CH 13: Gears-General

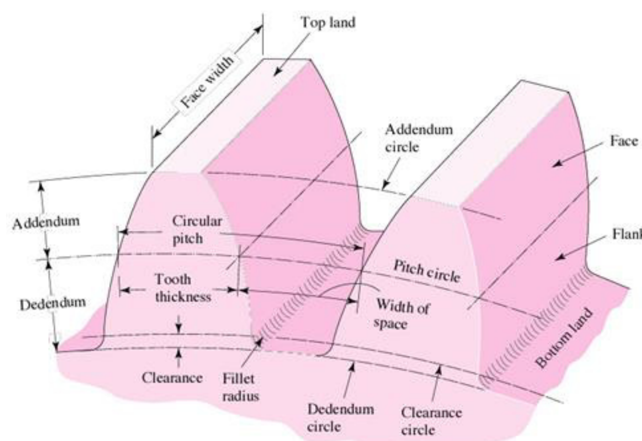
Types of Gears

There are four principal types of gears:

- *Spur gears*: (*Fig. 13-1*) the simplest type of gears. The teeth are parallel to the axis of rotation. They transmit rotation between parallel shafts.
- *Helical gears*: (*Fig. 13-2*) the teeth are inclined with respect to the axis of rotation. They transmit rotation between parallel shafts, same as spur gears, but they are less noisy than spur gears because of the more gradual engagement of the teeth during meshing. Because of the inclined teeth, thrust load is developed.
- *Bevel gears*: (*Fig. 13-3*) the teeth are formed on conical surfaces. They transmit rotation between intersecting shafts. The gear shown in figure 13-3 has straight teeth. In another type, the teeth form circular arcs and it is called spiral bevel gears.
- *Worms and worm gears*: (*Fig. 13-4*) transmits rotation between perpendicular shafts. The worm resembles a screw (it can be right handed or left handed). Worm gear sets are used when speed ratios are high (*3 or more*). Rotation is transmitted from the worm to the worm gear, but not the opposite.

Nomenclature

Since spur gears are the simplest type, it will be used for illustration to develop the primary kinematic relations. The figure illustrates spur-gears.



- *Pitch circle*: the theoretical circle upon which calculations are based and its diameter is called the "pitch diameter". Pitch circles of mating gears are tangent to each other.
  - The "smaller" of the mating gears is called the *pinion* and the "larger" is called the *gear*.
- *Circular pitch* *"p"*: the distance measured on the pitch circle from point on one tooth to a corresponding point on adjacent tooth. The circular pitch is equal to the sum of tooth thickness and width of space.

- *Dedendum and Addendum circles*: the circles defining the bottom and top surfaces of the teeth.
- *Addendum* "a": the radial distance from the pitch circle to the top surface of the teeth.
- *Dedendum* "b": the radial distance from the pitch circle to the bottom surface of the teeth.
- *Clearance circle*: the circle tangent to the addendum circle of the mating gear.
- *Clearance* "c": the distance between the tooth top surface and the bottom surface of a mating gear.
- *Diametral pitch* "P": the ratio of the number of teeth of a gear to the pitch diameter.

Diametral pitch *(teeth per inch)* → $$P = \frac{N}{d}$$ ← Number of teeth; ← Pitch diameter

Circular pitch → $$p = \frac{\pi d}{N} = \frac{\pi}{P}$$

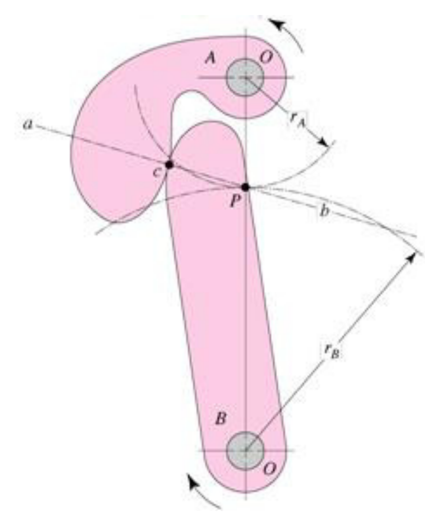
It should be clear that gears meshing with each other must have the same diametral pitch.

- *Module* "m": is the ratio of pitch diameter to the number of teeth *(it is the inverse of the diametral pitch and it is commonly used with the SI system of units)*.

$$m = \frac{d}{N} = \frac{1}{P} \quad \rightarrow \quad p = \pi m$$

Conjugate Action

The profile of gear teeth are designed such that they will produce constant angular velocity ratio during meshing, and this is called conjugate action. When one curved surface pushes against another, as seen in the figure, the point of contact occurs where the two surfaces are tangent to each other (Point *c*) and the forces will be directed along the common normal (line *ab*) which is also called the "line of action" or the "pressure line".



The line of action will intersect the line of centers at point "*P*" which also defines the point of tangency of the pitch circles of the two mating gears and it is called the pitch point.

The angular-velocity ratio is inversely proportional to the ratio of radii of the pitch circles of mating gear.

To transmit rotation at constant angular velocity the pitch point must remain fixed, meaning that all lines of action must pass through the same point "*P*".

To satisfy that, the profile of gear teeth are shaped as "involute profile".

With involute profile, all points of contact occur along the same line which is the line of action.

Involute Properties

The circle on which involute is generated is called "Base circle". An involute curve may be generated using a cord wrapped around the base circle, Fig 13-7.

** see text **

Fundamentals

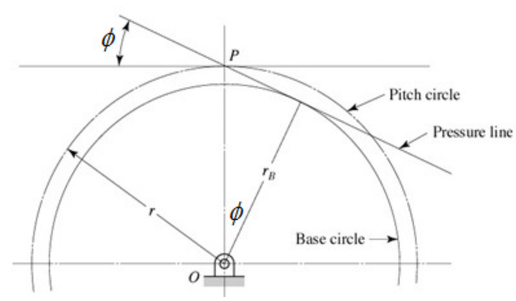
This section illustrates how to draw the teeth on a pair of meshing gears.

It's useful to study this section

- Pressure Angle "∅" is the angle between the pressure line (*line of action*) and the common tangent of the pitch circles of mating gears.
  - Common values of ∅ are *20°* or *25°*
  - Radius of base circle:

$$r_b = r\cos\emptyset$$

" r ": radius of pitch circle



  - Addendum: $a = \frac{1}{P}$
  - Dedendum: $b = \frac{1.25}{P}$

*See **Example 13-1** from text*

Contact Ratio

The contact ratio "m_c" defines the average number of teeth pairs in contact during meshing.

- If $m_c = 1$ it means that only one pair of teeth is in contact at a time.
- To reduce possibility of impact it is recommended that $m_c \geq 1.2$

Interference

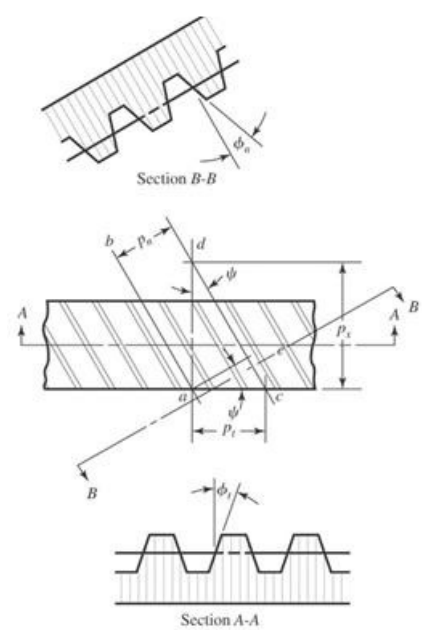
The contact of portions of tooth profiles that are not conjugate is called interference.

- Interference happens in gear pairs because the Dedendum circle is smaller than the base circle and thus the involute portion of teeth profile is small.
- Thus, to reduce interference larger pressure angles 20° or 25° are used (*i.e., making the base circle smaller*).

Parallel Helical Gears

Helical gears are mostly used to transmit motion between parallel shafts *(they can be used for non-parallel shafts as well)*.

- Both gears have the same helix angle, but one gear must have a right-hand helix and the other a left-hand helix.
- The shape of the teeth is an involute helicoid, and it can be produced by wrapping a piece of paper shaped as parallelogram on the base cylinder *(see fig. 13-21)*.
- Contact of the teeth starts as a point then extends into a diagonal line across the face of the tooth as teeth come into more engagement.
- Because of the gradual engagement of the teeth, helical gears can transmit heavy loads at high speeds.
- Helical gears produce both radial and thrust loads on the shaft.
- Two opposite-hand helical gears mounted side by side on each shaft can be used to cancel the thrust load.



The figure shows the nomenclature of helical gears.

- Helix angle: ψ
- Transverse circular pitch: p_t
- Normal circular pitch: p_n

$$\boxed{p_n = p_t \cos\psi}$$

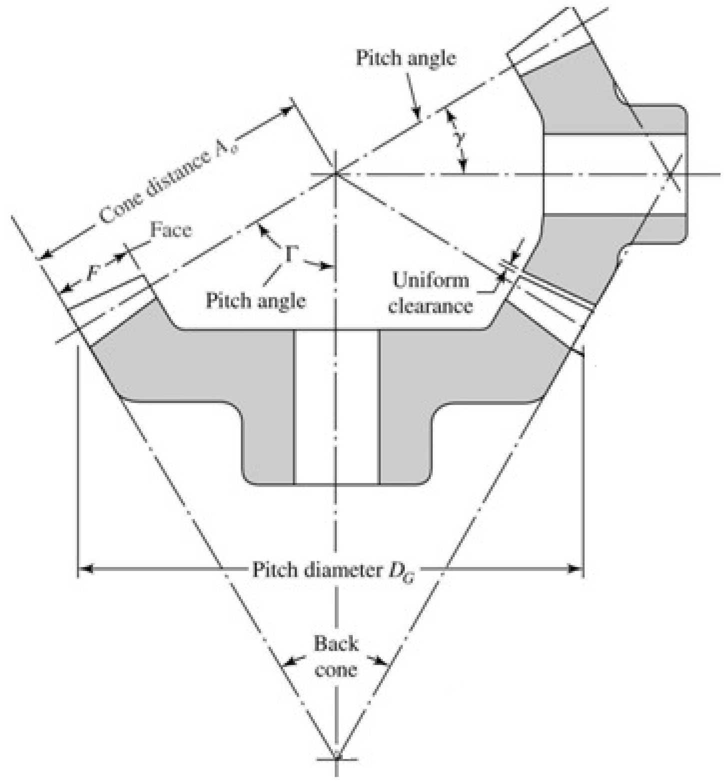
- Normal and transverse pressure angles ϕ_n & ϕ_t are related to the helix angle as:

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$$

Straight Bevel Gears

Bevel gears are used to transmit motion between intersecting shafts. The shafts usually make *90*° angle with each other, but also other angles are possible.

The figure illustrates the terminology of bevel gears.



- The pitch diameter is measured at the large end of the tooth.
- The circular pitch and diametral pitch are calculated the same as in spur gears.
- The pitch angles are defined as shown in the figure and they are related to the number of teeth as follows:

Pinion: $\tan\gamma = \frac{N_p}{N_G}$

Gear: $\tan\Gamma = \frac{N_G}{N_P}$

Standard straight-tooth bevel gears are cut using 20° pressure angle.

Worm Gears

- The worm and worm gear have the same hand of helix, but the helix angles are different.
- Helix angle for the worm, ψ_w, is large (see fig. 13-24) and usually the lead angle, λ , is specified instead.
- Lead angle for the worm, λ , is equal to the gear helix angle, ψ_G, (for shafts at 90°).
- Typically, the axial pitch "P_x" for the worm and the transverse pitch "P_t" for the gear are specified.
- The lead "l"and the axial pitch "P_x" of the worm are related as, $l = P_x N_w$ (where N_w: number of teeth of the worm).
- The lead "l" and the lead angle "λ" of the worm are related as , $\tan\lambda = \frac{l}{\pi d_w}$

Tooth Systems

A tooth system is a standard that specifies the relationships between the gear parameters such as addendum, dedendum, tooth thickness, pressure angle, etc.

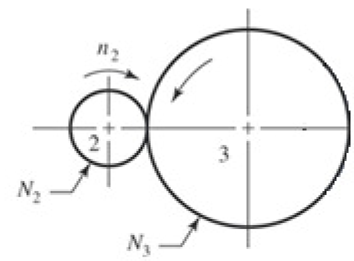
- The standards were developed to attain interchangeability of gears having the same pressure angle and diametral pitch regardless of the number of teeth.

❖ *Tables 13-1 to 13-5* give the standards for different types of gears.

Gear Trains

A gear train consists of more than two gears. In general, there can be several driving gears *(input)* and several driven gears *(output)*. Any gear that is not giving any input torque nor taking any output torque is called an "idler" gear.

Consider a pinion "2" driving a gear "3", the speed of the gear is:



$$n_3 = \left|\frac{N_2}{N_3} n_2\right| = \left|\frac{d_2}{d_3} n_2\right|$$

where

n: Angular velocity, N: Number of teeth, d: Pitch diameter

This equation applies to all types of gears (*spur, helical, bevel and worm*)

- *but note that $n_3 \neq \frac{d_2}{d_3} n_2$ for worm gear sets*

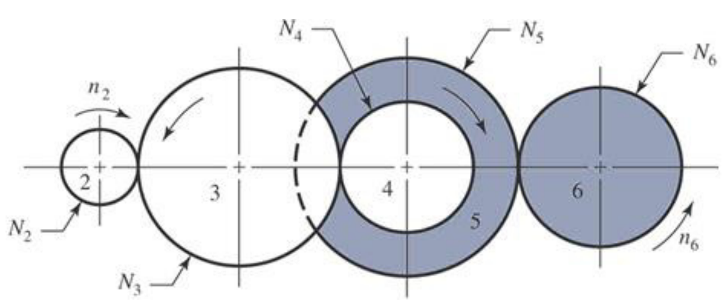
The absolute-value sign is to give freedom in choosing positive or negative directions.

- The sign convention follows the right-hand rule: $\begin{cases} + \text{ Counter clockwise} \\ - \text{ Clockwise} \end{cases}$

For the gear train shown, speed of gear "6" is:

The sign is decided manually.

$$n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6} n_2$$



- Note that gears 2, 3 & 5 are drivers while gears 3, 4 & 6 are driven

We define the *train value*, "e" as:

$$e = \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

The sign of "e" is positive if the last gear rotates in the same direction as the first, and negative if the direction is reserved.

Thus we can write:

$$n_l = e\, n_f$$

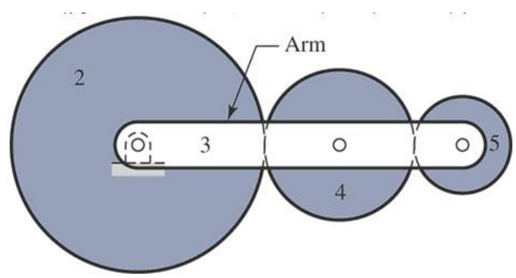
Where: n_l Speed of last gear
n_f Speed of first gear

Planetary gear trains

In this type of trains the axis of some gears rotates about other gears (see fig. 13-28).

- Planetary trains include: a sun gear, a planetary carrier or arm, and one or more planet gears.
- Planetary trains have two or more degrees of freedom and thus have two or more inputs.

For the gear train shown



→ Angular velocity of gear 2 relative to the arm

$$n_{23} = n_2 - n_3$$

→ Also, angular velocity of gear 5 relative to the arm

$$n_{53} = n_5 - n_3$$

→ Dividing we obtain the ratio of gear 5 to that of 2

$$\frac{n_{53}}{n_{23}} = \frac{n_5 - n_3}{n_2 - n_3} = e$$

In general we write:

$$e = \frac{n_l - n_A}{n_f - n_A}$$

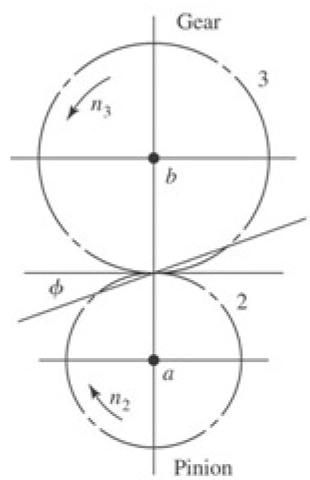
Ratio of last to first
n_A: Arm velocity

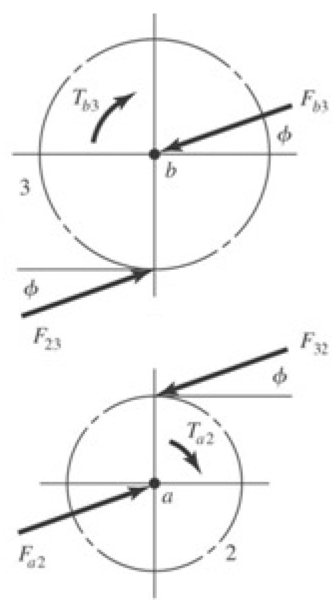
*See **Examples 13-3 & 13-5** from text*

Force Analysis- Spur Gears

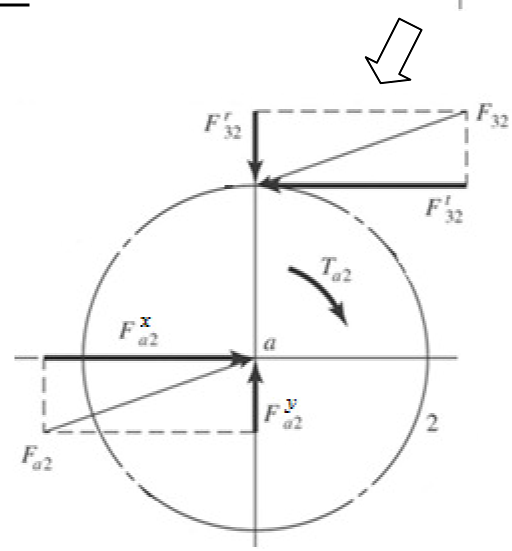
Notation:

- Number "1" corresponds to the frame of machine.
- The input gear is given the number "2" and then gears are numbered successively 3, 4, etc.
- Shafts are designated using lowercase letters; *a, b, c*, etc.
- Force exerted by gear "2" on gear "3" will be named F_{23}.
- Similarly, F_{2a} is the force exerted by gear "2" on shaft "a".
- Superscripts indicate the direction, directions are indicated using coordinates x, y and z for the shafts, radial and tangential, r & t for the gear teeth reactions.
  Example: F_{34}^{t} is the "tangential" component of the force exerted by gear "3" on gear "4".





- Consider gear "2" mounted on shaft "*a*" which rotates at velocity "n_2" and it drives gear "3" which is mounted on shaft "*b*" causing it to rotate at velocity "n_3".
  - A free body diagram of each gear will show the forces acting on it.
  - For "driver gears" the torque direction is same as the direction of rotation.
  - The reactions between mating gears will be directed along the pressure line, and its direction will be opposite to the direction of rotation of "*driver gears*".
  - The reaction forces can be further split into their components.
  - The only useful component of F_{32} is the tangentioal component, and it is defined as the " *transmitted load* " $W_t = F_{32}^{t}$
  - The applied torque can be related to the transmitted load as $T = \frac{d}{2} W_t$



If we designate the pitch line velocity as "V" where;

$$V = {}^{\pi d n}\!/_{12} \quad \textit{(feet/min)}$$

The power "H" can be obtained as:

$$H = \frac{W_t V}{33000} \quad \textit{(hp)}$$

Or in *SI units*

$$H = \frac{W_t \pi d n}{60(10^3)} \quad \textit{(kW)}$$

n: *(rev/min)*
d: (mm)
W_t: *(kN)*

*See **Example 13-7** from text*

Force Analysis-Bevel Gears

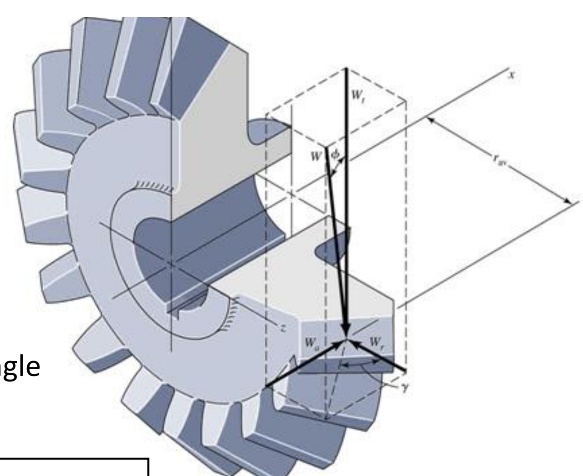
The point of action of the force is assumed to be at the midpoint of the tooth (*though it is really between the midpoint and the large end of the tooth*).

The transmitted load is found as

$$W_t = \frac{T}{r_{av}}$$

Where, T: Torque
r_{av}: Pitch radius at midpoint

The forces acting on the midpoint of the tooth are shown in the figure.



From trigonometry, the radial force W_r and axial force W_a can be found as:

$$W_r = W_t \tan\phi \cos\gamma$$

$$W_a = W_t \tan\phi \sin\gamma$$

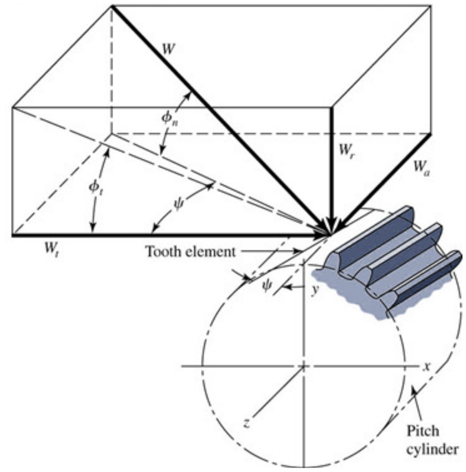
Where, ϕ: Pressure angle
γ: Pitch angle

*See **Example 13-8** from text*

Force Analysis- Helical Gears

The point of action of forces is assumed to be at the middle of the tooth.

Forces acting on the tooth are shown in the figure.



The transmitted load is found from the input torque same as in spur gears.

From trigonometry, the radial force W_r and axial force W_a can be found from the transmitted load W_t (tangential force) as:

$$W_r = W_t \tan \phi_t$$

$$W_a = W_t \tan \psi$$

Where, ψ : Helix angle
ϕ_t: Transverse pressure angle
ϕ_n: Normal pressure angle

And the total force

$$W = \frac{W_t}{\cos \phi_n \cos \psi}$$

*See **Example 13-9** from text*

Force Analysis - Worm Gears

With friction neglected, the only force acting on the tooth is the normal force W. The $x, y\ and\ z$ components of the force W^x, W^y & W^z are shown in Fig. 13-40.

Using subscripts W and G to indicate forces acting on the Worm and Gear

We note that

$$W^x = W_{wt} = -W_{Ga}$$
$$W^y = W_{wr} = -W_{Gr}$$
$$W^z = W_{wa} = -W_{Gt}$$

For gear axis in the x direction and worm axis in the z direction

In spur gears, the relative motion of mating teeth is primarily rolling thus friction is negligible. Where as in worm gears, the relative motion between the worm and worm gear teeth is pure sliding therefore friction is significant in worm gears performance.

Introducing a coefficient of friction " f " the frictional force (acting tangential to the surface of the tooth) " $W_f = fW$ " contributes in the force components:

$$W^x = W(\cos\phi_n \sin\lambda + f\cos\lambda)$$
$$W^y = W\sin\phi_n$$
$$W^z = W(\cos\phi_n \cos\lambda - f sin\lambda$$

Where,
ϕ_n:normal pressure angle
λ: lead angle

And the frictional force can be found as

$$W_f = fW = \frac{fW_{Gt}}{f\sin\lambda - \cos\phi_n\cos\lambda}$$

The tangential forces of the worm " W_{wt} " and the gear " W_{Gt}" can be related as:

$$W_{wt} = W_{Gt}\,\frac{\cos\phi_n \sin\lambda + f\cos\lambda}{f\sin\lambda - \cos\phi_n\cos\lambda}$$

The <u>efficiency</u> of force transmission by the worm set can be defined as:

$$\eta = \frac{W_{wt}(no\ friction)}{W_{wt}(with\ friction)} = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda}$$

- Using this equation it can be seen that the efficiency increases with increasing helix angle " " (i.e., decreasing λ), <u>see Table 13-6</u>.
- Experiments have shown that the coefficient of friction depends on the sliding velocity, " V_s ".
- The relative sliding velocity " V_s " can be found as, (see <u>Fig 13-41</u>)

$$V_s = \frac{V_w}{\cos\lambda}$$

V_w: Pitch-Line velocity of the worm

- The coefficient of friction " f " can be found as function of V_s from <u>Fig 13-42</u>.

See **Example 13-10** from text