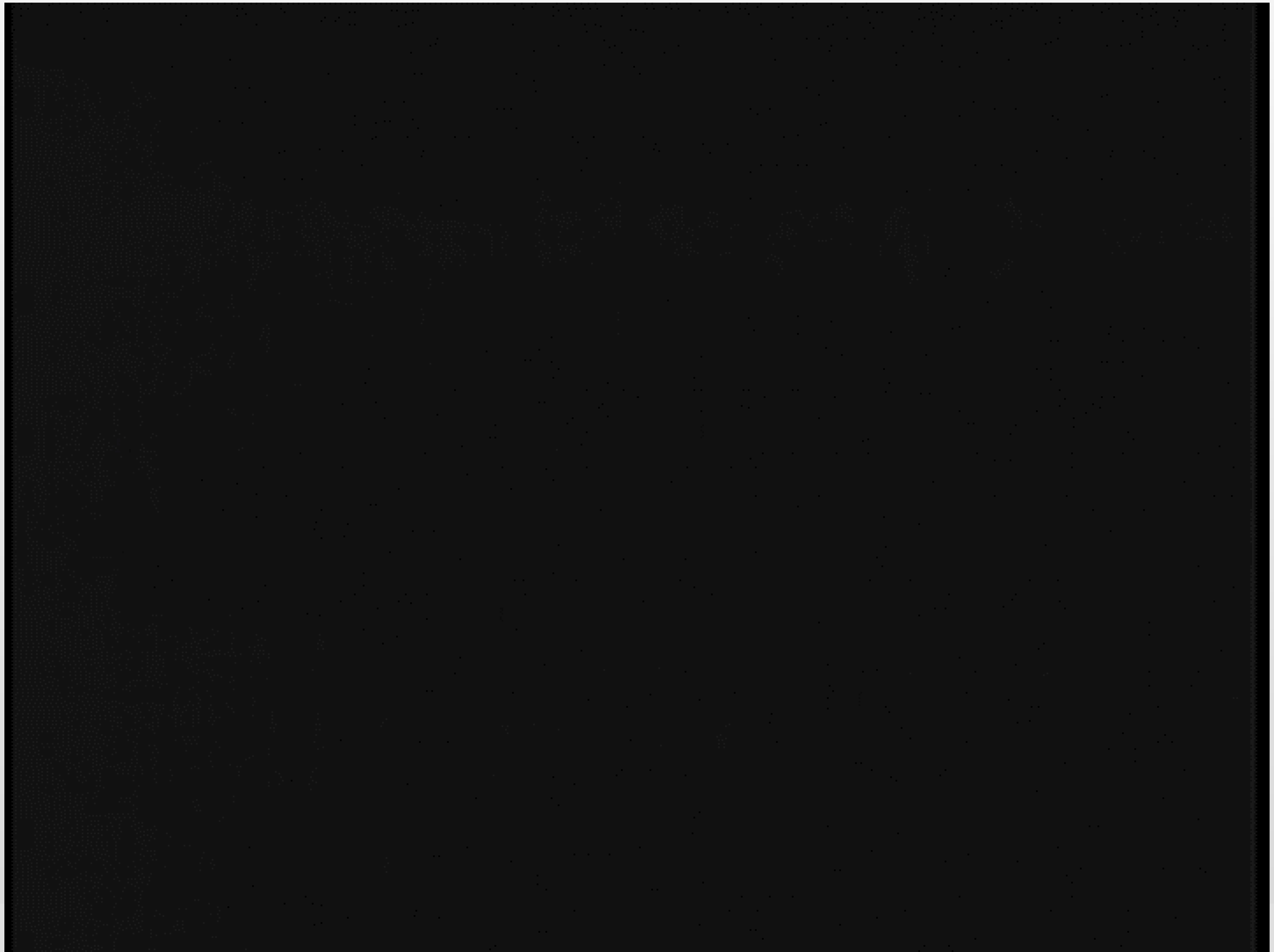
$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$





Projectile Motion



Projectile Motion

Symmetrical Trajectory of Projectile Motion

- Initial conditions ($t = 0$): $x_o = o,\ y_o = o$
- $v_{ox} = v_o \cos\theta_o$ *and* $v_{oy} = v_o \sin\theta_o$
- Horizontal motion:

$$x = 0 + v_{0x} t \quad \Rightarrow \quad t = \frac{x}{v_{0x}}$$

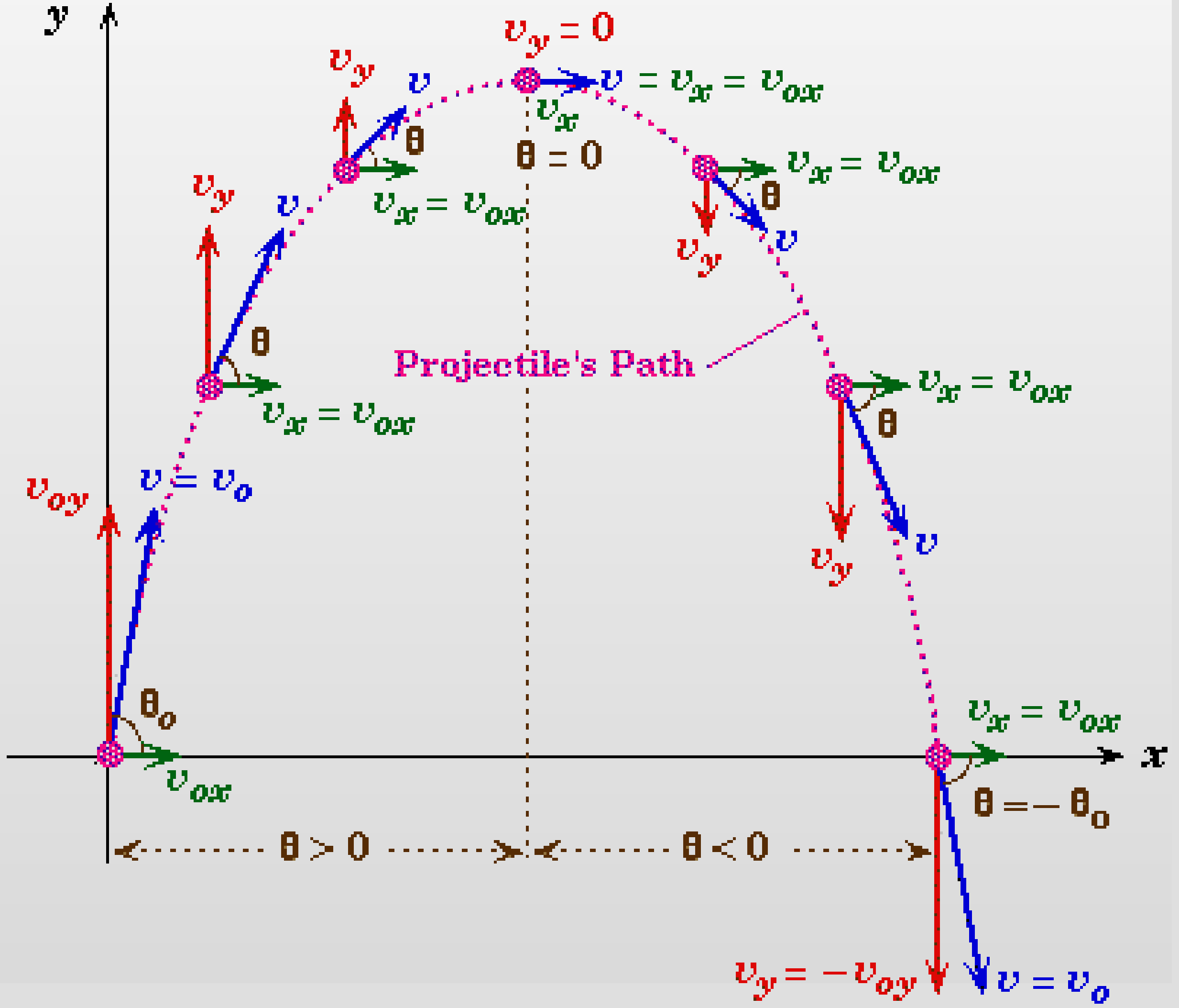
- Vertical motion:

$$y = 0 + v_{0y} t - \tfrac{1}{2} g t^2$$

$$y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \frac{g}{2}\left(\frac{x}{v_{0x}}\right)^2$$

$$y = x \tan\theta_0 - \frac{g}{2 v_0^2 \cos^2\theta_0} x^2$$

- Parabola;
  - $\theta_0 = 0$ and $\theta_0 = 90$?



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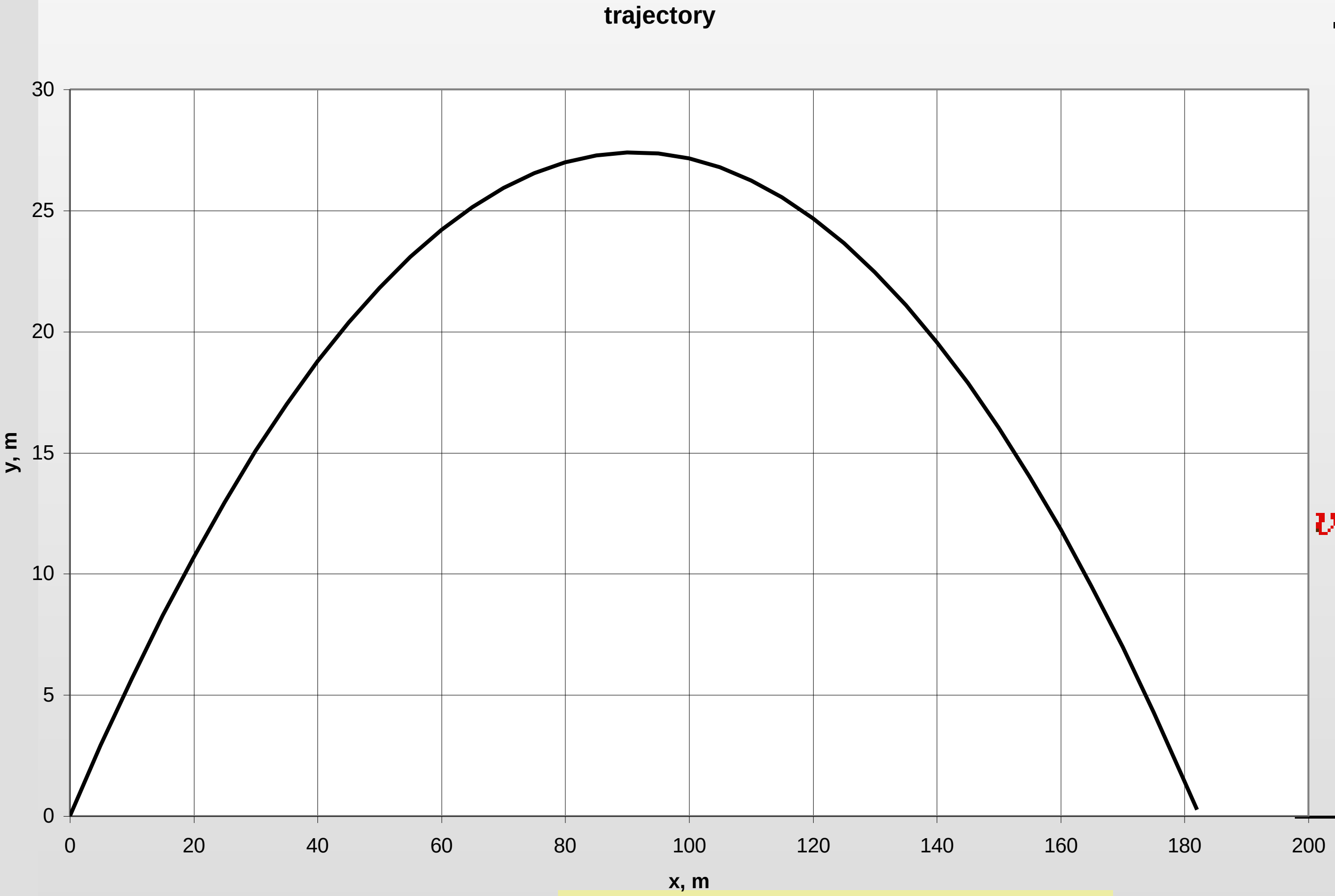
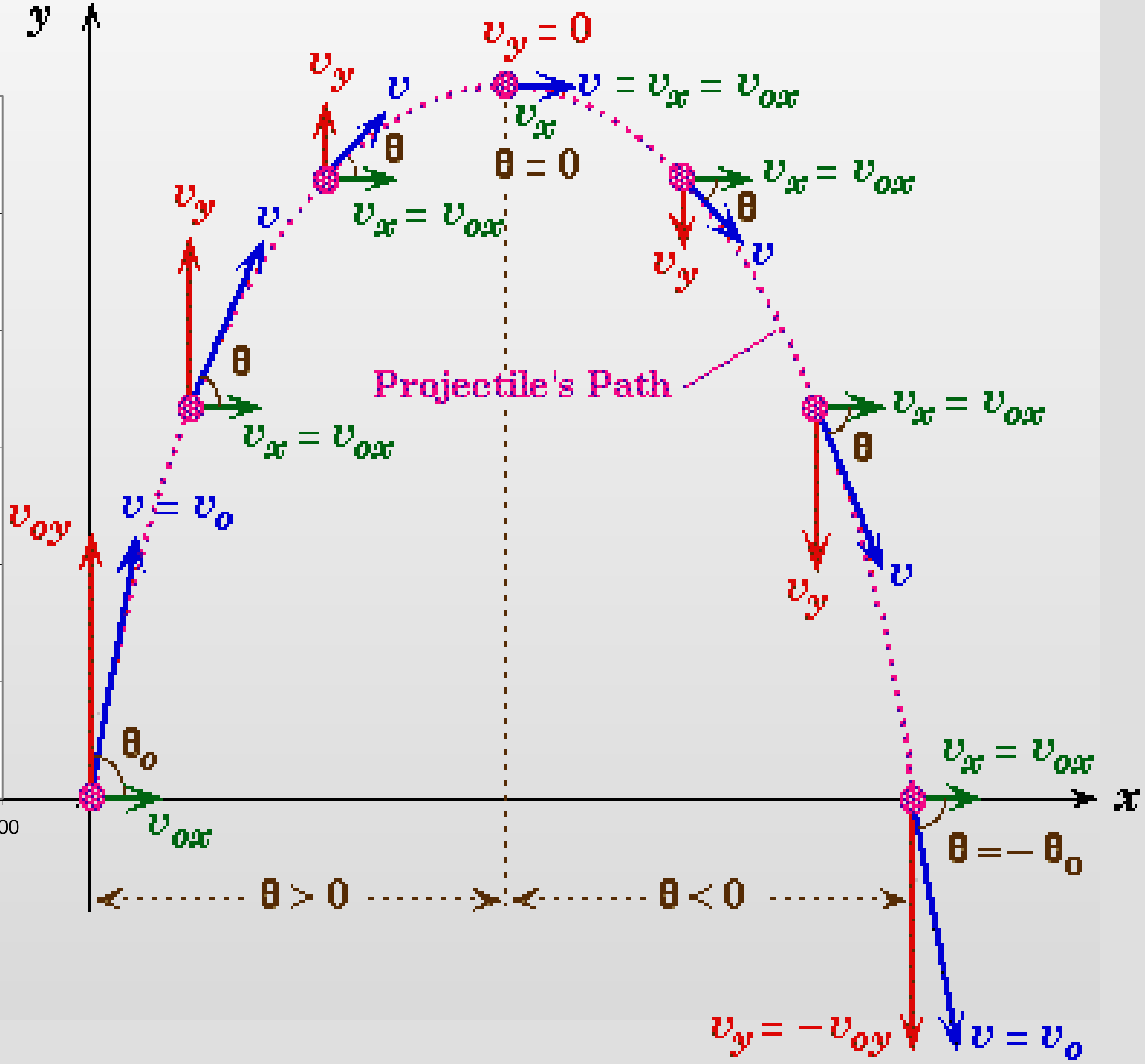
Equation of a symmetric Trajectory of Projectile Motion





- Parabola:

$$y = x\tan\theta_0 - \frac{g}{2v_0^2\cos^2\theta_0}x^2$$

What are horizontal range *R* and maximum height *h* ?

For a projectile of symmetrical trajectory ONLY

- Initial conditions ($t = 0$): $x_o = 0$, $y_o = 0$,

$v_{ox} = v_o \cos\theta_o$ and $v_{oy} = v_o \sin\theta_o$, then

$$0 = 0 + v_{0y}t - \tfrac{1}{2}gt^2$$

$$x = 0 + v_{0x}t$$

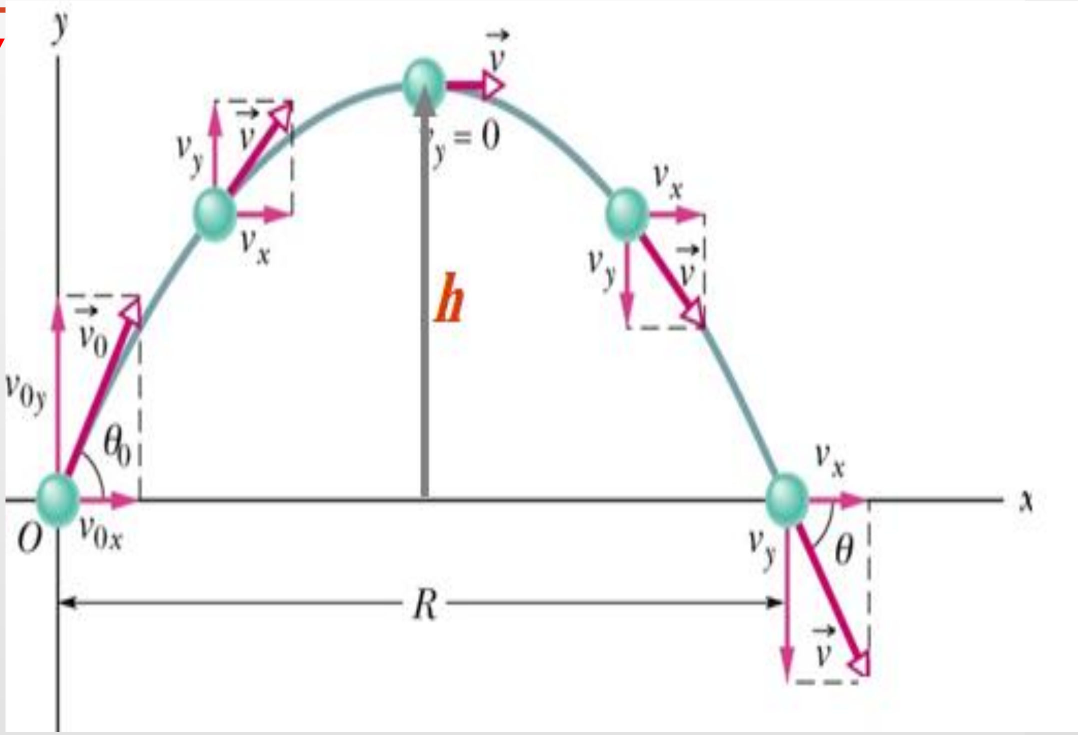
$$t = \frac{2v_{0y}}{g} = \frac{2v_0 \sin\theta_0}{g}$$

$$R = x - x_0 = v_{0x}t = \frac{2v_0 \cos\theta_0 v_0 \sin\theta_0}{g} = \frac{v_0^{\,2} \sin 2\theta_0}{g}$$

$$h = y - y_0 = v_{0y}t_h - \tfrac{1}{2}gt_h^{\,2} = v_{0y}\frac{t}{2} - \frac{g}{2}\left(\frac{t}{2}\right)^2$$

$$h = \frac{v_0^{\,2}\sin^2\theta_0}{2g}$$

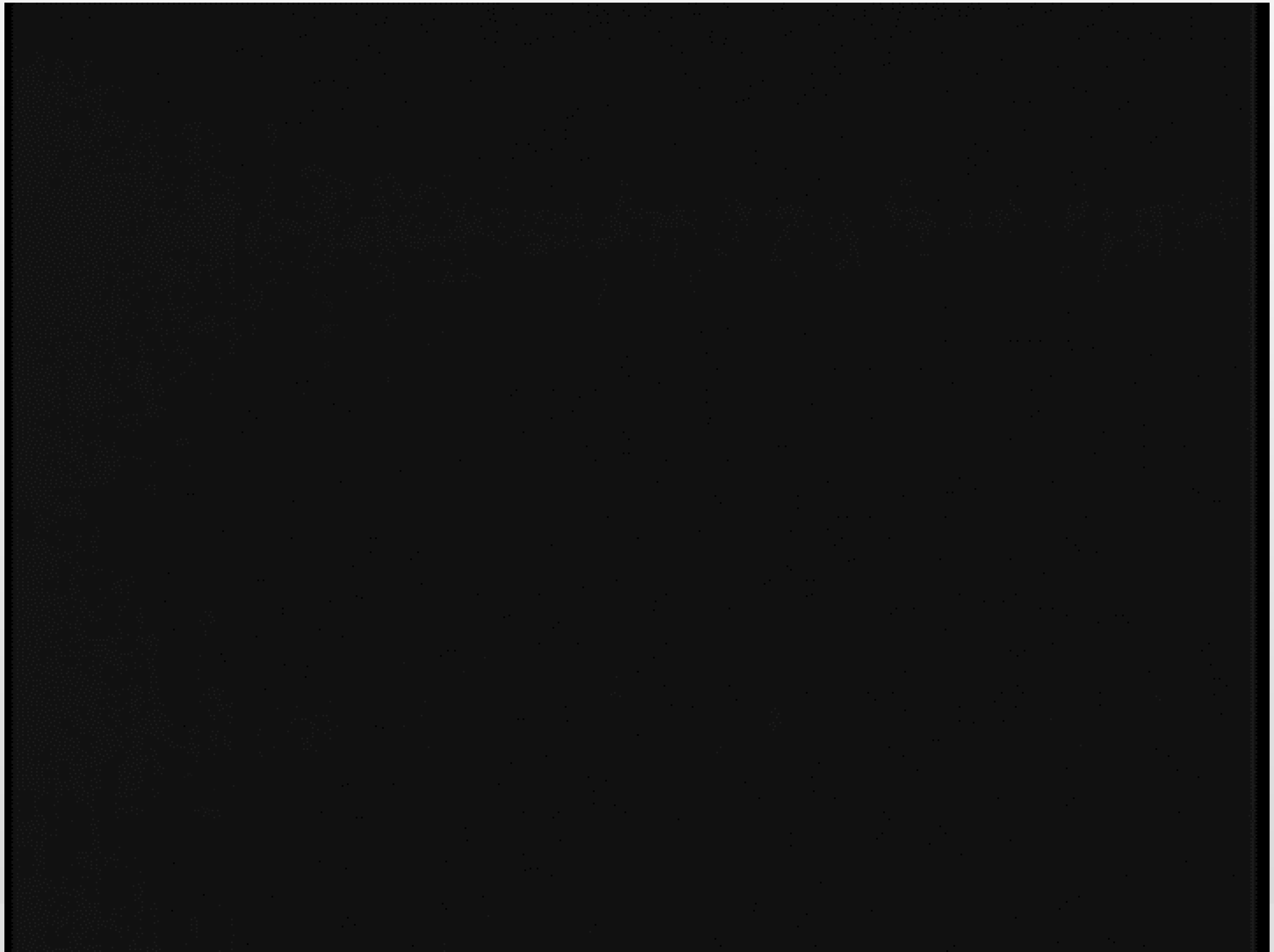
$$v_y = v_{0y} - gt = v_{0y} - g\frac{2v_{0y}}{g} = -v_{0y}$$



Horizontal	Vertical
$v_x = v_{0x}$	$v_y = v_{0y} - gt$
$x = x_0 + v_{0x}t$	$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$

Projectile Motion at Various Initial Angles

$$R = \frac{v_0{}^2 \sin 2\theta_o}{g}$$



Projectile Motion at Various Initial Angles

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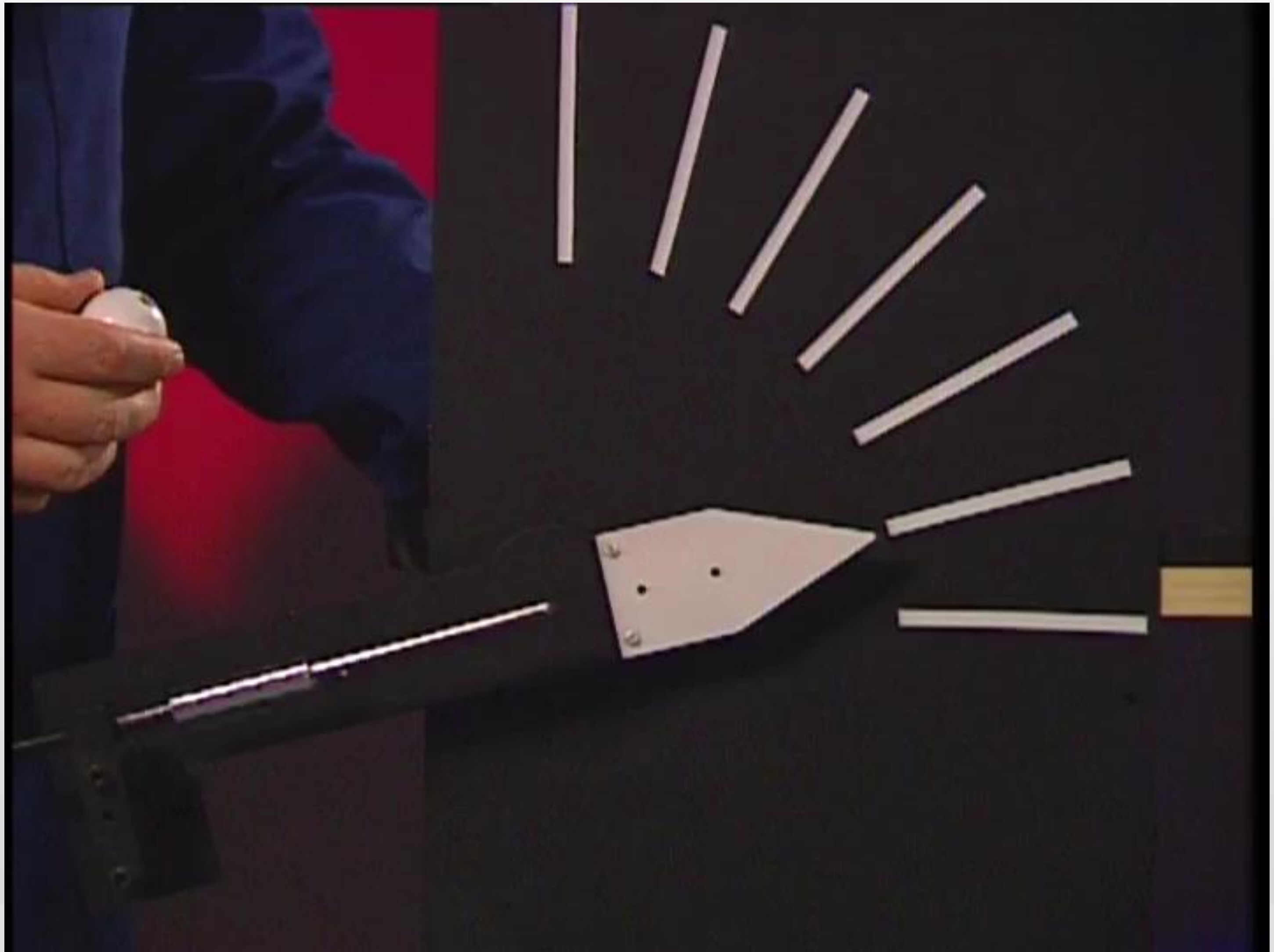

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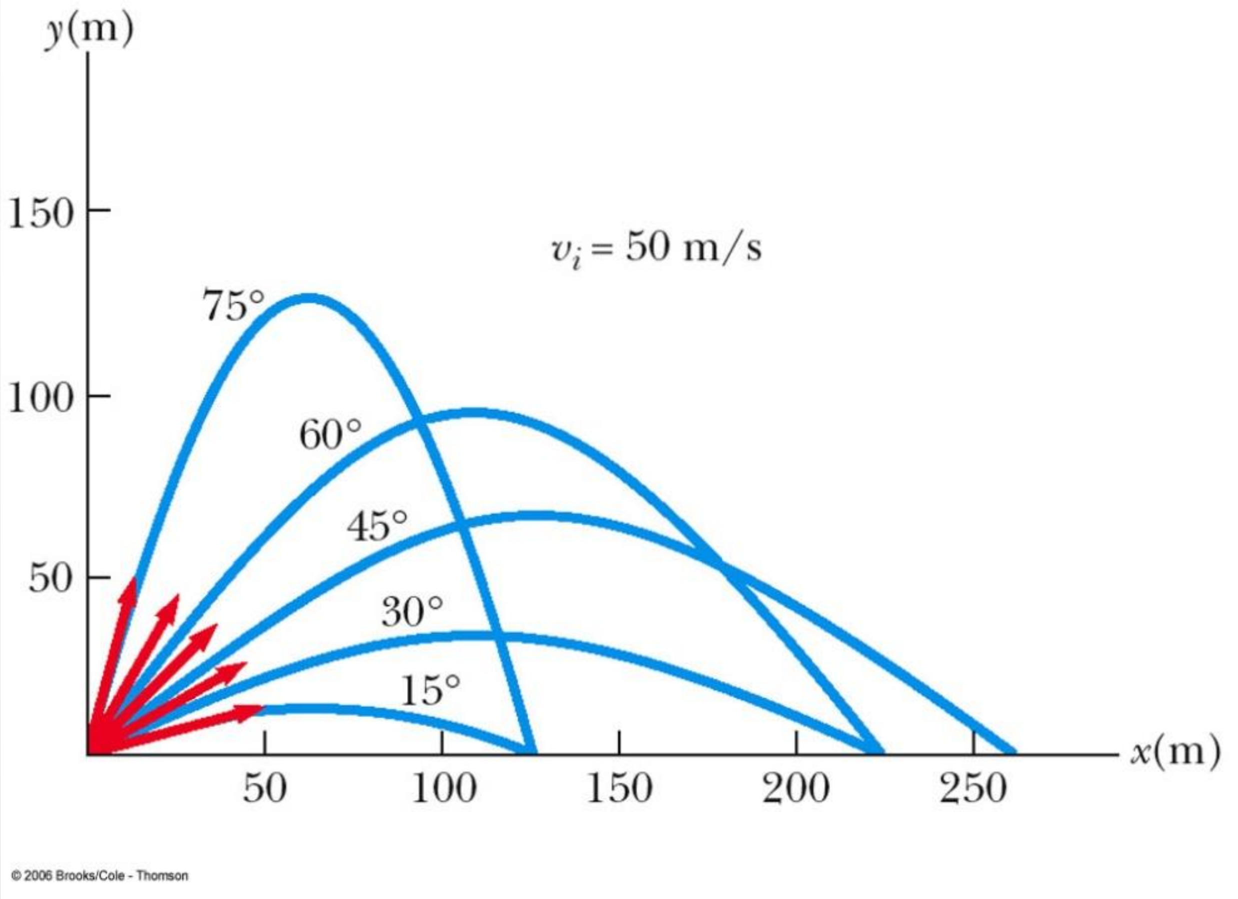
Projectile Motion at Various Initial Angles

$$R = \frac{v_0^{\ 2}\sin 2\theta_o}{g}$$

- Complementary values of the initial angle result in the same range
  - The heights will be different
- The maximum range occurs at a projection angle of 45º



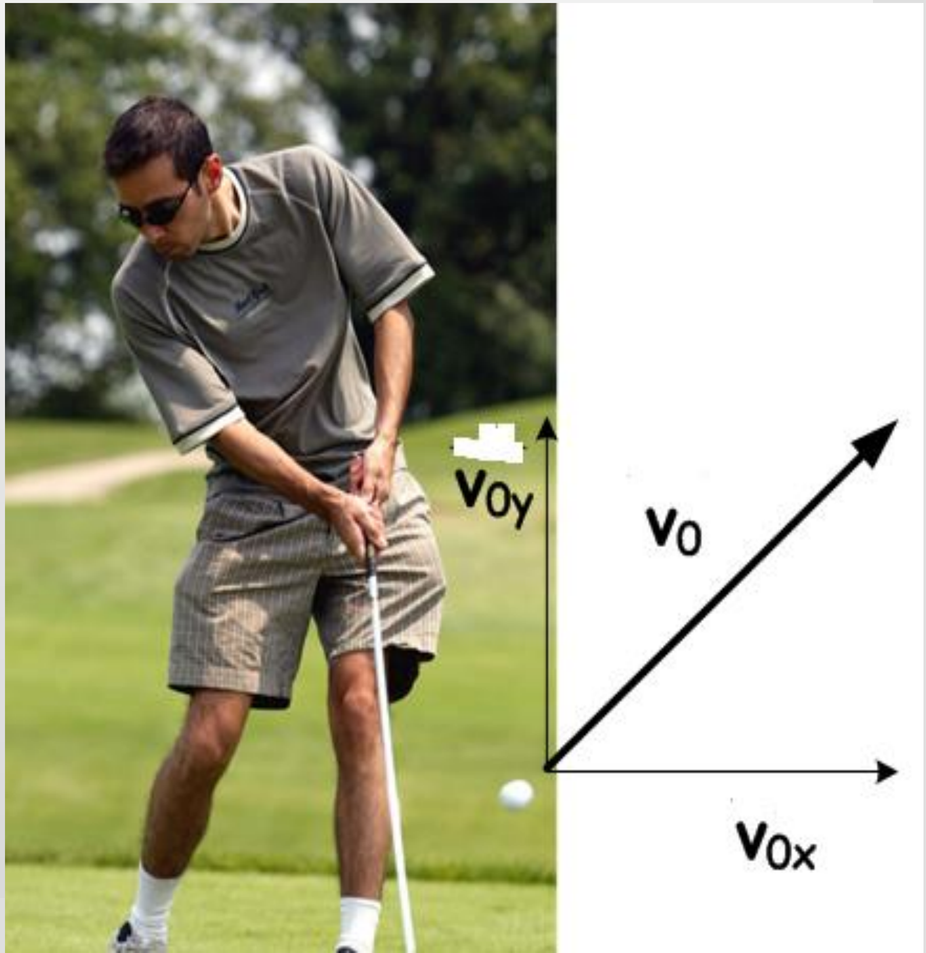
Example:

The golfer shown in the picture has launched his ball with an initial velocity of 45 m/s, 31° above the horizontal. (a) What is the total time the ball will be in the air?

(b) How high above the ground will the ball reach at its highest point?

(c) How far will the ball "carry" over level ground?

(d) Write the equation of trajectory



Hint: Take g=9.8 m/s²

Solution

(a) v_o =45 m/s, θ_o = 31° above the horizontal.

Resolve the ball's initial velocity into horizontal and vertical components.

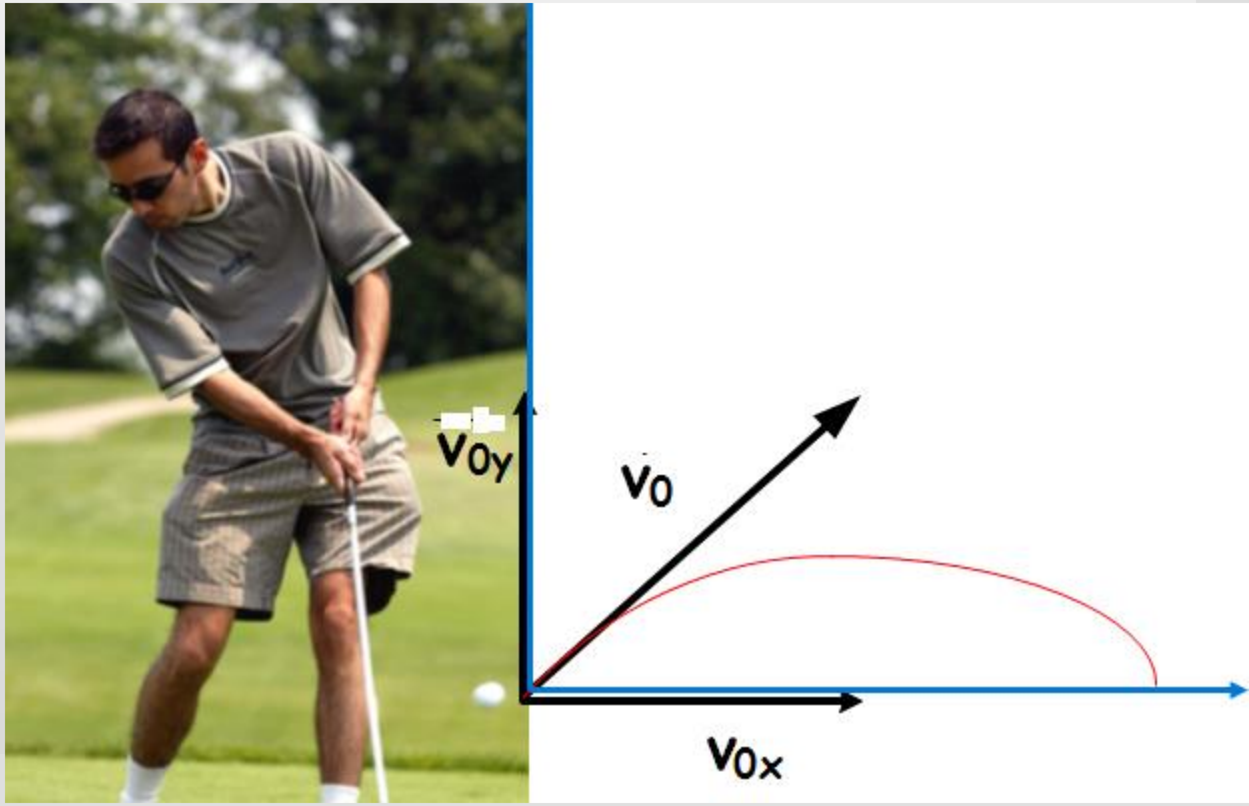
$$v_{0x} = v_0 \cos\theta_o = (45m/s)(0.857)$$

$$v_{0x} = 38.57m/s$$

$$v_{0y} = v_0 \sin\theta_o = (45m/s)(0.515)$$

$$v_{0y} = 23.17m/s$$

Note: Remember that v_{ox} will remain constant during the whole trip but that v_{oy} is not going to remain constant



Solution

(a) During the time the ball takes to rise vertically, stop vertically, and fall vertically back to the ground, while the ball moves horizontally with constant velocity.

We must determine just how long that time of flight is.

$$v_y = v_{0y} - g\left(\frac{1}{2}t\right) = v_0 \sin\theta_0 - g\left(\frac{1}{2}t\right) = 0$$

Solving for *t*, the total time of flight:

$$v_0 \sin\theta_0 - g\left(\frac{1}{2}t\right) = 0 \quad \Rightarrow \quad \frac{g}{2}t = v_0 \sin\theta_0$$

$$t = \frac{2v_0 \sin\theta_0}{g} = \frac{2(45\ m/s)(\sin 31^\circ)}{9.8\ m/s^2} = 4.730\ s$$

Note: The substitution of t/2 in above equation considers the rising part of the trajectory where the speed at maximum height is zero.

Solution

(b) To calculate this maximum height directly from the third kinematic equation:

$$y = v_{0y} t_h - \frac{1}{2} g t_h^2$$

$v_{0y} = 23.17 m/s$, as given ($v_o = 45 m/s$ and $\theta_o = 31^o$)

$t_h = \frac{t}{2} = \frac{4.73s}{2} = 2.35s$, where t time of flight as obtained from (a)

Put these values into the equation we get:

$$y = (23.17)(2.35) - \frac{1}{2}\left(9.8\,\text{m/s}^2\right)\left(2.35\,\text{s}\right)^2 = 27.1\,\text{m}$$

Note: The time we used was half the time-of-flight, *t*.

Solution

(c) To calculate the horizontal distance, you should remember that the acceleration is zero in horizontal direction and the initial velocity is constant along the whole distance. It was obtained as $v_{0x} = 38.57 m/s$

Use the equation: $x = v_0 \cos\theta_0 \cdot t$

Such that $x = (45\ m/s)(\cos 31°)(4.730\ s) = 182\ m$

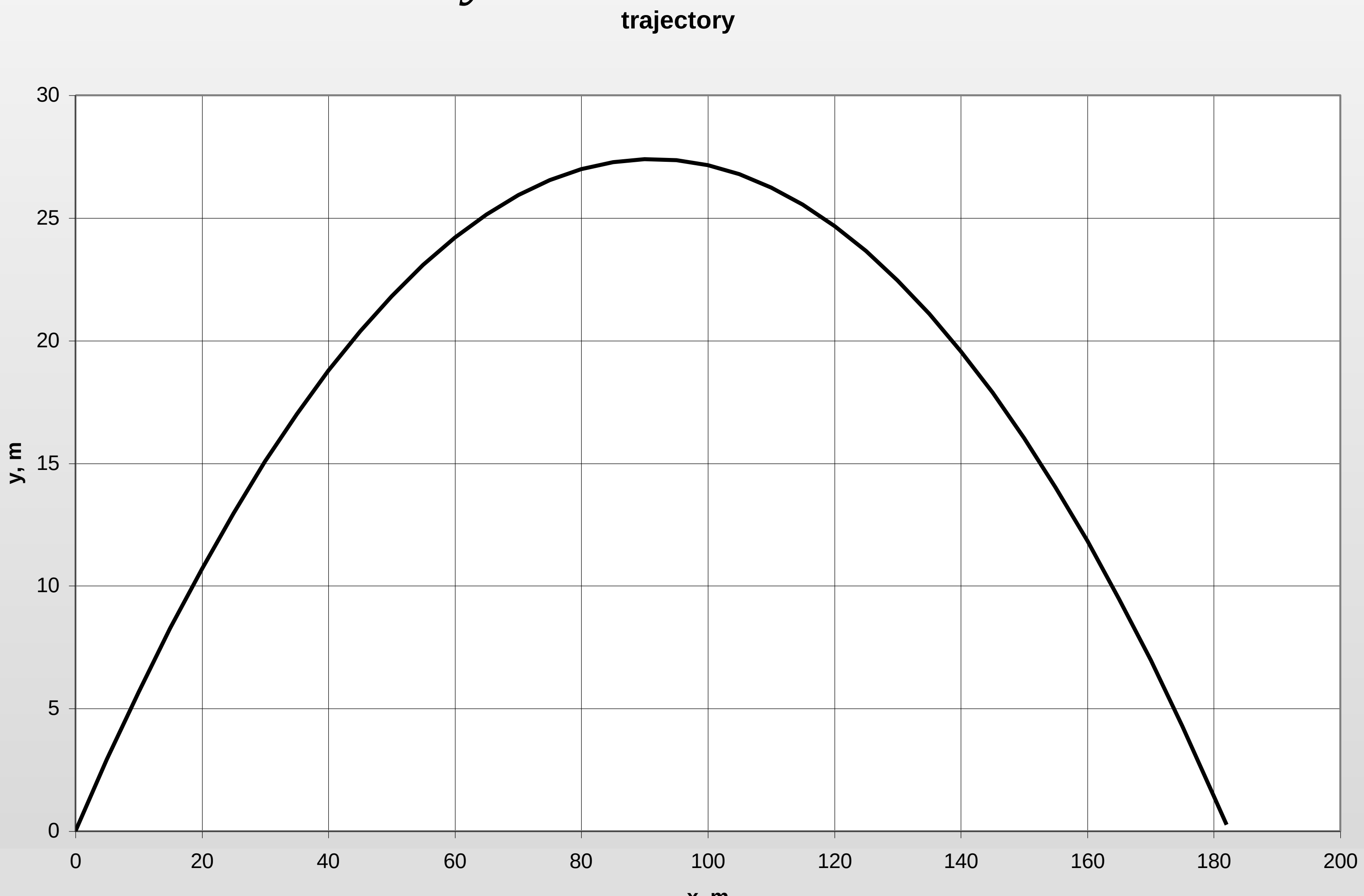
(d) To find the equation of trajectory, use the derived equation

$$y = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0}$$

$$y = 0.6x - 3.29x^2$$

Solution

$$y = 0.6x - 3.29x^2$$



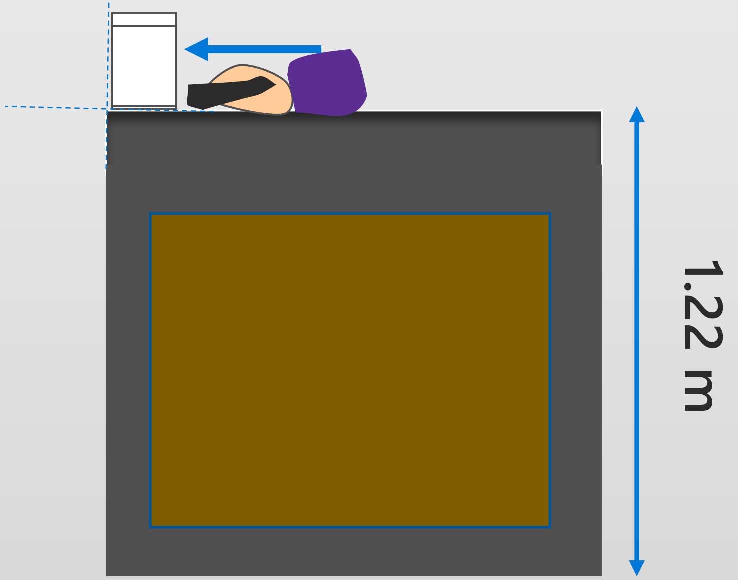
Example:

In a kitchen a girl slides a mug down the counter. The height of the counter is 1.22 m.

The mug slides of the counter and strikes the floor 1.4 m from the base of the counter.

(a) With what velocity did the mug leave the counter?

(b) What was the mug's velocity just before it hits the floor?



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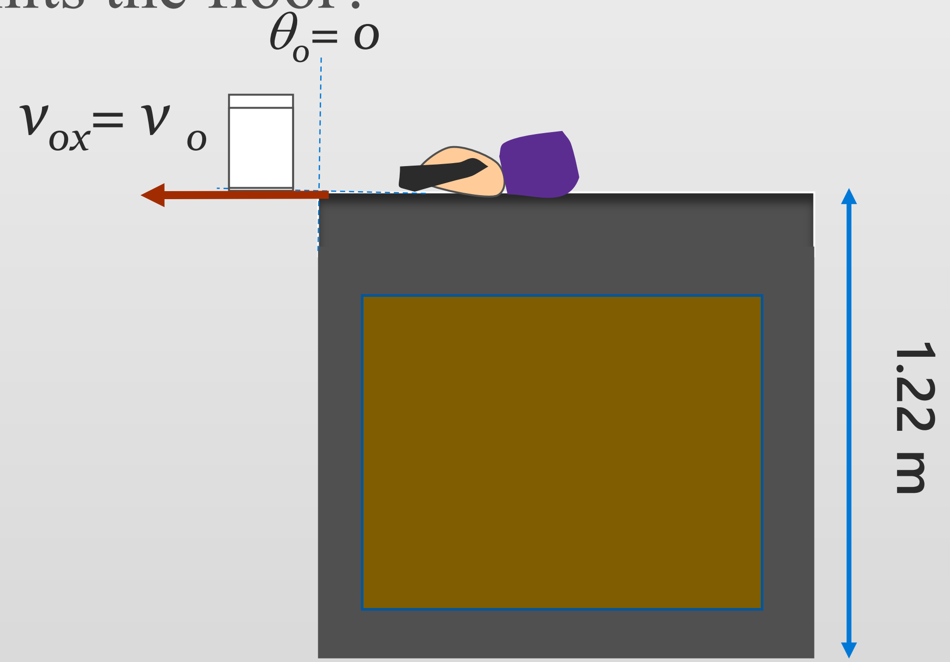

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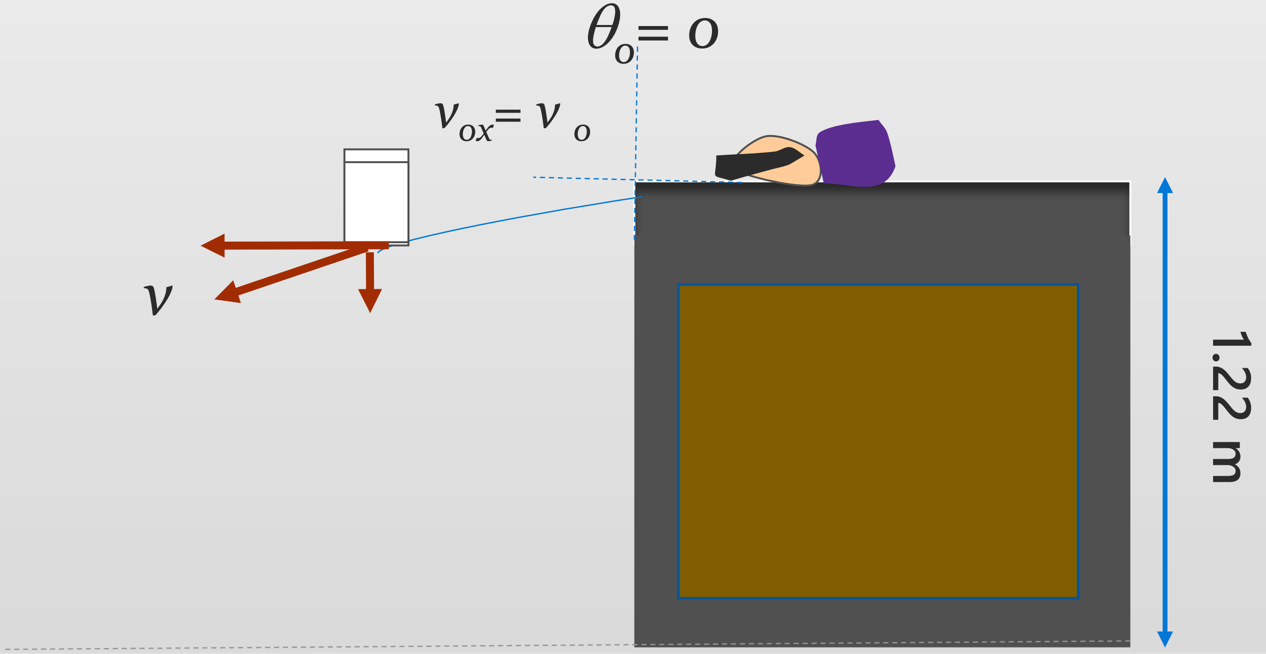

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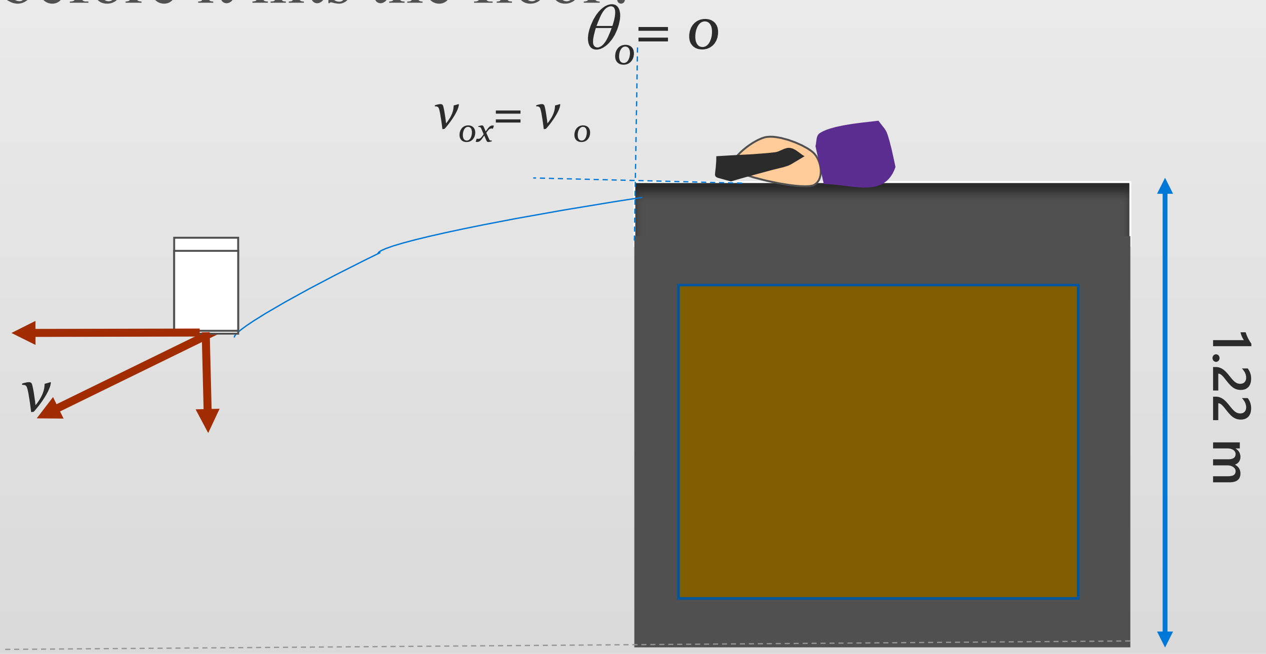
(a) With what velocity did the mug leave the counter?

(b) What was the mug's velocity just before it hits the floor?

Hint: Take g=10 m/s²



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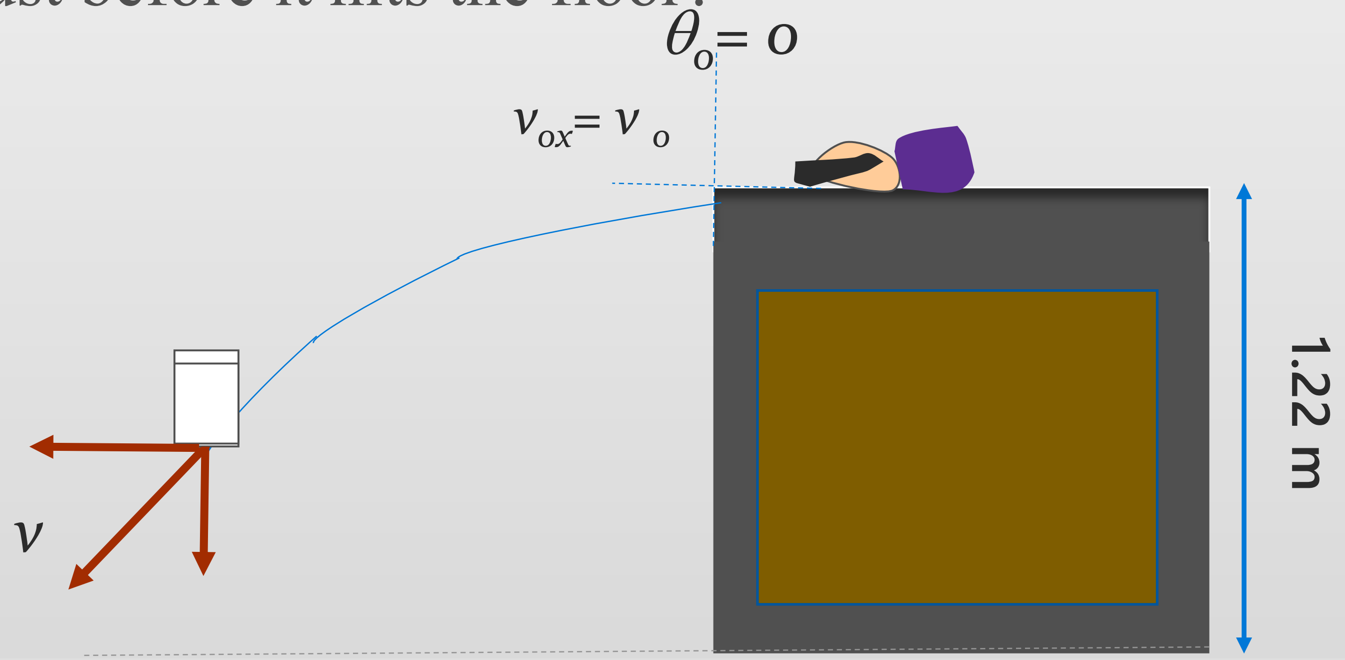


Solution

(a) To find the velocity of the mug when it left the counter:

$$x = 0 + v_{0x}t \quad \Rightarrow \quad v_{0x} = \frac{x}{t} = \frac{1.4m}{t}$$

But we do not have information about t, while we have Δy= -1.22 m and v_{oy} =0. Thus we use the equation of y-motion:

Hint: Take g=**10 m/s²**

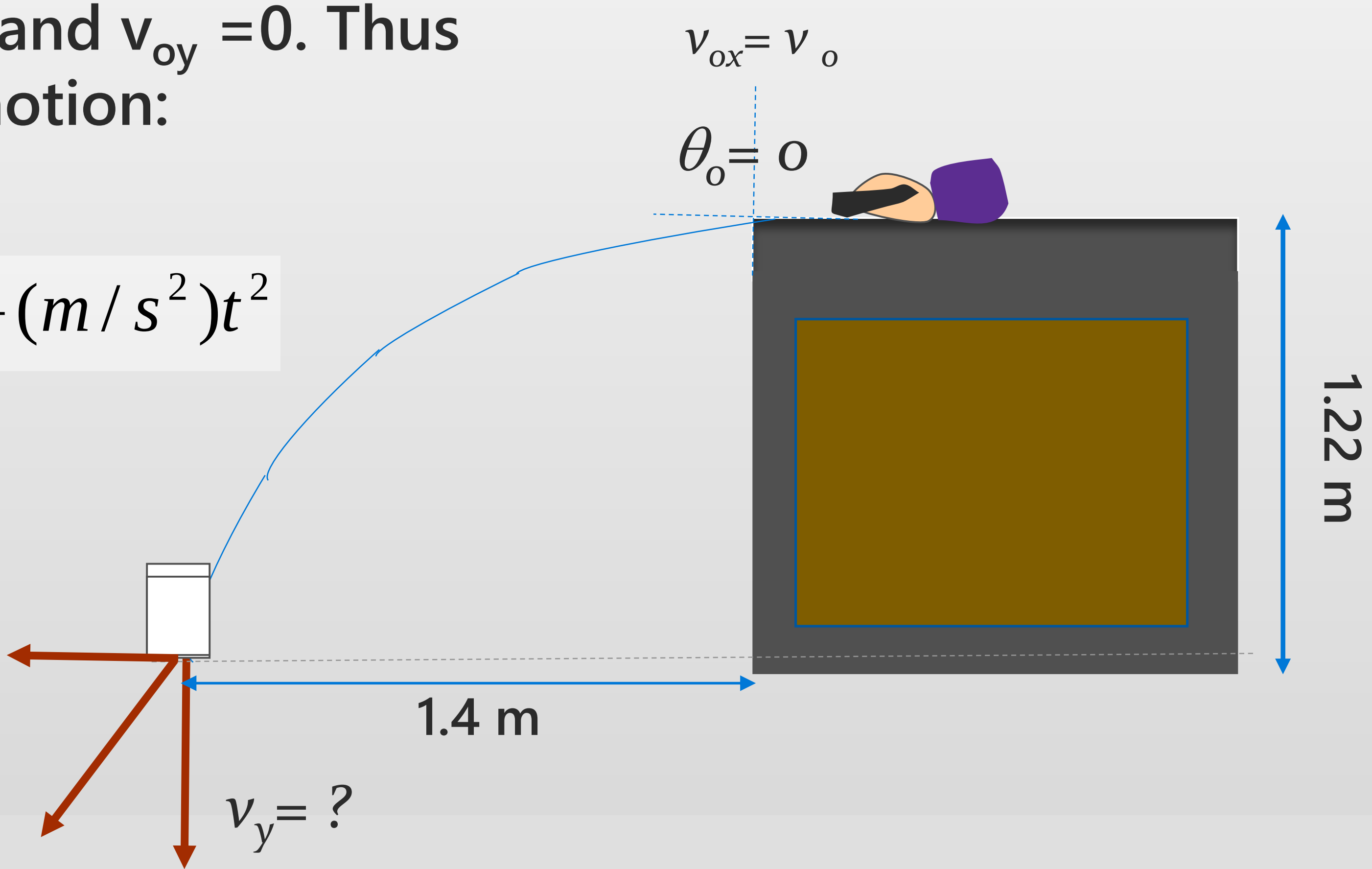
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

or $y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2 = 0 - \tfrac{10}{2}(m/s^2)t^2$

$$-1.22m = -(5m/s^2)t^2$$

$$\Rightarrow t = 0.49s$$

$$\therefore v_o = v_{ox} = \frac{1.4m}{0.49s} = 2.86m/s$$



Solution

(b) To find the mug's velocity just before it hits the floor:

$$v_y{}^2 = v_{0y}{}^2 - 2g(y - y_0)$$

We know that v_{oy} = 0 and Δy = - 1.22 m

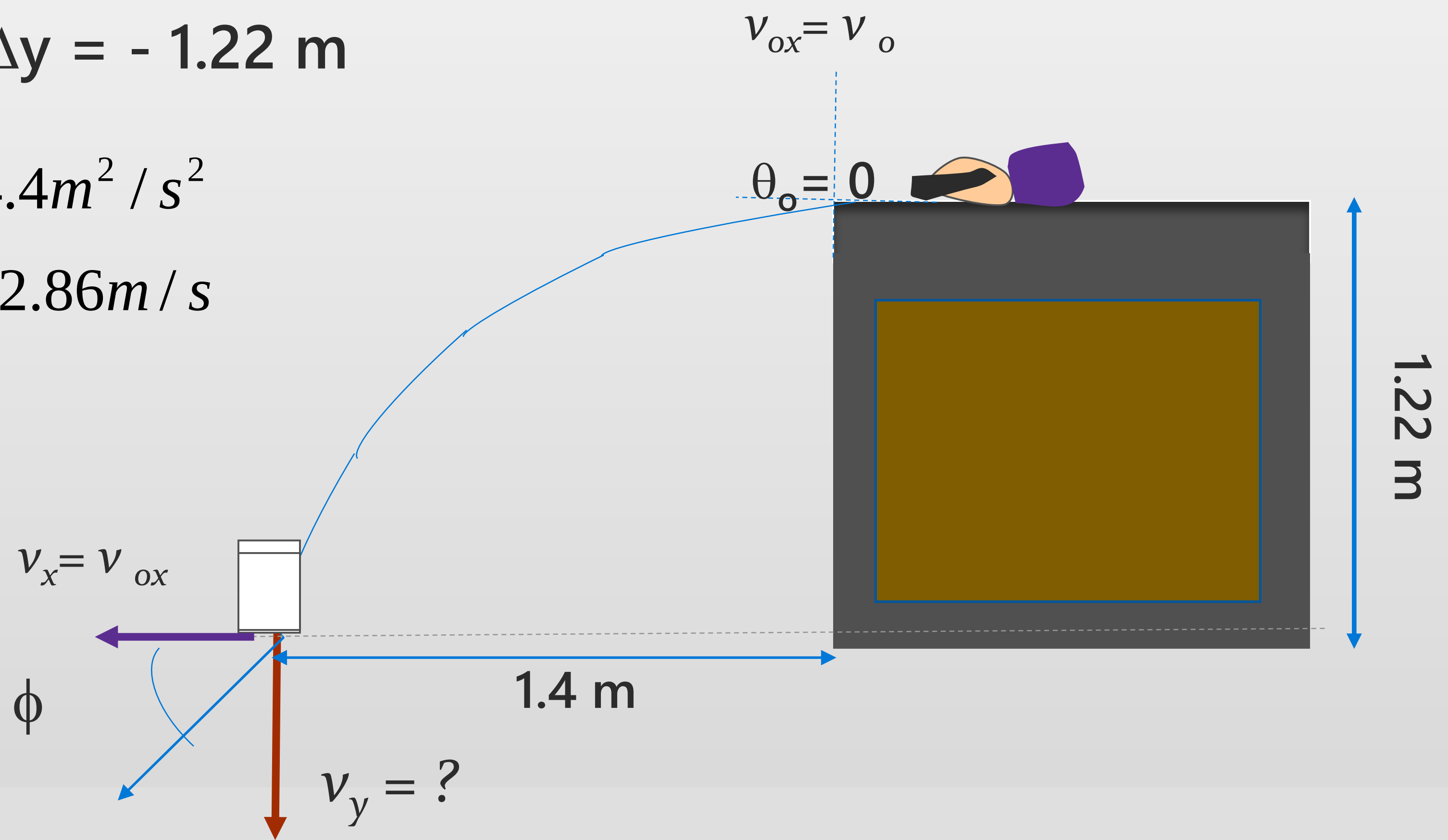
$$v_y^2 = -2(10m/s^2)(-1.22m) = 24.4m^2/s^2$$

$$v_y = -4.9m/s \qquad v_x = 2.86m/s$$

$$v = \sqrt{v_x^2 + v_y^2} = 5.7m/s$$

tan ϕ=4.9/2.86

ϕ=59.7°



Example:

A projectile is fired in such a way that its horizontal range is equal three times its maximum height. What is the angle of projection?

Solution

Given $R = 3h_{max}$ where, $h = \dfrac{v_0^2 \sin^2\theta_0}{2g}$ and $R = \dfrac{v_0^2 \sin 2\theta_o}{g}$

$$\Rightarrow \frac{v_o^2 \sin 2\theta_o}{g} = 3\left(\frac{v_o^2 \sin^2\theta_o)}{2g}\right.$$

Use the trigonometric identity $\sin(\theta_1 + \theta_2) = \sin\theta_1 \cos\theta_2 + \sin\theta_2 \cos\theta_1$

For $\theta_1 = \theta_2 = \theta_o \Rightarrow$ $\sin(2\theta_o) = 2\sin\theta_o \cos\theta_o$

$$\frac{v_o^2(2\sin\theta_o \cos\theta_o)}{g} = 3\left(\frac{v_o^2 \sin^2\theta_o)}{2g}\right.$$

$$\tan\theta_o = \frac{4}{3} \Rightarrow \theta_o = \tan^{-1}\left(\frac{4}{3}\right) = 53^o$$

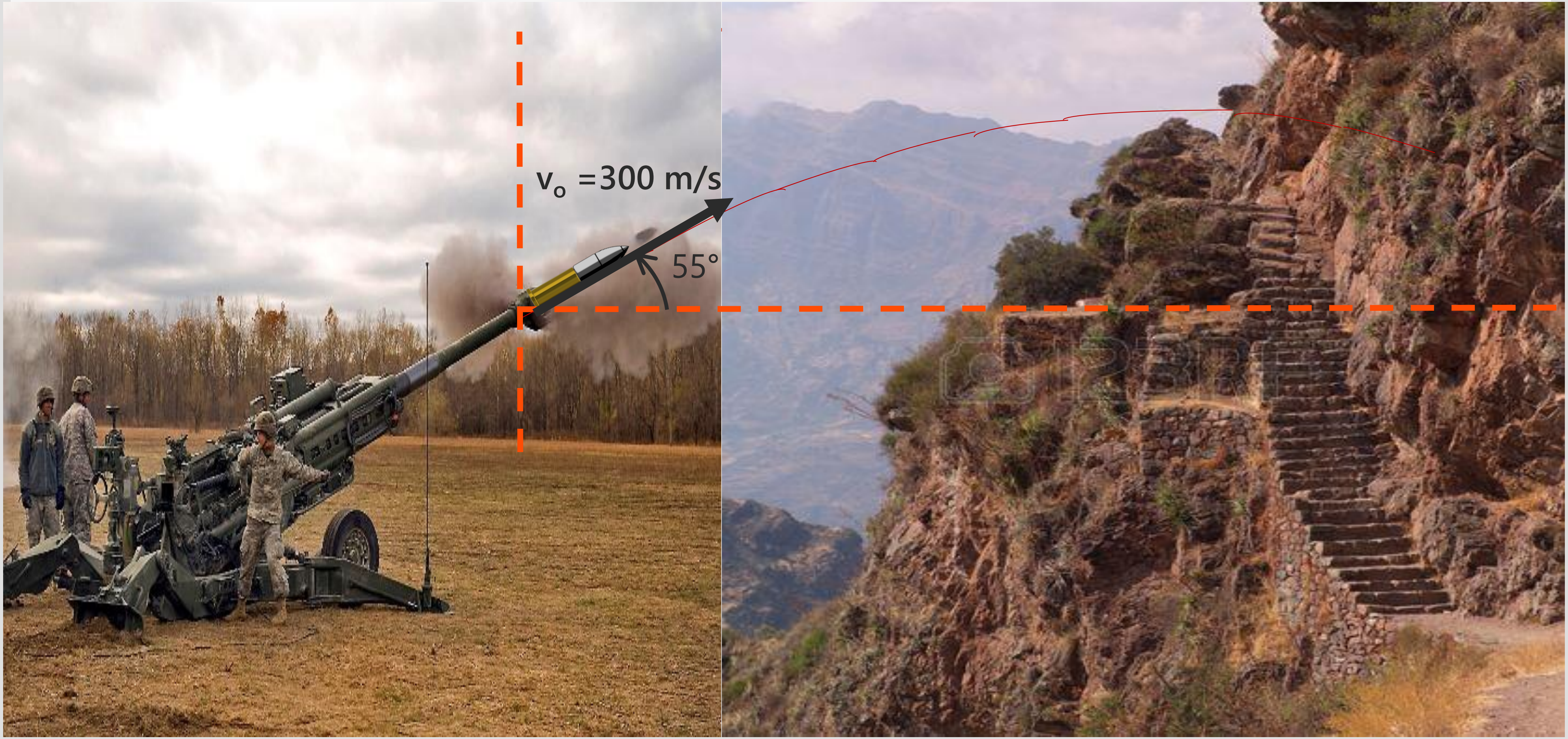
Example:

To start an avalanche on a mountain slope, an artillery shell is fired with an initial velocity of 300m/s at 55^{o} above the horizontal. It explodes on the mountainside 42 s after firing.

What are the *x and y* coordinates of the shell where it explodes relative to its firing point?

Hint: Take g=10 m/s^2

Example:



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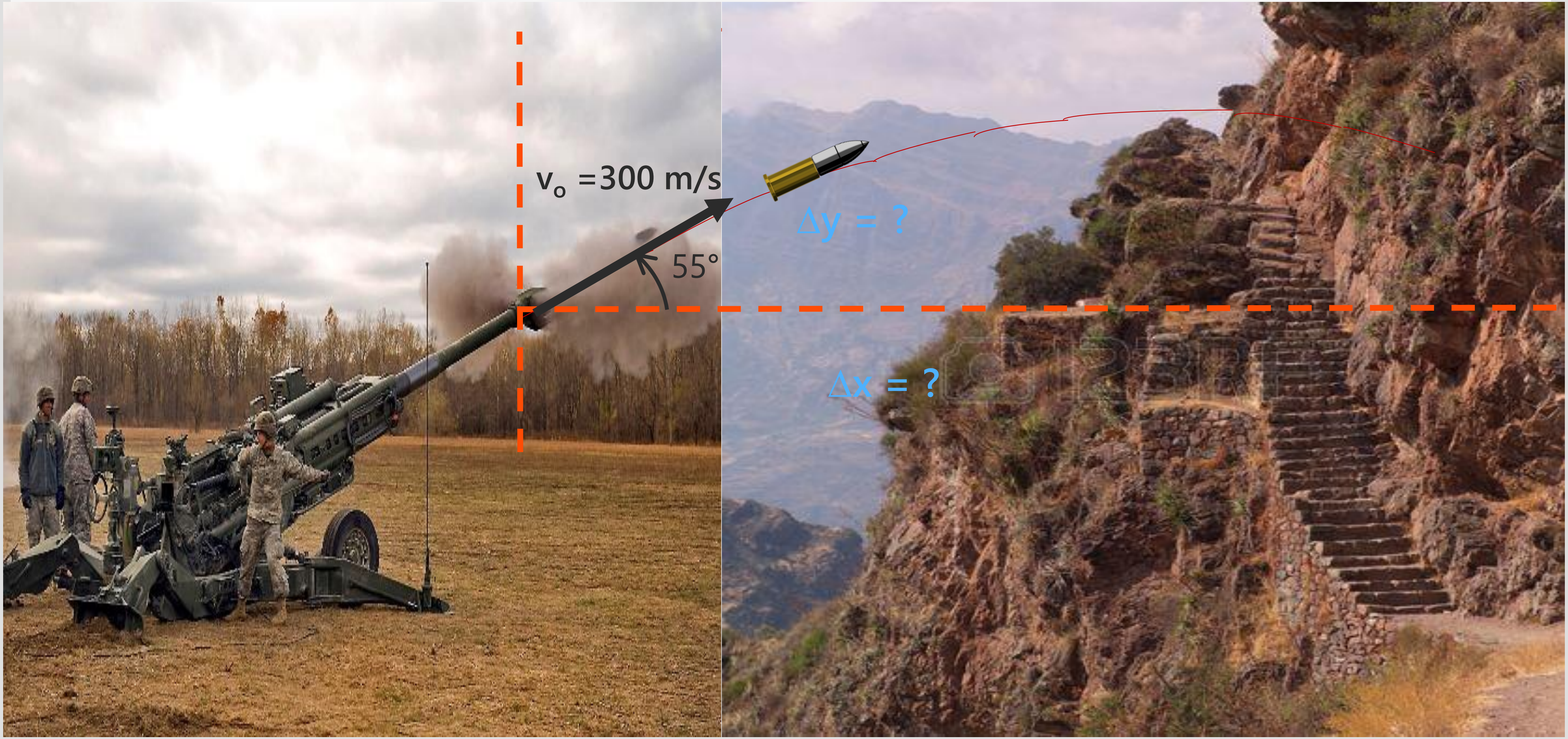
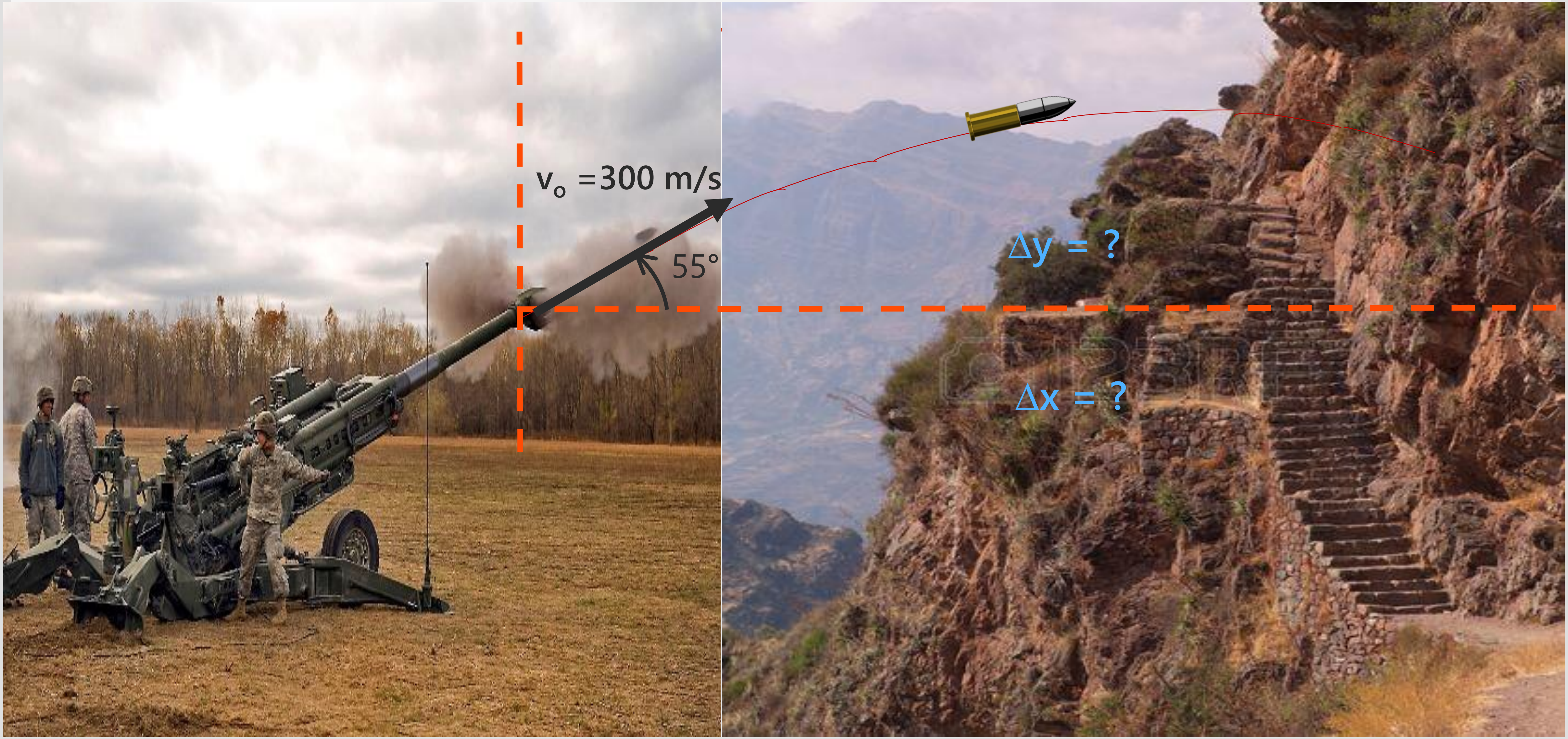
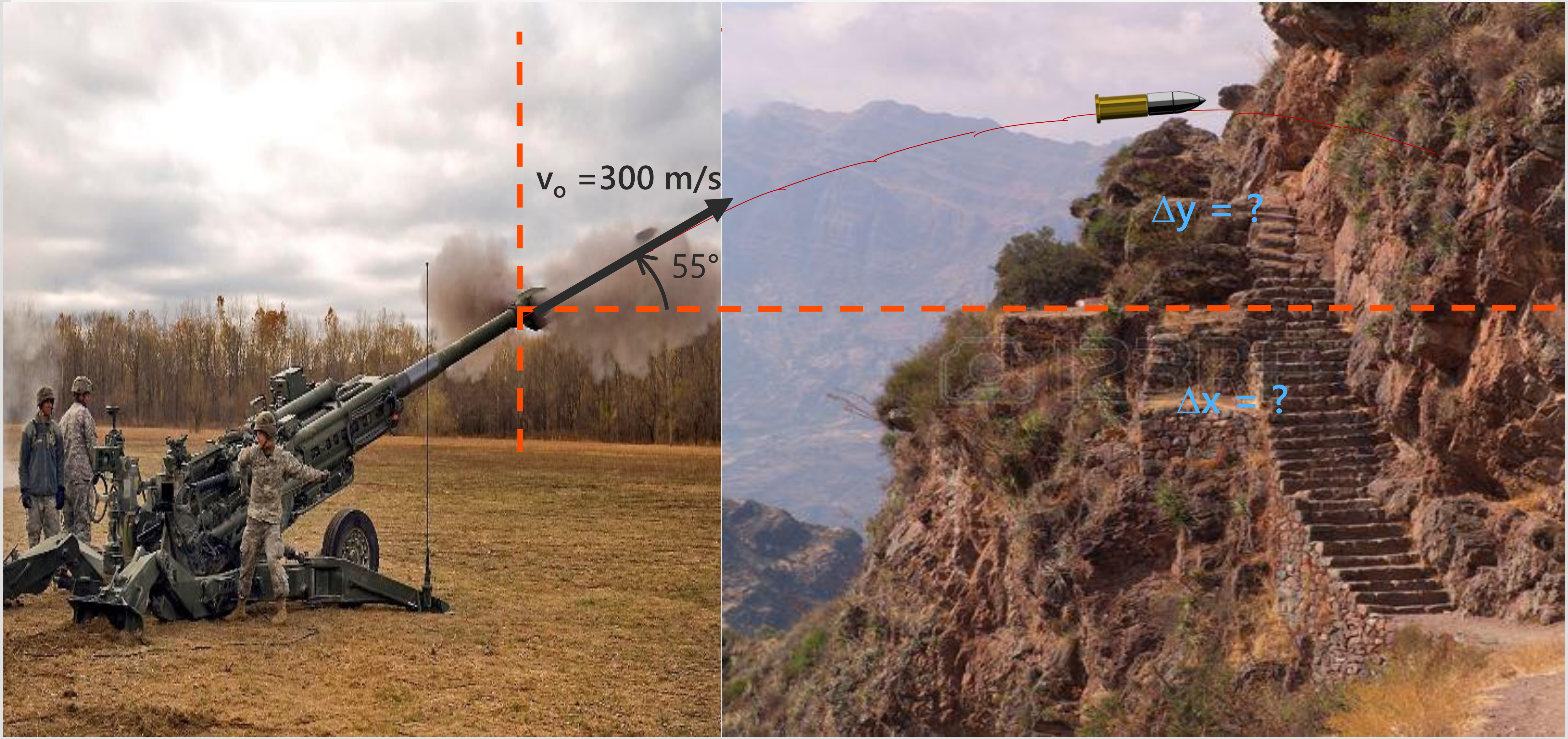
Example:



Example:



Example:



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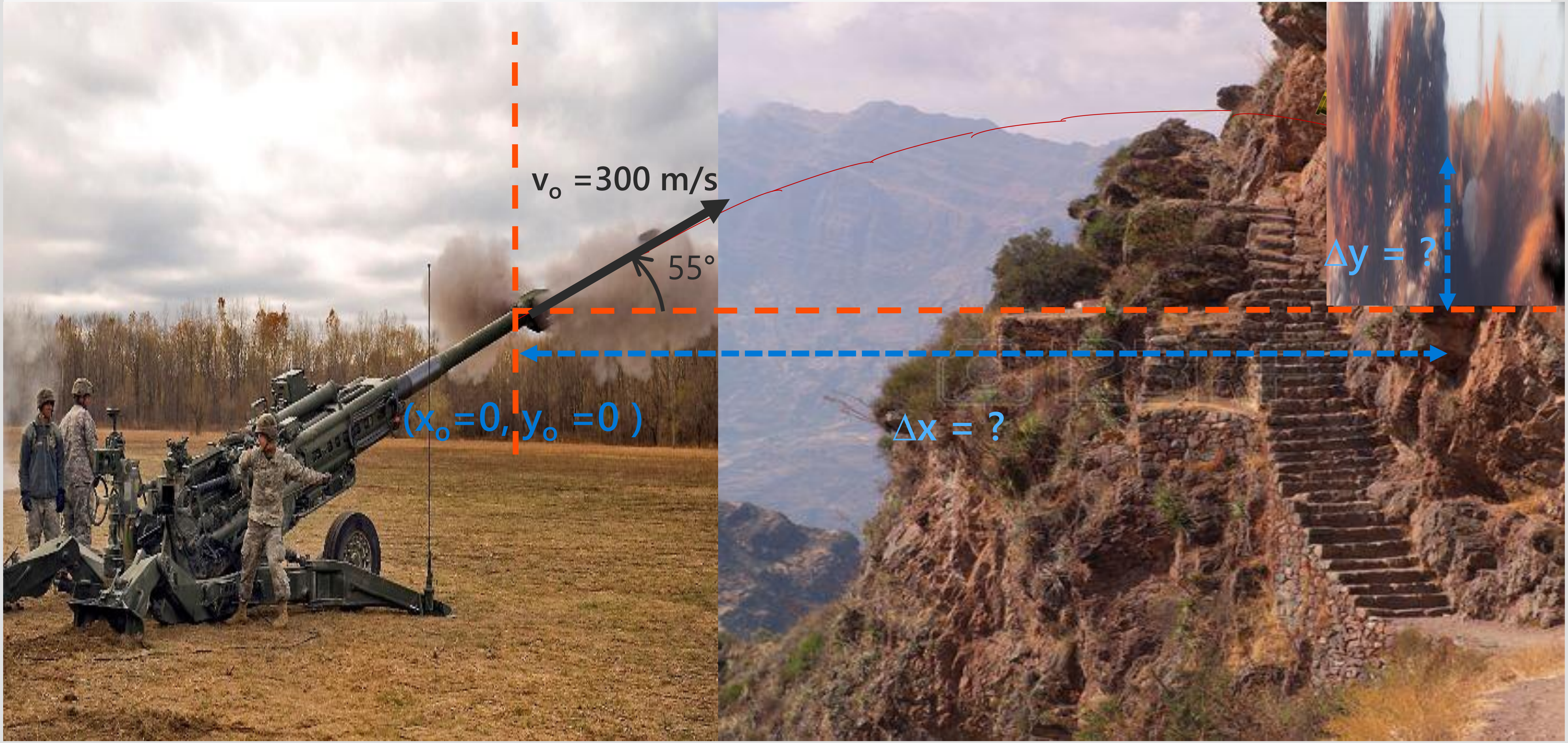
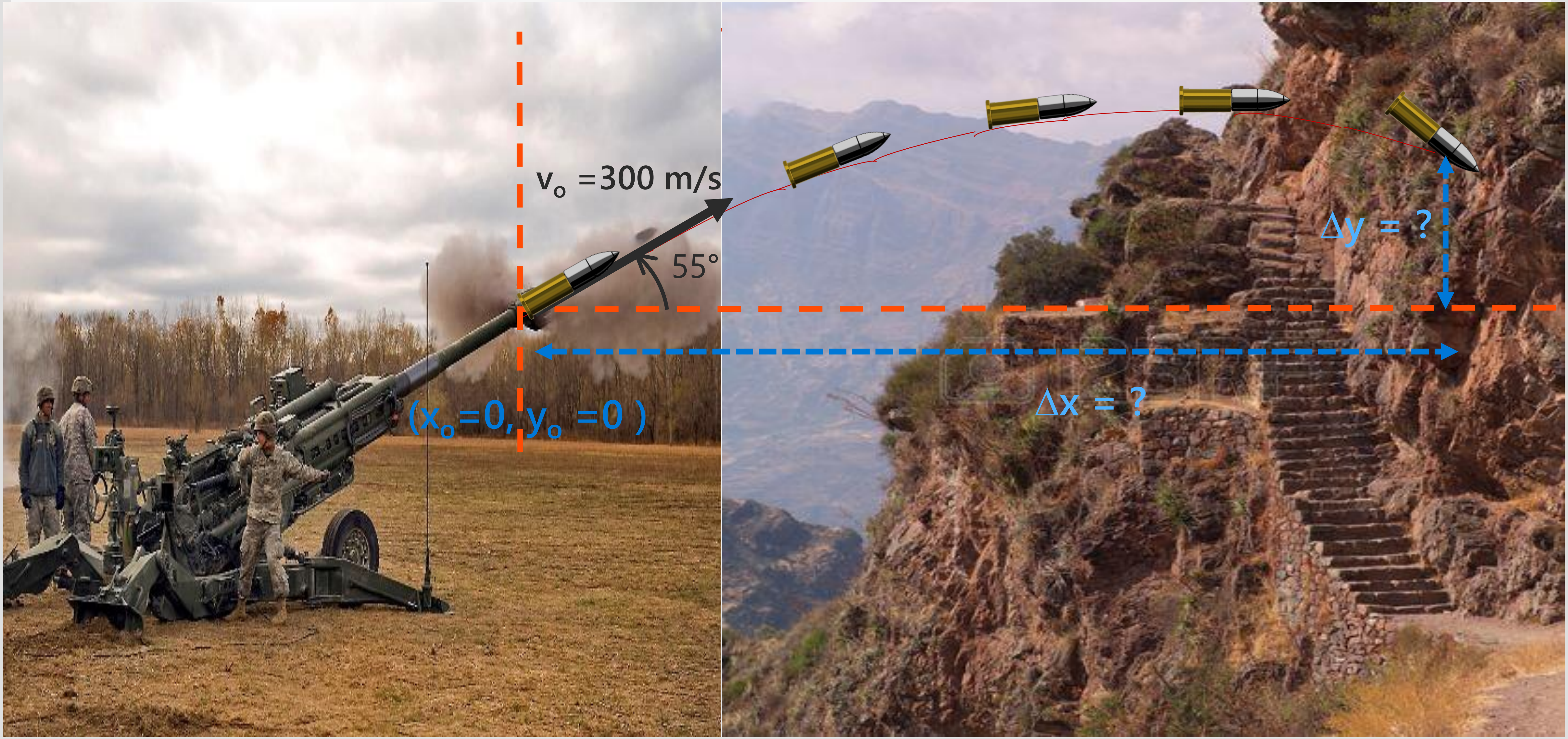
Example:

t=42 s



Example:

t=42 s



Solution

Firing point at $(x_o = 0, y_o = 0)$

Time of flight t = 42 s

x-motion: $v_{ox} = v_o \cos\theta_o$ = (300m/s)cos55º =172 m/s

y-motion: $v_{oy} = v_o \sin\theta_o$ = (300m/s)sin55º =245.7 m/s

x-motion: $\Delta x = v_{ox}\, t$ = (172 m/s)(42 s) = 7224 m

y-motion: $\Delta y = v_{oy}\, t - 0.5\, g\, t^2$ = (245.7m/s)(42 s) – 0.5 (10)(42)^2 =1501.3 m

Example:

A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8 m/s at an angle of 20^o below horizontal. It strikes the ground 3 s later.

(a) How far horizontally from the base of the building does the ball strike the ground?

(b) Find the height from which the ball was thrown?

(c) How long does it take the ball to reach a point 10 m below the level of launching?

Hint: Take g=10 m/s^2

Solution

(a) The horizontal distance from the base of the building can be obtained using $\Delta x = v_{ox}\, t_C$

$v_{ox} = v_o \cos\theta_o$ =(8m/s)cos 20º =7.5 m/s

$\therefore\ \Delta x$ =(7.5 m/s)(3 s) $\Rightarrow \Delta x$ = 22.5 m

(b) The height of the building can be found as follows: $\Delta y = v_{oy}\, t_C - 0.5\, g\, t_c^2$

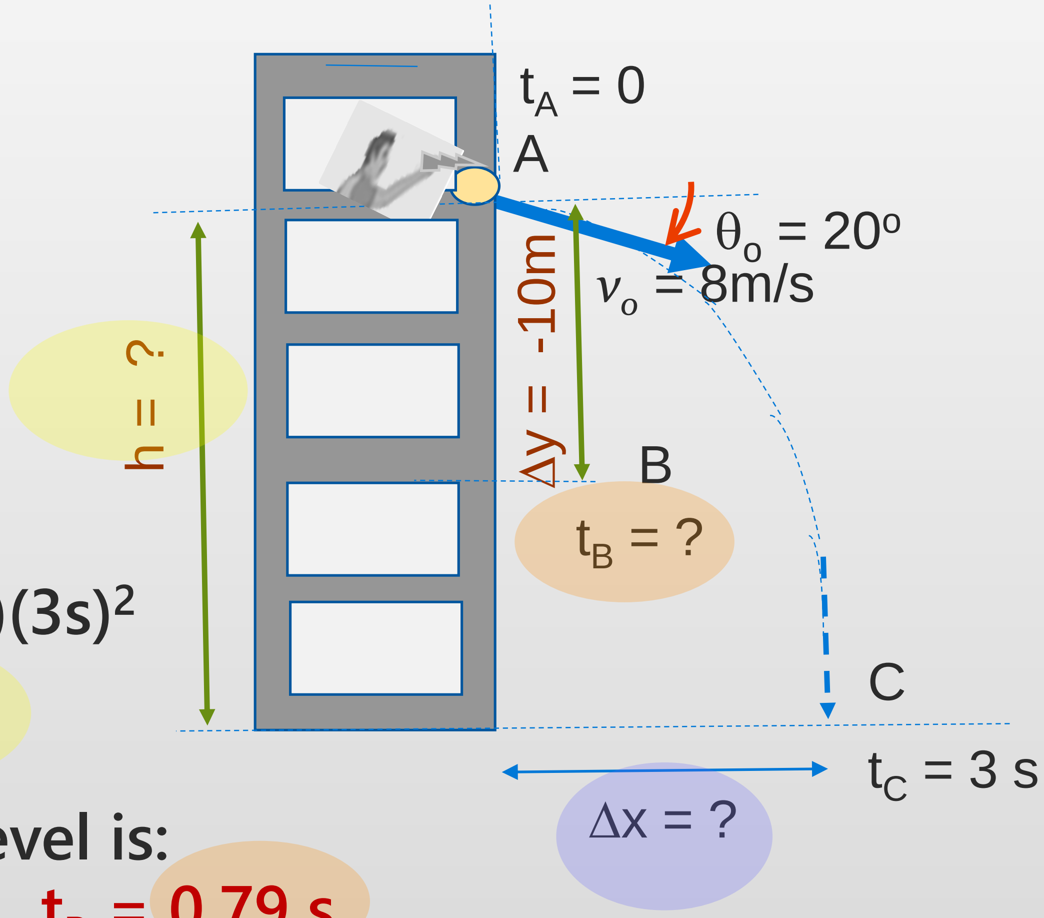
$v_{oy} = v_o \sin\theta_o$ = - (8m/s) sin 20º = - 2.74 m/s

$\Delta y = v_{oy}\, t_C - 0.5\, g\, t_c^2$ =(- 2.74 m/s)(3 s) – 0.5 (10m/s²)(3s)²

Δy =- 53.2 m $\Rightarrow h$ = 53.2 m

(c) The time it takes to be 10 m below throwers level is:

- 10 m= =(- 2.74 m/s)t_B – 0.5 (10m/s²)t_B^2 ➡ t_B = 0.79 s



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Exercise

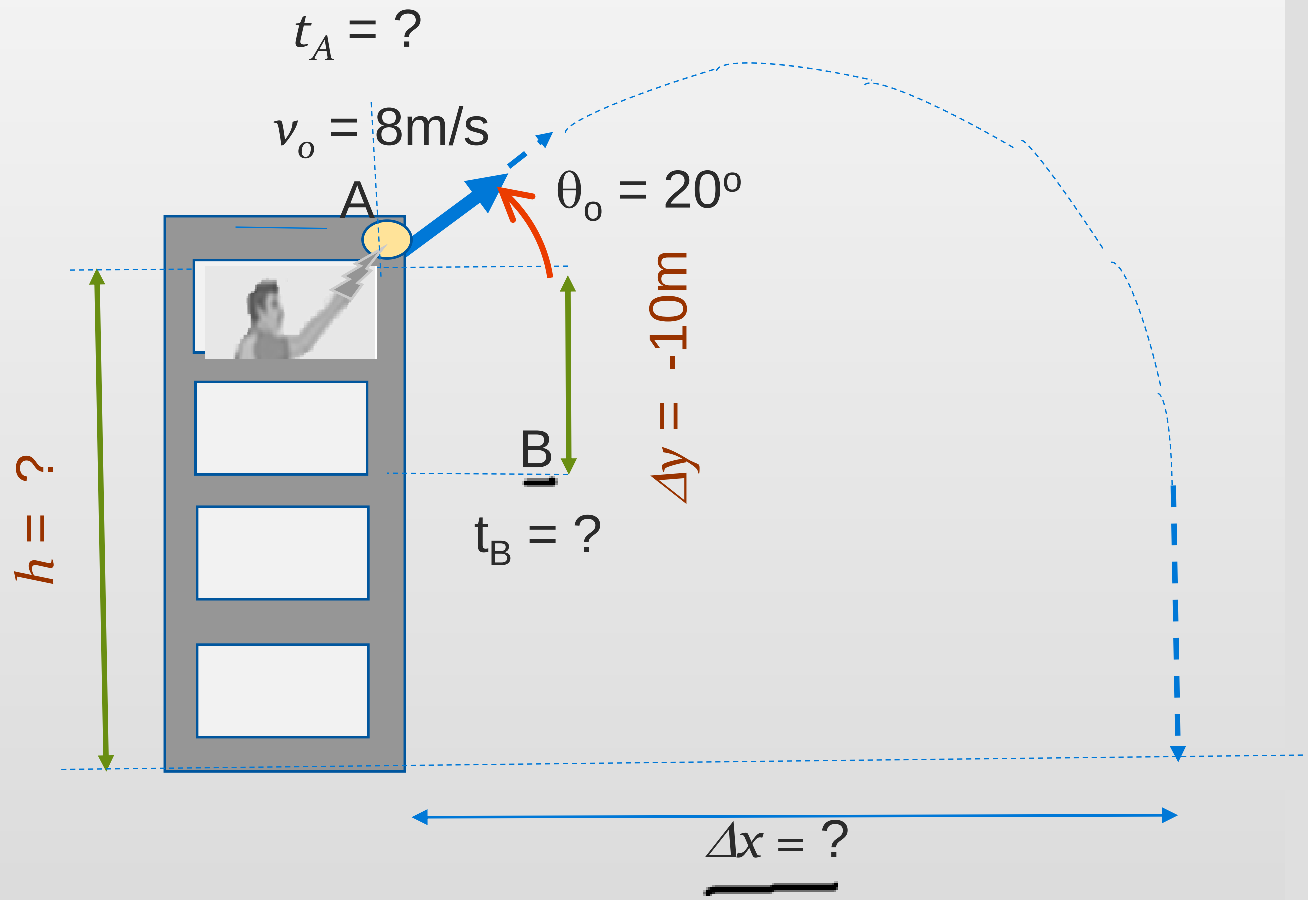
Hint: Take g=10 m/s²

A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8 m/s at an angle of 20º above horizontal. It strikes the ground 4 s later.

(a) How far horizontally from the base of the building does the ball strike the ground?

(b) Find the height from which the ball was thrown?

(c) How long does it take the ball to reach a point 10 m below the level of launching?



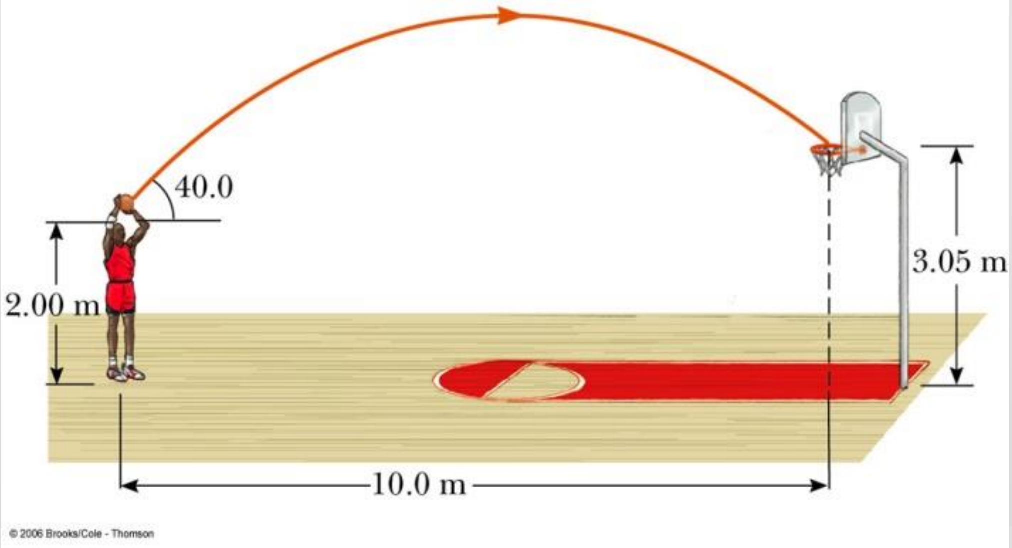
Answer: (a) 30 m, (b) 69 m, (c) 1.7 s

Exercise

A basketball player is standing on the floor 10 m from the basket as shown. The height of the basket is 3.05 m, and he shoots the ball at a 40º angle with the horizontal from a height of 2 m.

(a) What is the acceleration of the basketball at the highest point in its trajectory?

(b) At what speed must the player throw the basketball so that the ball goes through the hoop without striking the backboard?



Answer: (a) $-10\ m/s^2$, (b) $10.77 m/s$

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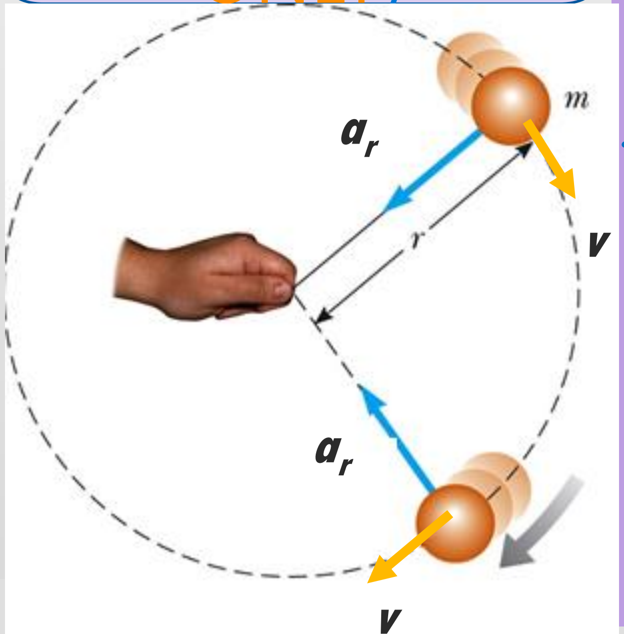
Circular Motion:Kinematics

Uniform Circular Motion (Centripetal Acceleration ONLY)

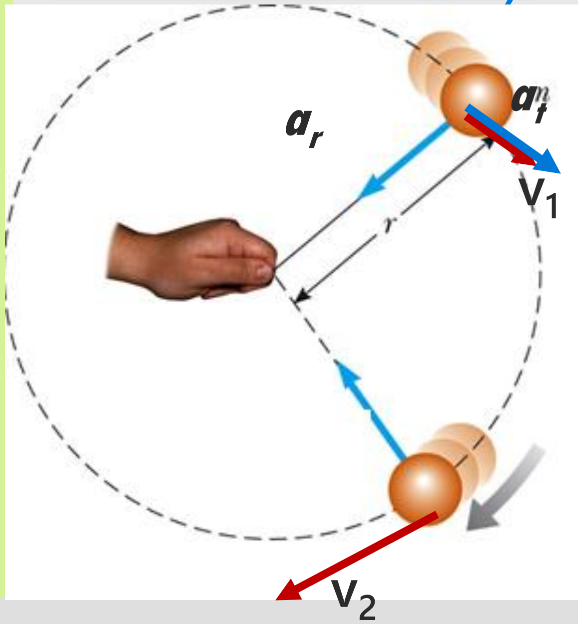
Constant speed, or, constant magnitude of velocity



Motion along a circle: Changing direction of velocity

Non-Uniform Circular Motion (Centripetal and Tangential Accelerations)

Changing speed, or, Changing magnitude of velocity

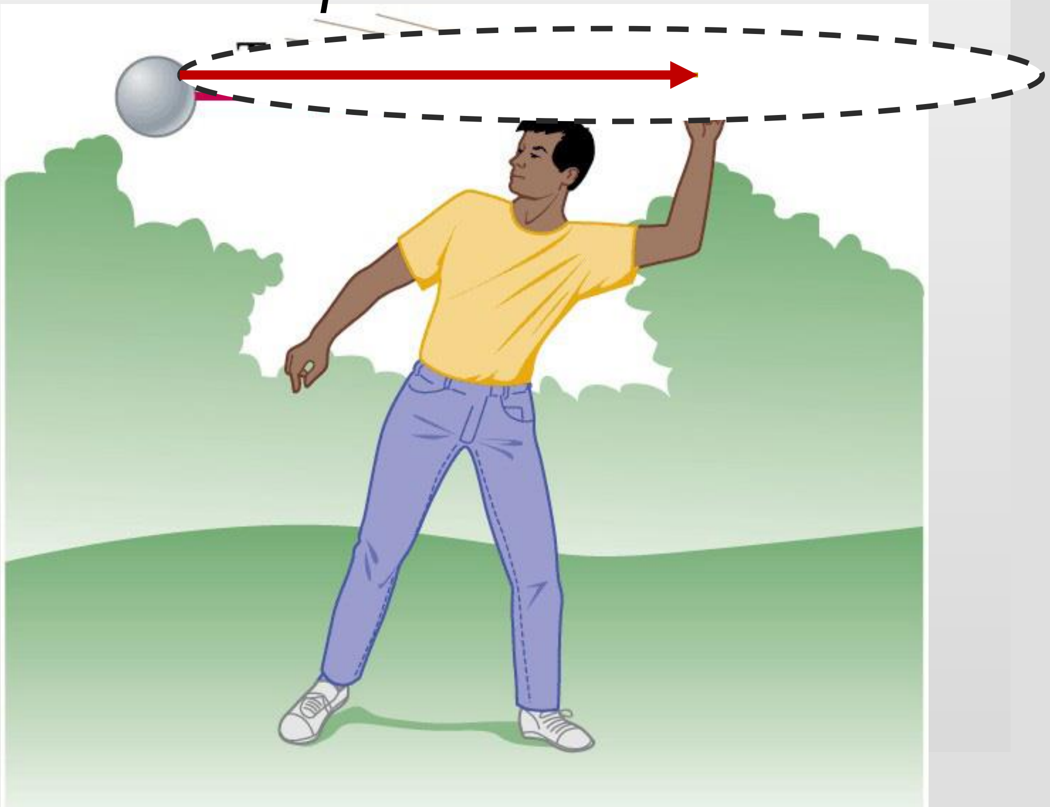


Motion along a circle: Changing direction of velocity

Uniform Circular Motion

- Object moving along a curved path with **constant speed**
  - Magnitude of velocity: same
  - Direction of velocity: changing
  - Velocity: changing
  - Acceleration is NOT zero!
  - **Net force acting on the object is NOT zero**
  - **"Centripetal force"**

$$a_r = -\frac{v^2}{r}$$



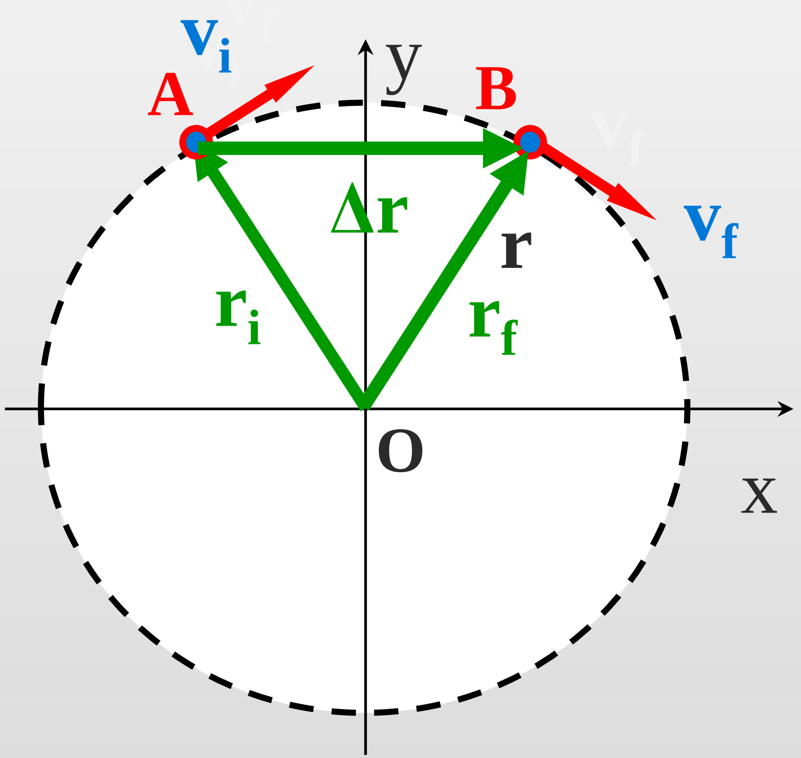
Uniform Circular Motion

- Centripetal acceleration

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \qquad so, \quad \Delta v = \frac{v \Delta r}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta r}{\Delta t}\frac{v}{r} = \frac{v^2}{r}$$

$$a_r = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$



- Direction: **Centripetal**

Uniform Circular Motion

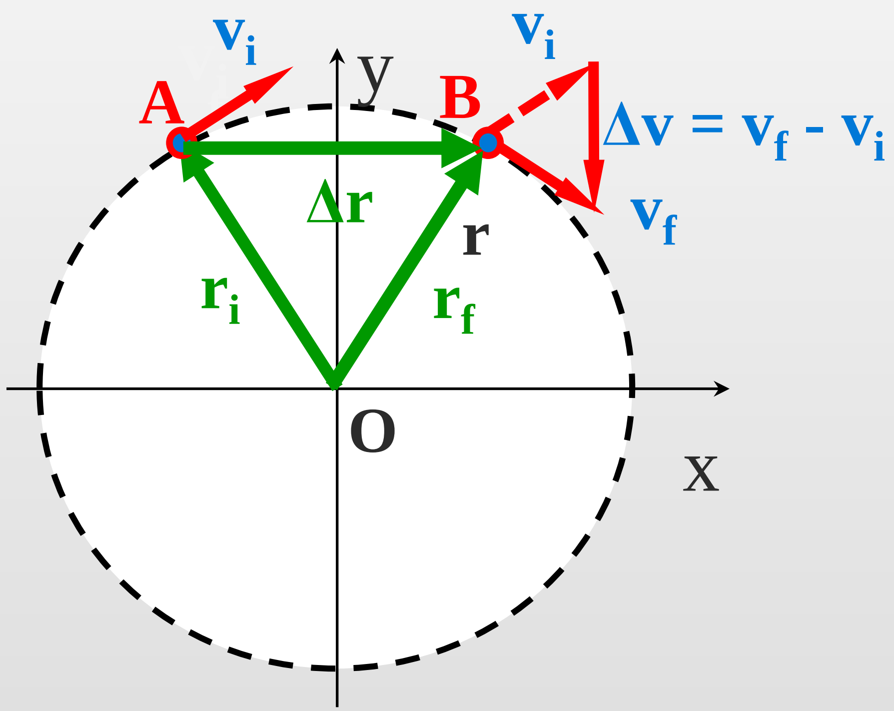
v_i is perpendicular to r_i and v_f is also perpendicular to r_f but $r_i = r_f = r$. The angle between r_i and r_f is same as the angle between v_i and v_f . Thus red triangle is similar to green triangle

- Centripetal acceleration

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \quad \text{so,} \quad \Delta v = \frac{v\Delta r}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta r}{\Delta t}\frac{v}{r} = \frac{v^2}{r}$$

$$a_r = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$



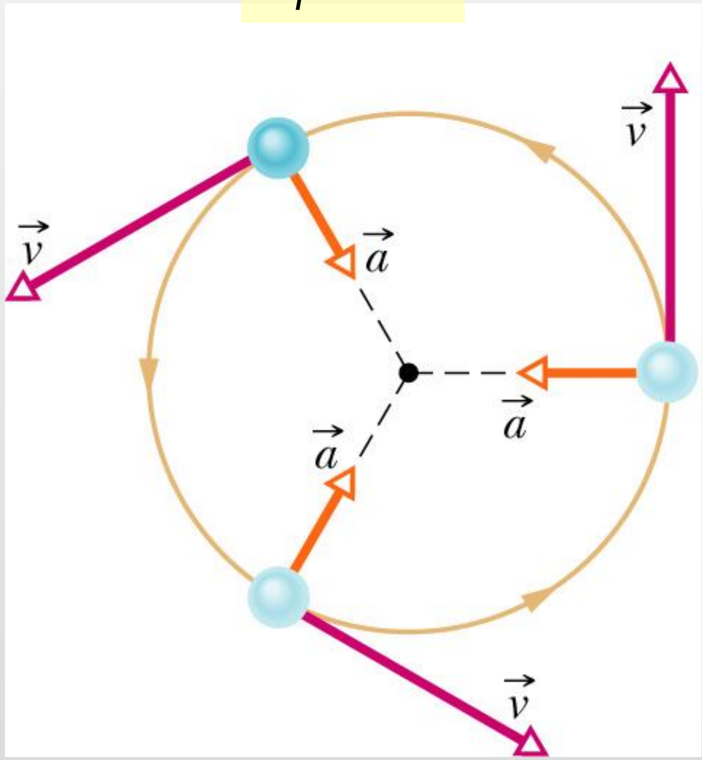
- Direction: **Centripetal**

Notes: Triangles are similar when corresponding angles are congruent and corresponding sides are all in the same proportion

Uniform Circular Motion

- Velocity:
  - Magnitude: constant v
  - The direction of the velocity is tangent to the circle
- Acceleration:
  - Magnitude: $$a_r = -\frac{v^2}{r}$$
  - Directed toward the center of the circle of motion as indicated by a minus
- Period:
  - Time interval required for one complete revolution of the particle $$T = \frac{2\pi r}{v}$$

$$\vec{a}_r \perp \vec{v}$$



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Uniform Circular Motion

Example:

A Tire-Balancing Machine

The wheel of a car has a radius of r = 0.29m and is being rotated at 830 revolutions per minute (rpm) on a tire-balancing machine. Determine the speed (in m/s) at which the outer edge of the wheel is moving.

The speed v can be obtained directly from $T = \frac{2\pi r}{v}$, but first the period T is needed. It must be expressed in seconds.

By Prof. Rashad Badran

Uniform Circular Motion

Solution

A Tire-Balancing Machine

830 revolutions in one minute

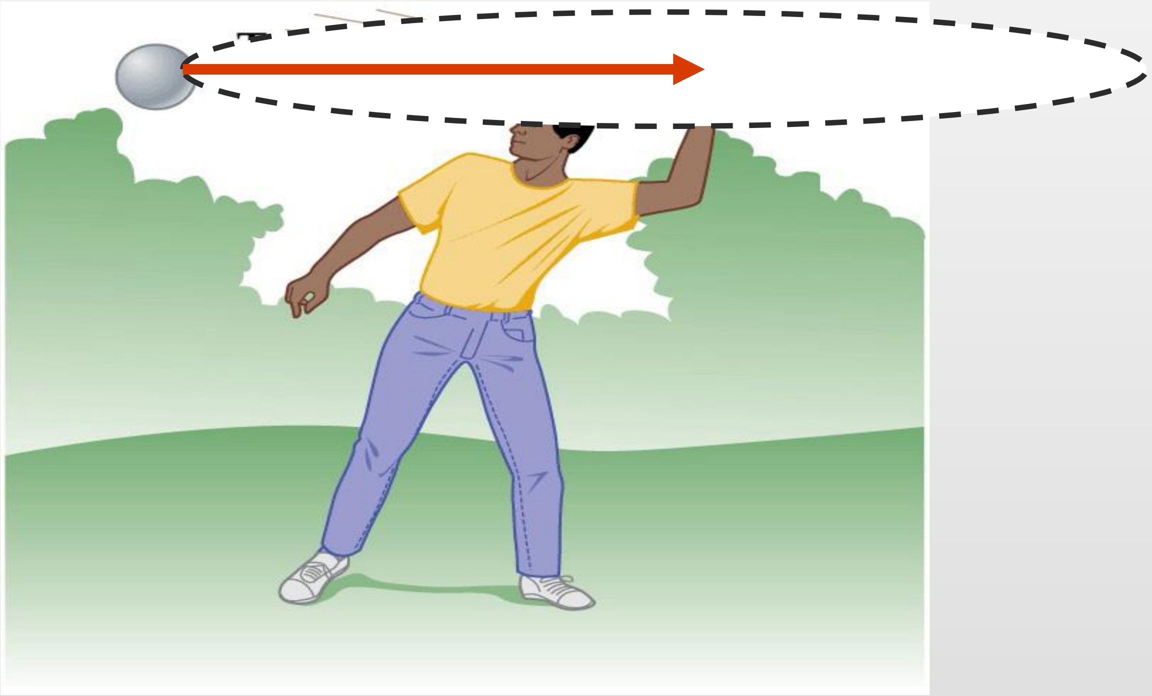
$$\frac{1}{830\ \ revolutions/\min} = 1.2\times 10^{-3}\ \min/\ revolution$$

T=1.2 ×10^{-3} min, which corresponds to 0.072s

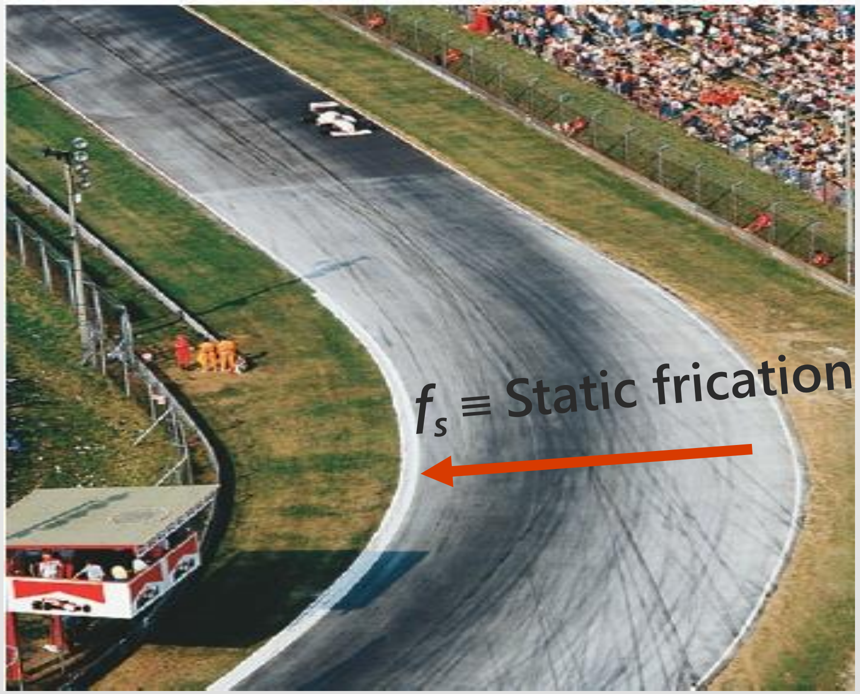
$$v = \frac{2\pi r}{T} = \frac{2\pi(0.29m)}{0.072s} = 25m/s$$

Uniform Circular Motion

T≡ Tension in a rope



For the mass attached to a rope, when the rope is cut the circular motion vanishes and the object tends to go in a straight line



On Highway curves, If the frictional force is insufficient, the car will tend to move more nearly in a straight line, as the skid marks show.

Non-Uniform Circular Motion (Centripetal and Tangential Accelerations)

This concept can be used for an object moving along any curved path, as any small segment of the path will be approximately circular.

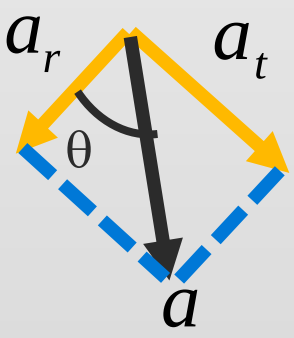
$$\vec{a}_r \perp \vec{v} \qquad \vec{a}_r \perp \vec{a}_t$$

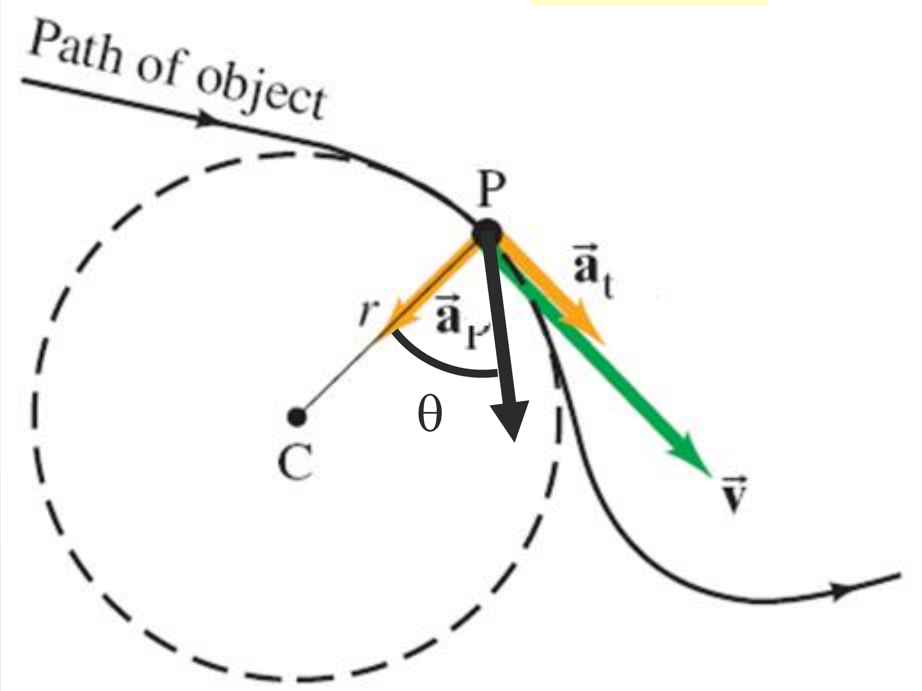
$$\vec{a} = a_r\hat{r} + a_t\hat{\theta}$$

$$a_t = \frac{dv}{dt} \qquad a_r = -\frac{v^2}{r}$$

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2}$$

$$\theta = \arctan\frac{a_t}{a_r}$$





Non-Uniform Circular Motion (Centripetal and Tangential Accelerations)

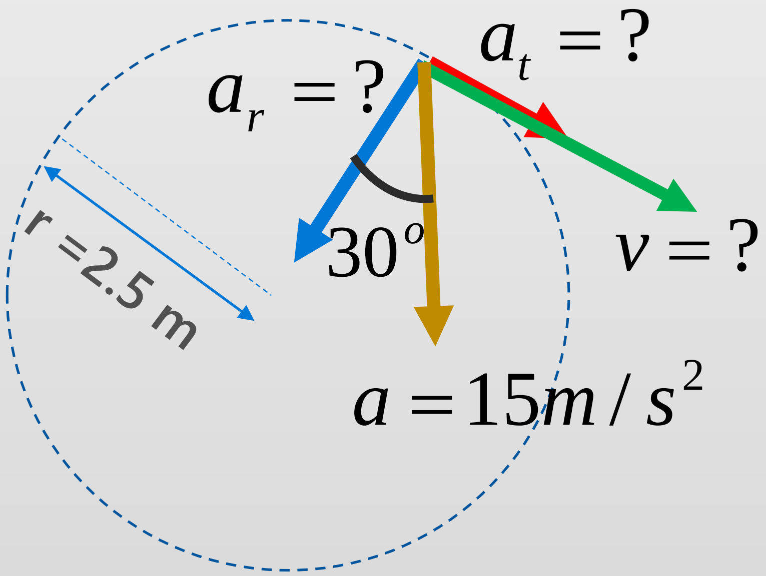
Problem:

The total acceleration of a particle moving clockwise in a circle of radius 2.5 m at a certain instant of time, is shown in the figure. For that instant, find

(a) The radial acceleration of the particle,

(b) the speed of the particle, and

(c) Its tangential acceleration



Non-Uniform Circular Motion (Centripetal and Tangential Accelerations)

Solution

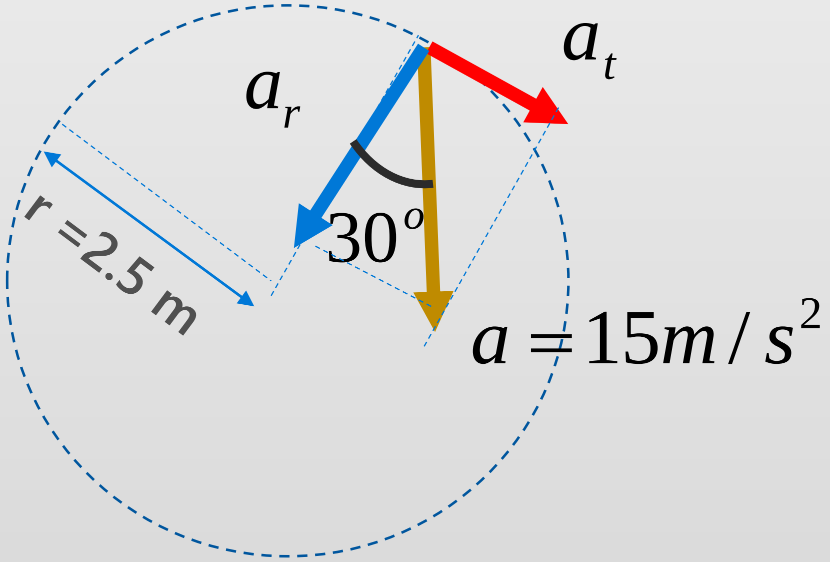
(a) To find the radial acceleration of the particle

$a_r = a \cos 30^o$ = (15 m/s^2)(0.866) = -13 m/s^2

(b) To find the speed of the particle

$$a_r = \frac{v^2}{r}$$

$$v^2 = ra_r = (2.5m)(13m/s^2) \Rightarrow v = 5.7m/s$$



(c) To find the tangential acceleration

$a_t = a \sin 30^o$ = (15 m/s^2)(0.5) = 7.5 m/s^2

Non-Uniform Circular Motion (Centripetal and Tangential Accelerations)

Exercise:

A train slows down as it rounds a sharp horizontal turn, going from 90 km/h to 50 km/h in the 15 s it takes to round the bend. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50km/h. Assume the train continues to slow down at this time at the same rate.

Answer: 1.48 m/s^2 and 29.9° backward

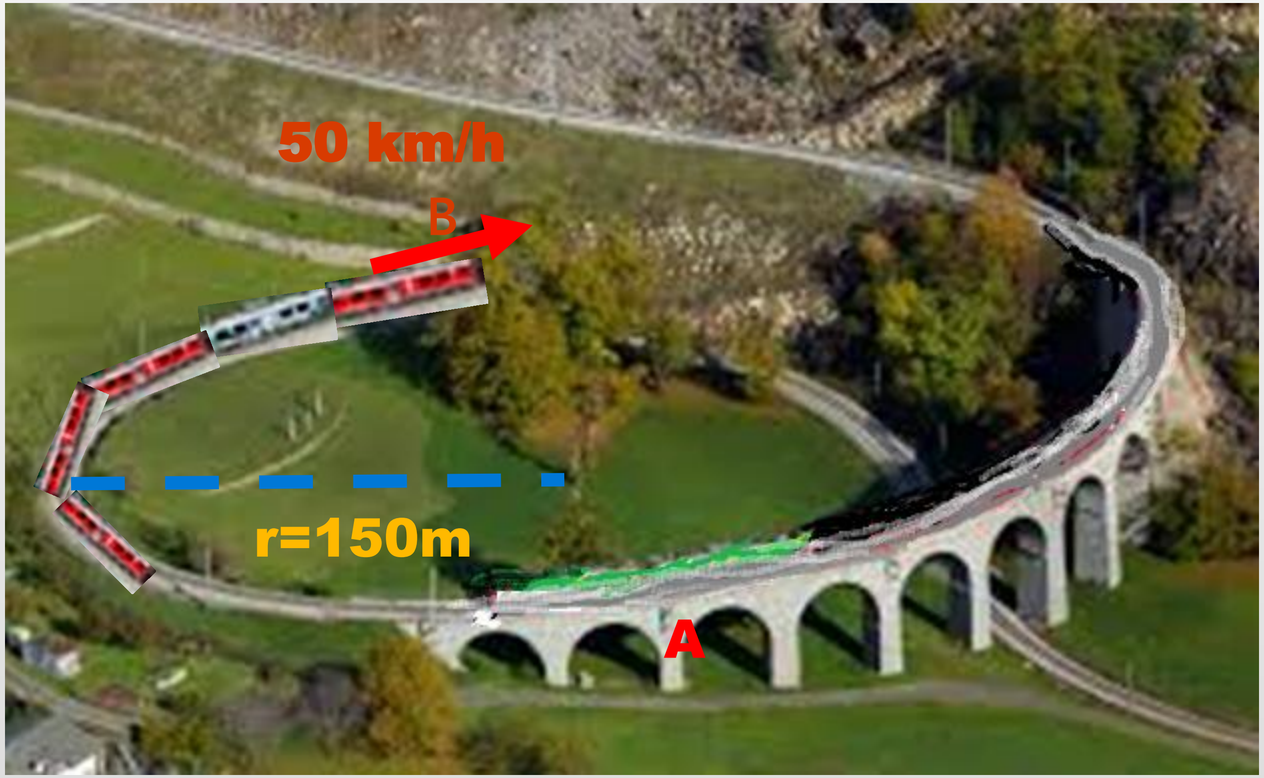
Non-Uniform Circular Motion (Centripetal and Tangential Accelerations)

Exercise:

Non-Uniform Circular Motion (Centripetal and Tangential Accelerations)

Exercise:

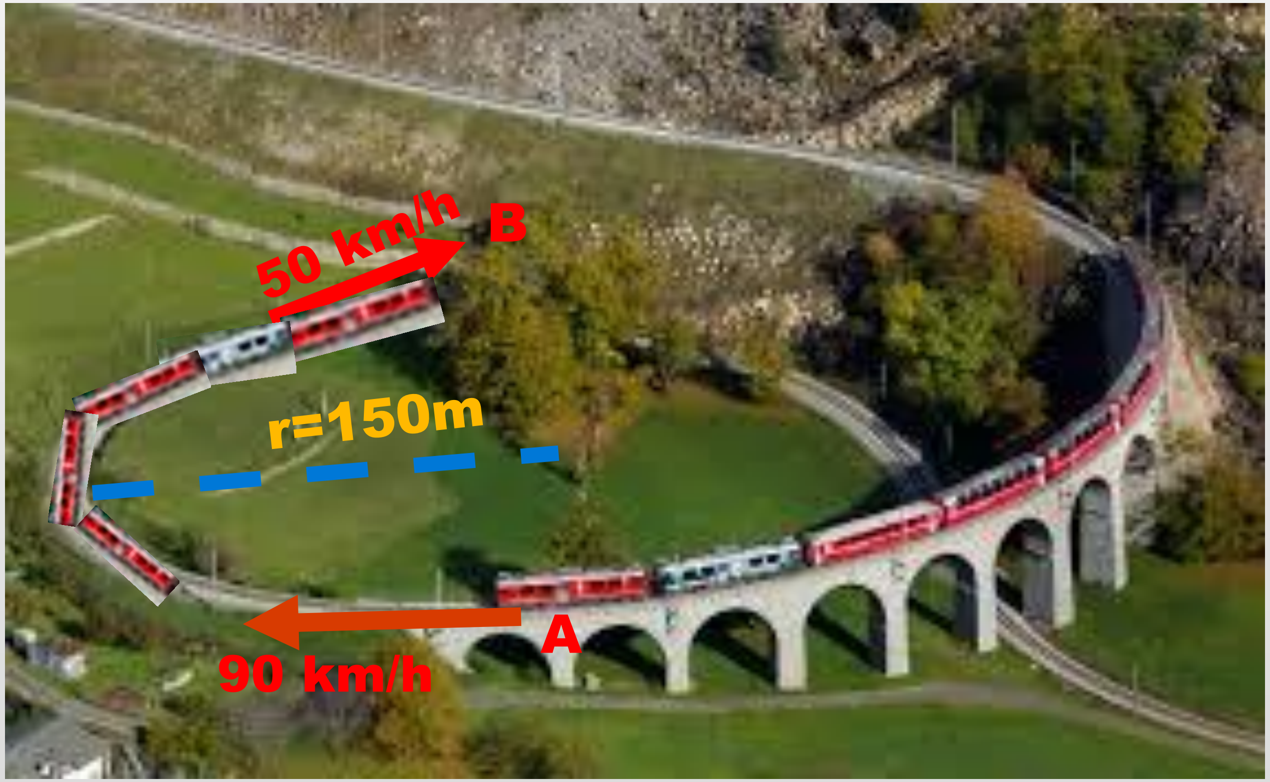
$t_{A \to B} = 15\ s$



Non-Uniform Circular Motion (Centripetal and Tangential Accelerations)

Exercise:

$t_{A \to B} = 15$ s



Non-Uniform Circular Motion (Centripetal and Tangential Accelerations)

Solution

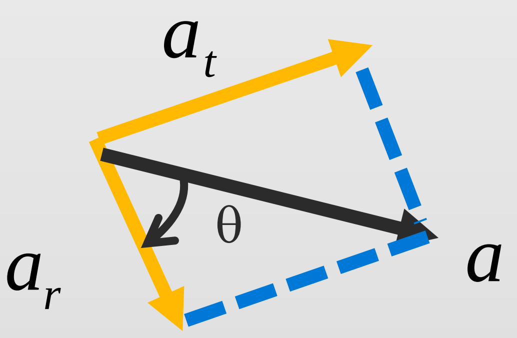
$$v_A = (90 km/h)(10^3 \frac{m}{km})(\frac{1h}{3600s}) = 25 m/s$$

$$v_B = (50 km/h)(10^3 \frac{m}{km})(\frac{1h}{3600s}) = 13.9 m/s$$

$$a_r = \frac{v^2}{r} = \frac{(13.9 m/s)^2}{150 m} = 1.29 m/s^2$$

$$a_t = a_{avg} = \frac{v_B - v_A}{t_B - t_A} = \frac{(13.9 - 25) m/s}{15 s} = -0.74 m/s^2$$

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{(12.9)^2 + (-0.74)^2} = 1.48 m/s^2$$



$$\theta_a = \arctan \frac{a_t}{a_r} = \arctan \frac{-0.74}{1.29} = -29.9^o$$

Note: The negative sign indicates that the direction of total acceleration is opposite to train motion