Bravais lattices (three-dimensional crystals):

There are 14 different Bravais lattice types. They are grouped into 7 lattice systems due to the similarity in the point group of symmetry elements. (See table 8 and figure 53).

We put here some emphasis on the cubic lattices.

1. Cubic Lattices: $|\vec{a}_1| = |\vec{a}_2| = |\vec{a}_3|$ and $\alpha = \beta = \gamma = \pi/2$

i) They are lattices with unit cells in cubes. They have the highest symmetry (see figure 54).

[i.e. the greatest number of symmetry elements of all lattice types (total of 23 symmetry elements)].

ii) 3 four-fold axes (tetrads) $\Rightarrow 3C_4$.

iii) 4 three-fold axes (triads) $\Rightarrow 4C_3$.

iv) 6 two-fold axes (diads) $\Rightarrow 6C_2$.

v) 9 mirror planes through the center of each cube. One inversion symmetry at body center.

2. Tetragonal lattices: $|\vec{a}_1| = |\vec{a}_2| \neq |\vec{a}_3|$ and $\alpha = \beta = \gamma = \pi/2$

a) Simple tetragonal lattice:

This lattice may be resulted by compressing or elongating a simple cubic lattice along one of its 4-fold axes. [Two of the 4-fold axes of the cube become 2-fold and the body diagonal are no longer symmetry axes].

Symmetry requirements: (See figure 55)

i) One 4-fold axes

ii) Four 2-fold axes.

iii) Five mirror planes through the center.

iv) Four 2-fold axes. (Two of them pass through centers of opposing rectangles; while the other 2-fold passes through centers of diametrically opposed rectangle edges).

v) Five mirror planes. (Two of them cut the square along diagonals plus two cut it from edge centers to edge center plus one parallel to the base and cuts rectangular cell sides at their mid points).

b) Body-centered tetragonal lattice:

Same lattice points as in (*a*) in addition to one lattice point at the body center.

The primitive lattice vectors of tetragonal lattice cell are:

$$\vec{a}_1 = \frac{a}{2}(\hat{x}+\hat{y}) - \frac{c}{2}\hat{z};\ \vec{a}_2 = \frac{a}{2}(-\hat{x}+\hat{y}) + \frac{c}{2}\hat{z} \text{ and } \vec{a}_3 = \frac{a}{2}(\hat{x}-\hat{y}) + \frac{c}{2}\hat{z}.$$

c) Face-centered tetragonal lattice: This describes the same lattice as a body-centered tetragonal cell (both bc and fc tetragonal are of the same Bravais lattice type).

3) Orthorhombic lattices: $|\vec{a}_1| \neq |\vec{a}_2| \neq |\vec{a}_3|$ and $\alpha = \beta = \gamma = \pi/2$

In this type of lattices all six faces of primitive orthorhombic unit cell are rectangles and cell sides which are perpendicular to the base. It can be resulted from a distorting tetragonal lattice along one or both of the 2-fold axes through rectangular face centers. The base is then distorted from a square to a rectangle, thus the 4-fold axes of previous lattice become three 2-fold axes.

Symmetry requirements: (See figure 55)

i) Three 2-fold axes which pass through centers of opposing rectangles.

ii) Three mirror planes, each of which is perpendicular to a 2-fold axis and passes through the cell body center.

Four types of Orthorhombic lattices:

a) Primitive Orthorhombic lattice.
b) Base centered Orthorhombic lattice.
c) Body centered Orthorhombic lattice.
d) Face centered Orthorhombic lattice.

4) Monoclinic lattices: $|\vec{a}_1| \neq |\vec{a}_2| \neq |\vec{a}_3|$ and $\alpha = \beta = \gamma \neq \pi/2$

It has a base as oblique parallelogram and rectangular sides perpendicular to the base.

Symmetry requirements: (See figure 55)

i) One 2-fold axis perpendicular to oblique base.
ii) One mirror plane parallel to oblique base and through the center of cell.

Types of monoclinic lattices:

a) Primitive monoclinic lattice.
b) Body centered monoclinic lattice.
c) Face centered monoclinc lattice. (Note: fc and bc monoclinic lattices describe each other and are of the same type).
d) Base centered monoclinic lattice: (It must be note here that primitive and base centered monoclinic lattices describe each other and are of the same type).

5) Triclinic lattices: $|\vec{a}_1| \neq |\vec{a}_2| \neq |\vec{a}_3|$ and $\alpha \neq \beta \neq \gamma \neq \pi/2$

No symmetry axes and no mirror planes exist in these lattices.

6) Hexagonal lattices: $|\vec{a}_1| = |\vec{a}_2| \neq |\vec{a}_3|$, $\alpha = \beta = \pi/2$ and $\gamma = 2\pi/3$

The base of a primitive unit cell is a 60° rhombus and the sides are rectangles perpendicular to the base.

Symmetry requirements: (See figure 55)

i) 6-fold axes parallel to the sides through the cell center.

ii) Six 2-fold axes: Each of which pass through the cell center and either a face or an edge center.

iii) Mirror plane perpendicular to each 2-fold axis and a mirror plane parallel to the base half way between the base and top of the cell.

7) Trigonal lattices: $|\vec{a}_1| = |\vec{a}_2| = |\vec{a}_3|$ and $\alpha = \beta = \gamma \neq \pi/2$

This is a distorted cubic unit cell along a body diagonal so that the cube edge move towards or a way from the diagonal.

Symmetry requirements: (See figure 55)

i) 3-fold axes.

ii) Three 2-fold axes: Each of which crosses the unit cell from the center of an edge to the center of the opposite edge.

iii) Three mirror planes: Each cuts across a face along a diagonal and passes through the cell center.

Table 8: The seven crystal systems divided into 14 Bravais lattices.

	System	Bravais lattice	Unit cell characteristics	Symmetry element characteristics
1.	Cubic	Simple Body-centered Face-centered	$\lvert\vec{a}_1\rvert = \lvert\vec{a}_2\rvert = \lvert\vec{a}_3\rvert$ $\alpha = \beta = \gamma = \pi/2$	Four 3-fold rotation axes (along cube diagonal)
2.	Tetragonal	Simple Body-centered	$\lvert\vec{a}_1\rvert = \lvert\vec{a}_2\rvert \neq \lvert\vec{a}_3\rvert$ $\alpha = \beta = \gamma = \pi/2$	One 4-fold rotation axis
3.	Orthorhombic	Simple Base-centered Body-centered Face-centered	$\lvert\vec{a}_1\rvert \neq \lvert\vec{a}_2\rvert \neq \lvert\vec{a}_3\rvert$ $\alpha = \beta = \gamma = \pi/2$	Three mutually orthogonal 2-fold rotation axes
4.	Monoclinic	Simple Base-centered	$\lvert\vec{a}_1\rvert \neq \lvert\vec{a}_2\rvert \neq \lvert\vec{a}_3\rvert$ $\alpha = \beta = \gamma \neq \pi/2$	One 2-fold rotation axis
5.	Triclinic	Simple	$\lvert\vec{a}_1\rvert \neq \lvert\vec{a}_2\rvert \neq \lvert\vec{a}_3\rvert$ $\alpha \neq \beta \neq \gamma \neq \pi/2$	None
6.	Hexagonal	Simple	$\lvert\vec{a}_1\rvert = \lvert\vec{a}_2\rvert \neq \lvert\vec{a}_3\rvert$ $\alpha = \beta = \pi/2$, and $\gamma = 2\pi/3$	One 3-fold rotation axis
7.	Trigonal (Rhombohedral)	Simple	$\lvert\vec{a}_1\rvert = \lvert\vec{a}_2\rvert = \lvert\vec{a}_3\rvert$ $\alpha = \beta = \gamma \neq \pi/2$	One 3-fold rotation axis

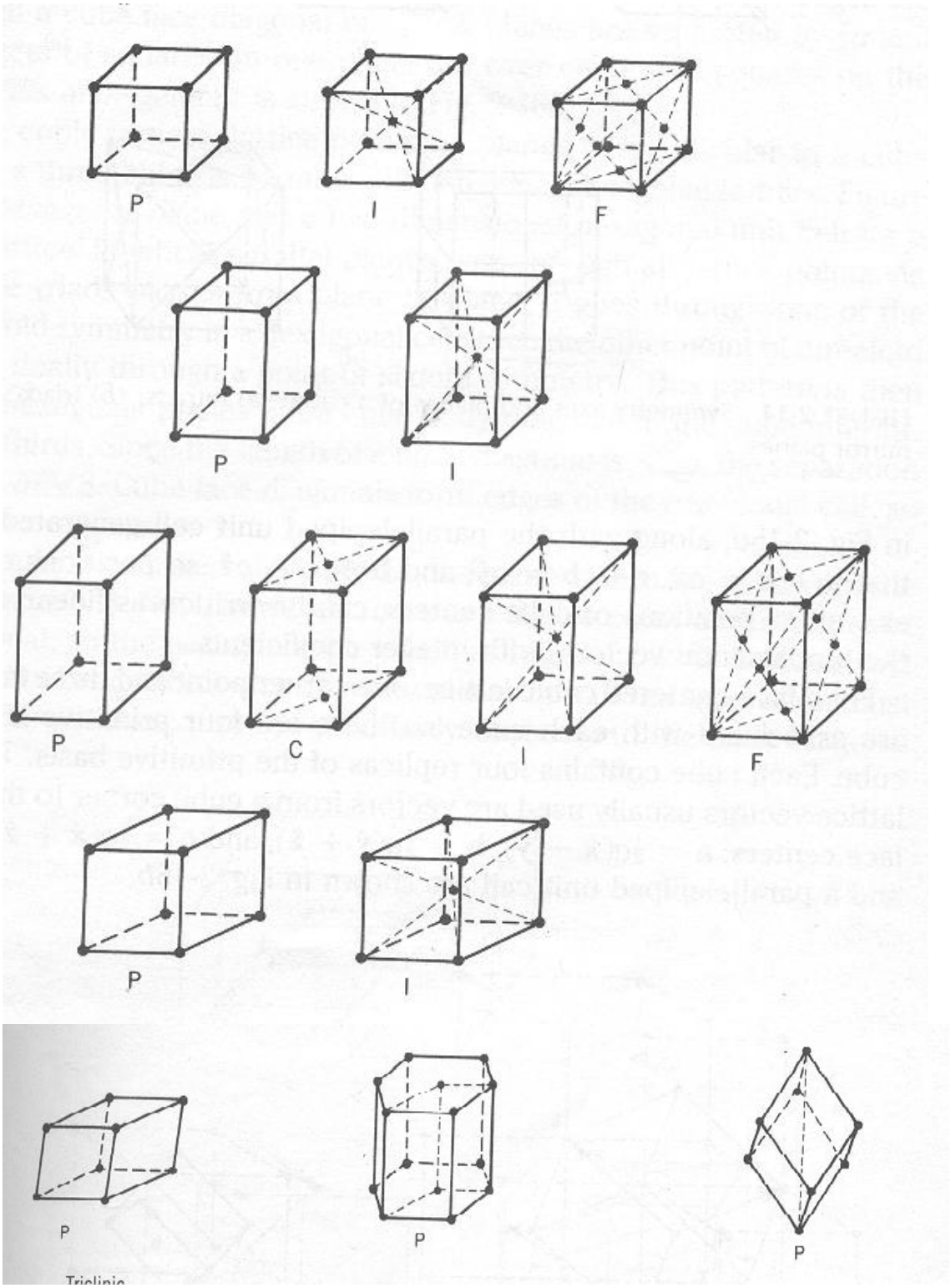

Figure 53: The 7 crystal systems divided into 14 Bravais lattices

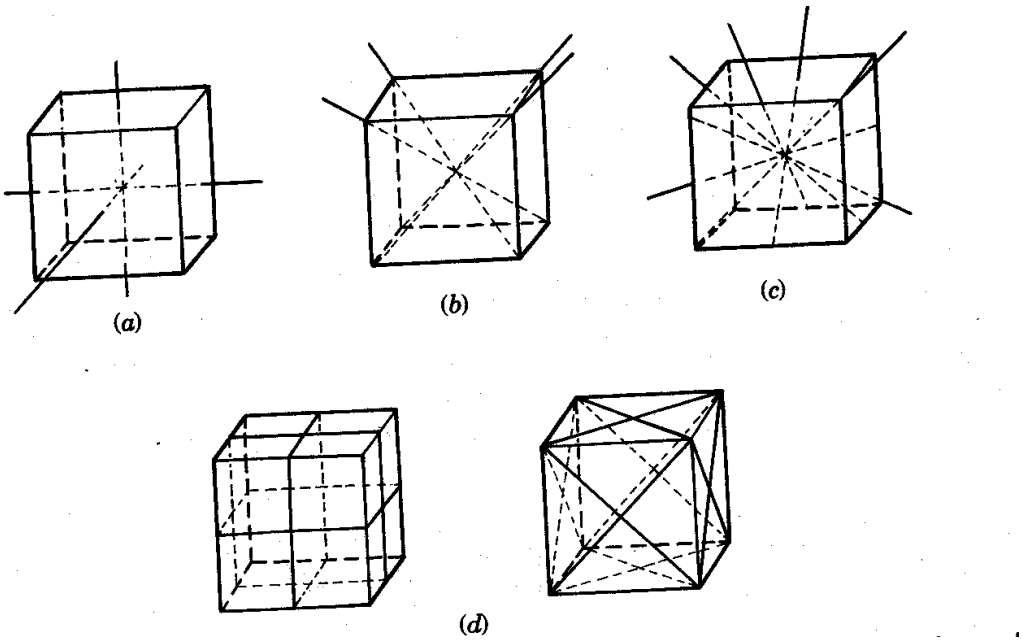

Figure 54: The symmetry axes and planes of a cube which include diads, triads, tetrads and mirror planes.

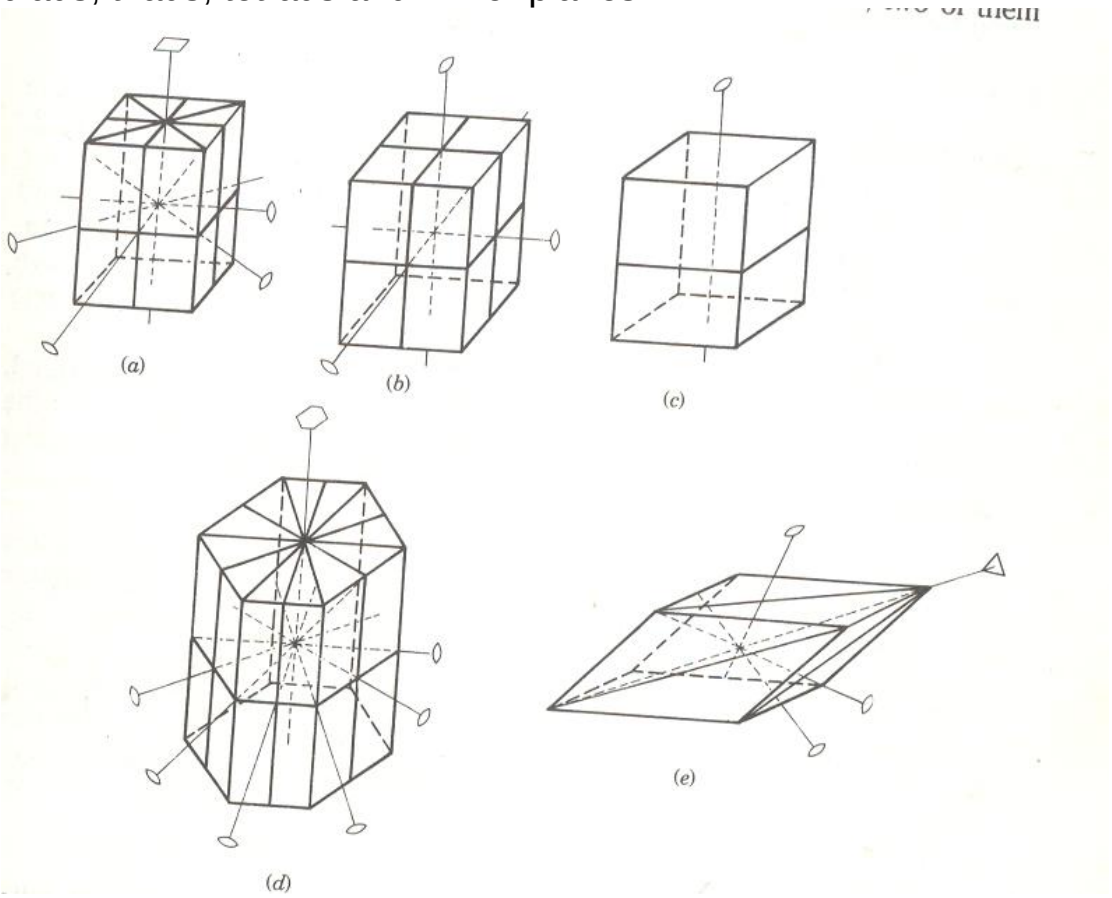

Figure 55: The symmetry elements of unit cells which include tetragonal, orthorhombic, monoclinic, hexagonal and trigonal.